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# Evaluation of Discrete-Choice Random-Utility Models as Practical Tools of Transportation Systems Analysis

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Travel decisions frequently entail choices among discrete sets of alternatives, such as frequencies, destinations, modes, and routes of travel, and a large proportion of travel behavior research and practical travel demand analysis is oriented toward predicting the outcomes of such choices. A convenient behavioral and mathematical framework for modeling these choices is provided by a class of mathematical models called discrete-choice random-utility models. These models are based on the assumption that an individual's preferences among the available alternatives can be described with a utility function and that the individual selects the alternative with the greatest utility. The utility of an alternative is represented as the sum of two components: a deterministic component that accounts for systematic effects of observed factors that influence choice and a random component that accounts for the effects of unobserved factors. The random-utility model then predicts the probability that a randomly selected individual with given values of the observed factors will choose a particular alternative (i.e., the probability that the utility of the particular alternative is greater than the utilities of all other alternatives). The multinomial logit (1,2), multinomial probit (3-5), and generalized-extreme-value (6,7) models are three well-known examples of discrete-choice random-utility models. [See the work of Domencich and McFadden (1) and Hensher and Johnson (8) for discussions of the behavioral foundations of these models in the context of travel analysis.]

Many of the fundamental concepts of random-utility modeling have been known for more than 50 years (9), and some of these concepts were used in travel demand analysis as early as 1962 (10). However, the development of random-utility modeling as a practical analytic technique did not begin until the late 1960s, and random-utility models were not brought forcefully to the attention of the transportation community until the early 1970s. It was argued then that random-utility models had several important theoretical and practical advantages over other available travel demand modeling approaches such as the well-known four-step process. The main advantages claimed for random-utility models were as follows:

1. They are based on an explicit principle of human behavior, whereas other available models are not;
2. They can treat a wider range of travel choices, interactions among travel choices, and policy variables than can other available models; and
3. They make more efficient use of data than do other available models.

It was argued that these advantages enable random-utility models to forecast travel demand more accurately and less expensively than do other models.

Since the early 1970s, there has been much research on theory and methods of random-utility modeling, and random-utility models have been used in a wide variety of practical travel demand analyses. As a result, the random-utility approach to travel demand modeling is understood much better now than it was 10 years ago. The purpose of this paper is to review current knowledge of random-utility modeling that affects practical travel demand analysis and to identify the implications of this knowledge for practice. Three broad questions related to this objective are addressed:

1. Do the initially claimed advantages of random-utility models still appear to be advantages and, if so, to what extent are these advantages being exploited in practice?
2. What additional issues affecting the use of random-utility models have arisen since the early 1970s, and what are their implications for practical travel demand analysis?
3. What important problems concerning the practical application of random-utility models remain unsolved?

The remainder of the paper is organized as follows. The next section reviews the main concepts underlying random-utility models and summarizes the methods normally used for developing empirical models; this section provides a basis for the subsequent discussion. The third section evaluates the initially claimed advantages of random-utility models, and the fourth discusses additional issues that have arisen during the subsequent 10 years. Concluding comments are also presented.

## BASIC CONCEPTS AND METHODS

Let an individual face a set of J mutually exclusive alternatives, one of which must be chosen. Let C denote the set of all available alternatives, and for each $j \in C$ let $\underline{Z}_j$ denote the vector of attributes of the individual and alternative j relevant to choice among the alternatives in C. The fundamental premise of random-utility modeling (as well as of many of other models of human behavior) is that there exists a function U of the attributes with the property that for any two distinct alternatives i and j in C, $U(\underline{Z}_j) > U(\underline{Z}_i)$ if and only if the individual prefers j to i. U is called a utility function.

It is generally recognized that the attributes of individuals and alternatives that are relevant to choices among travel options are not all known to analysts and that it is usually not feasible to observe (or measure) the values of all the known attributes. In random-utility models, this inherent uncertainty as to the identities and values of a potentially large set of attributes is dealt with by dividing the utility function into deterministic and random components. The deterministic component is a function of the observed attributes of individuals and alternatives and accounts for the systematic effects of these attributes on choice. The random component accounts for the effects of the unobserved attributes. Mathematically, the utility $U_j$ of an alternative j is written as follows:

$$U_j = V(\underline{X}_j, \underline{\theta}) + \epsilon_j(\underline{X}_j, \underline{\theta}) \quad (1)$$

where

- $\underline{X}_j$ = vector of observed attributes of the individual and alternative j,
- $\underline{\theta}$ = vector of constant parameters,
- V = deterministic function, and
- $\epsilon_j$ = random variable.

The notation $\epsilon_j(\underline{X}_j, \underline{\theta})$ signifies that the probability distribution of $\epsilon_j$ may depend on $\underline{X}_j$ and $\underline{\theta}$. In typical transportation applications, observed attributes of individuals might include income, automobile ownership, and household size. Unobserved attributes might include health, social status (except as indicated by income), occupation, and schedule commitments that affect travel choices. Observed attributes of alternatives typically include travel times and costs and, if the alternatives are locations, employment and population levels. Unobserved attributes of alternatives usually include reliability and comfort if the alternatives are modes and prices, quality, and variety of available goods and services (except as indicated by employment and population levels) if the alternatives are locations.

An individual chooses alternative $j \in C$ if $U_j > U_i$ for all $i \in C$ such that $i = j$ (i.e., if j is preferred to all other available alternatives). In random-utility models, choice cannot be predicted deterministically because the utilities are random (or, more fundamentally, because of the dependence of the utilities on unobserved attributes of individuals and alternatives). Rather, random-utility models give the probabilities that each of the available alternatives is chosen. Let $\underline{X}$ denote the matrix $(\underline{X}_1, \ldots, \underline{X}_j)$. Then the probability that a randomly selected individual chooses alternative $j \in C$ is

$$P(j|\underline{X}, \underline{\theta}, C) = \Pr(U_j > U_i \text{ for all } i \in C, i \neq j) \quad (2)$$

or

$$P(j|\underline{X}, \underline{\theta}, C) = \Pr[V(\underline{X}_j, \underline{\theta}) + \epsilon j > V(\underline{X}_i, \theta) + \epsilon i \text{ for all } i \in C, i \neq j] \quad (3)$$

An explicit functional relation between the choice probabilities and the deterministic components of utility can be obtained if the probability distributions of the random-utility components are known or assumed. The simplest assumption that leads to useful models is that the random variables $\epsilon_i$ $(i = 1,2,\ldots,J)$ are independently and identically distributed (IID) with the following distribution function (the Gumbel Type I extreme-value distribution):

$$F(\epsilon) = \exp[-\exp(-\epsilon)] \quad (4)$$

The choice probabilities are then related to the deterministic components of utility through the well-known multinomial logit model:

$$P(j|\underline{X}, \underline{\theta}, C) = \exp[V(\underline{X}_j, \underline{\theta})] / \sum_{i \in C} \exp[V(\underline{X}_i, \underline{\theta})] \quad (5)$$

Other distributional assumptions that yield useful choice models are that the $\epsilon$'s are multivariate normally distributed, thereby leading to the multinomial probit model (3-5), and that the $\epsilon$'s have a generalized extreme value (GEV) distribution, thereby leading to the GEV model (6,7). The sequential or nested logit model can be obtained as a special case of the GEV model by choosing appropriate values of the parameters of the GEV distribution.

The values of the constant parameters $\underline{\theta}$ rarely are known a priori in practice. They must be estimated by fitting the model to data consisting of observations of the choices and values of the attributes (or explanatory variables) for a random sample of individuals. The method of maximum likelihood usually is used for this purpose. [Discussions of the technical aspects of maximum-likelihood estimation of random-utility models may be found elsewhere (2,3,7,11).]

If the estimation data set contains observations of large numbers of independent choices corresponding to each set of values of the explanatory variables (i.e., independent repetitions of the observations), it is possible to use regression techniques to estimate the values of the parameters $\underline{\theta}$. The Berkson-Theil method for logit models (1,2, 12) is a well-known example of this estimation technique. Berkson-Theil and other regression-based estimation methods usually cannot be used when the estimation data consist of observations of real choices among real travel options since the necessary repetitions of observations rarely are available. However, the methods can be useful when the data are generated by design in laboratory environments, where repetitions can be incorporated into the experimental design, simulated by asking respondents to indicate the percentages of the time they would make various choices, or avoided entirely by asking respondents to assign cardinal rankings to their preferences among the alternatives. [See the paper by Louviere and others (13) for examples of the use of the BerksonTheil method to estimate logit mode-choice models from laboratory-generated data.]

Regardless of the estimation technique used, the computations associated with parameter estimation are simplified greatly when the deterministic component of the utility function is linear in the parameters, i.e., when

$$V(\underline{X}_j, \underline{\theta}) = \underline{\theta}' \underline{f}(\underline{X}_j) \quad (6)$$

where $\underline{f}$ is a known, vector-valued function. Accordingly, most practical applications of random-utility models are based on the assumption that V has this form. Note that linearity in parameters does not imply linearity in the explanatory variables $\underline{X}_j$ since the components of $\underline{f}$ can be, and in practice often are, nonlinear. There is no behavioral justification for the linearity-in-parameter assumption, but the assumption can be justified mathematically by the observation that any function satisfying certain broad regularity conditions can be approximated arbitrarily well by a polynomial, which is a linear-in-parameter form. In principle, this observation implies that only linear-in-parameter forms need to be considered in developing specifications for the utility functions of random-utility models. However, in practice an inconveniently large number of polynomial terms may be needed to achieve satisfactory approximations to functional forms that psychological theory or other considerations may suggest are useful. Accordingly, the specifications of the function $\underline{f}$ that are used in practice often include nonpolynomial components (e.g., logarithms or quotients of explanatory variables). There also are situations in which theoretical or empirical considerations may suggest the desirability of using nonlinear-in-parameter utility functions. Although parameter estimation with nonlinear-in-parameter utility functions is more difficult than parameter estimation in the linear-in-parameter case, it is by no means impossible. [For examples of random-utility models with nonlinear-in-parameter utility functions, see studies by Lerman and Louviere (14), Daly (15), and Koppelman (16).]

## INITIALLY CLAIMED ADVANTAGES

### Behavioral Basis

If there were convincing empirical evidence that the utility-maximization principle provides a correct description of travel behavior, random-utility models clearly would be superior to other models in both theoretical and practical terms. The use of other models would be justified, if at all, only if they were good approximations to random-utility models. Unfortunately, there is no empirical evidence either for or against the validity of utility maximization as a principle of travel behavior. It is worth noting, however, that in certain situations not related to travel it has been found empirically that some individuals' preferences are intransitive and therefore inconsistent with utility maximization (17,18). Although this finding has no immediate application to travel analysis, it does demonstrate that the utility-maximization principle is neither tautological nor unchallengeable, and it suggests the possibility that the principle may be testable empirically in travel-related contexts.

Since there is at present no empirical evidence concerning the validity of utility maximization as a principle of travel behavior, it is necessary to find other grounds for evaluating the principle as a basis for travel demand analysis. One possibility is to examine its intuitive plausibility. Some investigators have suggested that utility maximization lacks plausibility for reasons such as the following:

1. Utility maximization seems to preclude noncompensatory decision rules, although such decision rules may be used in some circumstances. For example, an individual may refuse to use a mode that is perceived as being unsafe, regardless of how attractive the mode is in other respects.
2. Utility maximization seems to ignore the possibility that some individuals' travel decisions may be constrained by schedule commitments or other factors, thereby precluding choice of certain seemingly attractive alternatives.
3. The utility-maximization principle seems to imply that individuals have knowledge of and evaluate all alternatives independently and simultaneously. However, in cases where there are large numbers of alternatives, some may be unknown to any given individual and others may be evaluated by using a sequential decisionmaking process. Sequential processes also may be used if the alternatives are perceived as belonging to groups such that the members of each group are similar to one another and the members of different groups are dissimilar.

Although these criticisms usually are directed at random-utility models as a whole, they are more appropriately applied to the particular forms of models that are in current use. It is not difficult to construct random-utility models that incorporate noncompensatory decision rules [e.g., the elimination-by-aspects model (19-21)]. Lack of knowledge of alternatives and constraints precluding the choice of certain alternatives can be dealt with by excluding unknown or precluded alternatives from an individual's choice set. Simple models of choice-set generation have been proposed by several investigators (22-24), and additional research may make it possible to relate choice sets to observed attributes of individuals and alternatives. [See paper by Landau, Prashkar, and Alpern (25) for some initial efforts in this area.] Finally, the elimination-by-aspects and sequential logit models are examples of sequential decision models that are consistent with utility maximization. With the exception of the sequential logit model, which is used relatively frequently (7,26), these variants of random-utility models have not entered practice, and most require further developmental work before they will be ready for practical application. Until this developmental work occurs, it will not be possible for practical random-utility models to exploit the ability of the utility-maximization principle to accommodate a wide variety of seemingly different and, in some circumstances, highly plausible behavioral principles and decision processes.

If utility maximization is a correct or approximately correct principle of travel behavior, one might hope that its use would bring certain practical benefits in addition to increased realism and accuracy of models. Examples of such benefits are

1. Guidance regarding the correct functional specifications of travel-demand models,
2. Transferability of models among geographical areas, and
3. Ability to forecast demand for alternatives not present in the estimation data set (e.g., new modes).

With regard to the first of these benefits, adherence to the utility-maximization principle does not prevent models from being misspecified. (Specification errors in random-utility models are discussed in greater detail in a later section of this paper.) However, the principle provides modest guidance on specification by excluding models that are clearly inconsistent with it (e.g., models that permit intransitive preferences). Depending on the tastes of the analyst, models whose consistency with utility maximization is questionable (e.g., sequential logit models in which the coefficient of the inclusive price term lies outside of the interval [0, 1]) also may be excluded. In addition, the utility-maximization principle provides analysts with a useful conceptual framework for thinking about how models should be specified.

The situation regarding geographical transferability and predicting demand for new alternatives is less encouraging. Among the few reported investigations of geographical transferability, two resulted in acceptance of the hypothesis that the models being considered could be transferred without changing their functional specifications or the values of their utility function coefficients (27,28). However, most investigations have resulted in rejection of the hypothesis of geographical transferability (29-32). In addition, the two available empirical studies of the ability of random-utility models to predict the demand for a new mode (both of which used the same data) suggest that at present such predictions cannot be made reliably (29,33).

There are many possible reasons for the failure of random-utility models to be transferable among geographical areas or to give reliable predictions of demand for new modes (34,35). Three particularly likely ones are as follows:

1. The probability distributions of the random components of models' utility functions may be different in different geographical areas.
2. Knowledge of the distributions of the random-utility components for existing modes may provide little information on the distribution of the random-utility component for a new mode, making it necessary to guess (perhaps erroneously) the new distribution.
3. Errors in measurements of explanatory variables (especially level-of-service variables) and the use of zonal aggregate or proxy variables (e.g., to represent the attractiveness of alternative destinations) may bias models in ways that differ among geographical areas and modes (31,33).

The first of these reasons may explain the observation that a model's geographical transferability sometimes can be improved greatly by using data from the new area to reestimate the values of any alternative-specific constants in the utility function (32). Reestimation of alternative-specific constants is equivalent to reestimating the means of the distributions of the random components of utility. Further improvements in geographical transferability may be achievable by using the data from the new area to rescale the remaining utility function coefficients (36). This is equivalent to reestimating the variances of the distributions of the random-utility components. Koppelman and Wilmot (104) have reported promising results with this procedure. Since reestimating alternative-specific constants and rescaling other utility function coefficients requires a smaller analytic and computational effort than does development of a completely new model, reestimation and rescaling may provide a practical method for obtaining satisfactory models in geographical areas where development of new models is not feasible. However, additional research is needed to determine the types of models to which this procedure is likely to be applicable (e.g., work-trip mode choice, nonwork destination choice) and the extent of the improvements in transferability that can be achieved.

## Range of Choices, Interactions, and Policy Variables That Can Be Treated

Travel and travel-related choices that have been treated with random-utility models include choices of mode (1,26,37-50,52,56,67), destination (1,28,31,37,43,45,48-50), travel frequency (1,28,31,40,47,48,50), multidestination travel or trip chaining (28,31,48-50,63), time of day of travel (1,44,51), residential location (52,53), retail location (i.e., a retail store owner's choice of where to locate) (54,55), number of automobiles owned (39,43,52,53,56), types of automobiles owned (57,59), and gap acceptance at intersections (60,62). Interactions among choices or joint choices that have been treated include mode and destination (37,40,43,45); mode, destination, and frequency (1,47); mode, destination, frequency, and trip chaining (48); destination and frequency (28); destination, frequency, and trip chaining (31); mode, destination, and trip chaining (49,50); automobile ownership and work-trip mode (39,56); residential location, automobile ownership, and work-trip mode (52,53); and work-trip mode and time of day of travel (44). Transportation policy variables that have been included in random-utility models include a wide variety of travel-time and cost components; various indicators of transit comfort, convenience, and reliability (44,51,64,67); carpool incentives (42,43,65,66); and households' gasoline allocations in a fuel-rationing program (68). (The foregoing reference citations are illustrative, not exhaustive. An exhaustive listing of transportation applications of random-utility models is beyond the scope of this paper.)

Many of the foregoing choices and some of the policy variables can be treated with models other than random-utility models. For example, the standard four-step travel demand modeling process treats choices of trip frequency, destination, and mode; and the destination and mode models used in the four-step process (and occasionally the trip-frequency model) normally include indicators of travel times and costs. However, no other class of models currently in use permits treatment of the variety and complexity of choices, interactions among choices, and policy variables that have been treated with random-utility models.

With the exception of mode-choice modeling, for which random-utility models now are widely used, most transportation applications of these models have been carried out by individuals who are either engaged mainly in research in travel behavior analysis or closely associated with such research. The ability of random-utility models to represent unusually broad ranges of travel choices and policies is not being exploited by wider applications in the community.

One aspect of travel demand analysis that has not yet been treated satisfactorily with random-utility models is equilibration of transportation system performance with such travel choices as frequency, destination, mode, and time of day of travel. Equilibration is important in practice because traffic engineering and other measures to improve transportation system performance may, through their effects on travel times and costs, induce significant changes in travel frequencies and the other choice dimensions. Because travel times and costs are determined jointly by the physical capabilities of transportation facilities and the choices of individual travelers, the effects of system improvement measures on travel times and costs cannot be determined independently of their effects on travel demand. This problem is difficult to treat with random-utility models for two reasons. First, depending on the models being used, it may be necessary to enumerate the routes between each origin and destination. This can be a task of substantial computational magnitude. Second, existing network-equilibration techniques operate with aggregate measures of travel demand (e.g., trip tables), whereas random-utility models produce disaggregate demand estimates. Although numerical procedures for aggregating random-utility models are well known (69), their use in connection with existing equilibration techniques entails substantial computational difficulty and expense. Analytic aggregation procedures are available for multinomial probit models (70), but the use of probit models is not feasible in many situations because of their complexity. When probit modeling is feasible, its use may help to reduce the computational magnitude of the equilibration problem. The use of probit models for equilibrating travel demand and transportation system performance has been discussed by Sheffi and Daganzo (71,72).

### Efficiency of Data Use

Econometric estimation and forecasting with random-utility models typically require data sets containing observations on several hundred individuals compared with the several tens of thousands of observations that can be required by other modeling approaches. The ability of random-utility models to operate with relatively small data sets makes these models particularly well suited to small-area studies, studies involving observations of travel behavior in several different time periods, and other applications in which acquisition or manipulation of large data sets is not possible.

Depending on the circumstances, additional economies in data acquisition and use may be available through use of choice-based sampling and estimation techniques (73,77). In choice-based sampling, the observations are stratified on the choice variable. For example, the estimation data for a mode-choice model might be obtained from roadside interviews of automobile travelers and on-board surveys of transit users. In binary choice situations, choice-based sampling yields considerably greater estimation efficiency than does simple random sampling when one of the alternatives is chosen by less than 5-10 percent of the population (77). The relative efficiency of choice-based and simple random sampling when there are more than two alternatives and the relative efficiency of choice-based and exogenous stratified sampling (i.e., stratification on the explanatory variables and not the choice variable) have not yet been investigated. The relative costs of data acquisition through choice-based sampling and other methods also have not been investigated, although it seems certain that some choice-based methods (e.g., roadside and on-board surveys for acquisition of mode-choice data) are less costly than some conventional methods (e.g., home-interview surveys). Choice-based sampling and estimation methods are not yet being used in practice for the development of random-utility models, possibly because these methods are relatively new and their merits in comparison with conventional methods are not yet well understood.

Another method that has the potential for greatly improving the efficiency of data acquisition and use is the collection of data through designed-choice experiments. In these experiments, individuals are asked to choose among alternatives whose attributes are specified by design. The main advantage of this method is that since the alternatives and attribute values are controlled, the experiments can be designed to include alternatives that do not yet exist, span wide ranges of attribute values, and achieve high levels of estimation efficiency. An important issue that must be resolved before this method can be applied with confidence is the extent to which models developed from designed-choice experiments can be used to forecast real choices among real transportation options. [Examples of the use of designed-choice experiments for developing models of travel and travel-related choices are described elsewhere (13,14,78-83).]

## ADDITIONAL ISSUES

### Specification Errors and Specification Testing

Random-utility models, particularly logit models, are subject to a large number of specification errors. Errors that can arise in logit models include

1. Misspecification of the choice set;
2. Use of an incorrect functional form for the deterministic component of the utility function (this includes as a special case the use of an incorrect set of explanatory variables);
3. Correlated deterministic and random components of utility;
4. Random taste variation, i.e., the parameters of the deterministic component of the utility function are not the same for all individuals;
5. Nonidentically distributed random components of utility; and
6. Correlated random components of utility.

There have been several theoretical studies of the magnitude of the errors in forecasts of choice probabilities that can be caused by specification errors (23,84-86). Although it is risky to attempt to draw general conclusions from this limited group of studies, the results seem to suggest that errors 2, 3, and 4 above are particularly serious. These can cause errors of more than 100 percent in forecasts of choice probabilities and thus are clearly capable of destroying a model's practical value. The remaining errors appear to be less serious, although they too can impair a model's usefulness. Errors of up to 50 percent in forecasts of choice probabilities have been reported from these causes. The limited empirical evidence that is available seems to confirm that specification errors can cause forecasting errors of these magnitudes (5,87).

The variety and severity of specification errors that can occur in random-utility models make it important to carry out specification testing as part of the development of empirical models. One group of specification tests that is used routinely in practice consists of examining the signs, t-statistics, and possibly ratios of the estimated utility function parameters for consistency with a priori expectations. These tests have the virtue of being very easy to carry out, and they can be useful for ruling out models with clearly unreasonable properties. However, the tests lack power. Models with specification errors that cause large errors in the choice probabilities can have parameters with satisfactory signs, t-statistics, and ratios and therefore escape detection with these tests (88).

A more powerful class of specification tests consists of formal statistical comparisons of models with different specifications (88-92). Depending on the models being compared, these comparisons usually are carried out by means of likelihood-ratio tests, Lagrangian-multiplier tests, or the likelihood-ratio index goodness-of-fit statistic. The limited evidence that is available suggests that many of these tests have high probabilities of detecting incorrectly specified models when the resulting errors in the choice probabilities exceed 10-20 percent (86,88,92). Moreover, the computational effort involved in carrying out these tests is modest. However, the tests are not yet in widespread use, possibly owing to their newness, the statistical formalism associated with them, and the lack of generally available computer software for implementing them. [Examples of applications of specification tests based on formal statistical comparisons may be found elsewhere (5,28,29,31,33,51,89).]

The probability of identifying an erroneously specified model when it is compared with one or more alternative models depends on the specifications of the alternatives. Hence, it is possible that comparison tests may fail to identify a highly erroneous model due to inadequacy of the alternatives. In addition, comparison tests do not provide direct indications of the forecasting errors that would be caused by use of an erroneously specified model. The possibility of inadequate alternatives can be avoided and direct measures of forecasting accuracy obtained by comparing a model's forecasts of aggregate-choice shares with observations of these shares (93). This method of testing models has obvious intuitive appeal in addition to avoiding complicated statistical formalism, and these characteristics may account for its relatively widespread use. However, the method also has some important difficulties. One difficulty is that predicted and observed aggregate shares both are subject to random-sampling errors. Depending on the sample sizes used in parameter estimation and aggregation and the magnitudes of the true (but unknown) choice probabilities, the sampling errors may be very large. Although most of the statistical theory required to estimate the magnitudes of these errors exists, software for carrying out the necessary computations does not. Thus, in practice it can be difficult to determine whether differences between predictions and observations should be attributed to random-sampling errors or treated as evidence that the model in question is misspecified.

A more serious difficulty is that it is not clear what population groups should be used for computing aggregate shares. For example, the population might be grouped according to income, location of residence, trip length, or location of workplace, among other criteria. The choice of group affects the differences between predicted and observed aggregate shares and the magnitudes of the sampling errors in these differences, and it can cause the differences to be irrelevant to the question of whether a model is correctly specified. For example, suppose a model is estimated from observations on individuals living in the northern suburbs of a city and is tested by comparing predicted and observed aggregate-choice shares of individuals living in the southern suburbs. Then large differences between the predictions and observations would indicate that the model transfers poorly to the new (i.e., southern) population, assuming that the effects of random-sampling errors are known to be small, but would not necessarily imply that the model is erroneously specified relative to the population from which it was estimated. (Of course, knowledge that a model transfers poorly to a new population can be of great practical importance, depending on the intended applications of the model, even if it is irrelevant to the question of whether the model is correctly specified.)

Because of these difficulties, comparisons of predicted and observed aggregate shares do not now provide as firm a basis for identifying erroneously specified models as do formal statistical comparisons of alternative models. [Examples of applications of the method of comparing predicted and observed shares may be found elsewhere (27,30,32,65,66,87,93-95).]

## Data Adequacy

Erroneous measurements of the explanatory variables are well-known causes of error in econometric models. In travel demand analysis this problem is particularly apparent in random-utility models, since these models typically include more explanatory variables than do other models.

There is evidence that the transportation level-of-service data contained in standard transportation data sets may be highly erroneous. These data typically are obtained from network models and represent, at best, average values for traffic zones. In an analysis of level-of-service data obtained in the San Francisco area, it was found that network-based data gave particularly poor estimates of the out-of-vehicle components of transit travel time (96). The network estimates were found to be biased, and the root-mean-square (RMS) differences between the observed and network values were comparable in magnitude to the mean observed values. The RMS errors in the network-based estimates of transit and automobile in-vehicle travel times varied from 25 to 50 percent of the mean observed values, depending on the mode.

Not surprisingly, erroneous measurements of level-of-service variables cause the estimated values of the parameters of travel demand models to be biased and can lead to highly erroneous forecasts (38,96). For example, in the analysis of the San Francisco data it was found that use of zonally averaged values of observed level-of-service data instead of network-based data reduced the error in a logit mode-choice model's prediction of Bay Area Rapid Transit (BART) patronage from 92 to 23 percent. The remaining 23 percent error is mainly the result of zonal averaging of the observations. Theoretical analyses have shown that zonal averaging of level-of-service or other explanatory variables can induce errors of roughly 100 percent in models' predictions of choice probabilities (85).

The large estimation and forecasting errors that can result from use of network-based level-of-service data suggest that in the future increased emphasis should be placed on measuring the values of level-of-service variables. The relatively small data sets required by random-utility models make this a considerably less onerous undertaking than it would be if data had to be obtained for a model requiring observations on tens of thousands of individuals.

Another aspect of conventional data sets that is deficient is their representation of locational attraction variables. These variables usually are limited to indicators of the population, employment, and geographical size of traffic zones. Population, employment, and size are, at best, crude proxies for the characteristics of locations travelers actually consider when making destination choices. Empirical evidence for the inadequacy of these variables, including the possibility that they may cause large errors in forecasts of destination choice, has been presented by several investigators (31,97,98). In the future, efforts should be made to acquire data on locational attributes more closely related to travelers' decision processes.

## Simplified Methods

Although random-utility models usually are presented in a context of elaborate mathematics and large computer systems, they also are amenable to simplified applications. Useful estimates of the effects of transportation policy measures on aggregate choice shares, fuel consumption, emissions of air pollutants, and costs of transportation services, among other impact variables, often can be obtained by hand with the aid of a desk calculator (99-101). In addition, the small data sets required by random-utility models make it possible to carry out more elaborate computations, such as parameter estimation, on microcomputers. Software packages for performing these computations are likely to become generally available in the near future.

## Interval Forecasts and Sensitivity Analysis

Most forecasts of travel demand are made in the form of point estimates with few or no quantitative indications of the potential magnitude of the errors that may be associated with them. However, it is generally agreed that these errors can be large and that it would be useful to have information on their potential magnitude.

There are three basic causes of error in travel demand forecasts:

1. Random-sampling errors that arise in the processes of parameter estimation and model aggregation,
2. Errors in forecasts of model's explanatory variables, and
3. Model-specification errors.

The effects of random-sampling errors on forecasts can be treated by using standard statistical methods, and confidence intervals for the forecasts can be obtained. Procedures for doing this have been described by several investigators (4,93,102,103). Many of these procedures require relatively modest computational resources. However, the mathematics associated with the procedures is relatively complex, the required computations usually cannot be performed by hand, and computer software for performing the computations is not generally available. Consequently, the procedures have been used only in a small number of illustrative applications (4,103). The results of these applications suggest that when logit models are estimated from data sets containing several hundred observations, the half-widths of the 95 percent confidence intervals for individual choice probabilities are roughly 15-30 percent of the estimated values of these probabilities. These results reflect only sampling errors in the values of the estimated parameters. Confidence intervals for aggregate shares, which also include sampling errors due to aggregation, are likely to be somewhat larger.

The errors in models' forecasts caused by erroneous forecasts of the explanatory variables and by specification errors can be considerably larger than those caused by random-sampling errors (85) but are, unfortunately, more difficult to evaluate. This is because there are no objective methods for estimating the magnitude of errors in forecasts of explanatory variables or model specification. Consequently, it is not possible to develop statistically meaningful confidence intervals for the effects of these errors. However, it is possible to evaluate the sensitivity of models' forecasts to judgmentally specified changes in the values of explanatory variables and specifications. The results of such sensitivity analyses provide qualitative indicators of the robustness of models' forecasts in the presence of errors in the values of explanatory variables and specifications.

There are two methods for carrying out sensitivity analyses of the effects of errors in forecasts of a model's explanatory variables. One method consists of varying the values of the explanatory variables singly or in groups over judgmentally determined ranges and observing the effects on the model's output variables. The other method consists of assuming a probability distribution over one or more explanatory variables and computing the resulting distributions of the outputs (4,93). The first method is simpler conceptually and easier to implement than the second, and it is used occasionally in practice. For example, travel demand forecasts sometimes are made for several different forecasts of a region's population or land use patterns. However, the first method can exaggerate the uncertainties in a model's output variables. For example, assigning all of the explanatory variables' values on the boundaries of their assumed ranges of uncertainty may produce large changes in the output variables, but it may be highly unlikely that the true (but unknown) errors in the values of the explanatory variables all would have their assumed maximum values simultaneously. The second method creates at least the appearance of avoiding this difficulty, since the assumed probability distributions of the explanatory variables can be specified so as to make the simultaneous occurrence of large errors in several variables unlikely. However, the results thus obtained depend on the assumed distributions and can be misleading if these distributions are not specified carefully.

To estimate the effects of specification errors on forecasts, it is necessary first to find a means of simulating these errors (i.e., of changing the model under consideration to represent the occurrence of specification errors). It is not clear at present how this can best be done, since the model adopted for use in forecasting presumably has the best of the specifications considered during the model development process. One possibility is to compute the range of forecasts resulting from the use of several differently specified models. This method may be particularly useful if it is possible to identify several models that fit the estimation data set roughly equally well but that give different forecasts of choice when the explanatory variables are assigned particular values of interest. However, the method may tend to exaggerate the consequences of specification error if the model adopted for use in forecasting provides a substantially better fit to the estimation data than do the other models used in the sensitivity analysis.

Another possible way of representing the effects of specification error is by varying the values of one or more parameters of the model in question. This method may be useful for estimating the effects of forecasting errors arising from random taste variations (assuming that this is not already accounted for in the model) or changes in individuals' tastes that may occur during the time period to which the forecast applies. It also may be useful in cases where a model's forecasts are determined mainly by a small set of parameters whose values are sensitive to specification. For example, the value of travelers' time implied by a model is likely to be sensitive to the specifications of the travel time and cost terms of the utility function. Thus, the effects of specification error on a forecast that depends mainly on the value of time might be investigated by varying the values of the parameters that determine the value of time.

[Examples of sensitivity analyses of the forecasts obtained from a set of random-utility models may be found elsewhere (68).]

## CONCLUSIONS

The main advantages claimed for random-utility models in the early 1970s when these models were first brought to the attention of the general transportation community were their basis in an explicit behavioral principle, their ability to treat a wide range of travel choices and transportation policy options, and their ability to make efficient use of data. In retrospect, the behavioral basis of random-utility models and the practical benefits associated with it seem to have been exaggerated. The utility-maximization principle is clearly useful for model development, but its validity remains uncertain, and even if it is valid, it does not guarantee that models will be behavioral, causal, or free of potentially serious specification errors. The other initially claimed advantages of random-utility models appear to be real and, quite possibly, even more important now than they were 10 years ago. In a period of limited resources and increased demands for nontraditional outputs from the transportation analysis process, the flexibility and efficiency of random-utility models are particularly valuable attributes. However, they remain largely unexploited by practitioners.

Several relatively recent improvements in random-utility modeling will be ready for widespread practical application as soon as computer software for implementing them becomes available. Examples of these are specification tests based on statistical comparisons of differently specified models and procedures for developing statistical confidence intervals for forecasts. The availability of the necessary software would make random-utility models the only class of operational travel demand models with systematic, easy-to-use procedures for specification testing and error analysis. Another attribute of random-utility models that may be of considerable practical value is their adaptability for use in simplified analyses.

There also are a variety of unsolved problems affecting the practical application of random-utility models. These involve such matters as alternative decision processes, geographical transferability, prediction of demand for new alternatives, computational procedures for equilibrating travel demand and transportation system performance, and the relative merits of choice-based and exogenous sample designs, among others. Further research on these problems would be highly desirable and could significantly enhance the already substantial practical advantages of random-utility models.

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