Pricing and Transportation Capacity

Herbert Mohring
University of Minnesota

Both consumer behavior in taking trips and the rules for optimizing the facilities on which these trips are taken are similar to the corresponding phenomena of relevance to any other commodity. In traveling, consumers act as if the price they pay for a trip is its direct money cost plus the time required times the value of travel time. Using travel facilities efficiently requires that consumers be charged tolls that force them to take into account the costs their trips impose on other travelers. Optimizing travel facilities requires that the sum of the direct resource costs and the value of consumer-supplied travel time be minimized. Failure to charge appropriate tolls yields welfare losses that are of substantial size on heavily congested facilities. Facilities that operate under the constraint that efficient tolls cannot be charged are developed for optimizing facilities. The implications of these rules for expressway travel are roughly quantified.

The basic purpose of this paper is to inquire into the role of price in the use and development of transportation facilities. More specifically, it seeks to answer the following questions:

1. What is the nature of the price that governs consumer decisions on the number, destination, and timing of trips?
2. What is the nature of the price that must be charged households if existing transportation facilities are to be efficiently used?
3. What price should govern social decisions regarding the expansion or contraction of transport facilities when the prices charged consumers are and are not those required for the efficient use of these facilities?
4. Finally, what costs does society incur as a result of its failure to price these facilities efficiently?

The prices of relevance to household travel decisions and to social decisions regarding the efficient use and expansion of transportation facilities are quite similar to those for any other commodities. This is true despite the fact that transportation is, in a way, qualitatively different from most of these other commodities. Decisions (whether efficient or inefficient) about the nature and pricing of transportation facilities have effects that pervade the economy. To explore these effects fully requires a truly monumental study. This analysis is therefore restricted to identifying the quantitative implications of inefficient pricing of highways from two very simple models.

HOUSEHOLD TRAVEL DECISIONS AND THE NATURE OF EFFICIENT PRICING AND INVESTMENT PROCEDURES

In benefit-cost analyses, time savings have long been regarded as a benefit of highway improvements. If an improvement cuts an hour from the time required for a truck round trip between here and there, a trucking company saves 1 hour's truck driver wages for each round trip it provides. It clearly seems appropriate to regard this saving as a benefit of the improvement.

Although the legitimacy of doing so is still occasionally questioned, the same sort of procedure is also commonly used in dealing with the benefits consumers derive from travel time savings. That is, if an improvement reduces the time required for a particular type of trip, this time saving multiplied by the number of trips of that type that are taken multiplied by some more or less arbitrarily selected value of private passenger vehicle travel time is normally recorded as a benefit of the improvement.

Two fundamental questions about this prevailing procedure for dealing with consumer travel time savings deserve answers: First, does the representative consumer value time savings in the same manner as the truck operator? That is, is the way in which he values these savings equivalent to multiplying the number of hours saved by an appropriate hourly value? Second, even if this is the way he takes them into account, might it not be appropriate to attach a different value to these savings in striving to allocate society's scarce resources efficiently? For example, an argument once commonly heard was that time saved in private passenger vehicle travel should be assigned a zero value inasmuch as this travel is not directly involved in any productive activity. Might not this or related views be valid?

As it turns out, the answers to these two broad questions are respectively yes and no. That is, utility-maximizing consumers do value travel time savings in essentially the same fashion as do truck operators. Furthermore, analysis of transportation improvements should take these consumer valuations into account. The validity of these assertions is demonstrated by analyzing two simple problems. First, consumer i, one of n individuals in a society, is assumed to derive utility from consuming a general-purpose commodity ("everything else") and a specific commodity—what happens there (be it work or recreation) when he takes bus trips from here to there and back. However, the consumer derives disutility from the time he spends taking these trips. He is subject to a budget constraint

$$I^i = a^i + Fb^i$$

where

I^i = his fixed dollar income,
a^i = his everything else consumption level (priced at \$1 per unit),
b^i = his bus trip consumption level, and
F = fare charged per bus trip.

The analysis demonstrates that, in maximizing his utility under these circumstances, the consumer acts as if the price he pays for a bus trip, P^i, equals $F + v^i t$ where t is the time required for a trip and v^i is his value of travel time, the amount of everything else he would be willing, at the margin, to give up in return for saving an hour's travel time. (This problem can easily be reinterpreted to account for any other transportation mode. For example, b^i could be interpreted as the number of automobile trips the consumer takes between here and there and F as the sum of his vehicle operating costs per trip and the highway user charges imposed on him. X, defined later as the number of bus-hours of service provided on the here-there route, would then become a measure of the capacity of the here-there highway.)

The second simple problem involves assuming that society wishes to maximize social welfare, a function $W(u^1, \ldots, u^n)$ of the utility levels, u^i, of its individual members. In doing so, it is subject to a constraint on total output that can be written

$$R = A + CX$$

where

A = total amount of everything else produced for all consumers,
X = number of bus-hours of service devoted to providing here-there trips, and
C = cost of an hour's bus services, i.e., the number of units of everything else given up to obtain these services.

The time required for a here-there round trip is a function $t(B, X)$ of X and the total number of trips taken, B. The analysis demonstrates two basic conditions for this maximization problem. First, the fare for a bus trip must be set equal to the cost that that trip imposes on other travelers

$$\Sigma b^i v^i \partial t/\partial B = BVt_B$$

the total number of trips taken ($B = \Sigma b^i$) times the weighted (by number of trips taken) average value of travel time ($V = \Sigma b^i v^i/\Sigma b^i$) times the change in the time required for a trip resulting from an additional trip being taken. Second, given that this efficient or benefit-maximizing fare is charged, maximization of social welfare requires that the quantity of bus service provided be that that minimizes $CX + BVt$, the dollar cost of bus service plus the value consumers attach to the time they spend traveling. This is essentially the same cost-minimization problem that would be faced by a truck operator in determining the total number of trucks to buy and the number of driver hours to hire in providing truck service between here and there (5).

In the first problem, consumer i's goal is to maximize his utility, $u^i(a^i, b^i, \tau^i)$, subject to the budget constraint, $I^i + a^i + Fb^i$, where $\tau^i = b^i t$ = total travel time. (Alternatively, the consumer's utility could be regarded as depending on time per trip, t, rather than total travel time, bt. Although the algebra would be more complicated, the conclusion reached would be unchanged.) Setting up the Lagrangian expression

$$z^i = u^i(a^i, b^i, \tau^i) + \lambda^i(I^i - a^i - Fb^i)$$

and differentiating with respect to a^i and b^i yield

$$z_a^i = u_a^i - \lambda^i = 0 \quad (1)$$

$$z_b^i = u_b^i + u_\tau^i t + u_\tau^i bt_B - \lambda^i F = 0 \quad (2)$$

as first-order conditions for utility maximization where subscripts refer to partial derivatives. It seems reasonable to suppose that consumer i does not take into account the effect his trips have on his own travel time. If so, $u_\tau^i bt_B$ can be ignored. Dividing Eq. 2 by Eq. 1 then yields

$$u_b^i/u_a^i + t\, u_\tau^i/u_a^i = F \quad (3)$$

In Eq. 3, u_τ^i/u_a^i is the ratio of the marginal disutility of travel time to the marginal utility of dollars in dollars per hour. It therefore seems reasonable to substitute for this ratio $-v^i$, the money cost consumer i attaches to his travel time. Doing so changes Eq. 3 to

$$u_b^i/u_a^i = u_b^i/\lambda^i = F + v^i t \quad (4)$$

This relationship says that consumer i will set the ratio of the marginal utility of bus trips to that of dollars equal to the fare plus the time cost of a trip.

In regard to society's welfare maximization problem, the economy is assumed to be subject to the constraint

$$R = A + CX$$

where

R = weekly flow of services from the available stock of resources,
A = weekly consumption of everything else, Σa^i, and
C = resource service cost of providing a bus-hour of services.

Implicit in this formulation is the assumption that the cost of a bus-hour is independent of both the carrying capacity of the bus and the speed at which it travels. Although clearly unrealistic, this assumption is not so bad as it may seem. An increase in bus capacity results in a less-than-proportionate increase in its capital, fuel, and related costs. More important, driver wages and fringe benefits account for about 70 percent of the total costs of typical urban bus operation. The cost of a driver-hour is independent of the size of the bus he operates. This implicit assumption has two consequences for the analysis. First, there is always room aboard a bus for an additional passenger. That is, no one must ever wait for another bus to come because the first bus passing his stop is loaded to its capacity. Second, the bus company incurs no direct cost as the result of serving an additional passenger. The additional passenger affects the system only by increasing the travel time costs incurred by other passengers. Modifying the analysis to allow for bus capacity constraints and costs imposed on the bus company by additional passengers would fundamentally not alter the conclusions reached.

Setting up the Lagrangian expression

$$Z = W(u^1, \ldots, u^n) + \eta(R - A - CX) \tag{5}$$

and differentiating with respect to a^i and b^i yield

$$W_i u_a^i - \eta = W_i \lambda^i - \eta = 0 \tag{6}$$

$$w_i(u_b^i + u_T^i t) + \Sigma W_j u_T^j b^j t_B = 0 \tag{7}$$

as first-order conditions where $W_i = \partial W / \partial u^i$ = the "marginal welfare weight" attached to individual i. The second equality in Eq. 6 follows from Eq. 1. By appropriately substituting Eqs. 3, 4, and 5, Eq. 7 can be shown to reduce to

$$\eta(F - BVt_B) = 0 \tag{8}$$

where V is the weighted (by number of trips taken) average value of travel time, $\Sigma b^i v^i / \Sigma b^i$. The Lagrangian multiplier, η, can be interpreted as the welfare gain resulting from a one-unit increase in available resource services. It is presumably positive. Hence, Eq. 8 says that, if welfare is to be maximized, the fare per trip must equal the difference between the marginal and the average time costs of a trip. That is, the fare must equal the additional time costs resulting from an additional trip minus those time costs incurred by the traveler himself.

Differentiating Eq. 5 with respect to X and making substitutions similar to those that led from Eq. 7 to Eq. 8 yield

$$-\eta(VBt_x + C) = 0 \tag{9}$$

This is the same expression that would result from selecting that value of X required to minimize $VBt(B, X) + CX$. This sum is the total time and dollar costs of B trips if an hour of consumer travel time is assigned its weighted average value.

This simple bus problem suggests that in transportation activities $F + v^1t$, the direct money cost of a trip plus the money equivalent of the time it requires, plays the same role in the consumer decision process as does the price of commodities usually dealt with in textbook discussions of consumer behavior. Furthermore, this simple problem suggests that it is efficient to take consumer travel time values, v^1, directly into account both in setting prices for transportation facilities and in designing them.

Expressed differently, the basic qualitative difference between transportation activities and the commodities that are commonly dealt with in textbook discussions is that, in transportation, consumers play a producing as well as a consuming role. In taking trips (whether by automobile, bus, or air), consumers both use transportation facilities and provide inputs—their own time, in particular—vital to the production process. The analysis indicates that, in the design and pricing of transportation facilities, it is appropriate to take these consumer-supplied inputs into account in precisely the same fashion as are inputs that are purchased on the open market for more conventional production processes.

COST OF INEFFICIENT USE OF EXISTING HIGHWAY CAPACITY

The following attributes of highway travel seem well established: The time required to complete a trip on a given street or highway increases with the number of trips travelers attempt to make on it. Starting from a low level, an increase in attempted trips is associated with an increase in the number of trips actually completed. However, once the capacity of the highway is reached, a further increase in attempted trips results in a reduced number of completed trips. In this range of highway utilization rates, the volume-capacity ratio is inversely related to trip demand. Indeed, as attempted trips continue to increase beyond highway capacity, the volume-capacity ratio approaches zero. That is, attained travel speeds become smaller and smaller, implying that the time required to complete a trip becomes indefinitely great.

The simplest functional relationship that yields these attributes can be written

$$N - 1 + a^2/4b = aS - bS^2$$

where

N = volume-capacity ratio = the ratio of completed trips T to highway capacity K and
S = speed.

[Adding $-1 + a^2/4b$ to the right of this expression is necessary to ensure that the maximum traffic flow occurs at a volume-capacity ratio of one. No empirical studies of speed-volume-capacity relationships have relied on this functional form. However, assigning reasonable values to a and b yields speed-volume relationships similar to those that have been reported in the literature (4).] If vehicle speed at a volume-capacity ratio of zero is twice that at a volume-capacity ratio of one, this relationship reduces to the simpler form

$$N = 4c(S - cS^2) \qquad (10)$$

Equation 10 can be solved for

$$t = 2c/[1 \pm (1 - N)^{1/2}] \qquad (11)$$

where t (=1/S) is the time required to travel 1 mile. In this relationship, the plus and minus signs hold when attempted trips are respectively above and below capacity levels. On an expressway, 60- and 30-mph speeds are reasonable round numbers to use for

Table 1. Travel times and marginal cost tolls per mile for various volume-capacity ratios.

Travel	Volume-Capacity Ratio	Average Time Per Mile (minutes) Below Capacity	Average Time Per Mile (minutes) Above Capacity	Time Increase for All Travelers, Below Capacity (minutes)	Marginal Cost Toll (cents/mile)
Expressway	0.0	1.00	∞	1.00	0.0
	0.1	1.03	38.97	1.05	0.1
	0.2	1.06	18.94	1.12	0.3
	0.3	1.09	12.24	1.20	0.5
	0.4	1.13	8.87	1.29	0.8
	0.5	1.17	6.83	1.41	1.2
	0.6	1.23	5.43	1.58	1.8
	0.7	1.29	4.42	1.83	2.7
	0.8	1.38	3.62	2.24	4.3
	0.9	1.52	2.92	3.16	7.7
	0.95	1.63	2.58	4.47	14.2
	1.0	2.00	2.00	∞	∞
City streets	0.0	2.40	∞	2.40	0.0
	0.1	2.47	93.53	2.52	0.2
	0.2	2.54	45.46	2.69	0.7
	0.3	2.62	29.38	2.88	1.2
	0.4	2.71	21.29	3.10	1.9
	0.5	2.81	16.39	3.38	2.9
	0.6	2.95	13.03	3.79	4.3
	0.7	3.10	10.61	4.39	6.5
	0.8	3.31	8.69	5.38	10.3
	0.9	3.65	7.01	7.58	18.5
	0.95	3.91	6.19	10.73	34.1
	1.0	4.80	4.80	∞	∞

Figure 1.

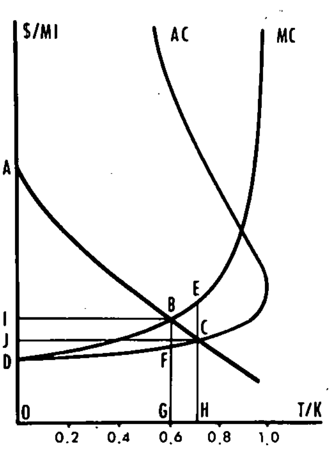


Figure 2.

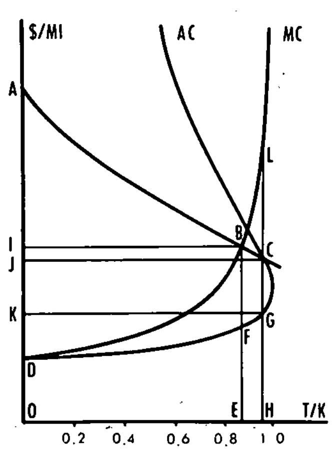
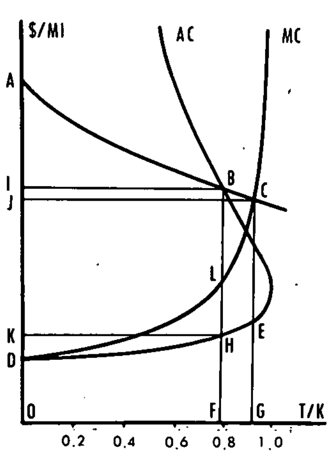


Figure 3.



travel rates respectively equal to zero and to highway capacity. Corresponding reasonable numbers for city streets might be 25 and 12.5 mph. With speed expressed in miles per minute, these values imply c equals 1 for expressways and 2.4 for city streets.

Multiplying Eq. 11 by T, the total number of trips taken, yields the total travel time required for these trips. Differentiation of tT with respect to T then yields marginal travel time per trip, the additional travel time resulting from an additional trip being taken. When demand is less than capacity, this expression can be written

$$\partial(tT)\partial T - t = N/\{[1 + (1 - N)^{1/2}]^2 (1 - N)^{1/2}\} \qquad (12)$$

In Eq. 12, t is the travel time input, per mile, of the marginal traveler himself and the right side is the number of minutes by which his 1-mile trip increases the trip time of all other travelers. If the effect of volume-capacity ratios on vehicle operating costs can be ignored, the analysis presented earlier indicates that efficiency would require charging each traveler (each traveler is, after all, the marginal traveler for the purpose at hand) a toll equal to the right side of Eq. 12 times an appropriate weighted average value of travel time. For alternative volume-capacity ratios, data given in Table 1 rely on Eq. 11 to determine the travel times per mile associated with efficient and inefficient use of expressways and city streets, inefficiency being involved when demand exceeds capacity. Table 1 also uses Eq. 12 to determine marginal travel time per mile and, if the appropriate value of travel time is \$3 an hour (1, 2), marginal cost tolls when the highway is operating efficiently.

The toll actually charged for highway use in the United States is user charges such as gasoline and other excise taxes. These currently appear to average about 1 to 1.5 cents per vehicle mile. This is a level appropriate for volume-capacity ratios on the order of 0.5 for expressways and 0.25 for city streets if the assumptions underlying Table 1 can be believed. However, for the purpose at hand, i.e., determining the social costs involved in not charging efficient tolls, it is useful to ignore any direct cash outlays by consumers for travel including those for vehicle operation. That is, it is useful to suppose that the only current cost of highway trips is the time they consume.

A commonly used measure of the gross consumer benefit derived from a commodity is the area under the demand schedule for it, the sum over all units consumed of the maximum amount that a consumer would be willing to pay for each unit. A commodity's net consumer benefit or, more commonly, its consumers' surplus is its gross benefit minus the amount that consumers actually pay for it. Given some restrictive assumptions, the efficiency conditions discussed earlier can be interpreted as requiring maximization of the difference between the gross consumer benefit and the cost of producing a commodity.

Figures 1, 2, and 3 show data from Table 1 and three alternative demand relationships plotted. Assume that the demand schedule for trips along a particular highway is ABC in Figure 1. The efficiency conditions would be satisfied at a trip consumption rate of OG, this being the rate at which the price some consumer would be willing to pay for a trip equals its marginal time cost (again, vehicle operating costs are being ignored). Consumers would take trips at this rate if they face a price of BG per mile. Because OG trips are associated with a time cost of FG per mile, this price could be effected by imposing a toll of BF per mile. The gross consumer benefit associated with OG trips is the area OABG. The net consumer benefit is area ABI, the difference between OABG and the sum of the time costs incurred by travelers and their toll payments.

Although the tolls travelers pay are clearly a cost to them of taking trips, the tolls are not the cost these trips impose on society as a whole. Rather, toll payments involve a transfer of income, of purchasing power, from travelers to whatever agency is responsible for collecting tolls. Payments such as these are commonly referred to as rents or producers' surpluses. In the case at hand, this rent can be interpreted as a reward to society for its provision of scarce, valuable road services. Payment of this reward serves two important purposes. First, it is essential if these road services are to be used efficiently. Second, it enables the reduction of other taxes collected for public purposes.

Now suppose that tolls are no longer imposed. The trip consumption rate would then increase to OH, the level at which the amount a consumer would be willing to pay for a trip equals its time cost to him. As a result of this increase in trip output, gross consumer benefits would change from OABG to OACH, an increase of GBCH in these benefits. However, these additional gross benefits come about at the expense of additional travel time costs aggregating to GBEH. (If OG trips are taken, total travel time costs equal FG dollars per trip mile times OG trips. FG times OG can be shown to equal the area, ODBG, under the marginal time cost of trips schedule in Figure 1. Similarly, if OH trips are taken, total travel time costs equal CH times OH. This quantity can be shown to equal the area ODEH in Figure 1. GBEH is the difference between ODEH and ODBG.) Thus, the cost increase that results from the additional GH trips exceeds the gross benefits generated by these trips by an amount equal to BEC. This difference between the increases in gross benefits and travel time costs can be interpreted as follows: In the demand situation depicted in Figure 1, elimination of the toll has the direct effect of making travelers better off. The increase from FG to CH in travel time per mile is more than offset by elimination of the toll. As a result, the price travelers pay for trips falls by IJ per mile. They therefore receive additional consumers' surplus benefits, which equal area IBCJ. At the same time, however, the government agency involved loses a source of revenue. Area BEC is the amount by which the taxes that would have to be levied to replace the toll revenues lost exceed the benefit consumers derive from not having to pay tolls.

Consider, now, demand schedule ABC in Figure 2. Efficiency would dictate that OE trips be taken. This trip output level could be effected by imposing a toll of BF. With such a toll in effect, total benefits from using the highway would equal ABD, which can be divided into a consumers' surplus of ABI plus a rent (equals total toll collections, BF times OE) of IBD.

In the absence of tolls, the system would be in equilibrium (if at all) at C in Figure 2. This equilibrium is inefficient in two senses. First, as with the equilibrium at C in Figure 1, an inefficiently large number of trips would be produced. A loss of BLC (which corresponds to BEC in Figure 1) in potential benefits results from this excessive output rate. Second, equilibrium at C involves producing OH trips at a cost per trip HC that is substantially greater than the minimum cost HG, at which this same quantity of trips could be produced. The total loss associated with the no-toll equilibrium in Figure 2, then, is area JCGK plus area BLC. As in Figure 1, this area equals the amount by which the taxes that would have to be levied to replace the toll revenues generated by equilibrium at B exceed the additional consumers' surplus benefits, IBCJ, derived from not having to pay tolls.

Finally, for demand schedule ABC shown in Figure 3, efficiency would dictate equilibrium at C with a toll of CE. In this equilibrium, total benefits would equal the area ACD. Elimination of tolls would lead to equilibrium at B. A pair of inefficiencies comparable to those of Figure 2 is involved in this equilibrium:

1. Producing an output different from that at which price equals marginal cost leads to a loss of BCL in potential benefits.
2. Inefficiency results from the loss of IBHK associated with producing OF trips at a time cost of BF per trip rather than the minimum possible cost HF.

Unlike the situations in Figures 1 and 2, however, the equilibrium trip price in the absence of tolls is higher in Figure 3 than that that would eventuate with tolls in effect. Thus, if the demand for trips along a stretch of highway is so great that an equilibrium eventuates like that at B in Figure 3, everyone involved could immediately be made better off by the imposition of tolls: The government agency would reap a formerly unharvested source of revenue, and travelers would consume more trips at a lower

Table 2. Percentage loss due to failure to charge marginal cost tolls as a fraction of travel expenditures when demand is inversely proportional to price.

Volume-Capacity Ratio				
Efficient	With No Toll	Loss 1[a]	Loss 2[b]	Total
10	10.3	0.04	–	0.04
20	21.1	0.17	–	0.17
30	32.6	0.45	–	0.45
40	45.0	0.98	–	0.98
50	58.2	1.95	–	1.95
60	72.4	3.77	–	3.77
70	87.0	7.52	–	7.52
80	98.9	17.06	–	17.06
90	82.1	1.68	59.45	61.13

[a]From producing inefficient number of trips.
[b]From producing trips in backward bending portion of average time cost schedule.

total cost per trip.

Just how consequential are the losses suggested by Figures 1 through 3? Some idea of the orders of magnitude that might be involved can be obtained by (a) assuming a specific functional form for the demand relationship, (b) assigning the alternative values to the parameter in this relationship necessary for it to intersect the marginal time cost schedule at different volume-capacity ratios of interest, and (c) determining the point at which this demand schedule intersects the average time cost schedule. With this information, the total benefits derived from use of the highway with and without tolls given alternative positions of the demand schedule could be computed. The difference between these two benefit levels divided by, say, the sum of the time and money expenditures on trips that would result if tolls were charged would then give a measure of the proportionate consequence of inefficient pricing.

Calculations of this sort are given in Table 2. The demand relationship underlying these calculations can be written PT = k. (This demand relationship is used for two reasons. The first is computational simplicity. The second is the fact that price changes that are not offset by changes in income must, on the average, result in offsetting proportional changes in consumption.) According to this relationship, total consumer expenditures on travel are independent of the price paid for a trip. An increase in trip price would result in an exactly offsetting reduction in the number of trips taken, thereby leaving total expenditures unchanged.

The absolute loss per mile traveled as a result of inefficient pricing is greater at any given volume-capacity ratio for high values of travel time than for low values and greater on city streets than on expressways. However, the nature of the demand and travel time-volume-capacity relationship underlying Table 2 is such that the relative loss measure tabulated depends only on the efficient volume-capacity ratio. It is independent of both the value of travel time and the specific type of trip under examination. Table 2 indicates that, if the demand for street or expressway services is so low that volume-capacity ratios of 40 percent or less would result when marginal cost tolls are imposed, complete elimination of these tolls would lead to neither significant increases in volume-capacity ratios nor appreciable welfare losses. As for the latter, if the efficient volume-capacity ratio is 40 percent or less, the ratio of the loss from eliminating tolls shown in Figure 1 to the trip expenditures that would result under efficient pricing is less than 1 percent. As efficient volume-capacity ratios increase above 50 percent, however, the effect of toll elimination on both volume-capacity ratios and losses becomes more and more significant. Indeed, if efficient pricing would result in a 90 percent volume-capacity ratio, the sum of the two losses shown in Figure 3 would amount to more than 60 percent of the total expenditures on trips. Such losses may well be common in major urban areas during morning and afternoon rush hours.

OPTIMIZATION OF HIGHWAY CAPACITY THROUGH INEFFICIENT PRICING

The model developed earlier dealt with a short-run situation. It asked, what are the social costs of inefficiently using a highway of fixed capacity? Given time, the capacity of any highway can be altered, which would partially offset the inefficiencies discussed. Now we will examine the rules for optimizing highway capacity when efficient prices cannot be used to ration highway services and determine rough orders of magnitude for the losses that result from inefficient pricing under these circumstances.

As stated previously, a common measure of the gross benefit consumers derive from taking trips is the sum over all trips taken of the maximum amount they would be willing to pay for each trip they take. If, for simplicity, each traveler is assumed to place the same value on his travel time, the inverse demand function involved in this summation can be written as

$$P = F + Vt(T/K) = g(T) \tag{13}$$

where

- V = average value of travel time,
- F = toll charged for a trip,
- $g(T)$ = inverse demand function, and
- t = travel time per trip, a function of T, the total number of trips taken, and K, the highway's capacity.

The net benefit derived from using the highway is the gross benefit to consumers minus the total cost of providing its services. In the long run, this total cost includes both the time costs borne by consumers (again, vehicle operating costs are ignored) and the costs of providing highway capacity borne by whatever highway authority may be involved. Suppose, albeit unrealistically, that the cost of providing the services of a unit of highway capacity is independent of the size of the highway. Then the net benefits afforded by the highway can be written as

$$B = \int_0^T g(\tau)d\tau - TVt(T/K) - P_K K \tag{14}$$

The integral in this expression is the area under the trip demand schedule, the gross consumer benefit. $TVt(T/K)$ is the value of travel time inputs, and $P_K K$ is the cost per time period of providing the capital invested in the highway.

A sensible objective would seem to be to establish those values of T and K that maximize Eq. 14. Differentiating with respect to T and K and rearranging terms yield

$$g(T) = F + Vt = Vt + TV\partial t/\partial T \tag{15}$$

$$TV\partial t/\partial K + P_K = 0 \tag{16}$$

Equation 15 will be satisfied if the toll per trip F is set equal to $TV\partial t/\partial T$. Equation 16 can be interpreted as saying that arterial capacity should be increased to the point where the cost, P_K, of the last unit of capacity added equals the saving in travel time afforded by that unit, $-TV\partial t/\partial K$. Further manipulation would reveal that, if Eqs. 15 and 16 hold simultaneously, total toll collections would just cover total capital costs. If construction of the highway involves increasing returns to scale (i.e., if a doubling of its capacity would result in less than a doubling of its total costs), however, simultaneous satisfaction of these two equations would result in toll revenues that fall short of total capital costs.

Suppose that it is impossible to charge the toll implied by Eq. 15 but possible to charge some other toll F_0 that may be greater or smaller than the one that would maximize benefits. With this toll in effect, the number of trips taken will be determined by

$$G(K, T) = T - f[F_o + Vt(T/K)] = 0 \tag{17}$$

A reasonable objective would seem to be to select K and T to maximize net benefits subject to this constraint. That is, the objective is that of maximizing

$$B^* = B + \lambda[G(K, T)] \tag{18}$$

where B is given by Eq. 14. Setting the derivatives of Eq. 18 with respect to K and T equal to zero and rearranging terms yield

$$F_o - TVt_T + \lambda(1 - f'Vt_T) = 0 \tag{19}$$

$$TVt_K + P_K + \lambda f'Vt_K = 0 \tag{20}$$

Dividing Eq. 19 by Eq. 20 and again rearranging terms yield

$$P_K + TVt_K = (F_o - TVt_T)[f'Vt_K/(1 - f'Vt_T)] \tag{21}$$

The left side of Eq. 21 is the difference between the price of an additional unit of capacity and the value of the travel time inputs that would be saved by adding that additional unit. In Eq. 21, $(F_o - TVt_T)$ is the difference between the arbitrarily specified toll and the increase in the travel time costs of other travelers that would result from an additional trip. In the second term, f' is the change in trip consumption rates that would result from a change in the full price of a trip and t_T and t_K are the change in time per trip that would result respectively from an additional trip and from an additional unit of capacity. The signs of f', t_K, and t_T are negative, negative, and positive. As a result, the second term has a positive sign. This means that the entire right side of Eq. 21 will be negative if F_o is less than TVt_T, which seems to be the prevailing state of affairs on urban arterial streets and expressways, at least during morning and afternoon rush hours. If F_o is less than TVt_T, Eq. 21 indicates that (constrained) efficiency requires establishing a capacity level such that P_K is less than $-TVt_K$, i.e., such that the price of the last unit of capacity added is less than the value of the travel time inputs saved by that unit.

In an attempt to provide a verbal rationalization for this finding, suppose that the toll component of the full price of a trip is, for whatever the reasons, less than the cost that trip imposes on other travelers. Also suppose that the road has been designed so as to minimize the sum of the cost of providing road capacity and the travel time costs incurred by those who use the road. The sort of loss described in discussion of Figure 1 would result from charging an inefficiently low toll. Contracting the capacity allocated to the road would increase travel costs by more than it would reduce capacity costs. At the same time, however, doing so would increase the travel time component of the price of a trip and thereby reduce the number of trips taken and, in turn, the size of the loss resulting from the inefficiently low money component of the price of a trip.

The prescription implied by Eq. 21 can be stated briefly. If the toll levied on trips is constrained to be less than that required to equate their price and their marginal costs, (constrained) efficiency requires building what is, in a sense, an inefficiently small road, i.e., a road that has a capacity less than that required to minimize the sum of the time and capacity costs associated with the number of trips being taken. Essentially the same reasoning applies when a too-high toll is charged. Then, constrained efficiency would dictate building an inefficiently large road—one for which the saving in travel time costs resulting from the last unit of capacity added to the highway is less than the cost of that unit of capacity.

It would be of interest to quantify both the capacity adjustments and the remaining inefficiencies that would result from application of Eq. 21. Unfortunately, the cost of an additional unit of city street capacity varies so much from situation to situation that this sort of quantification is impossible. Information enabling rough quantification of the implications in Eq. 21 for expressway travel is available, however.

Rough data developed by Meyer, Kain, and Wohl (3, pp. 200-211) can be interpreted as indicating that the annual cost at 6 percent interest of the captial invested in constructing 1 mile of an L lane urban freeway is approximately

$$Z = 11{,}200 + 2{,}500D + (7{,}200 + 300D)L$$

where D is the net residential density of the area through which the freeway passes, i.e., population per acre of land actually used for residential purposes. In addition, annual right-of-way costs are on the order of 0.005DZ. In the United States, net residential densities in urban areas range between about 10 and 300.

That a substantial proportion of both construction and right-of-way costs are independent of the number of lanes provided reflects the fact that a freeway includes paved shoulders on both sides of each set of traffic lanes, a median strip between opposing traffic streams, and buffer strips between the freeway and adjacent land. As a result, of the land area used by four- and eight-lane freeways, only about 33 and 50 percent respectively is devoted to freeway lanes themselves.

The capacity of a four- to eight-lane expressway is about 1,800 vehicles per lane-hour. Using this number to convert annual cost data into average and marginal costs per vehicle mile of capacity yields the data given in Table 3. These data suggest using 0.1 and 3 cents as lower and upper bounds in the illustrative calculations.

Suppose that the travel time-volume-capacity relationship described holds and that the average value of travel time is $3 per hour. Inserting these values into Eq. 16 yields a benefit-maximizing volume-capacity ratio of 24.6 percent for a highway with 0.1 cent per vehicle mile capacity costs and 78 percent for one with a 3-cent capacity cost. From Eq. 15, the corresponding efficient tolls are 0.41 and 3.85 cents per mile and the corresponding trip prices are 5.76 and 10.66 cents per mile.

As was suggested earlier, the prevailing user charge for freeway travel in the United States appears to lie in the 1- to 1.5-cent range. Suppose that the demand for trips takes the form TP = k where P is given by Eq. 13. Inserting a 1.25-cent toll, this demand relationship, and the values described in the preceding paragraph into Eq. 21 yields 23.4 percent as the constrained optimum volume-capacity ratio for the 0.1-cent capacity cost highway and 83 percent for that with a 3-cent capacity cost. The trip prices associated with these volume-capacity ratios and a user charge of 1.25 cents are respectively 6.58 and 8.33 cents. They would lead respectively to 12.5 percent fewer trips on the low-cost highway and 27.9 percent more trips on the high-cost highway than would be optimum if efficient prices could be charged.

Relative loss measures of the sort given in Table 2 can be constructed for these two examples by (a) evaluating Eq. 14 for optimum and constrained optimum trip consumption and volume-capacity levels and (b) dividing the difference between the resulting net benefit measures by the highway expenditure levels implied by the demand equation, TP = k. The results are 0.87 and 3.79 percent for the low- and high-cost highways respectively.

As is common in empirical research into the costs of inefficient pricing and investment policies, these numbers are less than earthshaking. In partial response to the obvious question, "Why bother?", two points are worth making: First, although

Table 3. Average and marginal capacity costs.

Net Residential Density	Marginal Capacity Cost (cents)	Average Capacity Cost (cents)		
		4 Lanes	6 Lanes	8 Lanes
10	0.07	0.13	0.11	0.10
100	0.35	0.98	0.77	0.66
200	0.85	2.47	1.93	1.66
300	1.54	4.56	3.55	3.05

3.79 is not a large percentage, 3.79 percent of the total time and money costs of urban trips is a large sum of dollars. Second, the constrained efficiency rules imply a grossly inequitable allocation of the burden of financing highways. To repeat, an efficiently designed and priced road would yield toll revenues just sufficient to cover its total capital costs if these costs are proportional to its capacity. However, the constrained optimum low-cost road would generate user charges equal to 2.91 times its capital cost. For the high-cost road, on the other hand, user charges would equal only 34.3 percent of capital costs. Thus, constrained optimization would, in effect, call for the users of rural roads to provide heavy subsidies to their urban counterparts. Elimination of these subsidies by limiting the costs of roads in a particular geographical area to user revenues generated in that area, for example, would lead to far greater aggregate losses than those that would result from following the rules developed here.

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