Soil Water Movement in the Film and Vapor Phase Under the Influence of Evapotranspiration

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Introductory Remarks by the Chairman

One of the most widely used mathematical formulations in science is the transmission equation which states that the time rate of transmission through a unit cross-section equals the product of a specific property factor of the material and the rate of change in concentration of whatever is to be transmitted in the direction normal to the area of transmission. In first approximation, this equation holds true for many different systems and materials or forms of energy to be transmitted. It represents a major part of the engineering application of the so-called energy concept of soil moisture, or rather the suction concept, since important energy considerations have often been neglected or overlooked by the more extreme practitioners of this concept. An equation, or a theoretical concept expressed by it, that has as wide a field of application as the transmission formula can be expected to possess severe limitations in a field of application in which the substance to be transmitted is able to form structures and shapes that affect the transmission process. For the case of soil and water, Dr. Hallaire investigates these limitations experimentally and theoretically in a splendid piece of work. His results are extremely important and illuminating; one of the main lessons taught is that in aqueous systems, possessing appreciable amounts of internal surface, the peculiar structural properties of water, and especially the cohesive strength of its films, must not be overlooked if one wants to develop valid and useful concepts.

GENERAL CONCEPTS

THE RATE of water diffusion in a direction z'z is usually given by

$$\frac{dQ}{dt} = \lambda \frac{d\phi}{dz}$$

(1)

in which $\frac{dQ}{dt}$ = the loss in a direction z'z (which can be given in mm/day or g/day per cm$^2$);

$\phi$ = the sum of the potentials acting on the water: the capillary potential $\Psi$, the hydrostatic pressure potential $\phi_1$, the gravity potential $\phi_2$; and

$\lambda$ = a conductivity coefficient which increases with increasing moisture content; that is, with decreasing capillary potential.

When the soil moisture content H (water per 100 g of dry soil) is lower than field capacity, then the capillary potential gradient $\Psi$ is usually much larger than that of the other two potentials $\phi_1$, and $\phi_2$, and Eq. 1 can be reduced to

$$\frac{dQ}{dt} = \lambda(\psi) \frac{d\psi}{dz}$$

(2)

$\lambda(\psi)$ being a decreasing function.

In the experimental or theoretical study of capillary diffusion, certain authors have...
tried to express the variation of $\lambda$ as a function of $\psi$. Others, considering a homogeneous soil and transforming Eq. 2 into

$$\frac{dQ}{dt} = \lambda(\psi) \frac{d\psi}{dH} \frac{dH}{dz}$$

or

$$\frac{dQ}{dt} = \Lambda \frac{dH}{dz} \tag{4}$$

with

$$\Lambda = \lambda(\psi) \frac{d\psi}{dH}$$

have endeavored to define the values of this new coefficient $\Lambda$ in front of the moisture gradient $\frac{dH}{dz}$.

There are experimental techniques in which water from a free water surface is made to rise into dry soil, and there are others in which continued water movement is assured by submitting a soil column to a definite pressure gradient. The results obtained by these methods are not always sufficient to explain and to interpret how water actually moves in a soil under natural conditions. Attempts to define the quantities of moisture that can be displaced by upward movement either to evaporate at the surface or to feed the vegetation and compensate for the transpiration losses show that the laws of moisture movement are much more complex than the previous equations indicate. The movement may be zero, despite an appreciable moisture gradient; on the contrary, a large movement $\frac{dQ}{dt}$ can be observed in the absence of a gradient or even against an existing gradient from relatively dry zones to zones of higher moisture content.

The purpose of this paper is to show the limits of the validity of the classical diffusion equations when the water movement in soil is caused by evapotranspiration. Two experimental studies are presented: (a) moisture profiles observed in the field under bare or cultivated soil during the drying period; and (b) drying of soil columns submitted in the laboratory to various conditions of evaporation.

**MOISTURE PROFILES AND CAPILLARY DIFFUSION OF WATER IN SOIL**

Moisture Profiles in Bare and Cultivated Soils

From a series of moisture profile determinations on different types of bare and cultivated soils during the drying period, it has been shown that the moisture content of the successive earth layers $H_1$, $H_2$, $H_3$, etc., remained in close correlation with respect to each other. This is the reason why moisture profiles, showing the change of moisture content with depth, present for each soil type a succession of characteristic forms (Fig. 1A), which can be easily demonstrated.

The moisture profiles shown in Figure 1A have been obtained on cultivated as well as on bare soils. If the cultivation causes a greater drying of the soil and thus leads to lower moisture contents, any discon-
tinuity between the series of profiles observed on bare or cultivated soils is not apparent.

These moisture profiles observed during the drying period correspond to an upward moisture movement to the surface of evaporation or to the root zone. If some show a very irregular form, or even an inversion of gradient, this is due to soil heterogeneity as a function of depth. The water diffusion is actually governed by the gradient dψ/dz, for which the moisture gradient dh/dz can be substituted only if the physical nature of the soil remains the same throughout the entire depth Z. (See Eqs. 2 and 4.) To interpret the results obtained, the variation with depth of the capillary potential ψ must be considered, not the moisture content H. However, it is easy to pass from one expression to the other if ψ has been determined as a function of H for each layer of the soil under consideration.

The new profiles expressing the variation of ψ with depth z (Fig. 1B) possess exponential form. Thus, and in contrast to the moisture content, the capillary potential ψ varies with depth in a continuous and regular manner. ψ tends asymptotically toward 1,000. Assuming that this particular value corresponds for every soil to the field capacity, this result signifies that the moisture content H tends with increasing depth to reach the field capacity that is characteristic for the layer under consideration. Experience shows that as the desiccation of the soil proceeds, a profile ψ(z) passes to another by a simple expansion of the corrected abscissas (ψ - 1,000). These results suggest a third representation in the form of the variation of log(ψ - 1,000) with depth Z (Fig. 1C). Instead of the previous exponential curves, quasi-linear curves are obtained and the transition from one to the other is no longer by expansion, but by simple translation; as a matter of fact, the straight lines are practically parallel to each other. There is, however, an anomaly in the vicinity of the surface of evaporation: the profiles show a break reflecting marked drying of the surface layers. This phenomenon, which can be seen on the average profiles shown in Figure 1C, is more apparent if the profiles on thinner soil sections (Fig. 2) are determined. This is, of course, a commonly observed phenomenon; namely, the formation of a crust of dry earth, a veritable natural surface mulch.

The results can now be summarized by stating that during the drying-out period, and neglecting the uppermost layers which may dry out to an extreme extent, the shape of a profile expressed as a function of log(ψ - 1,000) remains almost constant for a given soil, whatever be its depth and its state of desiccation.

\[ \frac{d \log(\psi - 1,000)}{dz} = C \]  

in which \( C = \text{C te} \) (for any Z or ψ). The results reported in this section have been verified on four different soils.

Hypothesis of the Permanent Regime

Assuming that the observed moisture profiles reflected a simple phenomenon in the upward movement of water, the initial hypothesis is that they corresponded with the conditions of the permanent regime. To say that the regime is permanent—or better, that the loss dQ/dt is the same at different depths Z, means that the loss is proportional to the gradient of log(ψ - 1,000) since this gradient according to Eq. 5 retains the same value at all levels. Accordingly,

\[ \frac{dQ}{dt} = A \frac{d \log(\psi - 1,000)}{dz} \]  

in which \( A = \text{C te} \).
Eq. 6, to which the proposed hypothesis leads, does not contradict the previously presented theoretical considerations. It shows simply that the coefficient $\lambda$ in Eq. 2 decreases as $\psi$ increases (at least within the range of variation of $\psi$ obtained in the experimental study: $1,000 < \psi < 15,000$) in accordance with

$$\lambda = \frac{A'}{\psi - 1,000}$$

where $A' = C \psi$.

It will be seen that introduction of Eq. 3a in Eq. 2 gives Eq. 6. An equation similar to Eq. 6 was obtained by Derjaguin on the basis of theoretical considerations concerning the displacement of liquid films along the solid wall formed by soil particles (3).

With this hypothesis, application of the indicated transformation gives Eq. 3a or Eq. 4; therefore, the coefficient $A$ appears to be almost independent of the moisture content if $\lambda$ varies in accordance with the proposed law (Eq. 3a). Thus, a result is obtained which several authors have verified in a more or less precise manner.

The hypothesis of the permanent regime, which has been advanced here for the interpretation of the form of the moisture profile, raises, however, the following questions:

1. Why do the moisture profiles respond to the conditions of the permanent regime when as a result of the continuous variations of evapotranspiration the water regime in the soil should be extremely variable?

2. If the hypothesis of the permanent regime is exact, this implies also that the loss remains constant while the soil is drying: the slope of the profiles $d \log(\psi - 1,000) / dz$, actually remains constant, not only while the depth increases, but also when with more marked desiccation the entire profile in $\log(\psi - 1,000)$ moves toward higher $\psi$ values; what then is the significance of this particular yield characteristic of each soil?

**Permanent Regime Expresses the Irreversibility of the Desiccation Process**

Starting with the proposed diffusion equation (Eq. 6) inquiry is made into the manner in which this equation applies in two distinct cases: that of soil desiccation and that of soil rehumidification.

It is specified first that the value of the capillary potential $\psi$ is that of soil desiccation, $\psi_s$. In the investigations cited here, it has been obtained by means of the cryoscopic method. A profile giving the variation of $\psi$ or of $\log(\psi - 1,000)$ as a function of the depth $Z$ (the profile is obtained by substitution of $\psi$ values for the directly measured moisture contents $H$) presents a true picture of the capillary potential if at every level under consideration the soil is actually in the drying phase; the picture is, however, false if the soil is in the remoistening phase, because the true potential corresponding to the respective moisture content is $\psi_h$, the capillary potential for remoistening, which is much smaller than $\psi_s$ (phenomenon of hysteresis).

Under conditions in which every thin soil section is actually in the drying stage under the influence of a sufficiently large evapotranspiration, the capillary potential corresponding to the measured moisture content is truly $\psi_s$; the profile in $\log(\psi - 1,000)$ represented on the graph depicts the real situation, and Eq. 6 gives an exact value of the loss or yield.

If, on the other hand, certain thin soil layers tend to take up moisture, the water losses in evapotranspiration being smaller than the yield given by Eq. 6, the water potential in the layers under consideration is no longer the desiccation potential $\psi_s$ considered in this study, but approaches the remoistening potential $\psi_h$. Then the profile in $\log(\psi - 1,000)$ as plotted gives an incorrect picture of the variation in potential and the diffusion formula no longer holds.

What then is the yield? The yield $dQ/dt$ at any depth $Z$ cannot be greater than (a) that yield value given by Eq. 6 for the entire depth between 0 and $Z$, and (b) the evapotranspiration. Otherwise, one would be dealing actually with the remoistening phase. Thus, in the extreme cases in which the evapotranspiration is zero, the yield itself is zero.
even in the presence of an appreciable moisture gradient, because the least displacement by upward movement of water would lead to a moistening of the overlying soil layers and hysteresis would intervene to stop all movement. As a matter of fact, remoistening of the soil is possible if the moisture comes from a body of free water. The capillary potential is then \( \psi_h \) at all levels. This phenomenon can intervene at distances of less than 50 or 100 m above the ground water level.

With respect to the conditions of the permanent regime (linearity of the profiles in \( \log(\psi - 1,000) \)) they correspond to a state limited by the conditions of desiccation where the profile is concave toward the right and where the soil dries in accordance with Eq. 6. On the other hand, they correspond to a state limited by the conditions of remoistening where the profile tends to be concave toward the left. At this moment, the phenomenon of hysteresis intervenes to invalidate the diffusion equation and to stop the development of the moisture profile. The permanent regime, therefore, depicts the irreversibility of the drying process.

The following experiment confirms the reasoning so far presented. The evolution of profiles on a lot maintained without vegetation and protected against rain was observed for several months (Fig. 3). Displacement of the profiles in the direction of increasing \( \psi \) proved that diffusion can function to insure the desiccation of the soil. However, when at the end of the season the evapotranspiration falls to a value of practically zero (in consequence of the drying out of the surface and the cooling of the atmosphere) there was no tendency of the soil to rehumidify—the gradient of \( \log(\psi - 1,000) \) is only an apparent one and does not indicate any upward movement of water.

**Characteristic Gradient Corresponds to a Certain Critical Yield**

The hypothesis of the permanent regime is advanced to explain the linearity of the profiles in \( \log(\psi - 1,000) \), but the yield remains the same whatever the state of desiccation of the soil. It was shown that the slope of the profiles in \( \log(\psi - 1,000) \) remained constant, although as the soil was drying the profiles were displaced toward increasing \( \psi \)-values. What then is the significance of this particular slope C? What, also, is the significance of this particular yield A.C.?

The experiment showed a critical yield that cannot be exceeded without causing an extreme desiccation of the soil which results in a marked reduction of the evaporation.

The top surface of a moist soil column is subjected to a certain evaporation E.
E is small (less than 1 mm per day in the case of a loam from Versailles), the surface remains moist for a long time and water conduction to the surface, under these conditions, compensates for the losses into the atmosphere. With all other factors the same, the evaporation remains constant. If, however, a high evaporation (for example, larger than 2 or 3 mm per day) is caused by means of electric lamps and ventilators, there is a rapid formation of a dry surface crust, a true natural mulch. Then the evaporation decreases rapidly and approaches a value of about 1 mm per day. Thus, the slope of the profile in log(\(\psi - 1,000\)) observed in the open field and under the surface crust of dry earth corresponds to this particular critical yield of the order of 1 mm per day for the loam studied.

MECHANISM OF SOIL DRYING UNDER INFLUENCE OF EVAPORATION

Experimental Technique and Methods of Measurement

Experimental Technique. Tubes (30 to 50 cm long and 6 to 7 cm in diameter) are filled with homogeneous soil possessing throughout the same initial moisture content of approximately field capacity. The tubes can be filled with either specimens taken with the sampler in the field or broken-up soil which is then compacted to the desired density. The bottom of the tube is closed and the surface of the earth cylinder is subjected in the laboratory to well-defined conditions of evaporation that can be controlled by means of electric lamps or fans.

Measurement of Evaporation. For a certain period of time, \(\Delta t\) (days), the evaporation \(E\) (mm/day) is given by

\[
E = \frac{\Delta P}{S \Delta t}
\]

in which

- \(\Delta P\) = the loss of weight of the soil cylinder (dg); and
- \(S\) = the cross-section of the cylinder (sq cm).

Total evaporation during the time interval \(t_1 - t_2\) is given by

\[
\int_{t_1}^{t_2} E \, dt = \frac{\Delta P}{S}
\]

Measurement of Quantity Yielded by Successive Thin Layers. To determine the moisture quantities yielded by successive thin soil layers a series of soil filled tubes are prepared in the same manner as the test samples and subjected from the start of the experiment to the same conditions of evaporation. At each weighing for the determination of \(E\), one or two tubes are sacrificed to determine, slice by slice, the new moisture content \(H\) (water per 100 g of dry soil). If \(H\) has decreased by \(\Delta H\) during a period \(\Delta t\) (days), the quantity of water yielded at the depth \(z\) and for a slice 1 cm thick is (in mm per day):

\[
u = \frac{\sigma \Delta H}{10 \Delta t}
\]

in which \(\sigma\) is the apparent density of the soil.

If the moisture content \(H\) is determined on soil slices of a thickness \(\Delta z\) (in cm), the quantity of water yielded by these slices is (in mm per day):

\[
U = u \Delta z
\]

Measurement of Yields at Different Levels. The measured evaporation \(E\) is simply the yield \(\left(\frac{dQ}{dt}\right)_0\) at the soil surface itself; that is, at the depth \(z = 0\). If \(U_1, U_2, U_3, \ldots\), designate the quantities of water lost by the successive soil slices of 0.6-cm thickness, for example, the yields are:

- at 0.6 cm \(\left(\frac{dQ}{dt}\right)_1 = E - U_1\)
at 1.2 cm \( \frac{dQ}{dt} = E - U_1 - U_2 \)

at 1.8 cm \( \frac{dQ}{dt} = E - U_1 - U_2 - U_3 \)

The error increases with greater depths.

Measurement of Moisture Gradient.

Knowing the moisture contents \( H_1, H_2, H_3, \) etc., of the successive soil layers, the moisture gradients \( \frac{dH}{dz} \) at the levels of the soil slices (for example, 0.6 cm, 1.2 cm, 1.8 cm) can be deduced.

Relation Between Yield and Moisture Gradient (3)

Eq. 4 gives the yield as a function of the moisture gradient under the condition that the soil be homogeneous and in the drying stage. It has been shown that \( A \) varies but little with change in moisture content.

If the yield could be measured in the soil columns at different depths, but during the same lapse of time, and if parallel determinations of the moisture gradient have been made it is possible to plot the variation of \( \frac{dQ}{dt} \) as a function of \( \frac{dH}{dz} \). The yield will decrease in a regular manner from the value \( E \) at the surface, approaching zero with increasing depth. Eq. 4, accordingly, permits anticipation of curves of the type of \( \Gamma \) or \( \Gamma' \), shown in Figure 5.

If \( \Delta \) is independent of the moisture content, the points must be located on a straight line (\( \Gamma \)) which passes through the origin and whose slope is exactly \( \Delta \) (see Fig. 5A).

If \( \Delta \) increases with increasing moisture content \( H \), since \( H \) itself increases with increasing depth, the \( \frac{dQ}{dt} / \frac{dH}{dz} = \Delta \) becomes greater with larger values \( z \). The points would be located on a curve \( \Gamma' \), which is concave toward the right (see Fig. 5B).

The actual curves (Fig. 6A) do not conform at all with the expected result: the upper part of the curve, which corresponds to the uppermost layers, has a remarkably linear shape obeying the equation

\[
\frac{dQ}{dt} = B + A(t) \frac{dH}{dz} \tag{11a}
\]

the slope \( A \) being a constant at a given moment, but decreasing systematically with time. One can thus distinguish in the surface yield \( \frac{dQ}{dt} = E \) the two components \( B \) and \( (E - B) \). The experiment shows that if one imposes a high degree of evaporation, the term \( B \) cannot exceed a value of the order of magnitude of 2 mm per day for the Versailles loam (Fig. 6B).

On the other hand, if the soil is simply poured into the tube and left in a non-coherent state, the term \( B \) is very small or equal to zero (Fig. 6C). Eq. 11a then approaches the form:

\[
\frac{dQ}{dt} = A(t) \frac{dH}{dz} \tag{11b}
\]
Relationships Between Yield and Moisture Gradient Explained by Depth Distribution of Water Losses

If one plots the variation of \(\log(u)\) as a function of the depth \(z\), the curve obtained is normally linear from the surface to a certain depth \(z_1\). At greater depths, \(\log(u)\) decreases at a lesser rate than if the law of variation remained linear (Fig. 7A). This means that the curve for \(u\) or \(\Gamma\) coincides at first with the exponential function \(\Gamma'\).
(Fig. 7), or

\[ u = u_0 \exp(-az) \]  

then it deviates at depths greater than \( z_1 \) (Fig. 7B).

In the case of non-coherent soil, the variation of \( \log(u) \) remains essentially linear (Fig. 7C); that is, \( u \) varies in accordance with an exponential law (Curve \( \Gamma \), Fig. 7D).

From these results, it is easy to find the relationship between the yield and the moisture gradient (Eqs. 11a and 11b). If a coherent soil of normal density corresponding to Figs. 7A and 7B is considered, it is seen that the yield \( \frac{dQ}{dt} \) at a level \( z = \int_z^{\infty} u \, dz \) corresponds to the area limited by \( \Gamma \) below the respective depth.

If \( \beta \) is the quantity of water (in mm per day) corresponding to the area comprised between the exponential curve \( \Gamma' \) and the actual curve \( \Gamma \), at any depth \( z \), located be-

Figure 7.
tween zero and \( z \), the yield is equal to the sum of \( \beta \) and the losses as defined by the exponential curve:

\[
\frac{dQ}{dz} = z \int_0^\infty u_0 \exp(-az) + \beta, \quad \text{or} \quad \frac{dQ}{dt} = \frac{u_0}{a} \exp(-az) + \beta
\]  

Besides, if for any level of the slice \( 0 - z_1 \) one seeks to evaluate the quantities of water \( q \) (in mm) yielded since time zero, when the moisture content was \( H_0 \), to the time \( t \), when the moisture content is \( H \), one obtains for a slice of 1-cm thickness one or the other of the expressions:

\[
q = \frac{\sigma}{10} (H_0 - H)
\]  

and

\[
q = \int_0^t u_0 \, dt = -a \exp(-az) \int_0^t u_0 \, dt
\]  

Equating Eqs. 14 and 15 and differentiating with respect to \( z \) gives

\[
-\frac{\sigma}{10} \frac{dH}{dz} = -a \exp(-az) \int_0^t u_0 \, dt
\]  

Finally, eliminating the term \( \exp(-az) \) between Eq. 13 and Eq. 16 leads to:

\[
\frac{dQ}{dt} = \frac{\sigma u_0}{10 a^2} \int_0^t u_0 \, dt \frac{dH}{dz} + \beta
\]  

Eq. 17a is the same as the experimental relationship (Eq. 11) inasmuch as the term in front of \( dH/dz \), a constant for a given moment but decreasing with time, can be written as \( A(t) \) and \( \beta \) is like \( B \), a constant as long as one is concerned with depths smaller than \( z_1 \).

In the case of non-coherent soil, for which \( u \) varies exponentially, calculation leads to

\[
\frac{dQ}{dt} = \frac{\sigma u_0}{10 a^2} \int_0^t u_0 \, dt \frac{dH}{dt}
\]  

which is identical with the experimental relationship (Eq. 11b).

Hypothesis of Diffusion in the Vapor Phase (4, 5)

The experimental results reported in the preceding seem to invalidate the classical laws of liquid diffusion in soils. As a matter of fact, according to the latter the relationship between the yield and the moisture content should normally be represented by curves of the type of \( \Gamma \) or \( \Gamma' \) (Fig. 5), which clearly differ from the actual curves given (Fig. 6).

If it is admitted that the water losses are due in part to an evaporation in the soil and diffusion in the vapor phase into the atmosphere, it seems possible to explain the experimental findings. It is, therefore, proper to examine this hypothesis and to explain why, after all, it cannot be retained.

When at a depth \( z \) a saturation deficit exists in the atmosphere of the soil pores (the actual vapor pressure \( f \) being lower than the maximum pressure \( F(t) \)), evaporation will take place at this level. The amount of water evaporated (in mm per day) and for the slice \( z, z + dz \), will be

\[
Y \, dz = M \left[ F(t) - f \right]
\]  

with \( M = C \, \text{te} \), and will represent a water vapor gain for the pore atmosphere. But the
same slice will lose at the level \( z \) a quantity of vapor equal to \( K\left(\frac{df}{dz}\right)_{z} \) while it will gain \( K\left(\frac{df}{dz}\right)_{z+dz} \) at the level \( z + dz \). Assuming establishment of equilibrium between vapor gain and loss, one obtains

\[
M \left[ F(t) - f \right] + K \frac{df^2}{dz^2} = 0
\]  

(19)

But if one passes from the variable \( f \) to the variable \( Y \), keeping Eq. 18 in mind, integration gives

\[
Y = Y_0 \exp(-az)
\]

(20)

with \( a = \sqrt{\frac{M}{K}} \). The term \( Y \), like \( u \), represents water losses in mm per day and per slice of 1-cm thickness, and varies, therefore, in accordance with an exponential law.

Returning to the variation of \( u \) as a function of depth (see Fig. 8), one may ask if the exponential function \( \Gamma' \) does not express the losses \( Y \) attributable to vapor diffusion. In this case, the area \( B \) or \( B' \), comprised between the curves \( \Gamma \) and \( \Gamma' \), would represent the losses by liquid diffusion proper. The components \( B \) and \( (E - B) \) shown in Figure 6 would then correspond, respectively, to a liquid flux evaporating at the surface and to a vapor flux (Fig. 8).

In the case where the soil is non-coherent (relatively dry and unpacked), the losses \( u \) varied at all depths in accordance with an exponential law, while the term \( B \) was practically zero. According to the proposed hypothesis, this soil would dry out only by simple vapor diffusion, which seems to conform to common sense.

This hypothesis of a vapor diffusion is in error. In the first place, if the term \( (E - B) \) actually corresponded to a vapor loss, the water vapor diffusion in the soil would have to be turbulent as has previously been proven. As a matter of fact, the values for the coefficient of diffusion \( K \) calculated from certain experimental data were considerably larger than the coefficient of molecular diffusion in a porous medium such as soil.

Secondly, the following experiment shows that the loss of liquid cannot be covered by the term \( B \) alone:

A solution of calcium chloride is sprayed in a homogeneous manner over a soil with which column evaporation experiments are to be made. Some of the cylindrical speci-
mens represent packed soil with an initial moisture content of about 25 percent of field capacity; these are subjected to a mild evaporation (E about 1 to 2 mm per day) in a manner that the term B represents approximately 90 percent of E. The other soil cylinders consist of soil of lesser density and lower moisture content (H₀ about 17 to 20 percent) and are subjected to more severe evaporation (E > 5 mm per day). The term B will be here practically zero. ρ is defined as the concentration in Cl (mg per 100 g of dry soil) in the top layer of 3-mm thickness and Δρ as the increase in concentration since the start of the evaporation. Reasoning shows that Δρ should be proportional to the quantity of water gained by the surface in the liquid state, that is,

\[ \int_0^t \frac{dQ}{dt} \, dt \]

and on the other hand to the term \( \frac{\rho_0}{\sigma H_0} \), in which \( \rho_0 \) = initial Cl concentration; \( \sigma \) = apparent density; and \( H_0 \) = initial moisture content.

The variation of Δρ is plotted as a function of

\[ \frac{\rho_0}{\sigma H_0} \int_0^t E \, dt \] (see Fig. 9), where \( \int_0^t E \, dt \) is the total evaporation in mm from the initial condition. The points group themselves along the same straight line irrespective of the soil condition (packed and coherent or loose) and irrespective also of the value B with respect to the evaporation E.

If it were true that the liquid loss could be covered by the term B, then the expres-
sion \[ \int_0^t E \, dt \] would considerably overestimate the quantity of water that reaches the soil surface in the liquid state, since \( \frac{B}{E} \) is small, and the corresponding points should lie lower than those relating to the higher \( \frac{B}{E} \) values. The vapor diffusion hypothesis is rejected, therefore, as an explanation of the experimentally found relationships.

Cohesive Forces of Water

Eq. 4 is derived, as previously shown, from the diffusion equation (Eq. 2), or its equivalent

\[ \frac{dQ}{dt} = \lambda(P) \frac{dP}{dH} \quad (2a) \]

where \( P \) represents the (negative) pressure of the water in the soil and is numerically equal to the capillary potential \( \psi \). In deriving Eq. 4 from Eq. 2a the transformation

\[ \frac{dQ}{dt} = \lambda(P) \frac{dP}{dH} \frac{dH}{dz} \quad (3b) \]

is effected. This operation is permitted, however, only if the water pressure \( P \) in the soil is rigorously defined by the moisture content \( H \). In the first part of this study, there have been set forth two restrictions on this manner of thinking, which consequently pertain to the validity of the diffusion equation as a function of the moisture gradient. In order that \( P \) may be defined as a function of \( H \) and of \( H \) only, it is required that (a) the character of the soil be the same at every point, and (b) every soil element be in the drying-out phase (in order to avoid the hysteresis effect). To these two conditions a third one must be added: in order that the pressure \( P \) depend only on \( H \), it cannot be modified by a tension due to the cohesion of the water which would be transmitted along the liquid films. Experience shows clearly that such a tension exists and that, therefore, at least under certain conditions, the actual pressure \( P \) of the water differs from \( P(H) \) in such a manner that in the absence of any liquid movement

\[ P \neq P(H) \quad (21) \]
Figure 11.
Experiments. A soil cylinder prepared so that the moisture content $H_0$ of the top layer (of 3 cm, for example) is greater than the moisture content of the underlying layers, is subjected to evaporation. If the pressure $P$ of the soil water were uniquely defined by $H$, the layers below the depth of 3 cm would not begin to dry out until the moment when the first layer had attained the moisture content $H_1$. Experiments, however, show that the layers in question are losing water from the start of the evaporation. The water which they lose moves toward the surface and passes through the more humid zones (see Fig. 10).

Examination of how the moisture content $H$ varies at different depths of the soil cylinders by plotting $\Delta H$ as a function of the amount of water evaporated, $\int_0^t E \, dt$ (Fig. 11), gives straight-line curves, all starting from the origin, a fact which has already been pointed out by Abramova (6). Thus, desiccation starts throughout the sample as soon as the surface is subjected to evaporation. If the water movement were bound to the existence of a moisture content gradient, desiccation would gain progressively in depth; the beginning of the desiccation would correspond to a total evaporation $\int_0^t E \, dt$, which would be greater with greater depth of the layer; and the straight lines found would cut the abscissa axis (Fig. 11) at a distance from the origin which would increase with increasing depth.

These two observations show that the displacement of water toward the surface of evaporation corresponds to a concerted movement of the water films, to which an internal tension assures a certain cohesion. This tension modifies the water pressure $P$ as defined by the moisture content $H$; hence, the yield $\frac{dQ}{dt}$ can not be expressed as a function of $\frac{dH}{dz}$.

However, when the water has been subjected for a certain time to a permanent regime, the moisture content tends for every depth toward such a value that the corresponding pressure $P(H)$ coincides with the actual pressure and $P = P(H)$. One can verify the law of variation with depth pertaining to the capillary potential $\psi$; that is, the pressure $P(H)$ (see discussion on variation of $\psi$ at the beginning of this paper, and Fig. 1). It is also under these conditions that a diffusion equation involving the moisture content can make any sense.

Distribution of Water Losses Throughout the Depth, and Probability of Rupture of the Liquid Films

Reexamination of the curve for the variation of losses $u$ (mm per day and per slice of 1-cm thickness) as a function of depth $z$ (Fig. 7B), shows that close to the surface, between levels zero and $z_1$, this curve $u(z)$ coincided with the exponential curve $\Gamma^\alpha$. At greater depth, the real curve $\Gamma$ deviates, at least when the soil has normal cohesion and density properties. It has been possible to show that every section of the curve $\Gamma$ can be expressed by an exponential function such as Eq. 12, but that the modulus $a$, which is constant between zero and $z_1$, decreases at depths greater than $z_1$. Furthermore, it has been possible to prove that this coefficient $a$ depends essentially on the yield. The curve for the variation of $a$ as a function of $\frac{dQ}{dt}$ for the Versailles loam is shown in Figure 12 (D for coherent and $D'$ for non-coherent soil).

It is proposed to show that the coefficient $a$, which is dependent on the nature and structure of the soil and on the yield $\frac{dQ}{dt}$, can be identified with the probability.
of rupture \( p \) of the water films.

If two successive soil layers (1) and (2) of the same thickness \( dz \), are considered, and \( p \, dz \) designates the probability of rupture of the water films in each of these layers and \( \left( \frac{dQ}{dt} \right)_1 \) represents the yield at the top of layer (1), the fraction of films ruptured in layer (1) is \( p \, dz \); the fraction of those not ruptured is, hence, \( (1 - p \, dz) \). (The probability of rupture can be considered as proportional to the thickness \( dz \) of the layer if it is of very small thickness and if the probability in question is correspondingly small.) It is the non-ruptured films which assure the circulation of water in slice (2), where the yield would then be, in first approximation,

\[
\left( \frac{dQ}{dt} \right)_2 \approx \left( \frac{dQ}{dt} \right)_1 (1 - p \, dz) \tag{22}
\]

Thus, the water losses in slice (1) will be

\[
u_1 \, dz = \left( \frac{dQ}{dt} \right)_1 - \left( \frac{dQ}{dt} \right)_2 \approx \left( \frac{dQ}{dt} \right)_1 p \, dz \tag{23}
\]

The water losses in slice (2) will accordingly be

\[
u_2 \, dz = \left( \frac{dQ}{dt} \right)_2 p \, dz \tag{24}
\]

which can be written

\[
u_2 \, dz = \left[ \left( \frac{dQ}{dt} \right)_1 - \nu_1 \, dz \right] p \, dz \tag{25}
\]

that is,

\[
u_2 = \nu_1 - p \, \nu_1 \, dz \tag{26}
\]

from which, finally,

\[
\frac{du}{u} = -p \, dz \tag{27}
\]

or

\[
u = \nu_0 \exp(-pz) \tag{28}
\]

Eq. 28 corresponds well with the experimental relationship (Eq. 12) if one identifies the probability of rupture \( p \) with the modulus \( a \) of the exponential expression. Figure 12 shows how \( a \) (that is, the probability of rupture \( p \)) varies with the structure and with the yield. It is logical to find that \( p \) is larger in the case of a non-coherent soil than in that of a coherent soil, and also that \( p \) tends toward zero as \( \frac{dQ}{dt} \) becomes smaller. On the other hand, one could expect that the probability of rupture \( p \) depended also on the moisture content \( H \). No evidence of such an influence of the moisture content could be found.

Relationship Between Yield and Rate of Drying at a Given Level

Every element of the curve \( \Gamma \), which gives the variation of \( u \) with depth \( z \) (Fig. 7),
can, therefore, be expressed at any depth by an element of the exponential curve (Eq. 12) as long as the yield at the respective level is given by
\[
\frac{dQ}{dt} = z \int_{z}^{\infty} u \, dz
\]  
(29)

If the coefficient \(a\) were constant, then the curve \(\Gamma\) would coincide completely with the respective exponential function and the integral giving the yield would be found immediately as
\[
\frac{dQ}{dt} = u_0 \int_{z}^{\infty} \exp(-az) = \frac{u}{a}
\]  
(30)

Thus, the speed of desiccation, characterized by \(u\), would be proportional to the yield at the level under consideration; that is,
\[
u = a \frac{dQ}{dt}
\]  
(31)

(see Fig. 13; the straight line D corresponds to \(a = Cte = 0.4\))

If \(a\) varies with the yield, the integration of Eq. 30 can only be approximate. It leads to a relationship between \(u\) and \(\frac{dQ}{dt}\) which for the Versailles loam in a coherent state is shown by curve \(D'\) in Figure 13. In this case, the rate of desiccation is very low as long as the yield is less than about 1 mm per day; above this, it grows very rapidly.

This result explains the conditions of mulch formation (Fig. 4). The latter requires an extremely long time to form when the evaporation \(E\) is just a little less than 1 mm per day and the surface losses \(u\) remain very small. The rate of mulch formation is practically proportional to \((E - 1)\).

Besides, these results should permit explanation of the conditions of temporary wilting of plants: curve \(D'\) indicates the actual rate at which the soil dries out at the root level, where the yield by upward movement tends to compensate for the losses by transpiration.

**SUMMARY AND CONCLUSIONS**

Study of moisture profiles in the field and their evolution with time, as well as more refined laboratory experiments on the mode of drying of soil columns, show clearly the limits of the validity of the diffusion equation
\[
\frac{dQ}{dt} = A \frac{dH}{dz}
\]

where the yield is expressed as a function of the moisture content gradient. The latter equation implies a simple and unique relationship between the pressure of the water in the soil and the moisture content. Hence, it is inexact in three cases, as follows:

1. When one deals with a succession of earth strata of different textures.
2. When certain soil strata tend to increase their moisture content at the expense of other strata that are losing water. The relationship between the pressure and the humidity is then different for the two cases (phenomenon of hysteresis); the diffusion equation given becomes false; therefore an appreciable moisture content gradient \(\frac{dH}{dz}\) may correspond to a zero yield.
3. At the start of soil drying. Actually, the displacement of water corresponds to a movement of the entire system of moisture films which bring the cohesive forces of water into play. The real pressure of the water is now modified by the tension that is transmitted along the films and is different from the pressure \(P(H)\) which is found to be defined as a function of the moisture content in the absence of any liquid movement. It is for this reason that the water in the deeper layers can be displaced toward the surface even if it must pass through zones of greater moisture content. Such a result evidently invalidates the diffusion equation presented, according to which water moves only in the direction of lower moisture contents.
The evaporation losses \( E \) (mm per day) correspond to the summation over all depths of the moisture losses \( u \) of the soil (mm per day per layer of 1-cm thickness).

It has been verified that at every level the curve of variation of \( u \) as a function of the depth \( z \) can be expressed by the exponential curve

\[
u = u_0 \exp(-az)\]

but in this equation, the coefficient \( a \) varies within the limits 0 and 1, depending on the soil structure (coherent or non-coherent) and on the water yield at the level under consideration.

It has been possible to show that this coefficient is expressive of the probability of rupture \( p \) of the water films in a soil layer of given thickness.

The curve establishing the relationship between the modulus \( a \) (or the probability of rupture \( p \)) and the yield \( \frac{dQ}{dt} \) is a fundamental characteristic of the soil, its structure, and its texture.

Finally, the relationship between the rate of drying and the yield \( \frac{dQ}{dt} \) at the same level easily may be deduced from the probability-yield curve.

This drying-yield relationship, which also is characteristic of the soil and its structure, is ideally suited to interpret the conditions of surface mulch formation and the influence, in this respect, of cultivation methods. Finally, it explains why, under various conditions of evapotranspiration, the upward yield of water is quite constant (about 1 mm per day for the loam studied). The relationship \( (u, \frac{dQ}{dt}) \) shows that \( u \), which is very small for yields below 1 mm per day, increases rapidly at greater yields. The yield, therefore, cannot exceed this value without causing rapid desiccation of the soil either at the surface or at the root zone. The consequences of this are a checking of the actual evapotranspiration and a definite self-regulation of water loss in soils.

REFERENCES