A Rationale for Analysis of Pavement Performance

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A hypothetical road test quite similar to the AASHO Road Test is used to illustrate a rationale for determining an association of pavement performance with variables that describe pavement design and applied loads. The rationale used for the illustration is essentially that used in the analysis of performance data for the factorial experiments at the AASHO Road Test.

The performance of a pavement is defined by a curve that shows its serviceability trend as axle load applications are increased. A mathematical form or model is selected for the association, then procedures are defined for fitting the data to the model so that all constants in the model are evaluated. When pavement design is given, the evaluated model can be used either to predict serviceability after a given number of specified loads has been applied, or to predict the number of specified load applications required to produce a given serviceability loss.

Many details of the rationale are given as the example proceeds from hypothetical data to the performance equation. Limitations and alternatives for the rationale are discussed, a table of discrepancies between predicted and observed performance is given, and the illustrative analysis ends with curves that may be used in the practical application of the derived equation.

● THE FIRST OBJECTIVE for the AASHO Road Test is to find significant relationships between pavement performance and certain characteristics of pavement design and and applied loads. To carry out this objective detailed specifications are needed in three areas. First, pavement performance must be defined so that performance data can be obtained for every test section in the investigation. Second, there must be experimental designs that give details for pavement design and load characteristics of the sections. Finally, it is necessary to set out definite procedures that lead to the required relationships. Several papers and talks have described Road Test specifications in the first two areas, and it is the main purpose of this paper to discuss specifications in the third area. However, the three sets of specifications are interrelated in that analytical procedures are determined to a large extent by the nature of the experimental designs and by the nature of the performance data. For this reason pavement performance and experimental designs are discussed before turning to a rationale for analysis. Use is made of a numerical illustration that differs from the AASHO Road Test pavement performance studies in certain details but not in principle. As a consequence, rationale for the illustration is applicable to the Road Test, and unless specific reference is made to the illustration, the following discussion pertains to the Road Test.

It is evident that there are alternatives for virtually every specification that may be given in any of the three areas; thus there are many possibilities for the total set of specifications. Because it may be supposed that a number of these possibilities are equally acceptable for meeting the first objective of the Road Test, it cannot be claimed that the rationale to be described represents the best, nor the only way to satisfy the objective, but it is assumed that any other acceptable rationale would produce essentially the same conclusions.

PAVEMENT PERFORMANCE DATA

Inasmuch as a rationale for analysis is rather meaningless unless the data that go into the anlayis are well defined, it is necessary to pin down the specific nature of performance data. The concepts and specifications to be described in this area have evolved after consideration of many alternatives.

It is supposed that the present serviceability history of a pavement section plays a very useful role in performance evaluation. At any particular time the section's present serviceability is a measure of its ability to serve high-speed, high-volume traffic, and in a previous paper (1) a system for the development of present serviceability index formulas was described. Separate formulas were presented for flexible pavements and for rigid pavements. When appropriate measurements of surface deformation and deterioration are made on day, t, substitution of the measurements into the index formula gives an index value, p_t', for the index day. The complete serviceability history of a pavement section consists of index values for a series of index days that begins when the section is first constructed and ends when serviceability loss is such that major maintenance or replacement is required. In both the illustration and the AASHO Road Test, serviceability index values are obtained for every section on bi-weekly index days, and the serviceability history of a section is considered to be completed if and when its index falls to 1.5 on a scale where maximum serviceability is 5.0. Although not all bi-weekly index values are plotted, Figure 1, which shows the serviceability histories of two sections used in the illustration, indicates a completed history for section 3212 after about 17 index days. As in the case of the AASHO Road Test, it is supposed that the illustrative road test is stopped after 55 index days with the expectation that at least some sections will still have high serviceability at the end of the test. One such section is shown in Figure 1, where section 3222 has a serviceability index of about 3.2 after 55 index days.

The general continuous pattern of a serviceability history is called a smoothed serviceability history. Smoothed histories for the two sections in Figure 1 are indicated by the solid lines. The smoothed history for a section is defined by a moving average that includes at least three (generally five) successive index values and that uses the end values for the history as end values for the smoothed history. Smoothed serviceability history values on index days are denoted by p_t.

A second element of performance for a pavement section is its history of load applications. Although theories (2) and procedures exist for dealing with mixtures of axle loads, reference in this paper to any particular number of applications implies that each application represents the same axle weight. For the illustration, Figure 2 gives both the number of axle load applications between successive index days and the accumulated number of applications for any index day. The respective notation for these two quantities is n_t and N_t. If more than one traffic lane is represented by n_t and N_t, it is assumed that lane-to-lane variation in n_t is negligible and n_t is averaged for all lanes before the accumulation, N_t. Whenever it is necessary to evaluate accumulated applications between index days, linear interpolation is performed between successive values of N_t.

Before specifications are given for performance data, one more history is discussed—a history that is associated with the general state of environmental conditions at any particular time. This history is called a seasonal weighting function. Relative to a specified norm, or base, it may be supposed that the conditions at any time or location are either normal, better than normal, or worse than normal. It is considered that the seasonal weighting function reflects serviceability loss potential, and that any particular section may or may not lose serviceability during a period when the weighting function is high. No specific formula for a weighting function is given in this paper, but it is supposed that such a formula has been evolved to give values, v_t, for every index period as shown in Figure 3. This function presumably depends in general on changes in moisture-temperature states, and has the value $v_t = 1.0$ for normal conditions. A value of zero is considered to be a lower bound at which no serviceability-loss potential exists for any pavement-load combination.

The seasonal weighting function shown in Figure 3 averages to be about 1.0 so that environmental conditions for the two years average to be normal even though there is much seasonal variation. Relative to the selected location, this index might not average to be 1.0 at a second location, whether or not the same seasonal variation occurred at the two locations.

For any index period, let the product of the weighting function value with axle load application be w_t, the number of weighted applications for the period. Thus $w_t = v_t n_t$ can be obtained by multiplication of index day ordinates from Figures 2 and 3. Also let W_t be the accumulation of weighted axle load applications through any index day. Graphs for both w_t and W_t are shown in Figure 4. If the weighting function were taken to be 1.0 on every index day, the curve of Figure 3 would be horizontal at unit height, and Figure 4 would be identical with Figure 2. Thus, N_t is a special case of W_t if v_t is always 1.0. In all the discussion that follows accumulated axle load applications are

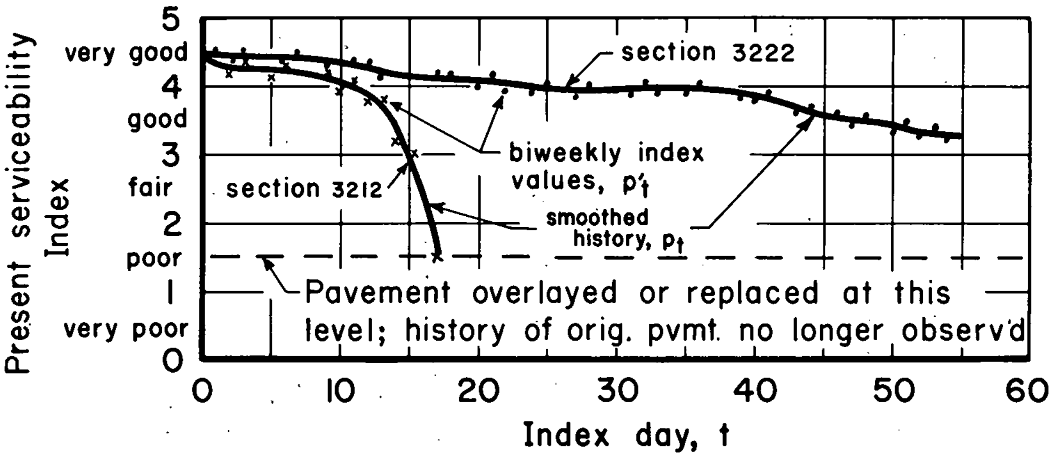

Figure 1. Present serviceability histories for two illustrative pavement sections.

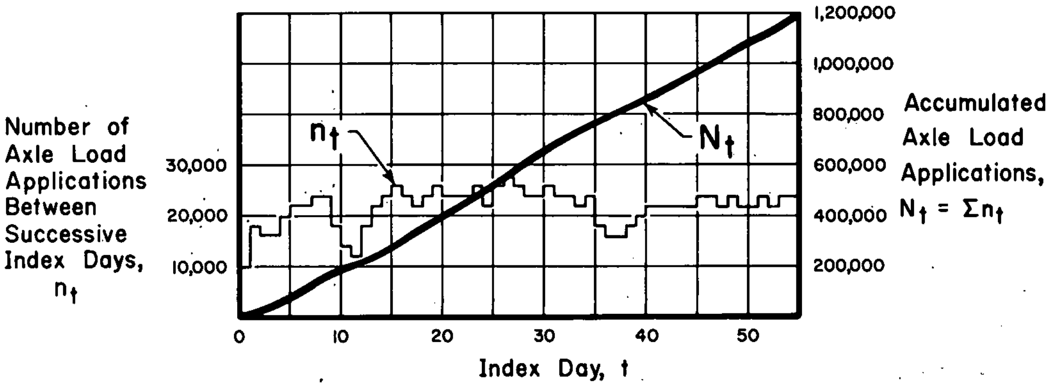

Figure 2. Axle load application history for the illustration.

represented by W, but it should be remembered that any difference between W and N depends on the values prescribed for v_t.

All of the variables just described have values that are observed and computed at points in time. If smoothed serviceability values for a pavement section are plotted against accumulated axle applications rather than against time, the resultant curve is called the section's serviceability trend. Coordinates of points on the serviceability trend are denoted by p and W, and the trend of p with W is defined to be the pavement's performance.

Trend plots for the two sections of Figure 1 are shown in Figure 5 for the case when applications are not weighted; that is, when $v_t = 1$. Coordinates for the trend curves in Figure 5 were obtained from ordinates of Figures 1 and 2 on common index days. Similarly, Figure 6 shows trend curves for the same sections when the seasonal weighting function of Figure 3 is used to obtain W. That is, coordinates for Figure 6 were obtained from ordinates of Figures 1 and 4 on common index days.

Summarizing the definitions of the various serviceability-time-applications relationships:

Serviceability history is the plot of observed values of serviceability, p_t', on a time scale;

Smoothed serviceability history is the plot of the five-point moving average of the serviceability history values on a time scale and smoothed history values are designated by p_t;

Serviceability trend is the plot of smoothed serviceability history values, p, on an

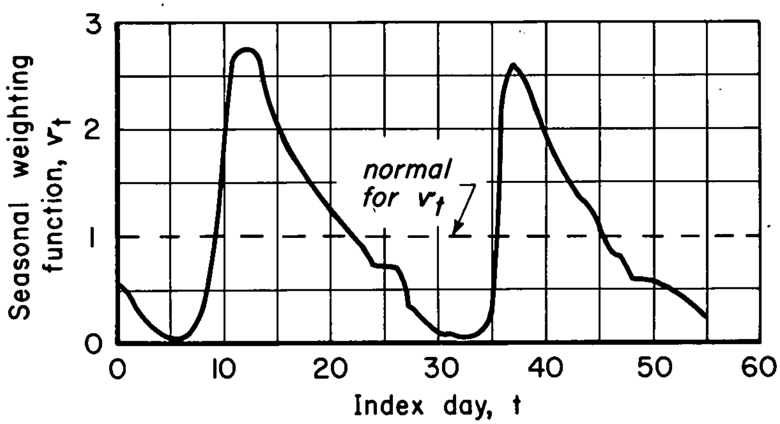

Figure 3. Seasonal weighting function for the illustration.

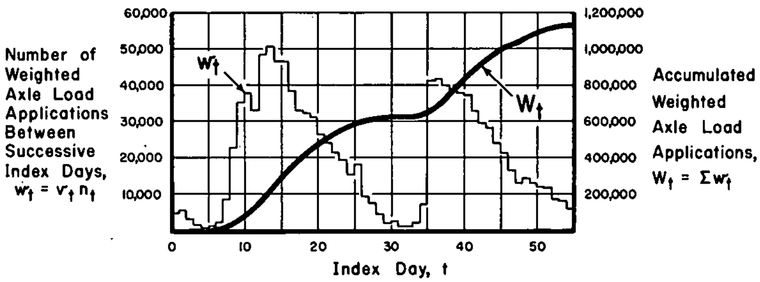

Figure 4. Weighted axle load applications for the illustration (seasonal weighting function from Fig. 3).

accumulated axle application scale, W, where axle applications may be weighted or unweighted; and the

Performance of a pavement is given by its serviceability trend.

The final step in the specification of performance data is to suppose that for numerical analysis a small number of pairs of coordinates from any trend curve can be selected to represent the curve satisfactorily. In the Road Test rationale five pairs of coordinates were selected from every trend. If the trend was complete (that is, p had fallen to 1.5), the trend was represented by five values that spanned the range of p. Specifically, W was noted when p was 3.5, 3.0, 2.5, 2.0, and 1.5. In the case of incomplete serviceability trends (p at the end of the Road Test was greater than 1.5) the observations were spanned by noting pairs of W and p at specific times (at 11, 22, 33, 44 and 55 index days). In both cases it is more convenient to record and use all W values in logarithmic form so that recorded performance data appear in the form p, log W. Thus if p = 2.5 when W = 200,000 applications, the recorded performance data would be 2.5 and 5.30 for p and low W, respectively.

In the example only three pairs of coordinates are used to represent serviceability trends. For the complete curves W is noted when p = 3.5, 2.5 and 1.5, and for incomplete trends W and p are noted at 15, 35 and 55 index days. For the two sections shown in Figure 1, Table 1 gives performance data as just defined, using both weighted and unweighted applications.

EXPERIMENTAL DESIGNS FOR PAVEMENTS AND LOADS

As details are given in the second area of specification for the illustration, the reader who is familiar with experimental designs at the AASHO Road Test will recognize that the illustration parallels in principle the main factorial experiments of the Road Test.

Suppose that the illustrative road test involves three rigid pavement tangents, 1, 2, and 3, each having two 12-ft traffic lanes, 1 and 2, on either side of its center line. Axle load specifications for the six traffic lanes will be: tangent 1, 4-kip single in lane 1, 8-kip single in lane 2; tangent 2, 16-kip single in lane 1, 30-kip tandem in lane 2; tangent 3, 24-kip single in lane 1, and 36-kip tandem axle vehicles in lane 2. Figure 2 shows the illustrative specifications for frequency of axle load applications over a two-year period.

Suppose the objective for the illustration implies that differences in pavement design for test sections are determined by only two factors, thickness of portland cement

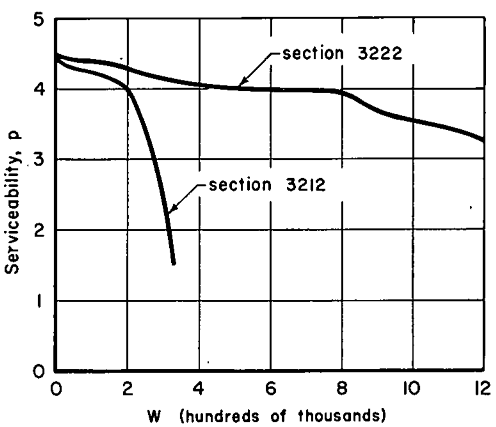

Figure 5. Performance curves for the two illustrative pavement sections of Figure 1 ($v_t = 1$).

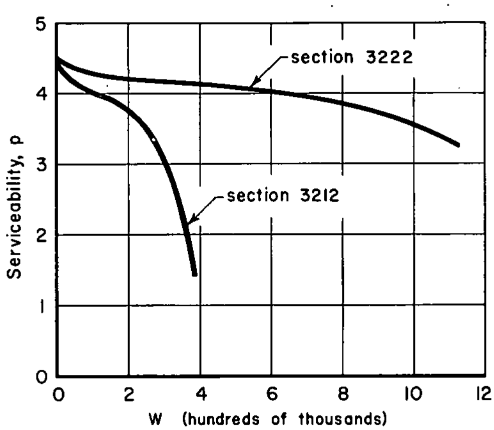

Figure 6. Performance curves for the two illustrative pavement sections of Figure 1 (seasonal weighting function from Fig. 3).

TABLE 1

PERFORMANCE DATA FOR TWO ILLUSTRATIVE SECTIONS OF FIGURE 1

Section No.	Section History	t[a]	Performance Data for $v_t = 1$: p	Performance Data for $v_t = 1$: log W	Performance Data for v_t from Figure 3: p	Performance Data for v_t from Figure 3: log W
3212	Com-	13.5	3.5	5.39	3.5	5.36
	plete	16.0	2.5	5.49	2.5	5.54
		17.0	1.5	5.52	1.5	5.59
3222	Incom-	15	4.2	5.45	4.2	5.48
	plete	35	4.0	5.89	4.0	5.81
		55	3.2	6.08	3.2	6.05

[a]Index day at which smoothed serviceability history equals p.

concrete surfacing and thickness of a granular subbase material. All other specifications for basement soil, pavement materials, and construction procedures are supposed to be identical for every test section.

Three fundamental principles of experimental design are balance, replication, randomization, and these principles are to be used in the design of the illustrative road test. The principle of balance is used to rule out undesired confusion among the effects of experimental factors on performance. By the effect of a factor is meant to change in performance that can be attributed to a change in the factor (for example, surface thickness effect is a change in performance that is clearly attributable to a change in surface thickness). It will be supposed that balance should be maintained in each test tangent for surface thickness and subbase thickness, so that the analysis can determine whether performance differences are due to one or the other of these factors or possibly to their interacting effect. In the absence of prior knowledge about their interacting effect, a sound experimental design for surface and subbase thickness is the complete factorial experiment that includes all possible combinations of levels selected for these two factors. In each tangent let each factor have three levels (that is, three values). Then the complete factorial experiment in each tangent requires 3 x 3, or nine, different pavement designs. As indicated in Table 2, levels for subbase thickness will be 3, 6, and 9 in. in each tangent, but levels for surface thickness are selected so that thicker pavements are used for heavier axle loads, there being one common surface thickness, 5.5 in., across all three tangents. Thus, although balance is maintained for surface and subbase thickness in each tangent and loads are balanced with subbase thickness across all tangents, load and surface thickness levels are unbalanced so that uninteresting surface thickness-load combinations will not occur. However, the load effect can be observed across the 5.5-in. surface thickness, and if there is no interaction between load and surface thickness effects, the load effect at 5.5-in. surface thickness could serve as the general effect of axle load on performance.

Replication (that is, repetition) of observations for controlled factor combinations provides a way to find how much the observations are influenced by residual variables that are uncontrolled. Replication can be performed in many categories. For example, the illustrative road test might be repeated in toto at a different location, or at a different time, or both. At a selected location and time, any tangent might be completely replicated by including a fourth tangent that has the same specifications as one of the tangents in Table 2. An axle load might be replicated in both lanes of the same tangent, or serviceability index values might be replicated for any index day. If there is sufficient replication in any category where conclusions are to be drawn about the effects of controlled factors within the category, it becomes possible to discern between per-

TABLE 2

LEVELS FOR THE EXPERIMENTAL FACTORS IN THE ILLUSTRATION

Tangent	Lane	Load[a] (kips)	Factorial Combinations: Slab Thickness (in.)	Factorial Combinations: Subbase Thickness (in.)	Factorial Combinations: Number of Test Sections	Replicated Combinations: Slab Thickness (in.)	Replicated Combinations: Subbase Thickness (in.)	Replicated Combinations: Number of Test Sections	Total Number of Test Sections
1	1	4S	2.5	3		2.5	6		
			4.0	6	9	4.0	6	2	11
			5.5	9					
	2	8S	2.5	3		2.5	6		
			4.0	6	9	4.0	6	2	11
			5.5	9					
2	1	16S	4.0	3		4.0	6		
			5.5	6	9	5.5	6	2	11
			7.0	9					
	2	30T	4.0	3		4.0	6		
			5.5	6	9	5.5	6	2	11
			7.0	9					
3	1	24S	5.5	3		5.5	6		
			7.0	6	9	7.0	6	2	11
			8.5	9					
	2	36T	5.5	3		5.5	6		
			7.0	6	9	7.0	6	2	11
			8.5	9					
Total					54			12	66

[a]S = single-axle load; T = tandem-axle load.

formance changes that can be attributed to controlled effects and those changes that must be attributed solely to uncontrolled or residual effects. For the latter effects, replication provides estimates needed to assess the reliability of controlled effects.

In the illustration, as in the Road Test, replication is provided only for certain pavement designs within each tangent. Table 2 indicates that two different pavement designs are to be once replicated within each tangent; thus there are to be eleven test sections in each of the six traffic lanes, or 66 test sections in all. More replication might be required if the illustration were an actual road test, as the number of replicates should be sufficient to obtain reliable estimates of residual variation (within tangents).

The third principle, randomization, is closely associated with the principles of balance and replication. As was stated, balance is necessary to prevent confusion among controlled factor effects, but it is also important that there be no confusion between controlled effects and residual effects on performance. If, for example, the sections in each tangent were constructed so that surface thickness increased from thin to thick along the tangent, and if an uncontrolled construction variable that could affect pavement performance, say humidity, also increased as the tangent was paved from one end to the other, any conclusion about surface thickness effect would be confused to an unknown degree with effects attributable to humidity during paving. It is well known that systematic uncontrolled variables operate during almost any experimental investigation, so randomization is necessary in order to minimize the risk that residual effects will be mistaken for controlled effects. As in any sampling situation, randomization is also necessary for obtaining proper estimates of residual variation. For example, if each replicate were constructed adjacent to its companion section, it might be expected that an underestimate of residual variation in the tangent would be obtained.

In the example, the eleven sections in each tangent are assigned a random order of occurrence within the tangent. As a result it can be expected that conclusions about surface and subbase effects are not biased or confused by the presence of systematic residual variation within any tangent.

The major performance studies in the AASHO Road Test have experimental designs that involve balance, replication, and randomization, in much the same way as described for the illustration. In addition, still other experimental designs appear in the Road Test in order to provide for special studies whose objectives are somewhat different from the first Road Test objective.

Specifications have now been given for pavement performance data and for experimental designs within which the performance data are obtained. It is rather obvious that many alternatives were available for nearly every specification. Nevertheless, the net result of the selected specifications for the example is a set of performance data as given in Table 3. The performance data consist of three pairs of p and log W values for each of the 66 test sections when the weighting function of Figure 3 is used. Table 3 includes data previously given in Table 2 for sections 3212 and 3222. In this paper section numbers are codes for factor levels. Section 3212 appears in tangent 3, lane 2, at the first surface thickness level and the second subbase thickness level. Similarly, section 3222 is in the second lane of the third tangent and has the second level of thickness for both surface and subbase.

Any section whose serviceability history was complete has p values of 3.5, 2.5, and 1.5 in Table 3. All remaining sections had incomplete histories.

For the AASHO Road Test, tables that correspond to Table 3 cover five tangents and involve five pairs of p and log W values for each of 284 flexible pavement sections and 264 rigid pavement sections.

PROCEDURES FOR ANALYSIS

The analysis consists of procedures that produce an empirical formula wherein performance is associated with load and pavement design variables. In order to use mathematical procedures it is necessary to assume some algebraic form, or model, for the association. In addition to the experimental variables the model involves constants whose values are either to be specified or to be estimated from the data. Thus, the analytical procedures are for the estimation of constants whose values are unspecified in the model—constants that indicate the effects of design and load variables on performance. The procedures also include methods for estimating the precision with which the data fit the assumed model.

In essence the model is an equation for serviceability trends as illustrated by Figure 5 or Figure 6. When pavement design and axle load are specified for a pavement section, the equation is supposed to predict the section's serviceability after a given number of applications. The equation should also be useful for predicting the number of applications the section will experience before reaching a pre-assigned serviceability level.

There are many different mathematical forms that could be used as models for serviceability trends, and a large number of these may fit the data with more or less the same precision. Only one of the numerous models investigated at the Road Test is used for the illustration in this paper. Further discussion of models is given after the illustration is completed.

Let p_0 denote the initial serviceability trend values for a particular test section. Then $p_0 - p$ is the serviceability loss experienced by the section when its trend value is at p. It will be assumed that p_0 does not depend on pavement design variables in the range of interest, and that the best estimate for p_0 is c_0, the average of all initial trend values for sections considered in the analysis. For the example, c_0 will be 4.5.

The assumption to be used for the nature of serviceability trends is that serviceability loss is a power function of axle load applications; that is,

$$c_0 - p = KW^{\beta} \qquad (1)$$

where β is a positive power and K and/or β may depend on load and design variables. Let $p = c_1$ be a serviceability level such that whenever p for a section falls to c_1, the section is "out of test" and no longer observed. Thus, the number of applications experienced by the section when $p = c_1$ may be called the experimental life of a section. For the example, as at the Road Test, $c_1 = 1.5$. Let ρ be the

TABLE 3

PERFORMANCE DATA (ILLUSTRATION), WEIGHTED APPLICATIONS

Tangent 1

Load[a] (kips)	Subbase Thickness (in.)	2.5-In. Slab p	2.5-In. Slab log W	4.0-In. Slab p	4.0-In. Slab log W	5.5-In. Slab p	5.5-In. Slab log W
4S	3	4.4	5.48	4.5	5.48	4.5	5.48
		4.1	5.81	4.4	5.81	4.4	5.81
		3.9	6.05	4.3	6.05	4.4	6.05
	6	4.5	5.48	4.5	5.48	4.5	5.48
		4.3	5.81	4.6	5.81	4.5	5.81
		4.0	6.05	4.2	6.05	4.5	6.05
	6[b]	4.6	5.48	4.5	5.48	-	-
		4.4	5.81	4.5	5.81	-	-
		4.1	6.05	4.4	6.05	-	-
	9	4.4	5.48	4.4	5.48	4.6	5.48
		4.4	5.81	4.4	5.81	4.4	5.81
		3.8	6.05	4.3	6.05	4.4	6.05
8S	3	3.5	5.37	4.5	5.48	4.5	5.48
		2.5	5.41	4.2	5.81	4.3	5.81
		1.5	5.41	3.9	6.05	4.4	6.05
	6	3.5	5.18	4.4	5.48	4.5	5.48
		2.5	5.41	4.4	5.81	4.5	4.81
		1.5	5.44	4.0	6.05	4.4	6.05
	6[b]	3.5	5.17	4.6	5.48	-	-
		2.5	5.28	4.3	5.81	-	-
		1.5	5.36	3.8	6.05	-	-
	9	3.5	5.35	4.5	5.48	4.4	5.48
		2.5	5.55	4.3	5.81	4.4	5.81
		1.5	5.60	3.9	6.05	4.3	6.05

Tangent 2

Load[a] (kips)	Subbase Thickness (in.)	4.0-In. Slab p	4.0-In. Slab log W	5.5-In. Slab p	5.5-In. Slab log W	7.0-In. Slab p	7.0-In. Slab log W
16S	3	3.5	5.20	4.5	5.48	4.5	5.48
		2.5	5.28	4.0	5.81	4.4	5.81
		1.5	5.30	1.8	6.05	4.0	6.05
	6	3.5	5.30	4.4	5.48	4.4	5.48
		2.5	5.31	4.3	5.81	4.3	5.81
		1.5	5.31	2.8	6.05	4.1	6.05
	6[b]	3.5	5.25	4.5	5.48	-	-
		2.5	5.35	4.1	5.81	-	-
		1.5	5.40	2.5	6.05	-	-
	9	3.5	5.28	3.5	5.75	4.5	5.48
		2.5	5.30	2.5	5.81	4.2	5.81
		1.5	5.31	1.5	5.86	4.2	6.05
30T	3	3.5	4.98	3.5	5.70	4.5	5.48
		2.5	5.00	2.5	5.80	4.2	5.81
		1.5	5.02	1.5	5.90	3.7	6.05
	6	3.5	5.02	3.5	5.78	4.5	5.48
		2.5	5.03	2.5	5.80	4.3	5.81
		1.5	5.04	1.5	5.82	3.9	6.05
	6[b]	3.5	5.08	3.9	5.48	-	-
		2.5	5.11	2.8	5.81	-	-
		1.5	5.15	1.7	6.05	-	-
	9	3.5	4.95	3.5	5.72	4.5	5.48
		2.5	5.01	2.5	5.80	4.3	5.81
		1.5	5.03	1.5	5.85	3.9	6.05

Tangent 3

Load[a] (kips)	Subbase Thickness (in.)	5.5-In. Slab p	5.5-In. Slab log W	7.0-In. Slab p	7.0-In. Slab log W	8.5-In. Slab p	8.5-In. Slab log W
24S	3	3.5	5.28	3.5	5.82	4.5	5.48
		2.5	5.31	2.5	5.93	4.3	5.81
		1.5	5.33	1.5	6.00	3.9	6.05
	6	3.5	5.30	3.5	5.70	4.4	5.48
		2.5	5.33	2.5	5.85	4.2	5.81
		1.5	5.33	1.5	6.03	3.8	6.05
	6[b]	3.5	5.20	3.7	5.48	-	-
		2.5	5.30	2.9	5.81	-	-
		1.5	5.38	1.8	6.05	-	-
	9	3.5	5.29	3.5	5.90	4.5	5.48
		2.5	5.31	2.5	5.95	4.4	5.81
		1.5	5.33	1.5	6.04	4.1	6.05
36T	3	3.5	5.47	3.5	6.00	4.6	5.48
		2.5	5.48	2.5	6.02	4.3	5.81
		1.5	5.49	1.5	6.03	4.1	6.05
	6	3.5	5.36	4.2	5.48	4.7	5.48
		2.5	5.54	4.0	5.81	4.4	5.81
		1.5	5.59	3.2	6.05	4.3	6.05
	6[b]	3.5	5.50	3.7	5.48	-	-
		2.5	5.55	3.5	5.81	-	-
		1.5	5.58	2.8	6.05	-	-
	9	3.5	5.44	4.6	5.48	4.5	5.48
		2.5	5.46	4.4	5.81	4.3	5.81
		1.5	5.47	1.9	6.05	3.9	6.05

[a]S = single-axle load; T = tandem-axle load.
[b]Replicate.
Note: Section used in Figure 1 and others.

value of W in Eq. 1 when $p = c_1$. Then $c_0 - c_1 = K\rho^{\beta}$, or $K = (c_0 - c_1)/\rho^{\beta}$, and Eq. 1 may be written in either of the forms.

$$c_0 - p = (c_0 - c_1) \left(\frac{W}{\rho}\right)^{\beta} \quad (2)$$

or

$$p = c_0 - (c_0 - c_1) \left(\frac{W}{\rho}\right)^{\beta}$$

where $c_1 \leqq p \leqq c_0$. (3)

For any particular section, β and ρ have fixed values, but it will be assumed that if β is not constant for all designs and loads, β decreases whenever ρ increases from one section to another. If $B > 1$, eq. 3 indicates that the serviceability trend will decline along a steeper and steeper curve as applications increase. If $\beta = 1$, the serviceability loss is linear with applications, and if $\beta < 1$, serviceability decreases along a curve that is concave upwards. Curves of Eqs. 2 or 3 are shown in Figure 7 for three different combinations of β and ρ. When $\beta = 2.0$ the trend is the right half of a parabola that opens downward, when $\beta = 1$ the trend is linear, and when $\beta = 0.5$ the trend is the lower half of a parabola that opens to the right.

In the first stage of the analysis, performance data for each section are used to obtain preliminary estimates of β and ρ for the section. If logarithms are taken on both sides of Eq. 2,

$$\log (c_0 - p) = \log (c_0 - c_1) + \beta (\log W - \log \rho) \quad (4)$$

or

$$\log\left[\frac{c_0 - p}{c_0 - c_1}\right] = \beta\left[\log W - \log \rho\right]$$

Calling the left side of Eq. 5 gives

$$G = \log\left[\frac{c_0 - p}{c_0 - c_1}\right] \quad (6)$$

where G is undefined unless $p < c_0$. G has a negative value whenever p is between c_0 and c_1, and G = O when $p = c_1$. Substituting Eq. 6 in Eq. 5 gives

$$G = \beta \left[\log W - \log \rho\right] \quad (7)$$

In G, log W coordinates, the graph of Eq. 7 is a straight line whose slope is β and whose intercept on the log W axis is $\log \rho$. Figure 8 shows curves of Eq. 7 for the β, ρ, combinations of Figure 7. Thus the curves in Figure 8 are linearizations of the performance curves shown in Figure 7. To show the connection between G and p, both scales are shown in Figure 8.

For each section, pairs of values for p and W are converted to corresponding values for G and log W, then a straight line is fitted to the G, log W points. Figure 9 shows transformed data and fitted lines for four sections whose data were given in Table 3. Sections 3212 and 3222 are the previously used illustrative sections, whereas sections 1133 and 1233 are included in Figure 9 in order to bring out certain rules used in the rationale. The fitted lines for sections 3212 and 3222 have slopes $\hat{\beta} = 1.97$ and $\hat{\beta} = 1.09$, respectively, and have log W intercepts $\log \hat{\rho} = 5.61$ and $\log \hat{\rho} = 6.44$. These estimates are determined by lines that minimize the sum of squared vertical deviations from the data for each section.

Section 1133 represents a case where not only is there no G value corresponding to the one p value that exceeds c_0 = 4.5 (Table 3), but the remaining two values also are the same, p = 4.4. Thus for this section $\hat{\beta}$ = 0 and log $\hat{\rho}$ is infinite. Section 1233 has p values 4.4, 4.4, and 4.3, so that little decrease in serviceability has been observed during the experiment. It is supposed that very little information about β and log ρ is given by data and graphs for sections whose serviceability loss is hardly outside the realm of measurement error. For such sections special rules are applied in order to obtain values of $\hat{\beta}$ and log $\hat{\rho}$. After examining all $\hat{\beta}$ values for sections that experienced an appreciable serviceability loss, a minimum value is assumed for β, and if the data for any section give $\hat{\beta}$ less than the assumed minimum, the minimum β is assigned as the section's $\hat{\beta}$. For the example, minimum β is taken to be 1.0, and both sections 1133 and 1233 are given the value $\hat{\beta}$ = 1.0. After this assignment, log $\hat{\rho}$ is obtained by fitting a line whose slope is 1.0 to the observed points. Using this rule, log $\hat{\rho}$ is 7.41 and 7.16, respectively, for sections 1133 and 1233, as indicated in Figure 9. If all p values for a section are equal to or greater than c_0 = 4.5, as for section 1132 in Table 3, not only is $\hat{\beta}$ assigned to be 1.0, but also log $\hat{\rho}$ is set at the median log $\hat{\rho}$ for all sections that differ only in subbase thickness from the section that has no G data. Table 4 gives $\hat{\beta}$ and log $\hat{\rho}$ values.

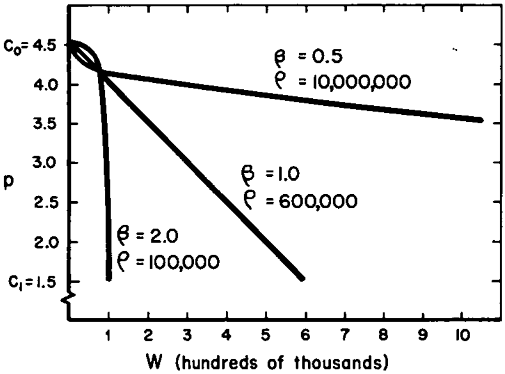

Figure 7. Graphs of $p = 4.5-3.0\left(\frac{W}{\rho}\right)^{\beta}$.

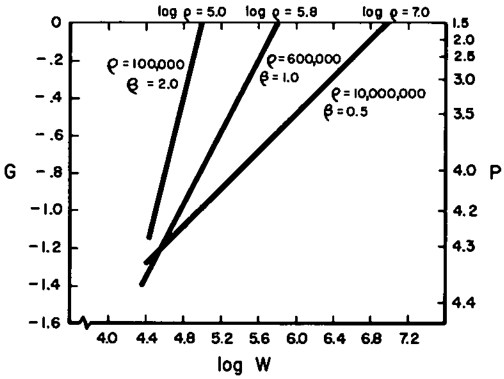

Figure 8. $G = \log\left(\frac{4.5-p}{3.0}\right) = \beta\left[\log W - \log \rho\right].$

After $\hat{\beta}$ has been determined for each section, the $\hat{\beta}$ values are plotted against pavement design and load variables, and an analysis of variance is made in order to infer the nature and extent of any dependence of $\hat{\beta}$ on design and load variables. Neither the plots nor the analysis of variance are shown here, but both proceed from the assumption that β, is related to design and load variables according to the model

$$\beta = \beta_0 + \frac{B_0 (L_1 + L_2)^{B_2}}{(b_1D_1 + b_2D_2 + b_3)^{B_1} L_2^{B_3}} \tag{8}$$

in which β_0 is a minimum value for β;
L_1 is the nominal load axle weight in kips (that is, load values as given in Table 3); L_2 is one for single-axle vehicles, two for tandem-axle vehicles; D_1 is the first pavement design factor, slab thickness, in inches; and D_2 is the second pavement design factor, subbase thickness, in inches.

The remaining symbols on the right side of Eq. 8 are positive constants whose values are either to be assumed (as is done for β_0) or to be estimated from the $\hat{\beta}$ values.

In general Eq. 8 implies that β increases as axle load increases and that β decreases as pavement design increases for a fixed loading. If there were three pavement design factors, as at the Road Test, the third factor, D_3, would have been introduced in the combination $b_1D_1 + b_2D_2 + b_3D_3 + b_4$. The constant term in the design combination (b_3 in Eq. 8) appears so that β is not necessarily infinite when there is no thickness for D_1 and D_2, and L_2 has been added to L_1 so that β does not necessarily approach β_0 as L_1 approaches zero.

For the example, graphs and variance analysis for $\hat{\beta}$ show little or no dependence of $\hat{\beta}$ on subbase thickness, so b_2 is taken to be zero. With only one variable, D_1, in the design combination, the effect of D_1 can be relegated to the power B_1 by assigning values to b_1 and b_3. For the illustration, let $b_1 = b_3 = 1.0$. Then, as β_0 has already been assigned to be 1.0, Eq. 8 is reduced to

$$\beta = 1.0 + \frac{B_O\,(L_1 + L_2)^{B_2}}{(D_1 + 1)^{B_1}\, L_2^{B_3}} \qquad (9)$$

in which only B_O, B_1, B_2 and B_3 remain to be estimated from the $\hat{\beta}$ data. Taking the logarithm of Eq. 9 gives

$$\log(\beta - 1.0) = \log B_O + B_2 \log(L_1 + L_2) - B_3 \log L_2 - B_1 \log(D_1 + 1) \qquad (10)$$

For each lane, Eq. 10 represents a straight line when $\log(\hat{\beta} - 1.0)$ is plotted against $\log(D_1 + 1)$, and linear regressions of $\log(\hat{\beta} - 1.0)$ on $\log(D_1 + 1)$ give lane estimates for the slopes, B_1. Omitting lane 1 of tangent 1, because the majority of $\hat{\beta}$ values in this lane were 1.0 by assignment rather than values obtained from performance data, the regression slopes are averaged to give $\hat{B}_1$, the final estimate for B_1. The average slope for the remaining lanes is $\hat{B}_1 = 5.90$. Transposition of Eg. 10 gives

$$\log(\beta - 1.0) + B_1 \log(D_1 + 1) = \log B_O + B_2 \log(L_1 + L_2) - B_3 \log L_2 \qquad (11)$$

where the left side of Eq. 11 is now estimated by $\log(\hat{\beta} - 1.0) + \hat{B}_1 \log(D_1 + 1)$ for every section. For any lane, average

TABLE 4

ESTIMATES OF β AND LOG ρ

Tangent 1

Load[a] (kips)	Slab Thickness, D_1 (in.)	Subbase Thickness (in.)	$\log(D_1 + 1)$	$\hat{\beta}$	$\tilde{\beta}$	$\log \hat{\rho}$	$\log \hat{\hat{\rho}}$	$\log \tilde{\rho}$
4S	2.5	3	0.54	1.39	1.20	6.51	6.63	6.60
		6	0.54	1.66	1.20	6.52	6.74	6.60
		6[b]	0.54	2.51	1.20	6.40	6.91	6.60
		9	0.54	1.39	1.20	6.64	6.78	6.60
	4.0	3	0.70	1.25	1.02	6.99	7.22	7.65
		6	0.70	1.00	1.02	7.05	7.03	7.65
		6[b]	0.70	1.00	1.02	7.53	7.49	7.65
		9	0.70	1.00	1.02	7.16	7.12	7.65
	5.5	3	0.81	1.00	1.00	7.41	7.40	8.42
		6	0.81	1.00	1.00	7.41[c]	7.40[c]	8.42
		9	0.81	1.00	1.00	7.41	7.40	8.42
8S	2.5	3	0.54	9.73	3.88	5.42	5.45	5.48
		6	0.54	1.64	3.88	5.48	5.40	5.48
		6[b]	0.54	2.52	3.88	5.36	5.33	5.48
		9	0.54	1.79	3.88	5.62	5.56	5.48
	4.0	3	0.70	1.25	1.35	6.61	6.56	6.53
		6	0.70	1.15	1.35	6.86	6.70	6.53
		6[b]	0.70	2.27	1.35	6.33	6.60	6.53
		9	0.70	1.99	1.35	6.40	6.62	6.53
	5.5	3	0.81	1.00	1.08	7.26	7.16	7.30
		6	0.81	1.00	1.08	7.53	7.42	7.30
		9	0.81	1.00	1.08	7.16	7.06	7.30

Tangent 2

Load[a] (kips)	Slab Thickness, D_1 (in.)	Subbase Thickness (in.)	$\log(D_1 + 1)$	$\hat{\beta}$	$\tilde{\beta}$	$\log \hat{\rho}$	$\log \hat{\hat{\rho}}$	$\log \tilde{\rho}$
16S	4.0	3	0.70	4.48	7.30	5.31	5.29	5.31
		6	0.70	38.83	7.30	5.31	5.34	5.31
		6[b]	0.70	3.16	7.30	5.40	5.36	5.31
		9	0.70	15.78	7.30	5.31	5.33	5.31
	5.5	3	0.81	3.05	2.34	6.08	6.11	6.09
		6	0.81	2.08	2.34	6.24	6.19	6.09
		6[b]	0.81	2.91	2.34	6.11	6.16	6.09
		9	0.81	4.38	2.34	5.86	5.90	6.09
	7.0	3	0.90	2.91	1.39	6.32	6.74	6.70
		6	0.90	1.05	1.39	6.90	6.62	6.70
		9	0.90	1.00	1.39	6.93	6.65	6.70
30T	4.0	3	0.70	11.93	13.73	5.02	5.02	5.06
		6	0.70	23.85	13.73	5.04	5.05	5.06
		6[b]	0.70	6.68	13.73	5.15	5.13	5.06
		9	0.70	5.74	13.73	5.04	5.01	5.06
	5.5	3	0.81	2.39	3.71	5.89	5.86	5.83
		6	0.81	11.94	3.71	5.82	5.86	5.83
		6[b]	0.81	1.19	3.71	6.05	5.87	5.83
		9	0.81	3.68	3.71	5.85	5.65	5.83
	7.0	3	0.90	1.78	1.80	6.37	6.37	6.44
		6	0.90	1.99	1.80	6.40	6.45	6.44
		9	0.90	1.99	1.80	6.40	6.45	6.44

Tangent 3

Load (kips)	Slab Thickness, D_1 (in.)	Subbase Thickness (in.)	$\log(D_1 + 1)$	$\hat{\beta}$	$\tilde{\beta}$	$\log \hat{\rho}$	$\log \hat{\hat{\rho}}$	$\log \tilde{\rho}$
24S	5.5	3	0.81	9.58	8.72	5.33	5.33	5.35
		6	0.81	9.58	8.72	5.35	5.35	5.35
		6[b]	0.81	2.66	8.72	5.36	5.32	5.35
		9	0.81	11.92	8.72	5.33	5.34	5.35
	7.0	3	0.90	2.66	3.27	6.00	5.98	5.96
		6	0.90	1.43	3.27	6.01	5.93	5.96
		6[b]	0.90	1.00	3.27	6.08	5.87	5.96
		9	0.90	3.23	3.27	6.03	6.03	5.96
	8.5	3	0.98	1.99	1.82	6.40	6.44	6.47
		6	0.98	1.48	1.82	6.48	6.35	6.47
		9	0.98	2.51	1.82	6.40	6.58	6.47
36T	5.5	3	0.81	23.93	6.91	5.49	5.51	5.50
		6	0.81	1.97	6.91	5.61	5.53	5.50
		6[b]	0.81	5.97	6.91	5.58	5.58	5.50
		9	0.81	15.79	6.91	5.47	5.49	5.50
	7.0	3	0.90	15.83	2.74	6.03	6.10	6.12
		6	0.90	1.09	2.74	6.44	6.04	6.12
		6[b]	0.90	1.00	2.74	6.21	5.94	6.12
		9	0.90	5.90	2.74	6.06	6.21	6.12
	8.5	3	0.98	1.25	1.63	6.75	6.56	6.62
		6	0.98	1.25	1.63	6.99	6.74	6.62
		9	0.98	1.99	1.63	6.40	6.50	6.62

[a] S = single-axle load; T = tandem-axle load.

[b] Replicate section.

[c] Use median value.

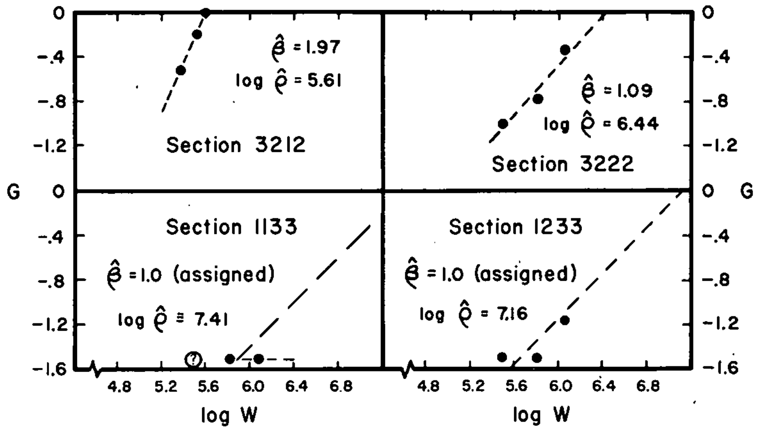

Figure 9. Illustrative estimates for β and log ρ from section data.

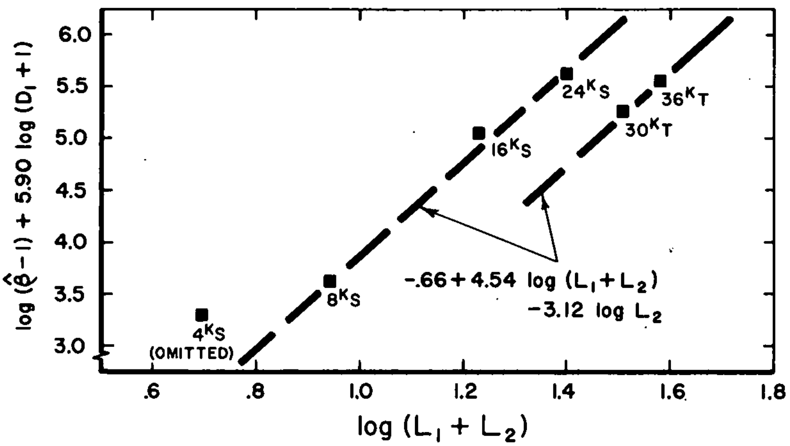

Figure 10. Adjusted mean log ($\hat{\beta}$-1) vs log (L_1+L_2).

log ($\hat{\beta}$ - 1.0) + average $\hat{B}_1$ log(D_1 + 1) is called an adjusted lane mean, and according to Eq. 11 the adjusted lane means depend linearly on log (L_1 + L_2) and log L_2. Figure 10 shows the six adjusted lane means for the example, and includes lines that are obtained from a linear regression analysis. The common slope of single- and tandem-axle lines is an estimate, $\hat{B}_2$, for B_2. The intercept of the single-axle line on the adjusted means axis is an estimate, log $\hat{B}_0$, for log B_0, and the difference between intercepts of the single-axle and tandem-axle lines produces an estimate, $\hat{B}_3$, for B_3. For the illustrative data, log $\hat{B}_0$ = 0.66 or $\hat{B}_0$ = 0.22, $\hat{B}_2$ = 4.54, and $\hat{B}_3$ = 3.12. Substitution of these values in Eq. 9 gives a new estimation formula for β,

$$\tilde{\beta} = 1.0 + \frac{0.22\ (L_1 + L_2)^{4.54}}{(D_1 + 1)^{5.90}\ L_2^{3.12}} \tag{12}$$

Values for $\tilde{\beta}$ from Eq. 12 are given in Table 4.

The second phase of the analytical procedures begins by using $\tilde{\beta}$ values to obtain new estimates for log ρ from the data for each section. The first estimates for log ρ were denoted by log $\hat{\rho}$ and were obtained as log W intercepts (Fig. 9) for lines whose slopes were $\hat{\beta}$. Using the same rules as for obtaining log $\hat{\rho}$, the new estimates, log

$\hat{\tilde{\rho}}$, are obtained as log W intercepts for lines whose slopes are $\tilde{\beta}$. Table 4 gives values for log $\hat{\tilde{\rho}}$ for each section in the illustration. In essence, the rationale supposes that estimates for β from Eq. 12 are better than estimates based only on individual section performance data, and that as a consequence, log $\hat{\tilde{\rho}}$ values represent better estimates for log ρ than do the log $\hat{\rho}$ values.

As was done with the $\hat{\beta}$ values, log $\hat{\tilde{\rho}}$ values are plotted against the design and load variables, and an analysis of variance is made to infer how and with what significance the log $\hat{\tilde{\rho}}$ values depend on design and load variables. The algebraic form for the association of log ρ with design and load variables is assumed to be

$$\rho = \frac{A_O\,(a_1D_1 + a_2D_2 + a_3)^{A_1}\,L_2^{A_3}}{(L_1 + L_2)^{A_2}} \tag{13}$$

in which A_O, A_1, A_2, A_3, a_1, a_2, and a_3 are positive constants. Eq. 13 implies that ρ increases with pavement design and decreases with axle load. The constant a_3 is included so that ρ is not necessarily zero in the absence of surface and subbase, and L_2 is added to L_1 in the denominator so that ρ is not necessarily infinite when L_1 is zero.

For the illustrative data, even casual inspection of Table 4 indicates little or no association between log $\hat{\tilde{\rho}}$ and subbase thickness, so a_2 is taken to be zero. As was done for b_1 and b_3 in Eq. 8, both a_1 and a_3 are set at 1.0. In logarithmic form, Eq. 13 thus becomes

$$\log \rho = \log A_O - A_2 \log (L_1 + L_2) + A_3 \log L_2 + A_1 \log (D_1 + 1) \tag{14}$$

Figure 11 shows how log $\hat{\tilde{\rho}}$ values vary with corresponding values for log $(D_1 + 1)$ for two lanes. Linear regressions of log $\hat{\tilde{\rho}}$ on log $(D_1 + 1)$ in each lane produce slopes that are estimates of A_1, and when the slopes are averaged for all but lane 1 of tangent 1, the estimate obtained for A_1 is $\hat{A}_1 = 6.79$.

Transposing A_1 log $(D_1 + 1)$ in Eq. 14 adjusts log ρ for surface thickness, and for each lane, average log $\hat{\tilde{\rho}}$ - average $\hat{A}_1$ log $(D_1 + 1)$ is an adjusted lane mean that should depend on log $(L_1 + L_2)$ and log L_2. Figure 12 shows adjusted lane means versus log $(L_1 + L_2)$ for single and tandem axles, and shows the lines obtained from the regression analysis. As indicated in Figure 12, log $\hat{A}_O = 5.98$, $\hat{A}_2 = 4.40$, and $\hat{A}_3 = 3.17$. Thus the procedures have produced a final estimation equation for log ρ,

$$\log \tilde{\rho} = 5.98 - 4.40 \log (L_1 + L_2) + 3.17 \log L_2 + 6.79 \log (D_1 + 1) \tag{15a}$$

or

$$\tilde{\rho} = \frac{10^{5.98}(D_1 + 1)^{6.79}\,L_2^{3.17}}{(L_1 + L_2)^{4.40}} \tag{15b}$$

Estimates for log ρ given by Eq. 15 are given in Table 4.

The results of the analysis can now be summarized. If it is desired to estimate p when W is given, Eqs. 3, 12 and 15 combine to give

$$\hat{p} = 4.5 - 3.0\left(\frac{W}{\tilde{\rho}}\right)^{\tilde{\beta}} \tag{16}$$

If it is desired to estimate log W when p is given, Eqs. 5, 12 and 15 combine to give

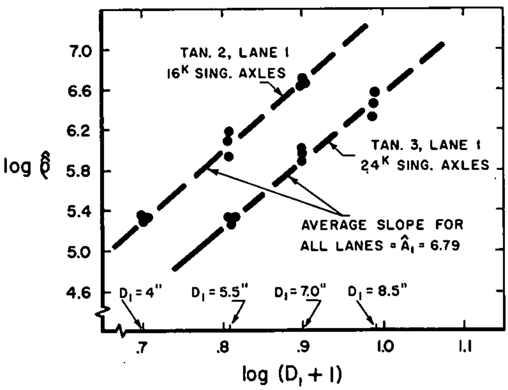

Figure 11. Log $\hat{\tilde{\rho}}$ vs log (D_1+1) for two lanes.

$$\log \hat{W} = \log \tilde{\rho} + \frac{\log \left[\frac{4.5 - p}{3.0}\right]}{\tilde{\beta}} \tag{17}$$

Eqs. 16 and 17 thus represent the first goal of the analysis—to associate the performance data with design and load variables.

The remainder of the rationale for analysis is concerned with summarizing the precision with which estimates can be made using Eq. 16 and 17 to estimate p and log W, respectively.

Values in parentheses in Table 5 are either serviceability estimates obtained from Eq. 16 for all sections whose serviceability did not fall to $c_1 = 1.5$, or are log W estimates obtained from Eq. 17 for all sections whose serviceability did reach 1.5. Estimates are given at 15, 35 and 55 index days in the first case and at p = 3.5, 2.5, and 1.5 for the second case. Differences between corresponding estimated values (Table 5) and observed values (Table 1) represent residuals that are not accounted for by Eqs. 16 and 17. Table 6 gives a summary of mean residuals and mean absolute residuals for both types of estimates, classified both by lane and by index day or serviceability level.

For all sections that were "in test" after 55 index days, the upper half of Table 6 gives the number of such sections in each lane, and the average residual $\hat{p} - p$, both algebraic and absolute. For the 31 sections involved, the average algebraic residual is shown to be 0.02, and the average absolute residual is 0.10. There does not appear to be any trend with respect to load, but in nearly all lanes the residuals increase with time or applications. However, even the largest mean residuals are only two or three tenths and it may be concluded that the p estimates are quite close to their respective observations.

In the lower half of Table 6, residuals in log W are summarized from differences obtained by subtracting log W values in Table 3 from corresponding log $\hat{W}$ values in Table 5 for all sections whose serviceability fell to p = 1.5. The mean algebraic residual in log W is nearly zero, while the mean absolute residual is about 0.04. These residuals appear to have about the same magnitude at each serviceability level and are not far from being equal in each lane. No log W residuals are shown for lane 1 of tangent 1 because no section was "out of test" in this lane.

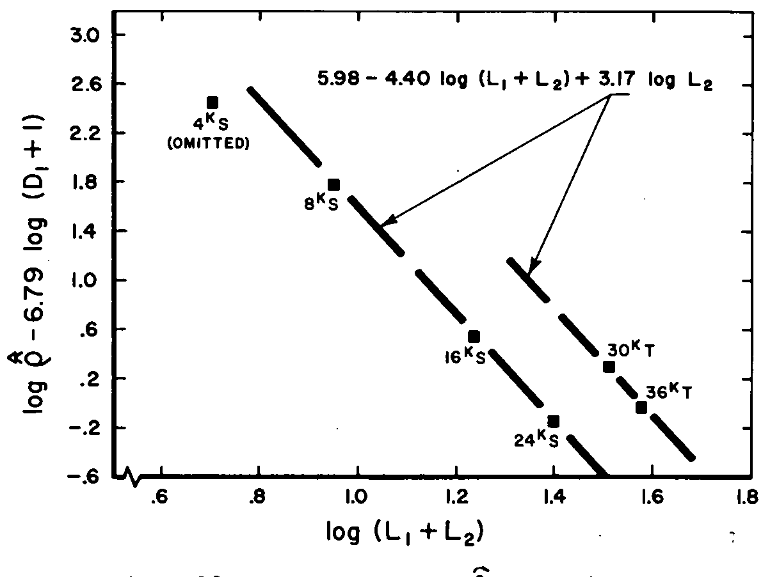

Figure 12. Adjusted mean $\log \hat{\rho}$ vs $\log (L_1+L_2)$.

A particular log W residual can be converted to the ratio of estimated to observed applications. For example, if $\log \hat{W} - \log W = 0.10$, $\frac{\hat{W}}{W} = 10^{0.10}$, or $\hat{W} = 1.26W$. If the log W residual had been -0.10, then $\hat{W} = 0.79W$. Through such conversions, average Log W residuals may be used to obtain an indication of the agreement between predicted applications and observed applications. For the example, the average absolute residual represents about a 10 percent deviation from the observed value. In the Road Test performance analyses, the corresponding average residuals are about 0.2 for log W estimates.

For both the present example and for the Road Test data, residuals in log W predictions compare favorably with differences in log W between replicate sections. From Table 3, the average absolute difference in log W between replicate sections is about 0.09, so that the average deviation of two replicate sections from their own mean is about 0.045. Thus, the predicted log W from Eq. 17 deviates about the same distance from log W as does log W from the mean of log W values observed for the same design and load. For this reason it is unlikely that any appreciable decrease in log W residuals can be made by fitting the illustrative observations with another model or by using another set of procedures.

Both Eq. 16 and Eq. 17 are rather complex and difficult to use in the form given. However, graphs of these equations can be made for whatever conditions may be useful. Figure 13 shows the curves of Eq. 17 for every combination of surface thickness and loading used in the illustration—all for the case that p = 2.5. For a particular load, the plotted curves shows the number of applications expected for any surface thickness at the time when the serviceability level has dropped to 2.5. Applications beyond 10^6 represent extrapolations for all curves, and each load curve has been extended beyond the lower and upper surface thickness level used in the experiment. Corresponding curves, of course, could be plotted for other values of p.

To show how close the observations are to the curves of Figure 13, Figure 14 repeats three of the curves from

TABLE 5

ESTIMATED PERFORMANCE DATA[a] (ILLUSTRATION), WEIGHTED APPLICATIONS

Tangent 1

Load[b] (kips)	Subbase Thickness (in.)	2.5-In. Slab: p or (p̂)	2.5-In. Slab: (log Ŵ) or log W	4.0-In. Slab: p or (p̂)	4.0-In. Slab: (log Ŵ) or log W	5.5-In. Slab: p or (p̂)	5.5-In. Slab: (log Ŵ) or log W
4S	3	(4.5)	5.48	(4.5)	5.48	(4.5)	5.48
		(4.2)	5.81	(4.5)	5.81	(4.5)	5.81
		(3.8)	6.05	(4.4)	6.05	(4.5)	6.05
	6	(4.5)	5.48	(4.5)	5.48	(4.5)	5.48
		(4.2)	5.81	(4.5)	5.81	(4.5)	5.81
		(3.8)	6.05	(4.4)	6.05	(4.5)	6.05
	6[c]	(4.5)	5.48	(4.5)	5.48	-	-
		(4.2)	5.81	(4.5)	5.81	-	-
		(3.8)	6.05	(4.4)	6.05	-	-
	9	(4.5)	5.48	(4.5)	5.48	(4.5)	5.48
		(4.2)	5.81	(4.5)	5.81	(4.5)	5.81
		(3.8)	6.05	(4.4)	6.05	(4.5)	6.05
8S	3	3.5	(5.35)	(4.5)	5.48	(4.5)	5.48
		2.5	(5.43)	(4.3)	5.81	(4.4)	5.81
		1.5	(5.48)	(3.8)	6.05	(4.4)	6.05
	6	3.5	(5.35)	(4.5)	5.48	(4.5)	5.48
		2.5	(5.43)	(4.3)	5.81	(4.4)	5.81
		1.5	(5.48)	(3.8)	6.05	(4.4)	6.05
	6[c]	3.5	(5.35)	(4.5)	5.48	-	-
		2.5	(5.43)	(4.3)	5.81	-	-
		1.5	(5.48)	(3.8)	6.05	-	-
	9	3.5	(5.35)	(4.5)	5.48	(4.5)	5.48
		2.5	(5.43)	(4.3)	5.81	(4.4)	5.81
		1.5	(5.48)	(3.8)	6.05	(4.4)	6.05

Tangent 2

Load[a] (kips)	Subbase Thickness (in.)	4.0-In. Slab: p or (p̂)	4.0-In. Slab: (log Ŵ) or log W	5.5-In. Slab: p or (p̂)	5.5-In. Slab: (log Ŵ) or log W	7.0-In. Slab: p or (p̂)	7.0-In. Slab: (log Ŵ) or log W
16S	3	3.5	(5.25)	(4.5)	5.48	(4.5)	5.48
		2.5	(5.29)	(4.1)	5.81	(4.4)	5.81
		1.5	(5.31)	(2.1)	6.05	(4.1)	6.05
	6	3.5	(5.25)	(4.5)	5.48	(4.5)	5.48
		2.5	(5.29)	(4.1)	5.81	(4.4)	5.81
		1.5	(5.31)	(2.1)	6.05	(4.1)	6.05
	6[b]	3.5	(5.25)	(4.5)	5.48	-	-
		2.5	(5.29)	(4.1)	5.81	-	-
		1.5	(5.31)	(2.1)	6.05	-	-
	9	3.5	(5.25)	3.5	(5.88)	(4.5)	5.48
		2.5	(5.29)	2.5	(6.01)	(4.4)	5.81
		1.5	(5.31)	1.5	(6.09)	(4.1)	6.05
30T	3	3.5	(5.02)	3.5	(5.70)	(4.5)	5.48
		2.5	(5.05)	2.5	(5.78)	(4.4)	5.81
		1.5	(5.06)	1.5	(5.83)	(3.9)	6.05
	6	3.5	(5.02)	3.5	(5.70)	(4.5)	5.48
		2.5	(5.05)	2.5	(5.78)	(4.4)	5.81
		1.5	(5.06)	1.5	(5.83)	(3.9)	6.05
	6[b]	3.5	(5.02)	(4.5)	5.48	-	-
		2.5	(5.05)	(3.5)	5.81	-	-
		1.5	(5.06)	Neg.[d]	6.05	-	-
	9	3.5	(5.02)	3.5	(5.70)	(4.5)	5.48
		2.5	(5.05)	2.5	(5.78)	(4.4)	5.81
		1.5	(5.06)	1.5	(5.83)	(3.9)	6.05

Tangent 3

Load[a] (kips)	Subbase Thickness (in.)	5.5-In. Slab: p or (p̂)	5.5-In. Slab: (log Ŵ) or log W	7.0-In. Slab: p or (p̂)	7.0-In. Slab: (log Ŵ) or log W	8.5-In. Slab: p or (p̂)	8.5-In. Slab: (log Ŵ) or log W
24S	3	3.5	(5.29)	3.5	(5.82)	(4.5)	5.48
		2.5	(5.33)	2.5	(5.91)	(4.4)	5.81
		1.5	(5.35)	1.5	(5.96)	(4.0)	6.05
	6	3.5	(5.29)	3.5	(5.82)	(4.5)	5.48
		2.5	(5.33)	2.5	(5.91)	(4.4)	5.81
		1.5	(5.35)	1.5	(5.96)	(4.0)	6.05
	6[b]	3.5	(5.29)	(4.5)	5.48	-	-
		2.5	(5.33)	(4.1)	5.81	-	-
		1.5	(5.35)	Neg.[d]	6.05	-	-
	9	3.5	(5.29)	3.5	(5.82)	(4.5)	5.48
		2.5	(5.33)	2.5	(5.91)	(4.4)	5.81
		1.5	(5.35)	1.5	(5.96)	(4.0)	6.05
36T	3	3.5	(5.43)	3.5	(5.94)	(4.5)	5.48
		2.5	(5.48)	2.5	(6.05)	(4.4)	5.81
		1.5	(5.50)	1.5	(6.12)	(4.2)	6.05
	6	3.5	(5.43)	(4.5)	5.48	(4.5)	5.48
		2.5	(5.48)	(4.3)	5.81	(4.4)	5.81
		1.5	(5.50)	(2.6)	6.05	(4.2)	6.05
	6[b]	3.5	(5.43)	(4.5)	5.48	-	-
		2.5	(5.48)	(4.3)	5.81	-	-
		1.5	(5.50)	(2.6)	6.05	-	-
	9	3.5	(5.43)	(4.5)	5.48	(4.5)	5.48
		2.5	(5.48)	(4.3)	5.81	(4.4)	5.81
		1.5	(5.50)	(2.6)	6.05	(4.2)	6.05

[a] Numbers in parentheses for "out of test" sections are estimates of log W when p is given. Numbers in parentheses for "in test" sections are estimates of p when log W is given.

[b] S = single-axle load; T = tandem-axle load.

TABLE 6

SUMMARY OF RESIDUALS

	Item		Lane 11	Lane 12	Lane 21	Lane 22	Lane 31	Lane 32	Average All Sections
		No. Sect.	9	6	5	3	3	5	
p residuals	Avg. $(\hat{p} - p)$	t = 15	0.00	0.01	0.02	-0.01	0.02	-0.03	0.00
		t = 35	0.01	0.01	0.03	0.09	0.08	0.07	0.04
		t = 55	0.06	-0.05	-0.08	0.09	0.05	0.03	0.01
		All	0.02	-0.01	-0.01	0.06	0.05	0.03	0.02
	Avg. $\lvert \hat{p} - p \rvert$	t = 15	0.04	0.04	0.04	0.01	0.04	0.15	0.05
		t = 35	0.09	0.08	0.12	0.09	0.09	0.12	0.10
		t = 55	0.10	0.08	0.25	0.09	0.13	0.35	0.16
		All	0.08	0.07	0.13	0.06	0.09	0.21	0.10
		No. Sect.	0	3	4	6	6	4	
log W residuals	Avg. $(\log \hat{W} - \log W)$	p = 3.5	-	0.05	0.02	0.00	0.01	-0.01	0.01
		p = 2.5	-	-0.03	0.04	0.01	0.00	0.00	0.01
		p = 1.5	-	-0.01	0.06	0.00	-0.03	0.01	0.01
		All	-	0.01	0.04	0.00	0.00	0.00	0.01
	Avg. $\lvert \log \hat{W} - \log W \rvert$	p = 3.5	-	0.06	0.07	0.04	0.04	0.04	0.05
		p = 2.5	-	0.05	0.06	0.02	0.03	0.03	0.04
		p = 1.5	-	0.08	0.06	0.03	0.04	0.05	0.05
		All	-	0.06	0.06	0.03	0.03	0.04	0.04

Figure 13 and includes appropriate data given as observations in Table 3. Data for only those sections whose p reached 2.5 can be shown in Figure 14. The dotted curves represent the band formed by $\pm$ 0.08; that is, plus or minus twice the mean absolute residual in log W. If residual deviations have a normal frequency distribution about the curves as shown in Figure 14, then plus or minus two mean absolute deviations should include about 90 percent of all individual residuals.

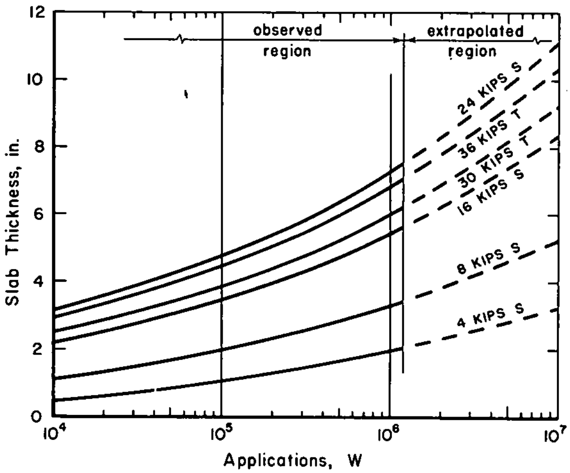

Figure 13. Performance equation (Eq. 17) for test loads (p = 2.5).

Another form of Figure 13 is given in Figure 15, which shows the associated surface thickness and load for selected applications expected when p = 2.5. Figure 15 serves to bring out the effect of load on surface thickness requirements.

When bands for residual variation are added to the graphs, either or both of Figures 13 and 15 constitute a summarization of the data given in Table 3, and presumably satisfy (for p = 2.5) the stated objective for the investigation.

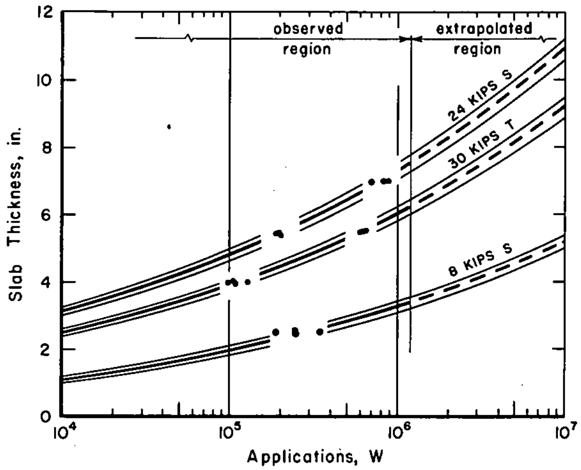

Figure 14. Performance equation (Eq. 17) for three selected loads (p = 2.5).

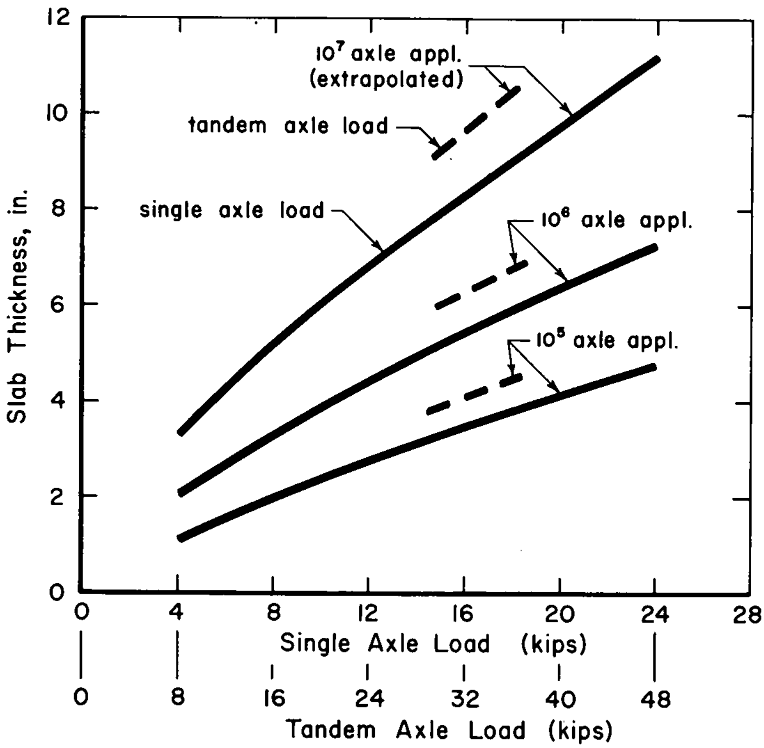

Figure 15. Performance equation (Eq. 17) for selected applications (p = 2.5).

SUMMARY AND DISCUSSION

The rationale just described defines both performance data and procedures for analyzing the data. The procedures included the selection of a general mathematical model for the association of experimental variables, the assignment of values to certain constants in the model, and rules for estimating values for all remaining constants in the model. The end result of the rationale was given by Eqs. 16 and 17, by a residual summary in Table 6, and by Figures 13, 14, and 15, which are meant to bring out the nature of the equations and their residuals.

If the experimental objectives are satisfied by curves such as in Figures 13 and 15, it is important to know how different the curves might be if the rationale were to be changed in one or more ways, say by the selection of a different model or by choosing different estimation rules.

Efforts to answer this question at the Road Test are represented by the investigation of many rationales, each of which produces a residual summary as illustrated by Table 6. A rationale is considered to produce an adequate fit to the data if mean algebraic residuals are near zero and if mean absolute residuals are of the same magnitude as deviations of replicate observations from their own mean. In terms of Figure 14, this criterion implies that the central curves should tend to pass through median observed points and that replicate observations should be scattered about the curves to the same degree as are unreplicated observations. If the first of these conditions is not met it may be supposed that the estimation rules have produced biased estimates for constants in the model, and if the second condition does not hold the model may not be appropriate. In either case the rationale can be modified until the criterion is met. It is noteworthy that existing theory for pavement performance gives little or no guidance in the area of model selection.

Changes in the rationale may also change the curves in the extrapolation regions of Figures 13, 14, and 15. Actual performance data for these regions would be necessary in order to appraise the validity of the extrapolations.