Workshop on Mathematical Structures and Uncertainty

Workshop Summary

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The focus of this workshop was various mathematical choice models representing travel behavior, based primarily on individual-choice theory. In conjunction with other methods, these tools produce information on travel flows on proposed transportation systems and other related alternatives. These techniques possess many advantages over existing procedures (covered extensively in the Workshop on Long-Range and Strategic Planning Methods) but are sometimes cumbersome to use and contain unknown errors. The state of the art was assessed, relating methods to practical and institutional problems. In background papers, Steven R. Lerman and Joel L. Horowitz summarized the state of the art and the state of the practice.

The workshop participants found that methods currently available can generally support decision-making for a wide range of planning problems. However, many planners and decisionmakers view these methods as unnecessarily cumbersome and irrelevant to their concerns in their current form. In addition, current models are deficient in ability to represent or predict travel behavior accurately for many options. Thus, two different areas were addressed:

1. Overcoming barriers to current use of the best available techniques for specific purposes and
2. Improving the behavioral content and accuracy of existing techniques.

A number of reasons were identified for failure to move techniques into practice. These are

1. Heavily technical descriptions of methods,
2. Excessive claims about unrealized advantages,
3. Lack of clarity of ways in which techniques respond to planner-identified problems,
4. Inadequate priority or time given to learning new techniques,
5. Failure to acknowledge the source of improved capabilities, and
6. Inadequate definition of current and future issues.

To deal with these concerns, it was suggested that gap-closing materials, such as methodological manuals, software support, instructional programs, and documentation of successful applications, be developed. Issues and planning areas most appropriate for analysis should also be identified. Numerous short-range projects, operating procedures, and pricing decisions are sample cases where simple applications of advanced models can be described. Selected regional-scale problems should also be studied through upgrading of current large-scale model systems. Emphasis should be on issues that cannot be addressed by current (traditional) methods and issues that can be addressed more efficiently by new methods. Planners' criteria for selecting and using models and procedures can be satisfied by new methods, which should be described. This includes simplified applications (such as pivot-point methods), improved (or new) issue sensitivity, higher levels of precision than traditional models (reduced uncertainty), and ability to apply model systems at different levels of complexity.

Options for adopting new models should be developed, varying according to level of sophistication, range of problems, and development of a new model or adoption of one from another environment.

Mathematical Models of Travel Demand: A State-of-the-Art Review

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The state of the art in modeling transportation systems has evolved at an extraordinarily rapid pace since the early highway studies of the 1950s. This rapid evolution has been the result of the confluence of several major factors, including

1. The growth of the federal government's involvement in the transportation sector and the government's willingness to fund both research and planning activities;
2. The extensive fertilization of transportation systems analysis from other fields, notably economics, statistics, and operations research;
3. The technological advances made in digital computers; and
4. The perceived demand for a rational decision process in making capital investment and operating decisions in the transportation sector.

These and other forces have all acted to produce major innovations in the field of travel demand analysis, which forms a large part of what has constituted transportation systems analysis. I intend to examine the various components of travel-demand modeling and describe what I believe to be the major changes in the state of the art. I will particularly emphasize innovations in the last few years in order to focus the attention of workshop participants on the progress we in the research community have collectively made since the last major international conference on travel demand, held in Eisbee, West Germany, in 1979. I will also present my personal evaluation as to whether the state of the art in different areas has advanced as rapidly in recent years as it has in the past and why any observed rate of change has shifted.

This review is restricted to what I term mathematical behavioral models, i.e., abstract, mathematical representations that purport to approximate in some way the processes underlying travel demand.

This eliminates other, more qualitative approaches such as focus groups, simulation games, and various exploratory data-analysis methods from the scope of the review. In addition, it eliminates models designed with the objectives of predicting travel demand as a function of a few aggregate correlates.

In reviewing the state of the art, I have intentionally adopted a very broad view of mathematical demand modeling. This view is intended to include far more than the technical aspects of the model structure and statistical inference.

In adopting a broad view, I hope to demonstrate that although the state of the art has made great progress along some dimensions, it has left a greater number of potentially fruitful avenues of research almost untouched. I will also propose the thesis that further efforts in the areas that we have historically emphasized will, in all probability, yield smaller returns than comparable efforts directed at some of these unexplored possibilities.

The remainder of this paper is organized as follows. The next section presents a subdivision of the travel demand modeling process into five distinct components. These are behavioral theory, measurement, statistical model structure, estimation, and forecasting. An assessment of the state of the art in each of these five areas is presented next. Finally, the progress made to date is summarized and some rough priorities for future research are suggested.

COMPONENTS OF TRAVEL DEMAND MODELING PROCESS

There are obviously a virtually infinite number of ways in which the process of travel demand modeling can be divided. However, for the purposes of this review, I have found it useful to divide the field into five distinct areas.

The first area is behavioral theory. This is intended to encompass all aspects of the behavioral theory underlying any particular travel demand model. It includes areas such as

1. The attributes considered by individuals making demand choices;
2. The particular choices represented in a model, including the type of choice (mode, destination, etc.), the period over which the decision is made (day, month, year, etc.), and the nature of the set of possible choices (continuous, discrete, or mixed);
3. The decision rule (compensatory, elimination by aspects, lexicographical, etc.); and
4. The assumption about the information available to the decisionmaker, including how information is acquired and used.

It is important to emphasize that the area I define as behavioral theory is quite distinct from the problems of how to make that theory operational. One can easily imagine an entirely reasonable theory of travel behavior for which there exist no corresponding data, statistical model, or estimation technique. For example, a theory of behavior in which travel decisions at any time depend structurally on all previous travel decisions might be entirely plausible but impractical to make operational.

The second area is what I term measurement. This includes all aspects of data collection such as how and which attributes are measured, how travel decisions are observed, how samples are taken, and whether data are cross-sectional or longitudinal. In addition, issues of whether so-called attitudinal or perceptual data are collected come under the general heading of measurement. Finally, whether the analyst measures actual choices (revealed preferences) or stated preferences to hypothetical choices is part of measurement.

The third area is model structure. This broad category incorporates the methods through which behavioral theory and data are combined to produce a statistical model of travel demand. For example, virtually all discrete-choice models have the assumptions of utility maximization as their underlying behavioral theory. Moreover, as commonly used, the measurement process is typically a cross section of individuals drawn at random and observed for some fixed period (usually 24 h); each observation consists of the actual travel choices and physically measured attributes such as time, cost, etc. In this subset of the field of travel demand analysis, the statistical model structure is usually some variant of the multinomial logit model, in which the analyst treats utilities as a vector of independent and identically distributed random variables, each Type I Extreme Value (or Gumbel) distributed. These assumptions, when combined with the theory and a set of observed attributes, provide the basis for specifying the probability distribution over the set of possible choices.

The fourth area is estimation. Conditional on the set of measurements taken and an assumed statistical process that generated them, estimation is the technique by which unknown parameters of the model are inferred. This area has been dominated in travel demand analysis by classical statistical inference by using the methods of least squares and maximum likelihood.

The final area is what I have loosely termed forecasting. Under this general heading I include all of the following:

1. Prediction of the values of independent variables;
2. Aggregation and/or disaggregation of demand forecasts;
3. Representation of policy changes in the models; and
4. Integration of demand forecasts with other components of the transportation systems analysis, including equilibration with other models in which demand is an independent variable.

Each of these five different components is reviewed in the following sections.

BEHAVIORAL THEORY

Despite the objections of innumerable critics, the vast majority of the literature in the field of travel demand analysis has been either implicitly or explicitly derived from the hypothesis of individual utility maximization. This class of models encompasses the entire spectrum of neoclassical economic models of consumer behavior, choice models, and large segments of the transportation-related research in marketing and psychology.

There are two reasons for the dominance of utility maximization as an underlying behavioral theory in travel demand models. First, there has been a strong influence of economics on the research community on travel demand forecasting dating from early work by Meyer, Kain, and Wohl (1). The second and more relevant reason has been the ease with which tractable, analytic results can be derived when one assumes that decisions on multiattribute services such as transportation can be reduced to the optimization of a scalar index of worth.

Many of the objections to utility maximization as

a behavioral theory can be dealt with by modifying other assumptions of the neoclassical economic model. For example, psychologists have often argued that subjects tend to rely on satisficing rules when they choose among many alternatives. Thus, individuals select alternatives that meet certain upper and lower thresholds along their various attributes rather than combining the attributes in some compensatory fashion [see, for example, a study by Golob and Richardson (2)]. This type of behavior, in which a decisionmaker behaves as though he or she possessed fixed levels of aspiration, can be approximated by a utility-maximizing individual who faces alternatives one at a time, where each alternative examined has associated with it some search cost. The resulting models, although based on the assumption of utility maximization, produce a behavior in which individuals examine alternatives until they find one with attributes leading to a utility level exceeding some fixed value. Examples of such models appear widely in the literature in statistical decision theory (3) and have more recently been extended in numerous works such as those by Weibull (4) and Hall (5).

Given the tractability of behavioral theories rooted in the assumption of utility maximization, it is not surprising that the state of the art in travel demand analysis has tended to move by extending that theory rather than by abandoning it altogether. These extensions can be viewed along the following dimensions.

Extension of Choice Sets

Travel demand models have moved steadily to encompass an ever-increasing range of potential choices. The earliest demand models that had any real underlying behavioral theory were exclusively models of mode choice [e.g., see early work by Lisco (6), Warner (7), Watson (8), Quarmby (9), Stopher (10), and Lave (11)]. Since those early discrete-choice models, work has been done on choice of destination, mode, automobile ownership, time of day of travel, frequency, activity duration, automobile type, housing, residential location, and workplace location. Much of this body of research, however, has added little to the area of behavioral theory; rather it has tended to focus on utilizing existing assumptions on different empirical problems. This is particularly true in much of the discrete-choice literature, where the emphasis on extending models to new empirical contexts has been on issues of specification of the statistical model structure.

Extension of Assumptions About Information

The simplest models of traveler behavior assume the existence of a perfectly informed decisionmaker who is aware of all of the available alternatives and who knows all the alternatives' attributes with certainty. Particularly when one is dealing with very large sets of feasible alternatives or travel behavior that is not routine, this assumption has been questioned. Recent work has explored the consequences of relaxing these assumptions in a number of different ways. For example, Lerman and Manski (12) have explicitly considered how the process of information acquisition can be incorporated into discrete-choice behavior. Their work treats information as obtained from three generic types of sources:

1. Direct experience,
2. Word-of-mouth communication from other informed members of the population, and
3. Media coverage transmitted through one or more mechanisms.

Other efforts have been directed toward relaxing the assumptions regarding the cost of obtaining information about alternatives. Hall (5), for example, considers the case of an active search for housing, where the decisionmaker must decide whether to accept a given alternative from the set he or she knows about or to incur some actual (or psychological) expense to obtain further alternatives. The resulting models are similar to those developed by Weibull (4) in the context of destination search.

Linking Travel Behavior to Activity Demand

Despite the consensus that transportation is a derived demand, it is only relatively recently that travel demand modelers have attempted to derive models of transportation choices from an underlying theory of the demand for activities in which individuals choose to participate. The early qualitative work by Hagerstrand (13), Lenntorp (14), and Chapin (15) produced a great deal of behavioral insight but virtually no mathematically structured theory. More recently, efforts to model time allocation by Bain (16), Jacobson (17), and others have produced models of the duration of activities but not how those activity decisions result in specific travel choices. Damm and Lerman (18) have modeled the choice of whether to participate in an activity at a given period of time during the day for workers jointly with the choice of the duration of the activity. However, there is still no single integrated theory that directly links choice of activities with tripmaking.

Interactions Among Household Members

Virtually the entire body of behavioral theory deals with the choices of a single decisionmaker, defined either as an individual or as a household. More realistically, there are many household decisions that result from interactions among household members, each of whom may have different objectives. To date, there has been almost no analytic theory of intrahousehold interactions, with the notable exception of empirical work by Jacobson (17) on the allocation of shopping activities between adults in the household. Such interactions are probably becoming increasingly important in determining certain types of travel behavior as traditional roles in households become less and less significant determinants of the allocation of household tasks between members of a married couple.

Choice-Set Determination

The existing theory has generally assumed that the choice set available to an individual is relatively large, limited in most cases only by resource constraints (e.g., budget constraints or time constraints) or physical availability (such as the unavailability of an automobile). The more qualitative literature of traveler behavior suggests that there may be other constraints operating, including some that may be attributed to issues such as lack of information.

Perhaps the best examples of constraints that we do not currently represent in existing theories is what Hagerstrand (13) terms "coupling constraints." Basically, these constraints arise when the decisions of two or more individuals must be coordinated for either one to make a trip. Most carpooling choices, which we now model as independent decisions of separate actors, are in reality constrained by the need for the members of the carpool to have matching schedules.

Development of Intermediate Constructs

The original behavioral theories typically treated utility as a function of a vector of observed, physically measured attributes. This approach was widely criticized as ignoring the process by which physically measured attributes are perceived and acted on by individual decisionmakers. It has been proposed that individuals assess alternatives by first constructing some intermediate variables and then evaluate their alternatives based on these intermediate constructs. Most of the efforts to explicitly capture these intermediate processes in explaining traveler behavior are the result of cross-fertilization of travel demand analysis by marketing research. Examples include the use of multistage models [such as the Lens model first proposed by Brunswik (19) in the marketing context] to study choice of a shopping center by Koppelman, Hauser, and Tybout (20).

In the Lens model, physically measurable attributes are translated into actual decisions in a sequence of steps. First, physical attributes of both an alternative and a decisionmaker produce perceptual attributes of the alternative. (These perceptions may be on a considerably smaller number of dimensions than the original attributes.) The perceived attributes then induce a set of preferences for the set of alternatives. These preferences are then further modified by situational constraints that limit the actual choice made.

Obviously, the Lens model is only one possible way in which intermediate constructs can be introduced into a behavioral theory. The important issue in developing such constructs is whether one can use them as a basis for an operational model. In particular, does the theory impose useful restrictions on the relationship between observable variables and the hypothesized constructs that help in developing more reasonable models?

The above review suggests that even within the paradigm of the utility-maximizing traveler there exist an enormous number of relevant areas for the extension of behavioral theory. Overall, one can probably substantiate the argument that in the area of behavioral theory, the field of travel demand analysis has evolved extremely slowly. This lack of progress is probably due to the great emphasis we as analysts have placed on models that are operational. Funding for research that has improvements in behavioral theory has been confined to just a few studies [notably NCHRP Project 8-13 (21)], and most of the progress in formulating better theories of behavior has come as a by-product of studies with more immediate objectives. This emphasis on developing behavioral theories that are simple enough to lead to tractable, immediately operational models that can be estimated and applied has produced a disinclination to explore behavioral theory for its own sake. Moreover, it has encouraged criticism of the basic paradigm of utility maximization as inadequately reflecting what many view as demonstrably nonmaximizing behavior. Without a judgment as to whether such criticism is in fact merited, it is probably safe to argue that those of us working with such models have failed to fully exploit their potential for explaining or approximating behavior that on the surface appears contrary to the hypothesis of utility maximization.

Measurement

Measurement includes an extremely wide range of subjects of relevance in travel demand modeling. In this section, I will restrict my comments to those measurement problems that relate directly to the development of mathematical models of tripmaking and ignore the issues that must be considered in more qualitative or exploratory travel demand analysis. This to some extent restricts the scope of the review, since many travel demand models are developed from data that were intended for other purposes.

For the purposes of exposition, the work in this area will be divided into four subareas. Each is considered below.

Attributes Collected

Data-collection efforts in travel demand studies have been dominated by the use of 24-h trip diaries, which are usually augmented by socioeconomic information about respondents and network-based data on the level of service provided by alternative modes. De facto, this domination has resulted in an emphasis on the use of a relatively limited subset of possible attributes and has traded off large samples for high levels of reliability in the data base.

Efforts to extend the range of attributes measured in travel demand analysis have been directed toward either generating data on different, physically measured attributes or measuring what have loosely come to be called perceptual or attitudinal data. Examples of the former include efforts by Small (22), Lerman and others (23), Abkowitz (24), and others to measure the reliability of modes, either by inferring the higher moments of the travel-time distribution from repeated observed trips or by associating distributions of travel times on links in the network and deriving network travel-time variances. Other extensions of the types of attributes used appear in the literature on destination choice (notably for shopping trips) where the typical land use measure such as employment by type and zonal areas devoted to different uses has been enhanced by measures of the number and variety of stores, parking spaces, mean walking times to parking, and measures of physical amenities such as enclosure of malls. One good example of this type of work is that of Kern and Parcells (25), where the Census of Retailing is used to measure a large number of attributes of different retailing centers.

The measurement of that which is not physically measurable, such as perceptions of quality, convenience, etc., has probably received somewhat greater attention. The early research results in this area by Spear (26) in the context of mode choice and by Kostyniuk (27) in the context of destination choice were somewhat mixed, particularly when these measures were used along with the physically measured attributes. Although these ambiguous results could be attributable to many causes, one might speculate that the most crucial problem was lack of a clear theory about how to use this type of data appropriately. The measures used were often the result of questions in which respondents imposed their own views on what the attributes labeled comfort or convenience were measuring. What is needed is a clearer structural theory of the process by which physically measured attributes and socioeconomic characteristics interact to form these intermediate constructs and a clearer theory of how the available psychometric techniques that purport to measure these constructs work. This more structural model would then provide some guidance as to how we should and should not use these types of variables and how the values of these variables can be modeled as functions of other attributes. This is apparently an active area of research in the marketing field (28) and may yield some useful results for building travel demand models that rely on these nonphysical attributes.

Sample Sizes and Sampling Strategies

With the development of statistical techniques for utilizing disaggregate data for modeling qualitative responses (notably the multinomial logit model) the research community consistently endorsed the collection of smaller samples that were broader in terms of the information collected from each respondent and better verified. The emphasis on using smaller, more reliable samples was reflected in early disaggregate demand studies (29-31). Recent data-collection efforts have followed these recommendations but have largely used the same sampling strategies as earlier studies.

Advances in the analysis of sampling techniques such as stratified sampling, choice-based sampling, and various hybrid strategies have provided a much richer set of alternative sample designs. Most of this literature, however, has focused on how these different sampling strategies can be used to estimate a model and not on how one should choose a sampling method.

Work by Lerman and Manski (32) opened up this issue without providing a great deal of specific practical results. Indeed, their key result was largely negative; i.e., the optimal sampling strategy for even simple problems cannot be determined without prior knowledge of the unknown parameters of the model to be estimated. Daganzo (33,34) noted this result and formulated the sample design problem as a nonlinear programming problem in which the parameters of the model were treated as known. In this model, the fractions of the sample taken from each of a finite set of possible sampling strategies are the decision variables; the objective function is a measure of the efficiency of the estimated parameters such as the trace or largest eigenvalue of the variance-covariance matrix of the estimator.

Subsequent empirical experiments by Sheffi and Tarem (35) have applied Daganzo's basic technique to sample design for logit model estimation. In their work, initial parameter estimates are obtained from a small, randomly drawn sample, and the resulting estimates are used to optimize a second-stage sample. Their results on both simulated and actual data suggest that substantial gains in efficiency are possible from use of this two-stage technique and that these gains are not particularly sensitive to the size of the first sample so long as it is greater in size than some minimal fraction of the total sample. Their work applies only to designing samples that are stratified on exogenous variables, leaving the optimal design of endogenously designed stratified samples almost entirely unresolved.

A sample design question that remains essentially unexplored is the development of optimal sample designs when the parameters are not viewed as known with certainty. (Daganzo's technique is derived by treating the parameters as fixed.) Potentially, one could treat the parameters as uncertain either because only estimates inferred from small samples are available or because the parameters are assessed subjectively from expert judgment. This would complicate the optimization problem substantially, and it is unclear whether the optimal strategies that would be derived would be significantly different from those based on deterministic a priori parameters.

Preference Data

For largely the same reasons that travel demand research has been dominated by the use of a limited number of physically measured attributes, we have historically relied on data on revealed preferences. Indeed, the issue of whether travel demand analysis should use data other than revealed preferences has been the focus of many of the longer debates in previous travel demand conferences. Most of this debate, unfortunately, has centered on the simple questions of what type of data should be used and not on what I perceive to be the most meaningful question of how to use different forms of data.

The types of alternative data on preferences that have been used include stated preferences (or rankings) of hypothetical alternatives, scaled measures of intensity of preference for either actual or hypothetical alternatives, and questions about trade-offs that individuals would be willing to make on particular attributes. Each of the above types of preference data has within it a myriad of distinct options, some of which involve detailed procedures such as the allocation of a fixed number of chips by a respondent in which the number of chips given to any alternative reflects the intensity of preference (36).

Proponents of these methods argue that they allow the analyst to extract vastly more information from each respondent, including multiple responses to different choice situations and reactions to combinations of attributes that are simply unobserved in revealed-preference data. These features of this type of data make it possible to model demand for new alternatives without making the strong assumptions on model structure and genericity of attributes that are required when revealed preferences for existing alternatives are used. In addition, because responses for a variety of choice situations can be elicited in collecting this type of data, it is possible to estimate demand models separately for any given individual in the sample. As demonstrated in a relatively simple case study by Fischer and Nagin (37), this greatly facilitates the diagnosis of the structure and causes of random taste variation in the population being modeled. Finally, the ability to construct a very wide range of attributes in the data makes it considerably easier to determine the appropriate functional form for demand models [see paper by Lerman and Louviere (38) for an example].

Those who argue for the exclusive use of revealed preference note that people often do not actually do what they say they would do under hypothetical circumstances. Thus, there may be a tendency to overcommit to hypothetical new alternatives in response to questionnaires, misleading the travel demand analyst into erroneous and often overoptimistic forecasts of the demand for such innovations. Given the uncertainty that exists about how people respond to hypothetical questions, revealed preferences provide the best basis for modeling demand.

Both the above arguments reflect extreme views. A more centrist (and in my view more reasonable) position is that although responses to hypothetical questions do indeed incorporate sources of error in the prediction of actual behavior, they do provide potentially valuable information about how people will actually behave. The key is to structure an explicit theory about how stated preferences map into actual behavior. This theory would allow us to use both revealed and hypothetical preferences within the same model; the model structure would be used to control for the fact that two distinctly different types of information are represented. One conceptual basis for such a theory has already been described by Koppelman, Hauser, and Tybout (20), who have adapted some of the work in the marketing field to the transportation context. However, there is a major, and to my knowledge still unfilled, gap between the conceptual theory and an operational model

that allows a synthesis of revealed and hypothetical preferences in a single model.

Choice-Set Availability

The FHWA-funded study of data-collection methods piloted in the Baltimore disaggregate data-collection effort had as one of its goals to measure the set of transportation alternatives available to a subset of the respondents for one trip. The results of this portion of the data collected showed that respondents reported having very few alternatives to their chosen travel mode and destination. This is in direct contrast to the assumptions made in many discrete-choice models of traveler behavior, where it is often assumed that the set of feasible alternatives is quite large. The question remains whether in fact the choice set actively examined by an individual is quite small or whether respondents chose to screen out many alternatives that they knew were available but were so decidedly inferior that they were ignored. In addition, the survey instrument used in this study was extremely lengthy, and it is conceivable that many respondents intentionally truncated their list of alternatives in an effort to shorten their interview.

The question remains, therefore, how the choices perceived by an individual can be measured. Part of the problem in addressing this question in any meaningful way is that we lack an operational definition of what really constitutes the availability of an alternative. Most of the models we now use treat availability as a binary issue; either an alternative is available to an individual or it is not. A more realistic model would recognize that there are degrees of availability, which range from alternatives that are used every day to those that are simply infeasible. In the middle of this spectrum lie alternatives about which the individual has only incomplete and potentially out-of-date information. This information may be so incomplete or hazy that the individual responds to questions about its availability by telling the interviewer that it is unavailable. In fact, circumstances such as the need to travel by modes other than those customarily used (as in the case of a breakdown of the family's private automobile) may trigger a process of active search for better information and subsequent use of the alternative.

STATISTICAL MODEL STRUCTURE

The field of model structure, particularly for discrete-choice models, has undergone major expansion since the development of the multinomial logit model by McFadden (39). Some of this expansion was the result of entirely new model structures, whereas other developments were the operationalization of models that had existed in theory for some time but were considered impractical due to what appeared to be insurmountable computational difficulties. As we shall discuss further in a comment below, the major area of progress has been the derivation of new models by altering the assumptions about the specification of the disturbances in random utility models. There has been considerably less active research in passenger demand analysis on new structural forms for continuous dependent variables. Each of the major research subareas is discussed below.

Logit-Based Extensions

Given the computational tractability of the conditional logit model and its extraordinary statistical properties when used to estimate different samples and subsets of alternatives, it is not surprising that there has been significant research activity directed toward generalizing the logit model while still attempting to preserve some or all of its most salient features. The earliest such work is due to Ben-Akiva (31), who first proposed what has come to be termed the nested logit model. His work led to both a practical extension of the logit model to allow for a limited class of nonindependent disturbances and a computationally tractable but not fully efficient estimation technique for that model. Efficient estimation via full information maximum-likelihood estimation has now been accomplished by Brownstone (40) and by Ben-Akiva and Lerman (41).

In subsequent work, McFadden (42) has shown that the now-standard conditional logit model and various forms of the nested logit model are both members of a large class of what he termed generalized-extreme-value (GEV) models. Members of this class can be derived straightforwardly by using McFadden's theoretical results that characterize the GEV model. However, to date there have been no new special cases of any great interest.

Another generalization of the logit model was derived by Sheffi (43). In this variant of the logit model, the probability of any given choice is represented as the product of a sequence of binary logit probabilities. In still another variant, Small (44) has derived what he calls a serial logit model with potential applicability to modeling cases where the choice set is logically ordered (e.g., integer outcomes).

Another area of logit extensions has been the evolution of the continuous logit model. In this work, the set of alternatives is treated as continuous, and the logit model is modified so that the denominator is the integral (rather than the sum) over the entire set of feasible choices. This model was first proposed without any formal derivation by Watanatada (45) and subsequently derived by assuming that the IIA property holds with respect to subsets of continuous alternatives (46). A formal derivation from random utility theory was constructed by Litinas (47), who also derived a variety of interesting and tractable analytic cases where the model could be applied. In particular, by assuming that the set of destination is a continuous plane and making specific assumptions about the distribution of level of service and potential destination over that plane, Litinas was able to derive closed-form expressions for vehicle miles of travel, mean trip length, and other travel summaries directly (48).

This underlying model was also used by Litinas and others (49) to model trip generation. In this work, the mean trip rate for an individual was represented by a continuous logit model defined over the nonnegative real numbers. In any given day, the number of trips made was represented as a stochastic process with integer outcomes conditional on the mean trip rate. For the case where the daily trips are Poisson distributed, they showed that the observed trips are negative binomial distributed.

Probit Models

The late 1970s were characterized by a significant growth in interest in forms of the multinomial probit model. Researchers had long been aware of the potential generality of multinomial probit; one can construct probit models that allow for random coefficients and disturbances with any arbitrary covariance structure (50). The key obstacle to the exploitation of the multinomial probit model had been the enormous computational burden associated with computing multinormal probabilities.

These problems are addressed by three distinct

computational approaches. The first was a method used by Clark (51) and applied by Daganzo, Sheffi, and others in a series of papers [e.g., one by Daganzo, Bouthelier, and Sheffi (52)]. The key to this method was the approximation that the maximum of two normally distributed random variables is normally distributed. (The reader can verify that this is in fact not true by considering the distribution of any two normally distributed random variables that are perfectly negatively correlated.) Early empirical experiments suggested that this approximation is relatively accurate for a large class of normal distributions, particularly if the normal variates did not have extremely different variances and were positively rather than negatively correlated [see study by Lerman and Manski (53)]. Moreover, the computational experience demonstrated that the method was easy to code, rapid to execute, and did not exhibit the exponential growth of computation times with increasing choice-set size that characterized many of the other approaches, particularly forms of numerical integration.

More recent evidence accumulated in extensive numerical experiments reported by Horowitz, Sparmann, and Daganzo (54) has generated the basis for some reconsideration of the early optimism about the value of the Clark approximation. Their work suggests that there are many instances where the approximation is unacceptably erroneous, particularly for problems with relatively high taste variation.

The second approach used has been the application of various types of series expansion (55). This approach led to the first actual empirical application of trinomial probit that allowed for non-IID disturbances [Hausman and Wise (56)]. Although quite accurate, the computational burden associated with series-expansion methods makes their use in choice problems with more than five or six alternatives infeasible for most purposes.

The third approach is a variant of Monte Carlo integration proposed by Lerman and Manski (53). In their work, the choice probabilities are computed by drawing realizations of the disturbances and exploiting the relative ease with which random number generators can be used to simulate normally distributed random variables. The fraction of drawings for which any alternative has the highest simulated utility is used as the basis for an estimate of the choice probability. Unfortunately, Lerman and Manski's simulation experiments suggest that multinomial probit models estimated in this way will require perhaps an order of magnitude more computational resources than comparable logit models.

At this point, it appears that trinomial probit models are the one case in which the computational requirements of model estimation have been reduced to what most researchers would judge to be reasonable levels. Sparmann (57) has developed a clever tabling scheme for the trinomial probit function that is parameterized in a way that reduces estimation costs significantly. Work with larger choice sets will require either acceptance of the Clark approximation (58), further work on faster codes (using any of the above techniques), the use of less general probit model forms that decrease the number of parameters to be estimated (59), and the complexity of the choice probability evaluation or simply larger computer budgets.

Truncated Dependent-Variable Models

There are certain instances in which the dependent variable either is restricted to a limited set of discrete outcomes or takes on some range of continuous values. The most common such instance in transportation is when activity duration is the dependent variable. Over the usual period of observation, the amount of time any individual spends is either zero (i.e., he or she does not participate in the activity) or a positive, real number. The methods used to deal with such problems have been generally adapted directly from the econometrics literature initiated by Tobin (60). Application of this method and more recent extensions to allow for simultaneous systems of truncated dependent variables appear in work by Bain (16) and Jacobson (17).

Models with Mixed Continuous and Discrete Variables

There are some travel problems for which the dependent variables are both continuous and discrete. For example, one might view a commuter's work trip choice as consisting of a decision on when to depart from home to work (a continuous dependent variable) and a decision on what mode of travel to use (a discrete choice). This type of problem was first explored in a travel demand context by Westin and Gillen (61). Their approach requires that the disturbances for both the continuous and the discrete decisions be normally distributed and that the utility of the discrete alternatives be linear in the continuous dependent variable. They derived a multistep estimation method based on work in the field of labor-force participation pioneered by Heckman (62). This method iterates between estimation of the continuous equation and the discrete equation, eventually leading to a consistent, although not fully efficient, set of parameter estimates. This was the basic approach adopted by Damm and Lerman (18) to model the decision of workers to participate in a nonwork activity during each of five periods during the day and, if the decision is to participate, for how long.

Variants of these types of models have been developed outside the transportation field by Dubin and McFadden (63), who use the multinomial logit model for the discrete choice. In other work, McFadden and Winston (64) have used full information maximum likelihood and estimated mixed continuous and discrete models of freight demand.

Time-Series Analysis

Because most of the data available to transportation analysts has been cross-sectional, the use of time-series approaches has not been emphasized in travel demand modeling. Most of the time-series methods used in travel demand analysis have been applications of well-known methods developed for the analysis of aggregate economic models, in which the dependent variable is continuous. These methods are considerably less applicable to microlevel transportation data, in which most of the observations will be a sequence of discrete trip choices. One approach used by Jacobson (65) in analyzing trip generation for a panel data set is to aggregate the data temporarily and to treat the resulting dependent variable as continuous.

The new methodological contributions in terms of modeling time series of discrete decisions have been largely developed by econometricians working on problems outside of transportation, particularly theoretical work in labor economics by Heckman (66). In the transportation literature, Daganzo and Sheffi (67) have developed a computational technique for applying probit analysis to a time series of discrete decisions. Their technique reduces the computational burden from exponential in the product of the number of time periods and alternatives to linear in the same product.

Lerman and Manski (12) have explored a time series of discrete decisions in which members of the

population are acquiring information about a change in the transportation system. Their model structure, however, leads to quite complicated expressions for the probability of observing a given sequence of choices.

Stochastic Choice Sets

Another extension to the standard discrete-choice model is derived from the assumption that the analyst no longer knows the decisionmaker's true choice set. In this case, a probabilistic model of choice-set generation can be hypothesized. This model typically has unknown parameters that must be estimated along with the parameters of the utility function.

Manski (68,69) has laid the theoretical foundation for this process. Particular cases have been explored by Pitschke (70), Pitschke and Lerman (71), and Ben-Akiva (72). Ben-Akiva has demonstrated that the Dogit model proposed by Gaudry and Dagenais (73) can be reinterpreted as a model where each individual has two possible choice sets. The decisionmaker is either captive to his or her chosen alternative or has the full choice set. A parameter in the Dogit model, when normalized, is the probability of captivity.

ESTIMATION

Of all the five areas covered in this review, there is perhaps the least to discuss in the field of estimation that is specific to travel demand analysis. Given the universality of the problem of inferring unknown parameters of statistical models, it is not surprising that most of the advances in methods used by travel demand analysts are in fact the result of advances in statistics and econometrics in general.

These more general advances, however, have had a significant impact on the state of the art in travel demand modeling. For the purposes of discussion, it is useful to divide the progress made in this area into three distinct subareas.

Maximum-Likelihood Estimation Under Alternative Sampling Strategies

The statistical theory of estimating a wide range of models under a host of alternative sampling strategies is now extremely well developed. The evolution of this theory is the result of a few key studies, each building on earlier developments. This literature began with the thorough exposition of maximum-likelihood estimation by McFadden (39). His work provided the basis for virtually all of the applications of discrete-choice analysis in the 1970s. Ben-Akiva (31) then extended the available estimators in developing a sequential, maximum-likelihood method for the nested logit model.

The first theoretical inquiry into the effects of endogenous sampling strategies on the estimation of discrete-choice models was by Manski and Lerman (74) for the case of pure, choice-based samples. Their work produced a computationally simple though not fully efficient estimator in which observations were weighted and then treated as exogenously drawn from the population. This weighting technique was appropriately termed weighted exogenous sample maximum-likelihood (WESML) estimation. Manski and Lerman also presented a proof originally due to McFadden that for the multinomial logit model with a full set of alternative-specific constants, choice-based samples can be treated as exogenously drawn without affecting the statistical properties of any of the coefficients except the constants; moreover, the constants can be consistently estimated by a simple correction to the exogenously estimated constants if one knows a priori the shares in the population choosing each of the alternatives.

Manski and McFadden (75) developed a number of estimation methods for what they term general-stratified samples, a very general class of sampling techniques that encompasses exogenous samples, choice-based samples, and various possible hybrid methods. Their work also considers how different forms of a priori information (such as knowledge of the distribution of exogenous variables and the population shares choosing the alternatives) can be used directly to increase the efficiency of parameter estimates.

A more recent contribution to this literature is that due to Cosslett (76). He extends the range of sampling strategies considered to include cases where choice-based and exogenously drawn samples are pooled (to form what he terms enriched samples) and where a usual sample is augmented by data that provide information about the distribution of attributes in the sample but not the observed choices (to form what he terms supplemental samples). Cosslett has also succeeded in unifying some of the earlier work by showing that the special results for the use of choice-based samples in the logit models not only yield consistent estimates but also provide asymptotically efficient parameters. Moreover, he has demonstrated the further property of the logit model that enriched samples can, for the purposes of estimation, be treated as exogenously drawn as long as the model has a full set of alternative-specific constants.

Lerman and Gonzalez (77) have analyzed a special class of endogenous sampling methods appropriate for the analysis of cases in which the choices take on integer values. Their specific example is where trip generation is represented as a Poisson process in which the mean is a function of attributes of the individual and the transportation system and the data are drawn so that the probability that any given individual appears in the sample is a linear function of the number of trips the individual made. They term this a proportionate endogenous sample (PES) and derive an extremely simple estimator for the case of Poisson-distributed dependent variables. Their work was generalized by Litinas and others (49) and by Lerman (78) to include cases where both the underlying process is not Poisson and the sample is drawn so that the probability of an observation is a general, not necessarily linear function of the dependent variable. Lerman's work also allows for the possibility that the function describing the sampling likelihood itself contains unknown parameters to be estimated.

It is important to note that although there is now a rich theoretical literature on how one can use data drawn by a host of sampling strategies to estimate discrete-choice models, many of the resulting methods have never been implemented. There is good reason to believe that for many of the estimation methods, major computational difficulties will be encountered as researchers attempt to write practical computer codes. For this reason, most of the empirical work with endogenous samples has relied on the tremendous simplifications that result when the underlying model is assumed to be multinomial logit. In cases where other model forms have been used, the less efficient but computationally convenient WESML estimator has been used (79).

Statistical Tests

A second major area in which substantial progress has been made is the development of statistical

tests that help diagnose failures of assumptions made in specific model form. This work has focused intensively on methods to test the validity of the multinomial logit model. McFadden, Tye, and Train (80) proposed a large number of tests in which the multinomial logit assumptions (particularly the property of independence of irrelevant alternatives) was treated as the null hypothesis. These various statistical methods were unfortunately often somewhat ad hoc, and serious questions were raised about the power of many of them. More recently, Horowitz (81) derived a form of Lagrange multiplier test in which a probit model with independent and identically distributed disturbances serves as an approximation to the null hypothesis of a logit model and the full spectrum of possible non-IID multinomial probit models is the alternative hypothesis. Although this method requires some reprogramming of existing software to obtain the test statistic's value, the added computational burden is quite small.

The most recent IIA test was derived by McFadden and Hausman as a special case of a powerful class of specification tests developed by Hausman (82). This test requires that the analyst reestimate a model by omitting one or more of the alternatives (as well as all the observations choosing the omitted alternative and the coefficients that cannot be estimated without data on the omitted alternative). The test statistic, which is chi-squared distributed, can then be computed solely as a quadratic function of the estimated parameters from the model estimated from the full and the restricted choice sets and the variance-covariance matrices of these two-parameter estimates. Analysis of the power of this test by McFadden suggests that this simple method is quite powerful, and recent empirical work has tended to use the method with increasing frequency.

Another distinct area of statistical tests has been developed by Horowitz (83). He notes that virtually all the tests developed and applied to data require that the null hypothesis be a restricted form of the alternative hypothesis. This had made it impossible to test whether different models were statistically better or worse than others unless one of them could be written as a restricted version of others. Horowitz's results testing a number of statistics indicates that one of the simplest, most widely used statistics for maximum-likelihood estimation, called the rho-squared statistic (39), when appropriately corrected for degrees of freedom is the basis for an extremely powerful test of nonnested hypotheses.

Robust Estimation

As noted above, most of the available estimation methods are based on either maximum-likelihood procedures or some variant that uses either a partial likelihood or what might, in the case of the WESML method, be appropriately termed quasilikelihood. Although maximum likelihood has the general advantage of being computationally tractable for many specific cases and can be shown to yield consistent and asymptotically efficient estimators for a very broad class of combinations of model form and sampling strategy, it has the distinct disadvantage of requiring that the distribution of observations be known as a finite vector of unknown parameters. Moreover, there is no reason to believe that maximum-likelihood methods are particularly robust with respect to either measurement errors or failures of the assumptions of the models to which they are applied. I and others have made simple numerical experiments that in fact suggest that for certain types of errors, maximum likelihood can in fact produce estimates that are extremely sensitive to just a few erroneous observations. (These cases are characterized by the miscoding of the observed choice, so that a few of the alternatives chosen in the sample have very low choice probabilities when evaluated at the true parameters.) Somewhat less empirical sensitivity has been shown for special cases in which the logit model is used even though the disturbances are not independent or homoskedastic. These experiments, reported by Horowitz (84), suggest that maximum likelihood is empirically (though not theoretically) robust to certain types of specification errors but sensitive to others.

Given the potential difficulties in using maximum-likelihood-based methods, there have been attempts to construct estimators that are more robust, i.e., that require less stringent assumptions yet still provide estimates with desirable statistical properties. Manski's maximum score estimator (85) is perhaps the first example of such a technique. In this method, the exact distribution of the disturbances need not be known; all that is required is that they have a set of general properties. This method results in consistent, though not fully efficient, estimates. More recently, Cosslett (86) has been exploring still more robust estimates for the case of binary discrete-choice models. His only key restriction is that the choice function be monotonic in the difference in the systematic utilities of the two alternatives. [As an interesting aside, Ben-Akiva and Lerman (87) present an example of a probit model that allows random taste variations in which this monotonicity property is violated.]

FORECASTING

The last area explored in this review covers the entire process through which a particular estimated model is used as part of a transportation systems analysis to make forecasts that are, it is hoped, of some use in assisting the parties involved in transportation decisions in making better choices among alternative policies. This area has received very little rigorous, theoretical treatment; in fact, many travel demand researchers have tended to treat the model rather than the uses to which it is put as the goal of demand analysis. This limited view may be acceptable in certain instances, such as when the objective is estimates of an individual's willingness to pay for the attributes of a transportation service (as in the value of time studies that were conducted both in the United States and abroad during the interval of large-scale cost-benefit analyses). It is, however, far too narrow a view for the entire profession to adopt. In this section, I attempt to summarize the research results in this broad field, with particular emphasis on recent advances made in equilibrating demand forecasts with other models of the transportation system.

Aggregation Across Alternatives

The one area of forecasting that has been heavily emphasized in the demand research community has been how models of individual behavior can be aggregated to obtain useful forecasts of population trip-making. This problem arises because most of the disaggregate models are nonlinear in attributes that vary across the population; merely substituting the means of the population attributes into the model estimated for a sample of individuals produces incorrect aggregate forecasts; in some instances, the resulting errors can be substantial.

The most comprehensive study of this problem is that by Koppelman (88), who divides possible procedures for aggregating across individuals into five categories.

1. Naive procedure: The analyst simply ignores the problem and hopes for the best. This works well if the population being studied is relatively homogeneous in their choice of relevant attributes and if the model is nearly linear in the area where most of the population falls.

2. Classification: The population is first divided into mutually exclusive, collectively exhaustive groups, and the naive procedure is applied within each group. The resulting group forecasts are then summed to contain the forecast for the entire population. The classification procedure works extremely well as long as the groups are chosen intelligently (i.e., so that within each group, the conditions for the accuracy of the naive procedure are approximated). The empirical evidence suggests that one can work with relatively few groups as long as the set of travel alternatives is the same within each group.

3. Statistical differentials: A Taylor-series expansion of the disaggregate model is used. Unfortunately, even though the expansion for the individual model may be accurate, when the series expansion is then used to aggregate across the population, the resulting forecasts can still have serious errors. This method is therefore not used in practice.

4. Explicit integration: If one assumes that the distribution of the attributes can be reasonably approximated by some tractable, known probability density function, there are certain instances in which a closed-form solution for the aggregation across individuals can be derived. Unfortunately, the combinations of model form and distribution of attributes that work out are quite small, making the available explicit integration methods of little practical use to date.

5. Sample enumeration: A sample from the population (appropriately weighted to reflect how it was drawn) is used to represent the complete distribution of the attribute within the entire population. Forecasts for the sample are summed (by using the sample weights), and the resulting estimate is used as the aggregate forecast. This method works well if the values to be forecast are for large population groups that are reasonably well represented in the available samples. It is inappropriate for forecasting values such as origin-destination modal splits, where the number of separate population groups for which forecasts must be obtained may greatly exceed the size of the sample.

As a summary note, it is worth mentioning that the problem of aggregation across individuals is viewed by most as largely solved. By this, I mean that the available techniques reduce the error associated with aggregating across individuals to levels that are sufficiently small so that research efforts are better directed toward other modeling problems. Very little new research has been done since Koppelman's comprehensive study, and few analysts are deeply concerned with the issue.

Network Analysis

Because the early work in the field on network equilibration was derived under the assumptions that total demand was known and inelastic with respect to level-of-service attributes, much of the research on equilibration was outside the travel demand field. These historical antecedents produced a separation of the two fields of research even after the relatively strong assumptions on demand were relaxed. As this point, significant progress has been made in establishing both the conditions for network equilibration and computationally practical algorithms for solving for equilibrium.

The early work in this area is derived from an hypothesis proposed by Wardrop (89). He postulated what is essentially a deterministic path-choice model; his rule for establishing an equilibrium in traffic networks was that one should always choose the path with the least time in a network and that all paths with travel times greater than the minimum available will never be chosen. This rule is the basis for a class of powerful computational algorithms in which the conditions for network equilibrium are achieved at a set of link volumes that maximize a convex, nonlinear program (90).

Extensions of this basic approach allowed for the use of origin-destination demand functions that were elastic with respect to level of service (91); however, these models still required the use of a deterministic rule for individuals' choice of path through the network. Other extensions allowed for multimodal networks, including the possibility that links in the network served more than a single mode (e.g., instances in which a street is used for both buses and private automobiles).

From the perspective of travel demand analysis, the more relevant work in this area is what has come to be termed stochastic user equilibrium. Basically, this work assumes that the choice of path through a network is explained by some known choice model, in which each path meeting some criterion is a feasible choice. Dial (92) developed the first computationally efficient algorithms for solving for such equilibria under the assumption of fixed link times and costs. More recently, there has been a rapidly expanding literature that includes the use of probit models for path choice (93,94). This class of models typically requires some restrictions on the choice models (particularly the restriction that there is no taste variation) but has led to some important proofs of the existence and uniqueness of equilibrium and to some efficient computational algorithms.

Nonnetwork Equilibration

In addition to the large body of work on establishing equilibrium in networks, there have been a few efforts at developing model systems outside the network context in which one or more attributes of the demand models are endogenous. Manski and Sherman (95,96) developed a model of the U.S. automobile market in which equilibrium in any given model year depends on a vector of endogenously determined prices for used cars. In the model, the market for each make, model, and year combination is a function of the prices for that automobile class; although new-car prices are assumed to be set exogenously by the manufacturers, used-car prices (and scrappage rates) are determined in a equilibrium framework.

A more general, theoretical treatment of this problem is given by Sheffi (97), who applies much of the methodology originally developed in the network context to equilibration without networks. He demonstrates that the equilibrium conditions can be formulated as a fixed-point problem and that there exists a convex, nonlinear program that, when solved, yields a solution to the fixed-point equations.

CONCLUSION

As discussed in the introduction, no review paper can be entirely comprehensive. In this work, I have focused my attention on what I believe to be the most important developments in the state of the art in mathematical modeling of travel demand.

The state of the art has moved quite rapidly in certain aspects and quite slowly in others. In my view, the progress has been greatest in the fields of estimation methods and model structure. At this time, the theory in these areas is quite rich, and most of the new developments are coming from non-transportation research. Although it is difficult to project how quickly new developments of relevance to travel demand analysis will occur, there is an often-voiced suspicion by many in this field that further revolutionary results such as the early developments in discrete-choice theory and the use of alternative sampling strategies are not particularly likely. It is more probable that we will see a period in which many of the theoretical achievements in previous years will be translated into operational tools. For example, it is likely that practical estimation codes for the use of different model structures and samples will become widely available, significantly reducing the costs of travel demand analyses.

In contrast to the rapid progress made in estimation and model structure, there has been very little shift in the state of what I have called behavioral theory. In part, this is probably due to the fact that travel behavior is extraordinarily complex and is therefore difficult to model. However, as discussed in the section on model structure, it is also attributable to the low emphasis the profession and funding sources have placed on this area.

The progress in measurement has been somewhat mixed. Certainly major contributions in the areas of sample strategies and optimal sample design have been made. However, there are still a vast array of psychometric measurement methods that we still do not know how to effectively integrate into the mainstream of the analysis of revealed-preference data. Given the potential benefits of being able to use data on actual and hypothetical choices within a single model structure that accounts for the underlying processes by which these types of data are generated, further research in this area would appear to have extremely high potential payoff.

In the area of forecasting, it is probably safe to argue that the problem of aggregating demand models across individuals is probably a major concern. However, there is a need to better draw together the work on equilibration with the mainstream of travel demand research. In particular, the questions regarding which of the assumptions now implicit in existing equilibrium formulations can and should be relaxed so that new demand model structures can be incorporated in more complete model systems remain open. For example, travel demand models typically include variables in which level-of-service attributes (e.g., time and cost) interact with socioeconomic attributes (e.g., income). In such cases, the contribution of link times and costs to the total utility of a given travel route through a network will vary across the population; in a sense, each traveler from a given origin to a given destination faces a different set of link times and costs depending on their individual attributes. Such situations are not represented in most of the equilibration techniques that have been developed to date.

As a final note, there remain serious questions regarding whether travel demand analysis as a field has lost much of the momentum that characterized it in the 1970s. These concerns arise from both the research community, where progress in the areas of model structure and estimation appears to be increasingly at the fringe of what is potentially applicable in the near term, and from the users of new methods who find recent results increasingly arcane. As a personal view, I believe that to some extent both views are justified, but both views miss a more central point. Research into model structure and estimation is only a small part of what constitutes travel demand analysis, and major innovations outside this area are not only possible but quite likely.

In order to perform this research, a continued commitment by funding agencies will be required. Obtaining this commitment will require that the research community better explain the value of the contributions of the research to date and better articulate the potential payoff of future research and development activities.

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Evaluation of Discrete-Choice Random-Utility Models as Practical Tools of Transportation Systems Analysis

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Travel decisions frequently entail choices among discrete sets of alternatives, such as frequencies, destinations, modes, and routes of travel, and a large proportion of travel behavior research and practical travel demand analysis is oriented toward predicting the outcomes of such choices. A convenient behavioral and mathematical framework for modeling these choices is provided by a class of mathematical models called discrete-choice random-utility models. These models are based on the assumption that an individual's preferences among the available alternatives can be described with a utility function and that the individual selects the alternative with the greatest utility. The utility of an alternative is represented as the sum of two components: a deterministic component that accounts for systematic effects of observed factors that influence choice and a random component that accounts for the effects of unobserved factors. The random-utility model then predicts the probability that a randomly selected individual with given values of the observed factors will choose a particular alternative (i.e., the probability that the utility of the particular alternative is greater than the utilities of all other alternatives). The multinomial logit (1,2), multinomial probit (3-5), and generalized-extreme-value (6,7) models are three well-known examples of discrete-choice random-utility models. [See the work of Domencich and McFadden (1) and Hensher and Johnson (8) for discussions of the behavioral foundations of these models in the context of travel analysis.]

Many of the fundamental concepts of random-utility modeling have been known for more than 50 years (9), and some of these concepts were used in travel demand analysis as early as 1962 (10). However, the development of random-utility modeling as a practical analytic technique did not begin until the late 1960s, and random-utility models were not brought forcefully to the attention of the transportation community until the early 1970s. It was argued then that random-utility models had several important theoretical and practical advantages over other available travel demand modeling approaches such as the well-known four-step process. The main advantages claimed for random-utility models were as follows:

1. They are based on an explicit principle of human behavior, whereas other available models are not;
2. They can treat a wider range of travel choices, interactions among travel choices, and policy variables than can other available models; and
3. They make more efficient use of data than do other available models.

It was argued that these advantages enable random-utility models to forecast travel demand more accurately and less expensively than do other models.

Since the early 1970s, there has been much research on theory and methods of random-utility modeling, and random-utility models have been used in a wide variety of practical travel demand analyses. As a result, the random-utility approach to travel demand modeling is understood much better now than it was 10 years ago. The purpose of this paper is to review current knowledge of random-utility modeling that affects practical travel demand analysis and to identify the implications of this knowledge for practice. Three broad questions related to this objective are addressed:

1. Do the initially claimed advantages of random-utility models still appear to be advantages and, if so, to what extent are these advantages being exploited in practice?
2. What additional issues affecting the use of random-utility models have arisen since the early 1970s, and what are their implications for practical travel demand analysis?
3. What important problems concerning the practical application of random-utility models remain unsolved?

The remainder of the paper is organized as follows. The next section reviews the main concepts underlying random-utility models and summarizes the methods normally used for developing empirical models; this section provides a basis for the subsequent discussion. The third section evaluates the initially claimed advantages of random-utility models, and the fourth discusses additional issues that have arisen during the subsequent 10 years. Concluding comments are also presented.