Separable Versus Simultaneous Travel-Choice Behavior

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After an initial discussion of the implications for travel modeling of alternative travel-choice behavior assumptions, 2 of these assumptions are discussed and analyzed in detail. The assumptions are separable and sequential travel choice versus simultaneous travel choice. Subsumed within this separable-simultaneous dichotomy are long-run activity location decisions and short-run travel decisions. Alternative methods of applying travel models based on the various assumptions are presented, and their strong and weak points are discussed.

The objectives of this paper are

1. To bring together and discuss the rationale for various strands of previous work in assuming separable-sequential and simultaneous travel-choice behavior in travel demand modeling and
2. To provide a common point of departure for discussion of extensions of present methods of modeling travel-choice behavior.

MODELING TRAVEL-CHOICE BEHAVIOR

Existing travel demand models are classified as short-run or long-run demand models according to whether short-run travel decisions (choices) are assumed to be made separately from long-run activity location decisions. The additional classification of direct and indirect demand models is used to describe whether the short-run travel decision is assumed to be one simultaneous "joint" choice or a series of separate choices (e.g., mode, destination, frequency). In this section, certain travel modeling implications of these behavioral assumptions are discussed.

Modeling Short-Run Travel Behavior

Travel-choice behavior modeled in direct demand models assumes that all attributes of an entire trip are known and considered simultaneously by the traveler. As shown in Figure 1, this behavior can be described as involving the simultaneous consideration of all the attributes normally associated with each of the 5 conventional descriptors of travel: frequency, time of day, destination, mode, and path. If each path through the travel decision tree is considered an alternative travel choice whose attributes are considered simultaneously "in competition" with the attributes of all the other travel choices, the models can become very complex. The number of choice combinations to be considered and modeled simultaneously is the product of the number of alternatives within each of the travel choices. For example, a simultaneous model of travel

that considers 3 modes, 2 times of day, 20 destinations, and 1 path requires the modeling of (3 × 2 × 20 × 1) or 120 travel choices for each origin. [This number may be reduced by eliminating zero-probability choices in calibrating models that satisfy the independence axiom (see next section).] The number of explanatory variables and the allowable interactions among variables that may be assumed to explain (model) simultaneous travel behavior can multiply very rapidly for realistic travel-choice situations in urban areas.

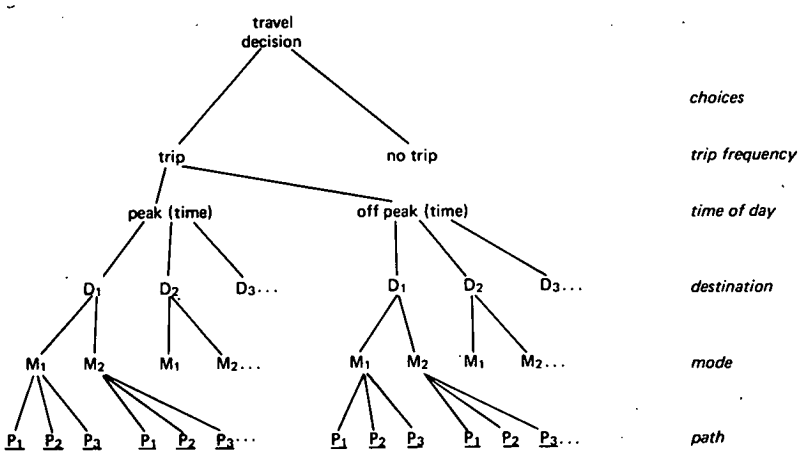
The need for "simple robust models" has been well articulated (2). Calibrating models for large numbers of alternatives (choices) with very low probabilities of choice is difficult in the extreme. Attributing properly the separate effects of large numbers of (possibly highly correlated) attributes describing complex choice environments (where calibration techniques often require certain assumptions, e.g., normality or homoscedasticity) boggles the mind. (One may speculate that the "number of variables required to predict probability of choice is finite and rapidly approaches the limit of human discrimination.") For these reasons, travel demand models must be reduced in complexity in some plausible way.

Restricting the choices available restricts the products or attributes the traveler is assumed to evaluate in making his travel decision. Restricting the choices that are presumed available to the traveler appears to be the way in which choice-specific travel demand models can be reduced in complexity. However, this involves making some important assumptions on the separability and the sequence of travel choices.

The assumption that travelers behave as though they sequentially consider (travel) choice-specific attributes (Fig. 1) means that there is a hierarchy of travel decisions in which certain travel decisions are made independently (separately) of others. In turn, other travel choices (e.g., higher level choices like destination, Fig. 1) are made given that lower level choices (e.g., mode) are predetermined.

There are 2 ways to model such sequential travel behavior. The first assumes that the relative valuation of choice attributes is constant throughout the set of travel choices. This requires that models of the independently made lower level travel decisions be calibrated based only on a subset of attributes describing those choices. The estimated (and preserved) utilities from the lower level choices are then added to a set of attributes on the basis of which the higher level choices are made. The traveler, it is assumed, makes some sequence of choices, and the earlier choices are based on independent and separate evaluations of personal utility (separate) from the "later" conditional or "constrained" choices. For example, the time of day (shopping purpose) choice was modeled (10) on the assumption that "there is a utility associated with the trip itself which is additive to the utility or disutility associated with the choice of time

Figure 1. Presumed hierarchy of travel choices.



of day, which is additive with the utility associated with the place to which the trip is made...."

Thus, the choice of mode is modeled separately and prior to the destination choice and is assumed to be independent of the overall number of trips between the origin and destination. Similarly, the choice of time of day is assumed to be made independently of the choice of destination.

The attributes that are assumed additive must map on the (sequential) choices. Otherwise, a choice-abstract model results. [Choice-abstract models assume that the attributes of travel choices are considered or perceived by the traveler independently of the objects or facilities that carry or support or propel the traveler (7).] If difficulty is encountered, either the travel choices can be redefined or the supply side description of choices (e.g., mode) can be abandoned and sequential choice-abstract models can be developed (7, p. 246).

The assumption of sequential travel choices, given that travelers perceive their choices as described by attributes inseparable from choices, is a difficult assumption to make. Yet it is an attractive strategy for reducing the complexity of travel demand models because it greatly reduces the number of interaction terms in the model. The other strategy is to reduce the number of independent variables that are assumed to influence travel behavior. That is, reduce the number of attributes the traveler is assumed to evaluate in his travel decision-making process without excluding interaction. Because the attributes that the traveler evaluates are identified with particular travel choices, this second strategy for reducing model complexity is more appropriate to choice-abstract models than to choice-specific travel demand models. [Choice-specific models assume that the attributes of travel choice are considered or perceived by the traveler together with the objects or facilities that carry or support or propel the traveler (7).]

A second way to model sequential travel behavior requires the still stronger (more difficult) assumption that some travel choices are made completely independently of other travel choices and that the relative valuation of choice attributes common to 2 or more travel choices is not necessarily the same in successive travel choices. This represents a third-level assumption regarding the consideration and valuation of the attributes (i.e., the relative marginal utilities) of the choice situation confronting the traveler. These 3 levels of assumptions are summarized in order from the weakest to the strongest (or most heroic) assumption.

1. All the attributes of the choice situation confronting the traveler are considered simultaneously. The complete trip is one decision. The relative valuation of the attributes is constant in any travel choice in the hierarchy shown in Figure 1.
2. There is a hierarchy of travel decisions in which certain travel decisions are made independently of other decisions. However, the relative valuation of choice attributes is constant in any complete travel decision (i.e., any single path through the travel decision tree shown in Fig. 1).
3. As in assumption 2, there is a hierarchy of travel decisions in which certain travel decisions are made independently of other decisions. However, the relative valuation of choice attributes common to 2 or more travel choices is not necessarily the same in successive travel choices.

The first assumption is the easiest to make. It requires the concomitant assumption of constant relative valuation of attributes in component travel choices of a complete travel decision.

The second (strict utility) assumption is made for ease of estimation (reducing the number of variables in the models to be estimated relative to the first and third assumptions). It requires some sequence of travel choices to be assumed for purposes of estimation as discussed above. Inclusive prices must be used to preserve the previously estimated utilities in strict utility models. The separately calibrated models using inclusive prices may be combined and applied simultaneously, or sequentially in any order.

The third assumption is the present assumption of UTP models that completely and

independently estimate the different travel choices with different valuations of the independent variables in each model. The traveler, nevertheless, must face the same values of the independent variables in more than one component travel choice. For example, "the costs of the various modes influence not only the choice of mode but also the selection of destination and the determination of whether the trip should be made at all" (10). The most damaging indictment of the third assumption is that the sequence of application of the models determines the results. That is, no unique equilibrium can be reached with these models so long as flow and congestion conditions and the resulting travel costs change in any way from those used to calibrate the models. That is, even if the conventional series of models (including trip generation) were system sensitive, the sequence of their application determines the network equilibrium reached after more than one iteration. In addition, of course, the third assumption poses the problem of what appropriate value to place on user benefits (e.g., time savings) in evaluation of transportation system alternatives when different valuations of the independent variables are assumed in each component travel choice.

From the above discussion, the conclusion may be drawn that the assumption is easier to make that travel choices are separable than that travel choices are made in some sequence. This assumption implies only that the marginal rates of substitution (trade-offs) among attribute variables that govern one travel choice do not vary among travel choices. Stated another way, this means that the trade-offs or ratio of "weighted" attributes that explain one travel choice are independent of the other choices.

It is with the last statement that 2 important results from separate disciplines can be joined. In mathematical psychology, this is a statement of separability property of the independence-of-irrelevance-alternatives axiom (21, 22). In economics (utility theory), at the conditions assumed at equilibrium (see Appendix), the ratio of the marginal utilities of 2 choices is equal to the ratio of their "weighted" attributes (i.e., their revealed "prices"). The relative marginal utilities of the attributes of a choice situation can be solved for (inferred from) observed data on the choices made (31).

Thus, the assumption of separable travel choices potentially allows complex travel choices to be broken down into simple travel choices whose relative marginal utilities can be inferred from observed data. However, a sequence assumption is necessary to determine which (separable) travel choice will be "simply" modeled, the inferred relative marginal utilities from which will be preserved in the remaining travel choices. Before the possible plausibility of any sequence and separability assumptions is discussed, the important properties and implications for travel demand modeling of the independence axiom will be described.

Independence-of-Irrelevant-Alternatives Axiom

The independence-of-irrelevant-alternatives condition (21) implies that, for any 2 alternatives i and j having a positive (nonzero) selection probability, the relative odds of choosing j over i in a set containing only the 2 alternatives are equal to the ratio of their probabilities of being selected from any larger set of alternatives containing both i and j. This can be expressed as (26)

$$\frac{P_{ji}}{P_{ij}} = \frac{P(j:A_1)}{P(i:A_1)} \tag{1}$$

where

P_{ji} = probability of selecting j in a 2-element set $A_1 = i, j$;
P_{ij} = probability of selecting i in a 2-element set $A_1 = i, j$;
$P(j:A_1)$ = (nonzero) selection probability of choosing j contained in any set A_1; and
$P(i:A_1)$ = (nonzero) selection probability of choosing i contained in any set A_1.

This condition states that the odds that alternative j will be chosen over i in a set containing both are independent of the presence of irrelevant "third" alternatives in A_1.

This is the separability property of the independence-of-irrelevant-alternatives axiom (21, 22).

"Strict utility" is defined by Luce (21) as being the function $h(Z_{ki})$ that satisfies Eq. 1 for the binary case i = 1, 2. That is, the relative odds of choice or share of, say, travel, P_i/P_j, between any 2 alternatives i and j are simply some function of the variables describing the 2-choice alternatives (and no others!).

$$\frac{P_i}{P_j} = \frac{h(Z_{ki})}{h(Z_{kj})} \tag{2}$$

where

P_i = probability of choosing i;
P_j = probability of choosing j;
$h(Z_{ki})$ = strict utility of i; and
Z_{ki} = (scale) variables, k, describing i.

The actual odds or probability P_i of choosing alternative i from a larger set of alternatives can vary, of course.

The binary-choice strict-utility model, Eq. 2, generalizes into a multiple-choice model only if the independence axiom holds, that is, only if the probability of a choice from a subset of alternatives is independent of what other choice alternative may also have been available. The resulting multiple-choice strict-utility model is (21)

$$P(i:A) = \frac{h\,(Z_{ki})}{\sum_{j \epsilon A} h\,(Z_{kj})} \tag{3}$$

for j = 1, ..., i, j, ..., where

$P(i:A)$ = probability of choosing i from a set of alternatives A;
$h(Z_{kj})$ = strict utility of alternative j in the set A, a monotonic function of the scale variables Z_k describing j; and
$j \epsilon A$ = complete set of alternatives between which a choice is made.

An exponential transformation of the strict utilities (and an abandonment of set notation) yields the multinomial logit formula:

$$P_i = \frac{e^{v(z_{ki})}}{\sum_{j=1}^{J} e^{v(z_{kj})}} \tag{4}$$

for j = 1, ..., i, j, ..., J.

Equation 4 says that the probability that a traveler will choose alternative i out of a set of J alternatives is directly proportional to its strict utility $V(Z_{ki})$ (a monotonic function of attributes k of the alternative i) and that the probabilities of choosing one alternative in the set of available alternatives, each with a nonzero probability of being chosen, must sum to one. ["Perhaps the most general formulation of the independence axiom is the assumption that the alternatives can be scaled so that the choice probability is expressible as a monotone function of the scale variables, k, of the respective alternatives" (35). This assumption is called simple scalability by Krantz (19).]

The function $V(Z_{ki})$ in Eq. 4 can, of course, be interpreted and estimated. In the language of the psychologist, it represents some function of the environment that stimulates a decision (33). In utility terms, it represents some function of the attributes of value to travelers of the alternative travel choices. A correct model specification is needed to capture appropriate effects on behavior of variables (attributes) describing

the choice situation. A constant term, θ, in an equation for $V(Z_{ki})$, e.g., $\theta_i \prod_k Z_{ki}^{a_{ki}}$, will include the effects of all attributes not explicitly included in the model.

Separability Property

The independence axiom is a general statement that has consequences that can be tested. For example, it says that, if alternative i is preferred to j in one context (choice situation), it is preferred to j in any context for which both are available. Furthermore, if the odds of choosing i over j are 0.7 in one context, those odds will be preserved in any choice situation. The traveler is assumed to exhibit transitivity in his behavior with respect to his "strict utility" $h(Z_{ki})$ versus $h(Z_{kj})$. That is, he values the attributes, Z_k, of any choice, i, the same (ratio scale) relative to choice j regardless of the context. Thus, the probability that an alternative (choice) will be chosen is exactly proportional to its strict utility (therefore, Eq. 3). And from Eq. 2, the relative odds that an alternative will be chosen from 2 alternatives is constant and a function only of the strict utilities of the 2 alternatives. This allows the introduction of new alternatives in a model application without calibration of the model, provided the previously estimated strict utilities are preserved.

In 1962, the author used the separability property of Eq. 3 to calibrate a share model of (multiple) choice among 4 access mode (walk, park-ride, kiss-ride, and feeder bus to line-haul rapid transit) alternatives being tested in Washington, D.C. The model was calibrated with paired aggregate modal-split data from a number of surveys because of the lack of data describing the relative usage of all 4 feeder modes together. This was allowable because of the "startling" behavior of the model (Eq. 3) that "the relative substitutability of any two sub-modes without the third being available is assumed equal to the relative attractiveness of the two in the presence of the third"(5).

McLynn and Woronka (28) used this property extensively to calibrate their "single pair" market share model developed for the Northeast Corridor project. In their model, automobile was used as the "base mode". When difficulties were encountered with certain nonsensical parameter estimates and the single-pair estimates, all single-pair equations were estimated simultaneously. From Eq. 2, it follows that such simultaneous estimation is irrelevant from the point of view of the behavioral grounding of the model, however much it may be desirable to constrain certain parameter estimates.

The property of "separability" of alternatives is not restricted to alternatives among modes. Alternatives can characterize the entire range of choices of trip frequency, destination, time of day, mode, and path, as already discussed. Thus, separate choice models can be calibrated separately and later combined into a travel demand model. However, behavioral assumptions as to the sequence of travel decisions are required, as already discussed. The separability property of the independence axiom was first explicitly recognized and used to calibrate a travel demand model by Charles River Associates (CRA) (8).

Share models have been used in travel forecasting without recognition of their separability properties for many years. For example, the gravity model of trip distribution (36) is a share model whose standard derivation is simple and general (12).

$$\left.\begin{aligned} V_{ij} &\sim G_i A_j Z_{ij}^k & G_i &= \sum_j V_{ij} = \sum_j C_i G_i A_j Z_{ij}^k \\ V_{ij} &= C_i G_i A_j Z_{ij}^k & G_i &= C_i G_{ij} \sum_j A_j Z_{ij}^k \\ & & C_i &= \frac{1}{\sum_j A_j Z_{ij}^k} \end{aligned}\right\} \quad (5)$$

$$V_{ij} = G_i \frac{A_j Z_{ij}^k}{\sum_j A_j Z_{ij}^k} \tag{6}$$

$$\frac{V_{ij}}{G_i} = \frac{A_j Z_{ij}^k}{\sum_j A_j Z_{ij}^k} \tag{7}$$

Equation 5 states that the volumes between zones i and j are proportional to the previously estimated trips generated, G_i, and attracted, A_j, and to the attributes, k, of travel between i and j. C_i is the constant of proportionality, which is solved for in the remaining equations. The result, Eq. 6, is the usual form of the gravity model, which is equivalent to a share model, Eq. 7, for the split fraction of total trips from a zone i destined to zone j. However, the previously estimated "strict utilities" that (may have) resulted in the estimation of the G_i and A_j are not normally preserved.

In fact, of course, no transportation attributes are normally used in the estimation of the productions, G_i, and the attractions, A_j. Empirical evidence to support the use of strict utilities is the juggling necessary to bring the V_{ij}'s into line with the G_i and A_j in any gravity model application. That is, the results of the separately calibrated trip-generation and -distribution models are not (internally) consistent with each other.

The separability property implies that the conventional gravity model should be calibrated only with subregional structures (partitionings) that define distinctly different destination alternatives with nonzero probabilities of being chosen from a particular origin by a particular traveler (type) for a particular trip purpose. This would considerably simplify calibration but would appear to complicate gravity model application, i.e., predicting trip distribution (see discussion in section on applying forecasting models). An understanding of the separability property may thus lead to substantially more effective gravity models. Empirical research is clearly needed.

The derivation of the gravity model (Eqs. 5, 6, and 7) from a simple proportionality statement can easily be generalized to derive any split fraction (e.g., fraction of total regional trips emanating from an origin zone, or fraction of total interzonal trips on each mode). Each split fraction is in turn dependent on the previously derived trip universe being split. The models can then be "solved," one in terms of the next, in one multiple-choice share model. The result is similar to Manheim's "general share model" (24):

$$V_{klmp} = \alpha \beta_k \gamma_{kl} \delta_{klm} \omega_{klmp} \tag{8}$$

where

V_{klmp} = travel between origin k and destination l by mode m and path p,
α = total (regional) travel,
β_k = split fraction of α from origin k,
γ_{kl} = split fraction of $\alpha\beta_k$ to destination l,
δ_{klm} = split fraction of $\alpha\beta_k\gamma_{kl}$ to mode m, and
ω_{klmp} = split fraction of $\alpha\beta_k\gamma_{kl}\delta_{klm}$ to path p.

Each of the terms on the right side of Eq. 8 is intended to be a function of activity system and transportation system variables in Manheim's model.

In summary, in the calibration of a travel demand model, the separability property of the independence axiom implies that the (marginal) probability distribution of choice of mode can be separately estimated and multiplied by the conditional probability distribution of another travel choice, e.g., P (destination, mode), to give the joint probability distribution of both:

$$P(M,D) = P(M)\, P(D|M) \tag{9}$$

provided the previously estimated strict utilities from the modal-choice model are

preserved. This operation requires 2 assumptions: (a) that destination choices are made conditional on mode choices and not the reverse, and (b) that the (dis)utility from the mode choice is additive to the utility from the destination choice. Thus, the mode choice is assumed to be independently made from the destination choice (in this case) but not the reverse. Given the separability and sequence assumptions, the choices can be separately modeled, assuming negligible income effects, and later recombined into one joint probability model by simple multiplication of the separately calibrated probability models, as in Eq. 9. Conversely, the joint distribution, P(M, D) must be estimated directly if the sequence and separability assumptions appear too strong.

Modeling Long-Run Activity (Household) Location Behavior

In travel demand forecasting, activity-location choices are assumed to take place in a much larger market than travel choices. Also, the time periods over which activity-location choices are made is assumed to be much longer. If activities are considered substitutes for each other in one market, this requires long-run demand models where activity locations and intensities are allowed to vary. The recent mixed success in land use modeling (20) testifies to the difficulty of describing the attributes of all the related choices in this larger market (which also includes travel choices). Thus, the present state of the art of travel demand forecasting with a few exceptions allows only amount of travel to vary, i.e., to be the dependent variable. [Some demand models have been formulated and calibrated that forecast (long-run) residential location, car ownership, and modal split in one equation set (1, 16). However, these models do not forecast quantity of travel. Nevertheless, the models provide a direction for further work.]

In modeling travel separately from activity location, the attribute variables describing the choice situation must be limited to those "highly" involved in the decision (i.e., close substitutes and complements). Indeed, a necessary condition for utilities derived from separately modeled travel decisions to be considered additive is that their components must be neither competitive (substitutes) nor complementary (23).

Trip purpose is the first way of describing the restricted set of choices that are said to be available to the traveler as an individual decision-maker. No substitution is assumed among trip purposes because the purpose of the trip corresponds to the activities at the trip destinations. The activities in place are taken as given in the partial equilibrium framework. If activities are taken as substitutes, a long-run demand (land use) model results.

The choice ordering implied by assuming that travel choices are made, conditional on activity locations, is represented in Eq. 10.

$$P(T, A) = P(T|A)\ P(A) \tag{10}$$

where

$P(T, A)$ = joint probability distribution of travel and activity location;
$P(T|A)$ = conditional probability distribution of travel, given activity location; and
$P(A)$ = marginal probability distribution of activity location.

Equation 10 implies the sequence assumption that activity-location choices are made first and precede travel choices. The sequence requires that the strict utilities inferred from activity-location behavior be used in the calibration of the travel demand model. This is, of course, not the way travel models are currently calibrated.

It is, of course, possible to assume that travel and activity location are independent. That is,

$$P\ (T|A) = P(T) \tag{11}$$

This is exactly the assumption that is made when one assumes that there is a sequence of travel-choice decisions in which mode and route choice precede destination choice. That is, these choices are assumed to be made solely on the basis of the (dis)utility of the trip itself. Making this particular assumption of travel-choice ordering (discussed in the next section) is at least consistent with Eq. 11.

In summary, although the logical conclusion of the theory of travel as a derived demand is to allow both short- and long-run travel activity to vary as complements in a general equilibrium framework (7), the assumption is made that we can eliminate the imposing structure this would require and model travel choices separately as an activity with a set of complements (activities) in place and fixed.

The resulting set of attributes needed to describe the choice environment for input to a travel-choice model is correspondingly (greatly) reduced. Further, the choice ordering implied by this assumption is that travel choices are adjusted much more quickly to a change in travel conditions than in residence and work-place location. Modeling the latter requires a dynamic model where changes are measured over relatively long periods. Thus, if a static travel model is assumed, the effects of changes in travel conditions on travel can be modeled (inferred), it is assumed, separately from their effects on activity location. This assumption and its implications are worthy of considerable research.

Aggregate Versus Disaggregate Models

The issue of aggregate versus disaggregate "probability" models permeates most current discussions in travel demand forecasting. The often-used term "disaggregate behavioral" models gives the impression that individual-choice models have a monopoly on incorporating travel behavior. That is clearly unfair, for travel demand models can be derived from behavioral assumptions independently of whether they will use aggregate or disaggregate data.

Choice behavior in disaggregate models must be interpreted as probabalistic. Deterministic choice (i.e., 0, 1 binary) behavior produces uninteresting results when aggregated over all individuals to describe aggregate behavior in a planning application. However, the probability process is assumed to be in static equilibrium and incorporates no time parameter in a behavioral sense; e.g., learning or experience does not change the probabilities (23). Disaggregate travel models should, therefore, be referred to as probabilistic and not stochastic if they are used with cross-sectional data.

The generally strong arguments for using disaggregate models usually include data efficiency arguments. That is, more information on travel-choice situations and behavior is usually available with disaggregate data than with aggregate data. For example, Fleet and Robertson (13) showed that aggregation of trip data to zones reduced the variation in trip-making (trip generation) between observations to only 20 percent of the value at the dwelling unit level. In the process of aggregation, nonlinear relations may also be lost by using averages of explanatory variables. However, disaggregate travel models have not yet demonstrated practical superiority in providing travel information to decision-makers. In fact, we have as yet a way to go in getting models based on individual-choice behavior into the field. [Disaggregate models of some of the conventional UTP steps (i.e., trip generation) will be easy to introduce "in the field" (17).]

However, there is little doubt that the emerging techniques (34) for using travel models based on the behavior of individuals and not the behavior of aggregate numbers of trips will accelerate our understanding of travel-choice behavior. The empirical results of the next few years should greatly improve the travel behavior assumptions discussed in the next section.

Travel Behavior Assumptions

Travel forecasting procedures must have a basis in behavior if planners and

decision-makers are to be able to understand and interpret the results of the forecasts. This is true for many reasons. The forecasts that result depend on the behavioral assumptions. Behavioral models are needed for transferability (in space and time) to situations other than those for which the models were calibrated. Behavioral models are needed also for evaluation if the (usual) assumption is to be made that the trade-offs between time and money in a travel-choice situation are valid for user benefit calculations.

Transportation planning concerns itself with making, or contemplating making, changes to or affecting the transportation system. Our interest is in describing the behavior of travelers as they respond to travel choices and the changes in travel choices that confront them. The ability to predict the amount and distribution of travel in any situation is, therefore, only as good as our understanding of the underlying perceptions that travelers have of the choices that confront them.

In travel demand forecasting, therefore, we must confront squarely the validity of our theories that describe relations between people and their locations on the one hand and travel on the other. This involves consideration in particular of how and in what sequence, if any, people view the transportation system that connects or potentially connects their origins and destinations.

Separable Travel Choices

The open question is, What does the traveler perceive in his evaluation of his travel alternatives? Modeling travel directly as a simultaneous decision means including the attributes of every conceivable alternative to a specific choice in any model of that choice. By modeling long-run demand separately from short-run travel, we exclude moving the traveler's residence and work-place location as alternatives to his travel choice. However, such alternative choices remain as traveling to activities at varying locations as an alternative to staying put (destination choice versus no-trip choice); an automobile trip at a different, say, off-peak, time of day as an alternative to a transit trip at the peak hour; and so on.

As noted earlier, the conventional breakdown of individual travel choices is to separately model trip frequency, trip destination, time of day, mode choice, and route choice. Such a breakdown involves a stronger set of assumptions than the assumption of simultaneous travel decisions. The trade-off is generally between a stronger set of assumptions but less complex models and weaker assumptions but more complex and difficult-to-calibrate models. The unanswered questions are, How difficult to calibrate are models that combine travel decisions, and how difficult are they to forecast with?

At least 2 of the conventional travel choices might plausibly and relatively easily be combined, at least for purposes of empirical testing. That is, combining trip frequency and trip destination into 1 set of alternative choices appears theoretically plausible and convenient. Zero-trip frequency is the equivalent of no change in traveler location. Other combinations may also be speculated on. However, some appear more difficult than others, not because of the difficulty in assuming that travel-choice behavior is a simultaneous decision, but because of the separability property of most existing travel models. For example, combining mode and route choice into one decision may be difficult because of the similar characteristics of alternative routes within modes and the overly strong separability property in this situation. [The evidence is that "the addition of an alternative to an offered set 'hurts' alternatives that are similar to the added alternative more than those that are dissimilar" (35).]

Because the basis of calibrating travel demand models using the separability property is to constrain some decisions on the basis of attribute (utility) evaluations made in decisions modeled earlier in the chain, a discussion of travel-choice-separation assumptions cannot proceed far without including consideration of the ordering of the separate choice assumptions.

Choice Ordering

The assumed order of the travel decisions, given a separation, determines which choice situation is used to estimate the initial strict utilities. Empirical testing with alternate orderings and breakdowns can provide some evidence as to "natural" orderings, given the underlying assumption of "conditional" choice behavior. Is there a logical or natural ordering of travel choices? If there is any separation at all, hypotheses can be attempted for specific orderings of the choices. The following hypotheses are some that support the assumption that travel choices are separable and proceed in some sequence or order.

1. Sequential choice ordering based on timing. Traveler decision-making proceeds from the latest to the earliest decisions in time. For example, for a particular trip purpose (choice-of-destination activity), the traveler may be hypothesized to have some notion of the conditions on the available modes and routes when choosing his destination. That is, he has already considered the modes and routes that are available to him. He anticipates and makes choices on routes and modes that may then limit or constrain his available destinations and departure times. (Within a mode, he is apt to have anticipated the conditions on the alternative routes within the mode when he makes his mode choice. This suggests that mode-choice decisions are made after path decisions as opposed to both decisions being made simultaneously.) This implies a logical order of travel-choice decisions running counter to their sequence in time.

The possibility of a logical order of decisions running counter to their sequence in time in the case of travel decisions was discussed already by Beckmann et al. in 1955 (3). This reverse order also gets us around the practical difficulties (probably impossibility) of having to compute supply-sensitive system characteristics (travel attributes) on an area-wide basis for input to (disaggregated!) trip-frequency decisions made at a point (or zone), or for input to a modal-split model that precedes trip distribution. Production functions g(x) for, say, travel times, are well known on a link and route within modal basis (15).

2. Sequential choice ordering based on adjustment time. Models that assume some choice ordering in a sequence could rest their plausibility on the time it takes to adjust behavior to a change in policy. Some decisions (e.g., route choice) can be adjusted more quickly by an individual than others (e.g., an origin change involving a house purchase or a mode change involving a car purchase) because they involve less commitment to their former situation. Thus, sequential choice models that involve adapting to changes in supply considerations can be considered in this sense dynamic or stochastic (4). Conversely, simultaneous-choice assumptions result in models that are in this sense static. Unfortunately, only cross-sectional data exist at present to empirically test most travel demand models.

3. Sequential choice ordering based on experience. Traveler decision-making proceeds from those choices on which there is the most experience to those choices on which there is the least experience. Most, if not all, current travel demand models are based on or can be shown to be equivalent to rational "economic man" assumptions. These yield plausible (if normative) descriptions (models) of travel behavior, but they demand more of man's capabilities than he can generally "deliver." In addition, they assume that the traveler's values, and the choices he confronts, are constant over time. Conversely, there are other descriptions of behavior that assume less (or a bounded set of) knowledge on the part of the individual decision-maker. These provide alternate but as yet largely unexplored bases for modeling travel behavior, and the dynamics of commitment to old and selection of new travel choices as families move spatially and socially over time.

Important theoretical support for separate and sequential choice modeling comes from the theory of decision-making called "satisficing" (25). This theory rejects the notion that there exists a rational economic man who is perfectly knowledgeable and perceptive about all the possible alternatives that confront him and who can compare all possible alternatives with one another to find his optimal choice by manipulating

stored criteria describing the alternatives. Satisficing substitutes for this true or complete rationality a hypothesis of bounded rationality. This implies sequential search and limited sets of criteria used for evaluation. That is, in place of simultaneous (or separable and transitive) comparison of all alternatives, alternatives are examined sequentially according to satisficing. And rather than being compared to one another on the basis of a set of (interval scale) operational criteria, the alternatives are compared to a simpler set of minimal criteria until an alternative is found that satisfies the decision-maker. Alternatives are discovered or searched sequentially until a satisfactory alternative is encountered. No attempt is made to exhaust all possible alternatives. Moreover, search for new alternatives will only occur if the traveler perceives a discrepancy between his level of aspiration and his level of reward from the existing behavior.

This "model" in its general formulation can be interpreted as supporting models of sequential travel behavior. Travelers can be considered to evaluate sequentially well-defined travel alternatives in terms of the objects that provide the travel service (modes) and in terms of the benefits from the travel service (destinations). Conversely, the traveler may sequentially apply a limited set of criteria that are used to reject alternatives that do not meet threshold levels of those criteria. (This latter interpretation provides support for choice-abstract sequential models.) In both cases there is support for the hypothesis of choice behavior that involves sequential examination of choices.

We may describe the present trip of a traveler as one path through the tree shown in Figure 1 (assuming he presently makes a trip). If he is dissatisfied with any aspect of his present trip or, if confronted by a new alternative with a promised or expected improved level of service, does he sequentially examine "near" alternatives at only one level of choice? Or does he reconsider many paths involving changes throughout the hierarchy? Or does he simply consider only the new alternative if available and accept it or reject it?

According to the theory of satisficing, there is generally a conservative bias in the system of choice. That is, over time, levels of aspiration tend to adjust to levels of achievement. (It is the difference in the levels that is said to motivate search for new alternatives.) A new alternative may or may not change the traveler's perception of difference between present and possible (future) alternative states if he changes his travel behavior. We clearly need to better understand what those perceptions of difference are, at what level in the hierarchy they occur, in what sequence they occur, and how their relative requirements of adjustment time may operate to eliminate certain choices from the sequence.

The above hypotheses that support sequential travel decision-making are not made as a matter of idle speculation. The current conventional procedure of travel forecasting assumes sequential travel choice and a very particular choice ordering. The choice ordering is allowed to vary only slightly in practice. For example, the place of modal split in the order of trip-choice decisions has been called "the most actively debated issue in modal split" (37). The context of this statement referred to whether modal split should precede or follow trip distribution. The alternatives can be represented by the following 2 model structures (probability statements in this case):

$$P(M,D) = P(D|M)\,P(M) \tag{12}$$

$$P(M,D) = P(M|D)\,P(D) \tag{13}$$

where M = mode, and D = destination. If Eq. 13 were true and Eq. 12 false, destination choice would be independent of the availability of a mode (say, automobile) to reach the destination. This does not seem plausible except possibly in the case of work trips. (In such a case, the car is assumed to be purchased if not available and if necessary for reaching the destination.) In the reverse case (Eq. 12 is true, and Eq. 13 is false), the choice of mode is assumed to be made independently of the choice of destination. For example, the automobile, if available, might be selected for the trip, and the destinations that can be reached by automobile are then considered by the traveler. This appears somewhat plausible (say, for convenience shopping trips), at least more plausible than the reverse sequence. (If this is true, at least for some important trip pur-

poses, it augers badly for transit usage. That is, choice of mode, e.g., transit usage, would be independent of origin-destination transportation system characteristics, including origin-destination pairs in larger cities where transit service may be excellent.)

There is an alternative model structure that poses a way out of the above dilemma if the order of travel behavior is not stable or must be subjected to further empirical testing. Equations 12 and 13 may be rewritten in the following form (11):

$$P(M,D|MEX_D) = P(D|M)\ P(M|MEX_D) \tag{14}$$

$$P(M,D|DEX_M) = P(M|D)\ P(D|DEX_M) \tag{15}$$

where X_D is the set of all decisions made prior to the choice of destination, and $P(M,D|MEX_D)$ is, therefore, the conditional probability that M and D will be chosen if mode choice precedes destination choice. Analogous statements apply to Eq. 18. Because MEX_D and DEX_M are mutually exclusive, Eqs. 14 and 15 can be added together to yield

$$P(M,D) = P(D|M)\ P(M|MEX_D) + P(M|D)\ P(D|DEX_M) \tag{16}$$

This is an exact expression for P(M, D). Equation 19 is equivalent to Eq. 12 or 13 only if mode choice always precedes destination choice or vice versa. It is also possible to expand Eq. 16 to include all aspects of travel decision-making.

Unfortunately, a solid case cannot be made for many trip-choice sequence assumptions. Our theory is weak, and we must look at whatever empirical evidence is available. Ben-Akiva (4) showed empirically that mode choice, assumed before or after destination choice, or the 2 travel choices modeled jointly all lead to different valuations (relative marginal utilities) of the trip attributes, (e.g., time and money costs of travel). (But this is insufficient evidence to lead to the conclusion that both sequences are wrong or that the separation assumption is incorrect.) His work on estimating the joint probability of mode and destination choice directly is the first demonstration that disaggregate data can be used for simultaneous travel-choice models, though not all travel choices were included. [The first simultaneous choice model using aggregate (zonal) data was by Kraft in 1963. The trip-generation and mode-choice decisions were combined and modeled simultaneously. Again, not all travel choices were included.] By combining choices and modeling them simultaneously, the need for sequence assumptions, but not separability assumptions (except when applying the model directly), is avoided. That is, the separability property of any formula satisfying Eq. 3 (e.g., multinomial logit) allows travel choices to be separated while still preserving the strict utilities. The separability property allows the conditional and marginal probabilities of the travel choices to be computed from the joint probability distribution estimated from the simultaneous model. Thus, for forecasting purposes, models satisfying Eq. 3 may be separated and applied sequentially (indirectly) or combined for application in a direct model (see later discussion of alternative methods).

When travel-choice models are calibrated separately, the alternatives allowed are determined by the conditional probabilities. That is, in Eq. 12, the only alternatives allowed are the destinations that are available or can be reached by mode m. The estimated strict utilities from this set of choices are then assumed to be independent of the choices as soon as the separability property of Eq. 3 is used in travel forecasting (see later discussion of definition of alternative choices).

The hypothesis of simultaneous (i.e., not conditional) travel choices can be easily tested by using standard chi-square tests for differences between marginal and conditional distributions of the same random variable. If there are no differences, the hypothesis of no relation between, say, mode and destination could not be rejected. Because it is relatively easy to show a relation by the chi-square test with large sample sizes, an inability to reject no sequence might be considered evidence that the decisions are being made simultaneously. (However, the power of the test is low.)

Theories of choice that consider different choice-abstract aspects of travel attended to at difference times and in some specific order were discussed earlier. Aspects of

travel can overlap with the definitions of travel choices because attributes in the definitions of each are often common to both. Some arguments against transitive value (strict-utility) models can be used in part to advance the case for assuming sequential travel choices and thus advantageous use of the separability property to calibrate demand models.

Similarly, arguments against a logical ordering of travel-choice decisions argue also for strict-utility travel-choice models because such arguments are consistent with assuming a single monotonic function of the scale variables of the alternatives and the single estimation of joint probability distributions of simultaneous travel choices (i.e., "direct" demand models). Therefore, uncertainties as to whether travel choices can be assumed to be separable and occur in some logical order do not point to abandoning strict-utility models. They may point to combining choices and making less use of the separability property in model calibration.

In summary, there may be some clear-cut travel-choice ordering that can be assumed from the standpoint of travel behavior and, thus, lead to the conclusion that probability models for combined choices should be calibrated directly wherever possible. Fewer sequence assumptions can lead to improved use of the separability property for combining separately modeled choices into a demand model. Because the independence axiom excludes, in any event, alternatives with zero probability of being chosen, the data requirements for estimating strict-utility models of combined travel choices can be greatly reduced. Simultaneous (direct) demand models rather than sequential choice models seem indicated from a behavioral point of view, although the discussion cannot be closed in view of the above hypotheses.

COMBINING STRICT-UTILITY SEQUENTIAL TRAVEL-CHOICE MODELS

CRA (8) used the separability property of the independence axiom to calibrate a series of shopping-trip travel models in the following assumed sequence: mode choice, destination choice, time-of-day choice, and trip frequency (including whether to make the trip). Data at the individual traveler level were used. The relative marginal utilities of modal attributes revealed (estimated) in the mode-choice decision were preserved in the next choice modeled, namely, trip destination, by weighting the attributes of travel by mode to each destination by the probability that the mode would be chosen, given the selection of the destination. The weighting and aggregation are done with the estimated parameters from the previous (mode-choice) decision. The previously estimated strict utilities or "inclusive prices" are preserved. A proof is given that this method of combining separately calibrated travel-choice models is consistent with the assumption of additive utilities. There is no summation over the estimated number of trips because the choice of mode is assumed to be independent of the number of trips between an interzonal pair. "Tastes about modes are (assumed) independent of tastes about trip frequency" (8).

The method can be schematically portrayed for the 4 sequential shopping-trip decisions as follows:

$$\left.\begin{aligned} P(\text{mode}) &= f_m(p, s) \\ P(\text{time of day}) &= f_t(\hat{p}, s) \\ P(\text{destination}) &= f_d(\hat{\hat{p}}, s) \\ P(\text{frequency}) &= f_r(\hat{\hat{\hat{p}}}, s) \end{aligned}\right\} \tag{17}$$

where

p = vector of travel attributes,
$\hat{p}$ = previously estimated strict utility = "inclusive price,"

$\hat{p}, \hat{\hat{p}}$ = inclusive prices previously estimated, and
s = vector of socioeconomic variables.

This is the logical conclusion of the assumption of transitive tastes. (Strict utility suggests that "behavioral time values" have a legitimate place in transportation benefit measurement, assuming transitive tastes!

In summary, the assumption of individuals' evaluating choices such that their probability of choice is expressible as a monotonic function of the choice-specific attributes of all the alternatives (simple scalability or strict utility) has been shown to be the expression of the independence-of-irrelevant-alternatives axiom. This means that the relative probability of choice between 2 alternatives is independent of the attributes of other alternatives in the offered set of alternatives. The transitive nature (strict utility) of the resulting choice behavior results in multinomial, multivariate probability or share models. The separability property of the independence axiom and its resulting multiple-choice share models allow big, complicated travel decisions (e.g., those modeled in direct demand models) to be broken up into smaller, more easily modeled choices. However, these models may be separately calibrated only if separation and sequence assumptions are made. The separately calibrated models can then be linked through their previously estimated parameters into a demand model (i.e., a direct or one stage-pass demand equation). To do so requires use of probabilities (or relative frequencies), not summation of numbers of trips from the prior travel choice in the assumed sequence.

There is, in addition, a set of travel-choice models based on the strong assumption that the choice probabilities are expressible as a function of attributes of subsets of travel choices making up one complete travel decision. This requires the assumption of sequential and completely independent travel choices where the relative valuation of attributes common to 2 or more travel choices, making up one trip decision, is not constant throughout the hierarchy of travel choices (Fig. 1). These models (e.g., the present UTP models) cannot be combined into one direct demand model, but must be applied sequentially in the order in which they have been calibrated, as discussed in the next section.

APPLYING TRAVEL FORECASTING MODELS

Alternative Methods

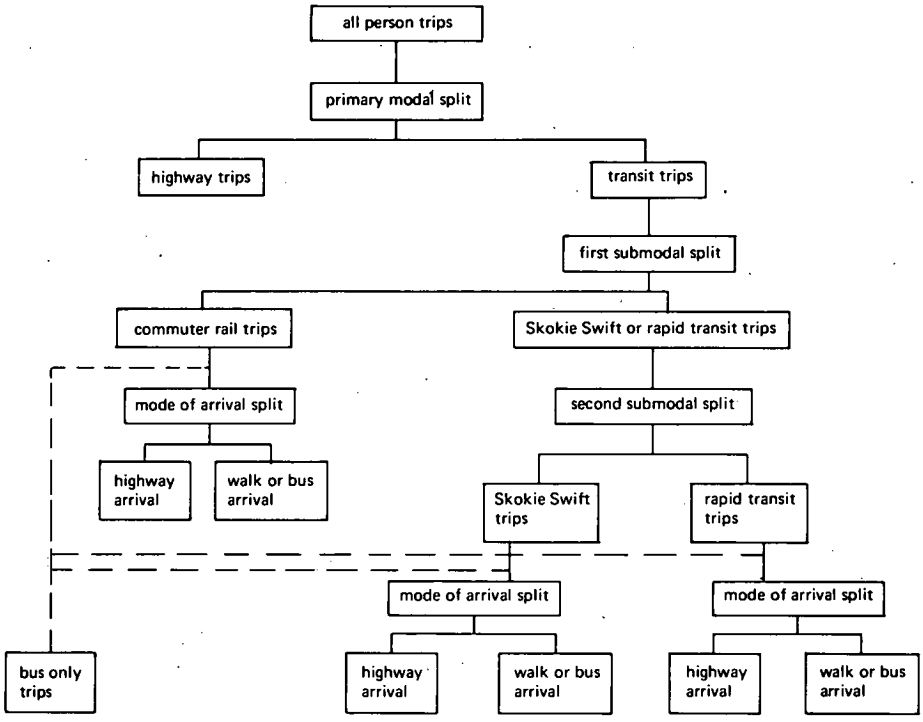
The question remains of how to apply travel forecasting models. Five alternative methods are apparent.

1. Apply the models in chains in their usual UTP order (i.e., trip generation, trip distribution, modal split, traffic assignment);
2. Apply the models in chains as travelers are assumed to order their choices;
3. Link sequentially calibrated travel-choice models parametrically and apply them in one stage (i.e., as a direct demand model);
4. Apply simultaneously calibrated travel models in one stage (i.e., as direct-demand models); or
5. Apply sequentially the conditional and marginal probabilities of separate travel choices derived from the joint probability of a simultaneously calibrated model.

In the first (conventional) strategy of chaining independently calibrated travel-choice models with different relative valuations of independent variables common to 2 or more choices, the sequence of application determines the results. In such cases, the separability property of the independence axiom does not apply among choices. For example, in the application of binary-choice modal-split models in a chain, shown in Figure 2 (32), the results (i.e., splits) calculated higher in the chain are preserved lower in the chain. And in conventional UTP, the trips calculated higher in the chain are normally preserved lower in the chain on any pass through the chain.

The critical problem in method 1 is how to input the system characteristics (attri-

Figure 2. Modal-split chain for commuter travel.



butes) of the choices lower in the chain at points higher in the chain. For example, how in trip generation-trip frequency can the system characteristics for the entire region be aggregated to a single point or zone for input to this first step? The choice attributes can either be summed over (weighted by) trips calculated lower in the chain (e.g., potential functions or gravity-model weighted sums) and brought "up" to be input to higher models in the chain. Or the estimated parameters common to all the ordered-choice models can be used to probabilistically aggregate the choice-specific attributes from the lower level choices. The latter method, as noted before, is the only method consistent with the assumption of additive utilities from sequentially calibrated separable multiple-choice travel models.

If sequential models are derived and calibrated consistently with the (implicit or explicit) behavioral assumptions of preservation of strict utilities in separable multiple-choice models, there is no difference among methods 1, 2, and 3 in the resulting computed network-equilibrium travel patterns. That is, the same separable model may be applied sequentially in a series of separate travel-choice forecasts, or the joint probability distributions of choices may be calculated directly by parametrically combining the separately calibrated choice models as per the independence axiom. However, the sequential application of the models in this case can actually be in any order including methods 1 and 2. The estimated strict utilities are independent of the choices, as per the original behavioral assumption implemented by using the separability property of Eq. 3.

Conversely, from a simultaneously calibrated model satisfying the independence axiom, the conditional and marginal probabilities of travel choice may be derived, and the separate submodels of travel choice may be applied sequentially. Submodels so derived may be applied in any order, including methods 1 and 2. Joint estimation of the choice probabilities eliminates the need for the sequence assumption, but not the separation assumption, for models based on or consistent with the independence axiom.

Models based on or consistent with the independence axiom are separable multiple-choice models. Preference for any method of application is a matter of convenience, control, and purpose of the transportation systems analysis. For example, it is often desirable to be able to compute travel in sequential steps (generation, distribution, and so on) in order to be able to check the intermediate results and exert control over the forecasting process in some way. A direct application of the parametrically combined or simultaneous model may be appropriate if the user is confident of his results and wants to save time and money. If the model has been derived in a fashion consistent with its behavioral assumptions, both methods will produce the desired output for calculating the flow volumes on links in a transportation network. The choice of method should be based on the requirements of different planning environments.

Because the aggregate of trips, not the probabilities, are assigned to a network, a complete run through the sequence will be required to produce the joint probability distributions of travel (including trip-frequency probabilities) needed for aggregating over the total number of individual trip-makers to calculate the aggregate demand. Assignment of trips must also be made to update link and path supply functions for computation of an appropriate network equilibrium. Network equilibration can proceed either through incremental (fractional) loading or by iterating.

Defining Alternative Travel Choices

In the application of separable, multiple, choice-specific travel models (models having the separability property of the independence axiom), great care must be taken in choosing alternatives in order that the separability property not be too strong for the application. The strict utilities in these models are estimated in choice-specific situations even though the separability property of Eq. 3 allows travel choices to be separated for forecasting purposes while still preserving the strict utilities. Truly independent and distinct alternatives as perceived by travelers should be chosen in the application of separable multiple-choice share models. A black bus following the same route as a yellow bus, when chosen as an "independent" alternative, has the effect of reducing the use of automobile (the third choice) in order to preserve the relative odds of choosing automobile over either of the bus alternatives taken singly. This is a misapplication of the separability property because the property would appear to be too strong in this application. In model calibration, the color of the bus does not usually specify or identify a choice, so this seems perfectly clear. The black bus running on a different route from that of the yellow bus between the same origin and destination would have the same effect; and again this effect appears too strong, unless the strict utilities are clearly identified as route (choice) specific. If the yellow bus were now changed to yellow rail transit, and if the multiple choice-specific model were calibrated specifically with rail and bus transit parameters, as well as with automobile parameters, the separability property would appear not to be troublesome. Caution, however, is certainly advised.

Alternative destinations are rarely if ever defined in such a way that choice-specific strict (destination place) utilities are estimated for each destination. That is, the use of socioeconomic variables to describe the (static) trip-end activities amounts to the behavioral assumption of choice-abstract destination-place attributes embedded in an otherwise choice-specific travel demand model. Even more troublesome for the use of separable travel models are the implications of changing the destination alternative set from a small set of alternatives used for model calibration, each having nonzero probabilities of choice, to the usual large number of alternatives, among which trips are forecast in order that a high degree of resolution may be obtained for traffic-assignment purposes. In such cases, forecasting should probably be a 2-step process. That is, forecasts of trips should be made to large aggregations of zones, grouped on the basis that they are distinctly different and real (known) alternative destinations to travelers at the origin. Such grouped destinations might be based on a hierarchy of increasingly regionally oriented work or shopping places for the type of worker or shopper in each zone. Destinations not likely to be known to travelers at each origin

would be eliminated from consideration. Forecasts to these zonal aggregations would then be allocated in some way to the small component zones for traffic-assignment purposes (e.g., based on employment share). Another possible way of forecasting is simply to truncate to zero trips to low (calculated) probability destinations, just as low or zero probability destinations were excluded from the data used in model calibration, as per the separability property of the independence axiom.

In summary, in an application of a separable multiple-choice share model (Eq. 3) within a hierarchical level (e.g., mode choice), the implication of the independence axiom is that the introduction of an additional transit alternative (mode or submode other than one for which the choice-specific strict utilities were estimated) will change the probability of choice (modal split) for all the existing modes. The relative share of all the existing modes included up to then in the analysis will be preserved because of the independence axiom. This also means that the cross elasticity of the modal fraction for each old mode with respect to an attribute of the new mode is the same for each of the old modes. For example, the cross elasticity of modal fraction on the old modes with respect to fare on a new transit submode will be equal for all automobile and transit alternatives considered thus far. This precludes a pattern of differential substitutability among modes and, in effect, implies a (mode) choice-abstract model with respect to the modal fraction, but not with respect to aggregate demand, however (8, 28).

A number of specific examples, such as the above black and blue bus versus the yellow and red bus, can be and have been used as criticisms of the overly strong separability properties of the independence axiom in many instances. Much practice will be required in defining alternatives before multiple-choice share models are usable in any but the most straightforward mode-choice situations in which they have thus far been applied with apparent success (e.g., by Rassam, Ellis, and Bennett, 30). One set of arguments in certain situations consists of citing examples where the relative odds of choice in a binary-choice situation are unlikely in fact to be preserved when new choices are offered [i.e., the black and yellow bus argument, or a second Beethoven record added to an original Debussy and Beethoven binary choice (9)]. Luce and Suppes (23) state:

> We cannot expect the choice axiom to hold over all decisions that are divided in some manner into two or more intermediate decisions. It appears that such criticisms, although usually directed towards specific models, are really much more sweeping objections to all our current preference theories. They suggest that we cannot hope to be completely successful in dealing with preferences until we include some mathematical structure over the set of outcomes that, for example, permits us to characterize those outcomes that are simply substitutable for one another, and those that are special cases of others. Such functional and logical relations among the outcomes (alternatives) seem to have a sharp control over the preference probabilities, and they cannot long be ignored.

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