to-one and that will account for other variations in usage such as average weekday, Saturday, and Sunday travel;

3. Development of demand and supply models for detailed analysis that are behavioristically oriented and, thus, sensitive to profiles of users and nonusers and their attitudes toward the system's attributes; and

4. Development of an optimization procedure that will integrate the demand, supply, and cost models to identify operating parameters that will maximize ridership, give the most favorable arrangement of vehicle supply, and minimize cost of operation.

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Analytic Model for Predicting Dial-A-Ride System Performance

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Previous development work on dial-a-ride (DAR) has focused principally on defining the supply side of the system. Detailed computer simulation models that have been developed at M.I.T. and the Ford Motor Company (1, 2) relate the quality of service to the number of vehicles operating and the level and distribution of demand for the service. At the early stages of development and investigation of the general potential of the system, this was appropriate because detailed and realistic simulation was necessary to determine these fundamentals of operation. During this phase, different assignment algorithms were tested and, for the best set, calculations were developed between level of service as a key output measure and number of vehicles, vehicle speed, pickup and delivery time, ridership, and distribution of ridership as key input parameters. This basis that was then formed for detailed costing of DAR systems related cost per vehicle, cost per operator-hour, control costs, and operating costs to important output measures such as cost per passenger trip and cost per passenger-mile.

At this stage the supply side of the system was quite well defined, and the analyst was able to make reliable statements such as, "If a dial-a-ride system is to be implemented to serve 200 passengers per hour at a mean level of service of 2.5 in a 10 square mile area, then x vehicles will be required and the average cost per trip will be y." This was clearly an important step in establishing feasibility of the concept, but it did not include the demand side of the picture. Work on the demand side has been much more limited, and until now no model has been developed that includes both the demand and the supply sides. This paper presents such a model, which is designed so that a transit planner can quickly and inexpensively explore a variety of design and policy options for a proposed dial-a-ride system.

The model predicts the equilibrium operation of a dial-a-ride system and the outputs, including gross and net revenue, total cost, total ridership, and quality of service, that the designer is most concerned with when configuring a system. Equilibrium is computed by 3 components: supply model, cost model, and demand model (3). The supply model is an analytic model that is based on the operation of the system and has been calibrated with simulation model experiments. This enables a much less expensive supply model to be used that retains much of the accuracy of the full simulation. The model requires a minimal data base, which is generally available in any metropolitan area, and no significant additional data must be collected.

This equilibrium model for the first time allows a transit planner to test a wide range of service areas, vehicle fleet sizes, and fare policies to select the best set of options for given objectives, which may be couched in terms of realizing a net revenue, providing a given quality of service, or attaining a given level of ridership. Relations such as the effect of fare on ridership and net revenue and of fleet size on service level and ridership can now be explored in given areas by using the proposed equilibrium model.

Table 1 gives the exogenous and endogenous variables in each submodel as well as the notation that is used for these variables.

SUPPLY MODEL

The supply model considers each passenger's trip to consist of 2 parts: a wait time, t_w, and a travel time, t_T. Each vehicle is modeled as a queue. Passengers arrive at the queue at the moment they are picked up and leave the queue when they are dropped off at their destinations. While on board, they wait in the queue to be served, and the average in-vehicle time, t_T, is their average time in queue.

The rate at which a vehicle can serve a passenger depends on the time needed for a passenger to board and exit from the vehicle, t_E and t_P respectively, and the average distance between stops, D. Although on the average each demand served results in 2 stops, one of these is to pick up passengers and can, therefore, be ignored for model-

Table 1. Model variables.

Variable	Notation	Exogenous or Endogenous	Models in Which Variables Appear		
			Supply	Demand	Cost
Average vehicle speed	SPEED	Exogenous	x		x
Average trip length	L	Exogenous	x	x	
Total time of service per day	T	Exogenous	x	x	x
Factor input prices	—	Exogenous			x
Size of service area	AREA	Exogenous	x		
Total daily internal trips in service area during DAR operating time	GENR	Exogenous		x	
Vehicle boarding time	t_p	Exogenous	x	x	x
Vehicle exit time	t_E	Exogenous	x	x	x
DAR modal split	MS	Endogenous	x	x	x
DAR fare	f	Exogenous		x	x
Vehicle fleet size	v	Exogenous	x	x	x
Average DAR wait time	t_w	Endogenous	x	x	
Average DAR travel time	t_t	Endogenous	x	x	
Automobile access time	CART	Exogenous		x	
Daily net cost of DAR	TC	Endogenous			x

ing the travel time component of a trip. Therefore, the average time needed to serve a passenger, $1/\mu$, is

$$\frac{1}{\mu} = \frac{D}{SPEED} + t_E + t_P \tag{1}$$

The value of D will clearly be a function of a number of variables, including the pattern of origins and destinations in the service area, the dispatching algorithm, and size of the service area. However, this problem was greatly simplified by treating D as a linear function of the average trip length and the rate at which demands arrive at the vehicle. Simulations were run, and the results were used to estimate an equation for D by using ordinary least squares. The results are

$$\begin{array}{rccc} D = & 1.109 & + \; 0.036L & - \; 6.12\lambda \\ & (0.109) & (0.056) & (0.737) \\ & (10.162) & (5.483) & (-8.299) \end{array} \tag{2}$$

$R^2 = 0.877$, and $F(2, 15) = 53.513$. The coefficient of L is positive, reflecting the difficulty associated with creating efficient tours when a service area is characterized by long trips. The variable λ reflects the average number of origin-destination pairs available for putting together tours and, therefore, has a negative coefficient indicating lower interstop distances with higher demand rates per vehicle.

If values of λ and μ are measured in the same units, in this case demands serviced per minute, a number of possible queuing models can be applied to predict the travel time, t_T. The simulation results were used to test the M/M/1 and the M/G/1 queues. Both models tended to underpredict travel times for highly congested systems. However, in general the M/M/1 model resulted in predictions that better matched the simulated data. Furthermore, the M/G/1 specification requires a prediction of the variance of the service rate. Therefore, the M/M/1 model shown in Eq. 3 was selected.

$$t_T = \frac{1}{\mu - \lambda} \tag{3}$$

Even though the model underpredicted travel time for certain dial-a-ride systems, the range of model validity was relatively well defined. It was found that, unless Eq. 4 held, the model was likely to be seriously in error. Equations 2 and 6 were solved by using only simulated data within the range of model validity.

$$8.82\lambda - v/AREA \leq 0.250 \tag{4}$$

The wait time could in theory be treated as a queue. However, a much simpler method was used in the supply model to reduce the complexity of the equilibration process.

Given the value of the average in-vehicle travel time t_T and the exogenously determined average trip length, it is possible to determine an average effective velocity, V_{EFF}, in any given direction. This velocity corresponds to the effective rate at which a vehicle moves along a tour toward any passenger's destination. The wait-time equation simply assumes that the same effective velocity applies to a vehicle moving to pick up a passenger as to a vehicle heading toward a passenger's destination. If the average distance from the vehicle to a demand is L^W, then the expected wait time, t_W, can be expressed as

$$t_W = \frac{L^W}{V_{EFF}} = \frac{L^W}{L} t_T \tag{5}$$

As with D, an equation for L^W was developed by using the results of the simulation runs. Equation 6 presents the results of this estimation. The vehicle density, v/AREA, reflects how far from the demand's origin a vehicle is likely to be. The demand density, λv/AREA, reflects the degree of system congestion that is likely to make assignment of a very close vehicle to be inefficient. As expected, both coefficients have the proper sign.

$$\begin{array}{llll} L^{w} = & 1.622 & + \; 4.98\lambda v/AREA & - \; 0.892\, v/AREA \\ & (0.104) & (1.227) & (0.140) \\ & (15.613) & (4.061) & (-6.386) \end{array} \qquad (6)$$

$R^2 = 0.753$, and $F(2, 15) = 22.829$.

DEMAND MODEL

Because there are few comprehensive data about the demand for dial-a-ride services, a relatively simple incremental demand model was selected (4). This model assumes that total daily travel within the service area is fixed and that DAR modal split, MS, is a function of only the expected wait time, t_w, the fare, f, and the ratio of DAR travel time to that of automobile travel time, TTR. It is assumed that a base-point modal split, MS°, is known, which corresponds to a base wait time, t_w°, a base fare, f°, and a base travel-time ratio, TTR°. Given this base point, the demand model is

$$MS = MS^{\circ}\left[e_w\left(\frac{t_w - t_w^{\circ}}{t_w^{\circ}}\right) + e_{TTR}\left(\frac{TTR - TTR^{\circ}}{TTR^{\circ}}\right) + e_f\left(\frac{f - f^{\circ}}{f^{\circ}}\right)\right] \qquad (7)$$

where e_w is the elasticity of modal split with respect to wait time, e_{TTR} is the elasticity of modal split with respect to the travel time, and e_f is the elasticity of modal split with respect to fare.

The base point of 2 percent modal split for a wait time of 15 minutes, a fare of $0.60, and a travel-time ratio of 2.0 were used based on the records for early months of operation in the autumn of 1971 in Batavia, New York.

The elasticities selected were derived from the attitudinal survey by Golob and Gustafson (5). They derived a set of demand curves that in light of existing operational experience gives modal-split values that are far too high (6). However, the elasticities implied by those curves seem quite reasonable for DAR demand. These elasticities are

$$e_w = -0.3;\; e_{TTR} = -0.3;\; e_f = -1.1 \qquad (8)$$

The wait-time elasticity is somewhat lower than that usually used for demand analysis and is roughly equal to travel-time ratio elasticity. This probably reflects the fact that dial-a-ride passengers generally wait in their homes rather than in a transit station. The fare elasticity is quite high, perhaps reflecting the high proportion of low-income, elderly, and young persons using dial-a-ride service.

Automobile out-of-vehicle time, denoted as CART, was assumed to be 2 minutes. Average vehicle speed for automobile travel was assumed to be equal to that for dial-a-ride.

EQUILIBRIUM

At equilibrium, Eqs. 1 through 7 are all satisfied simultaneously. This condition, after substantial algebraic manipulation, results in the following polynomial equation of the endogenous variable λ.

$$\left[a_1\ \mathrm{SPEED} + \frac{a_2 d_1}{q_1} + \frac{a_2 a_4\ \mathrm{SPEED}}{q_1} + \frac{a_3 \ell_1 d_1}{L} + \frac{a_3 \ell_1 a_4\ \mathrm{SPEED}}{L}\right]$$

$$+ \left[-a_1 d_1 - a_1 a_4\ \mathrm{SPEED} + \frac{a_2 d_2}{q_1} + \frac{a_3 \ell_1 d_2}{L} + \frac{a_3 \ell_2 d_2}{L} + \frac{a_3 \ell_2 a_4\ \mathrm{SPEED}}{L} - \mathrm{SPEED}\right]\lambda$$

$$+ \left[-a_1 d_2 + \frac{a_3 \ell_2 d_2}{L} + d_1 + a_4\ \mathrm{SPEED}\right]\lambda^2 + d_2\lambda^3 = 0 \qquad (9)$$

where

$$a_1 = \frac{\mathrm{GENR}}{v}\mathrm{MS}^\circ\left[1 - e_{TTR} - e_w + e_f\left(\frac{f - f^\circ}{f^\circ}\right)\right];$$

$$a_2 = \frac{\mathrm{GENR}}{v}\mathrm{MS}^\circ\frac{e_{TTR}}{\mathrm{TTR}^\circ};$$

$$a_3 = \frac{\mathrm{GENR}}{v}\mathrm{MS}^\circ\frac{e_w}{t_w^\circ};$$

$a_4 = t_\epsilon + t_o$; and

$$q_1 = \frac{L}{\mathrm{SPEED}} + \mathrm{CART}.$$

Equations 2 and 6 have been simplified so that

$$D = d_1 + d_2\lambda$$

$$L^w = \ell_1 + \ell_2\lambda \qquad (10)$$

The positive, real solution of this third-order polynomial in λ, the demand arrival rate, can then be used to determine the travel time and wait time directly from the supply model equations.

NET COST MODEL

To predict the net cost of service requires that both costs and revenues be calculated. The DAR system was assumed to be computer dispatched by the use of available minicomputer technology. Costs were considered in 4 general categories:

1. Customer communications (handling and processing incoming calls for service),
2. Vehicle operation (capital and operating costs),
3. Dispatching (computer rental, space), and
4. Overhead.

The 4 categories were further disaggregated into space, labor, phone rental, and other requirements. Wage rates for various job categories and other factor input prices were taken from a number of sources and represent reasonable values for a typical northeastern city with unionized labor.

In the cost analysis, true demand-responsive service was assumed to operate only during off-peak hours and more efficient subscription bus service was assumed to operate during the peak hours. Thus, a portion of total cost was allocated to peak-hour service. Similarly, only a typical weekday was modeled, and a portion of fixed costs was allocated to weekend and holiday DAR service.

Revenues were calculated from the demand arrival rate. However, on the average each demand corresponds to slightly more than 1 passenger; therefore, 1.1 passengers per demand were used.

MODEL RESULTS

The model was used in a hypothetical parametric test case in which a range of sensitivity analyses was performed by systematically varying average trip length, size of service area, demand elasticities, base modal splits, fares, and vehicle fleet sizes. The model was used successfully to analyze several thousand different configurations and demonstrated the complex interrelations between design parameters.

For systems characterized by both high fares and high fare elasticities, no positive equilibrium solution could be found, probably because the incremental demand model is inadequate at fares or service times that are much larger or smaller than the base values.

Occasionally, when high fare-high fare elasticity systems resulted in a positive equilibrium solution, the results were completely unreasonable in that dial-a-ride travel time was less than that for automobile travel time. However, these systems were a small fraction of the thousands of configurations tested and were generally characterized by input values far beyond the range over which the supply model was calibrated. The development of a much larger data set on which more sophisticated expressions for L^w and D could be calibrated might eliminate much of this difficulty.

No dial-a-ride system examined resulted in a profitable operation. This is consistent with existing operational experience and seems reasonable when one considers that only the off-peak hours were considered. Efficient peak-hour subscription bus service for work trips would probably offset at least some of this deficit.

CONCLUSION

The need for effective dial-a-ride system planning tools other than expensive, supply-oriented simulation will become more acute as more and more small communities consider the implementation of dial-a-ride service. Analytic models that capture both supply and demand effects within an equilibrium framework offer an alternative that can aid the design process in small communities and be used in conjunction with simulation in large-scale planning problems. Although still untested in an actual design problem, the model presented in this paper seems to offer reasonable potential for meeting an important planning need.

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