A Model for Traffic Delay and its Convenience and Wage Costs

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One of the large problems facing engineers in the economic analysis of public projects is the estimation of comfort and convenience costs and of lost wage costs. Comfort and convenience costs have generally been either ignored or treated as a linear function of the trip delay time; wages lost to tardiness have also generally been assumed to be linear when they have been considered at all. But personal experience argues against the idea of a linear comfort and convenience cost—except in unusual circumstances, short delays in the completion of a trip have negligible actual costs. Similarly the wage loss due to tardiness is recognized to have a zero value when the tardiness is below some threshold value, and this threshold value is recognized in most union contracts. This paper treats the problem of calculating comfort and convenience costs analytically and proposes a probability distribution that is easily used to estimate the value per person trip of both such costs. This technique is then applied to cost estimation where the delay is caused by snow and ice on a roadway.

In attempting to quantify the benefits from a proposed transportation system improvement, transportation engineers frequently are vexed with the problem of quantifying comfort and convenience benefits and benefits to industry because of reduced tardiness. It seems difficult to assign a dollar value to the comfort and convenience of travelling rapidly along a well-plowed road rather than one where the traffic does the plowing, yet some measure of this benefit is clearly necessary before an adequate benefit-cost analysis of roadway plowing can be performed. Complicating the problem is the observed fact that a small amount of delay is not regarded as significant. The problem of wages lost due to tardiness is another problem that is not as simple as it first appears. Most union contracts contain a clause on tardiness whereby a person is not docked in pay unless he or she is more than a specified number of minutes late. Due to many shop practices regarding tardiness, the average threshold is probably greater than that specified in the union contract, somewhat complicating the problem of assigning a dollar value to this component of delay. These two problems were addressed in two reports on snow and ice removal completed as part of a project funded by a twelve-state consortium and DOT (1,2) and are the subject of a Transportation Research Board report to be published in the summer of 1978 (3). Although the senior author was a consultant on that project and a co-author of all three reports referred to, the analytical material presented below is new and was not prepared as part of the consulting contract.

Consider again the problem of estimating comfort and convenience costs. A study performed by Stanford Research Institute (4) by examining the tolls people were paying in New Jersey to avoid delays of certain known time durations came up with a series of curves such as that shown in Figure 1. Eight different curves were found, for eight different income categories. Note that the concept illustrated by this curve is far more intuitively appealing than would be the concept of estimating a fixed amount per vehicle minute—being five minutes late may not cause any inconvenience, and the additional cost of being 25 minutes late rather than 20 minutes may not appear as significant as the difference between being seven minutes late rather than twelve.

Figure 1. Comfort and Convenience Cost

Cost per Person

Minutes of Delay

6.5 15 25

6.65 9.25

8.6
To estimate cost due to tardiness, it is necessary to know the threshold at which wages are actually docked and to know the actual wage. For any given industry segment, both of these numbers can usually be either obtained or estimated. The graphical representation of this cost component is shown in Figure 2, using an average wage of $3.50 and a threshold of six minutes.

Figure 2. Tardiness Cost

<table>
<thead>
<tr>
<th>Minutes of Delay</th>
<th>Cost per Person</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>4.70</td>
</tr>
<tr>
<td>12</td>
<td>3.50</td>
</tr>
</tbody>
</table>

Estimating the comfort and convenience costs and the lost wage costs is further complicated by the easily overlooked fact that the delay for a vehicle is a random variable. The magnitude of the delay for any one vehicle is given by

\[ \text{Delay} = \text{Trip Length} \times [\frac{1}{\text{snow speed}} - \frac{1}{\text{normal speed}}] \]

The speed under snow conditions will vary from vehicle to vehicle, as will the speed under normal (dry-road) conditions. Thus, the delay will also vary and the way it varies depends upon the probability distributions of snow speeds and normal speeds. Trip-length also varies but will be assumed constant at this time. Dry road speeds have been observed to generally follow a normal distribution and it was assumed that speeds under snow conditions would follow this same distribution. Unfortunately, the probability distribution consisting of the difference between the reciprocals of two normally distributed random variables is not known. If it were known, however, it could be integrated to obtain the expected value of the comfort and convenience cost and the lost wage cost per person, using the functions shown in Figures 1 and 2. If \( f_D(w) \) is the density function for delay, then the expected value of comfort and convenience cost is given by

\[
\int_0^\infty C(w) f_D(w) \, dw
\]

(1)

Where \( C(w) \) is the equation of the function consisting of zero and the two joining line segments as shown in Figure 1. The corresponding value of the expected cost due to lost wages is given by

\[
W \times [\text{TH}(K) \times (1 - F_D(\text{TH}(K))) + \int_{\text{TH}(K)}^\infty w \, f_D(w) \, dw]
\]

(2)

The first term in this equation is caused by the fact that the person is docked for the first part of the hour if he is late beyond the threshold time, e.g., if the threshold time is 10 minutes and the person is 11 minutes late, he is docked 11 minutes rather than 1 minute.

\( W \) = wage in dollars per hour

\( \text{TH}(K) \) = the threshold time in hours

\( f_D(w) \) = density functions of delay \( D \)

\( F_D(w) \) = cumulative distribution functions of delay \( D \)

The computations associated with calculating the density function described above and with performing the given integrations were treated in the original project, but the density function was very complex and was difficult to use in standard numerical integration routines. At the suggestion of Mr. David Minsk, of the U.S. Army Cold Regions Research and Engineering Laboratory, the authors undertook to find a more tractable solution technique for the computational part of the above problems.

The fundamental problem is to ascertain the type of probability distribution associated with

\[ Y = \frac{1}{S} - \frac{1}{N} \]

where \( S \) is the random variable snow speed assumed to be normal with mean \( \mu_S \) and standard deviation \( \sigma_S \) and where \( N \) is the random variable of normal speed with mean and standard deviation of \( \mu_N \) and \( \sigma_N \) respectively.

A small simulation program was written for the HP-97 calculator in order to get an idea about the shape of the distribution for various normal distributions associated with snow and normal speed. Four examples are shown in Figures 2, 4, 5, 6.

Figure 3.

Based on 200 Points

<table>
<thead>
<tr>
<th>Frequency</th>
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</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>18</td>
</tr>
<tr>
<td>50</td>
<td>23</td>
</tr>
<tr>
<td>60</td>
<td>22</td>
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<tr>
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<td>6</td>
</tr>
<tr>
<td>90</td>
<td>3</td>
</tr>
<tr>
<td>100</td>
<td>1</td>
</tr>
</tbody>
</table>

\( M_N = 57.5 \quad M_S = 30.0 \quad \sigma_N = 7.0 \quad \sigma_S = 7.0 \)

Based on 200 Points

\( M_N = 57.5 \quad M_S = 35.0 \quad \sigma_N = 7.0 \quad \sigma_S = 7.0 \)
Based on these histograms and because of the nature of $Y$, a log-normal distribution appeared to be the ideal type of distribution for the $Y$ variates. The general equation of the log-normal is given by the probability function

$$f_Y(y) = \left\{ \begin{array}{ll} \frac{1}{\sqrt{2\pi} \xi^2} \exp \left[ -\frac{(\ln y - \lambda)^2}{2\xi^2} \right] & y \geq 0 \\ 0 & \text{elsewhere} \end{array} \right.$$

(3)

where $\xi^2 = \ln [1 + \frac{\sigma^2}{\mu^2}]$  

(4)

and $\mu$ and $\sigma$ are the mean and standard deviation respectively of $Y$.

In order to see how good a fit the log-normal function choice was for the distribution, a Kolmogorov-Smirnov goodness of fit test was performed on the case $\mu Y = 57.5$, $\sigma Y = 7$, $\mu S = 30$, $\sigma S = 5$. In order to perform the goodness of fit test, one must calculate $\xi$ and $\lambda$ as given by Equations (4) and (5). Thus, the mean and standard deviation are needed. It was decided to expand $Y = 1/S-1/N$ into a Taylor series about the mean and use the series to estimate $\mu_Y$ and $\sigma_Y$ from the distributions of $N$ and $S$.

Expanding $Y$ in a Taylor Series about the means of $N$ and $S$ one gets,

$$Y = \left( \frac{1}{\mu_S} - \frac{1}{\mu_N} \right) \left( S - \mu_S \right) + \left( \frac{1}{\mu_N} \right) \left( N - \mu_N \right) + \left( \frac{1}{\mu_S} \right) \left( \left( S - \mu_S \right)^2 \right) + \left( \frac{1}{\mu_N} \right) \left( \left( N - \mu_N \right)^2 \right) + \ldots$$

(6)

To approximate $\mu_Y$ using up through second order terms,

$$\mu_Y \approx \left( \frac{1}{\mu_S} - \frac{1}{\mu_N} \right) + \frac{1}{\mu_S} \sigma_S^2 + \frac{1}{\mu_N} \sigma_N^2$$

(7)

To approximate $\sigma_Y$, using up through first order terms,

$$\sigma_Y^2 \approx \frac{1}{\mu_S} \sigma_S^2 + \frac{1}{\mu_N} \sigma_N^2$$

(8)

Now $\mu_Y$ and $\sigma_Y$ will be used to calculate $\lambda$ and $\xi$ for use in testing the assumption of the log-normal distribution. These equations applied to the case being considered yield:

$$\mu_Y \approx \left( \frac{1}{30} - \frac{57.5}{25} \right) + \frac{1}{30^2} \cdot (25) - \frac{1}{(57.5)^2} \cdot (25) = 0.0166102$$

$$\sigma_Y^2 \approx \left( \frac{1}{30} \right)^2 + \frac{1}{(57.5)^2} \cdot (25) = 0.0000353467$$

and

$$\xi^2 = \ln [1 + \frac{\sigma_S^2}{\mu_S^2}] = \ln [1 + \frac{0.0000353467}{(0.0166102)^2}] = 0.012054784$$

or $\xi = \sqrt{0.012054784} = .3472$

and $\lambda = \ln \mu - \frac{1}{2} \xi^2 = \ln (0.0166102) - \frac{1}{2}(0.3472)^2

= -4.158$

Now using the Kolomogorov-Smirnov Test to see if the data reasonably fit a log-normal distribution with $\lambda = 4.158$ and $\xi = 0.3472$, we get the following table.

<table>
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<tr>
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<tbody>
<tr>
<td>&lt; 0.010</td>
<td>79</td>
<td>0.095</td>
<td>0.099</td>
<td>.004</td>
</tr>
<tr>
<td>0.010-0.015</td>
<td>65</td>
<td>0.420</td>
<td>0.453</td>
<td>.033</td>
</tr>
<tr>
<td>0.015-0.020</td>
<td>52</td>
<td>0.68</td>
<td>0.76</td>
<td>.08</td>
</tr>
<tr>
<td>0.020-0.025</td>
<td>44</td>
<td>0.90</td>
<td>0.911</td>
<td>.011</td>
</tr>
<tr>
<td>0.025-0.030</td>
<td>9</td>
<td>0.945</td>
<td>0.949</td>
<td>.007</td>
</tr>
<tr>
<td>0.030-0.035</td>
<td>6</td>
<td>0.975</td>
<td>0.985</td>
<td>.014</td>
</tr>
<tr>
<td>&gt; 0.035</td>
<td>5</td>
<td>1.000</td>
<td>0.9965</td>
<td>.0035</td>
</tr>
<tr>
<td>200</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Based on the maximum deviation of .08, compared to the K-S value of .096 at the five percent level of significance we see that the assumption of the log-normal is not unreasonable.

Because of these results, the log-normal approximation has been used for the calculations of the integrals in Equations (1) and (2). The results compare favorable to those obtained as part of the
earlier project, but there is a significant advantage to the new system—the procedure can be implemented on an HP-97 (or presumably other) programmable calculator. It is thus possible to estimate the costs per person under a wide set of differing conditions, with a minimum of inconvenience, without the necessity of obtaining access to a computer.

References


(4) Thomas, C. Thomas and Thompson, Gordon I., The Value of Time Saved by Trip Purpose, SRI Project MSU-7362, October 1970.