A Theory for Transforming AASHO Road Test Pavement Performance Equations to Equations Involving Mixed Traffic

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The general equations to be developed at the AASHO Road Test relating pavement performance, pavement design and axle load will describe the behavior of these pavements when subjected to repeated applications of an axle load of a given type and weight. The theory proposed herein can be used to derive from each such equation a corresponding equation which, by hypothesis, will predict the behavior of these pavements if subjected to mixed traffic of any given composition. The theory may be of use in the application of Road Test results to the design of highway pavements to carry a known distribution of axle loads.

I. General Description, Application to Pavement Design

MIXED VS SINGLE-LOAD TRAFFIC

INCLUDED in the official reports of the AASHO Road Test research will be two general pavement performance equations, one applying to flexible and the other to rigid pavements. Each equation will describe the performance of a pavement of a given design when subjected to repeated applications of an axle load of a given weight and type (single or tandem). Such an equation will be referred to hereafter as a "single-load" equation.

Neither of the two equations will directly yield information as to the behavior of pavements acted on by mixed traffic; that is, normal highway traffic composed of both single and tandem axles of a variety of weights.

Without question the usefulness of the forthcoming Road Test single-load equations will be enhanced if they can be transformed to multiload equations applicable to mixed traffic. A technique for accomplishing such a transformation is described herein so that interested highway engineers may, if they wish, apply the transformation procedure to the AASHO Road Test equations when they are published. In the interest of brevity, the procedure will be referred to hereafter as the "mixed-traffic theory."

DISCUSSION OF ASSUMPTIONS

Certain assumptions are being made in connection with the derivation of the AASHO Road Test single-load equations. They may be stated as follows:

1. At the moment a new pavement is opened to traffic, damage to the pavement begins to accumulate, and continues to accumulate throughout the life of the pavement.
2. Accumulated damage can be measured, and the measurement expressed as a single number. (Occasionally the letter g will be used hereafter to represent this number.)
3. A mathematical relationship exists between accumulated damage on the one hand and pavement design, traffic and environment on the other.

The mixed-traffic theory rests on the preceding assumptions as well as two additional assumptions, the first of which is:

4. Two or more pavements with the same value of g will react to a given axle load in the same manner, if the pavements are of the same design and exist in the same environment.

Before stating the fifth assumption, certain implications of the first four will be illustrated by applying them to the graphical solution of a problem involving mixed traffic of a particularly simple kind. The problem may be stated as follows:

Two test pavements, designated Pavements 1 and 2, respectively, exist in the same environment and are of the same design. Pavement 1 has been extensively tested under an axle load, X, and Pavement 2 under an axle load, Y. Measurements of accumulated damage, g, have been made on both pavements at frequent intervals, and a curve of accumulated damage versus axle applications has been plotted for each pavement, as indicated by the solid lines in Figure 1.

A new pavement, designated as Pavement 3, is to be subjected to 1,000,000 applications of load X, followed by 1,000,000 applications of load Y. Pavement 3 is of the same design as Pavements 1 and 2, and exists in the same environment. Two questions bearing on the performance of Pavement 3 are to be answered by the use of the assumptions; namely, (a) how may a curve of g versus axle applications be plotted for Pavement 3, short of actually measuring the accumulated damage, and (b) what is the accumulated damage of Pavement 3 after the 2,000,000 applications?

For the first 1,000,000 applications, the required curve tracks the curve for Pavement 1, ending at point A (see Figure 1). From point A it parallels segment UV of the curve for Pavement 2 for the next 1,000,000 applications (AU and BV being parallel to the applications axis), and terminates at point B. The required curve is OAB and the accumulated damage is the g coordinate of point B. A comparison with the assumptions previously stated will show that the procedure followed in arriving at these answers is consistent with the assumptions.

But if the loads X and Y had been applied to Pavement 3 in the reverse order, curve OA'B' of Figure 1 would have resulted, and the total accumulated damage would have been the g corrdinate of point B' instead of B as in the previous case. It can be seen at once that both the shape and the terminal point of the damage-application curve for Pavement 3 depend upon the order in which the two loads are applied.

If, on the other hand, the curves of Pavements 1 and 2 had been straight lines, as shown in Figure 2, the curve for Pavement 3 would have been the connected line segments, OAB, if load X had been applied first, or OA'B, if load Y had been applied

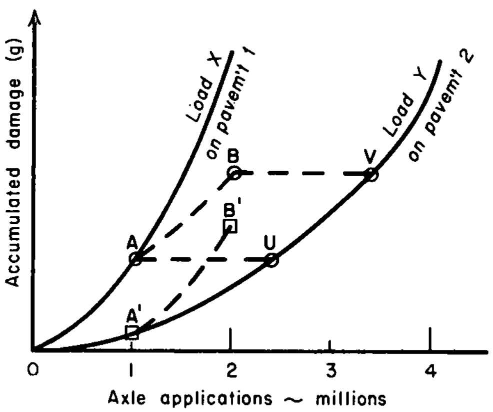

Figure 1. Accumulated damage as a curvilinear function of axle applications.

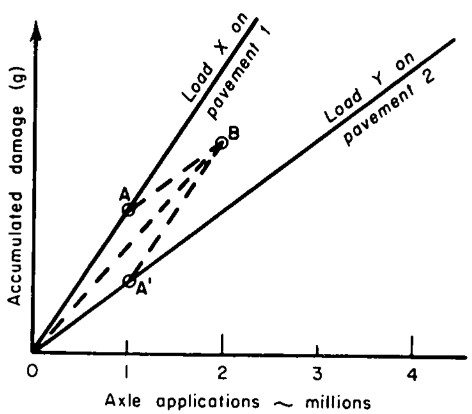

Figure 2. Accumulated damage as a linear function of axle applications.

first. For this case, then, the shape, but not the terminal point, of the damage-application curve depends upon the order in which the loads are applied.

One is forced to the intuitive conclusion that a complete and unique answer to a mixed-traffic problem cannot be obtained by a technique based on the first four assumptions alone; in addition, the order of application of the loads must be known or assumed. Rather than make such an assumption in solving practical problems, the issue is avoided by assuming, instead, that the traffic is well mixed. This, the fifth and last assumption may be stated as follows:

5. Whenever a pavement is subjected to more than one weight and/or type (single or tandem) of axle load, it is assumed that a representative sample of this traffic passes a fixed point in the pavement within a period of time so short, when compared to the life of the pavement, that this period may be treated as a differential quantity without excessive error, and the traffic is said to be "well mixed."

If the traffic applied to Pavement 3 had been well mixed, for the straight-line case illustrated in Figure 2 it can be shown that the curve of g versus applications for this pavement would have been the straight line OB. For a solution to the curved line case shown in Figure 1, it is necessary to turn to the calculus (see Part II).

All five assumptions may be summarized in a single equation which has proved to be useful in the development of the mixed-traffic theory. The equation is stated as:

The rate of change of accumulated damage, g, with respect to applications of an axle of a given weight and type = a mathematical function of pavement design, environment, axle weight, axle type, and g (1)

As will be shown later, any Road Test single-load equation can be reduced to the special differential form previously given. Given this differential relationship, development of the corresponding multiload equation is a matter of straight-forward mathematics (see Part II) which is deferred for the present, to pass, instead, directly to a discussion of the treatment of practical problems.

PRESENT SERVICEABILITY INDEX

A measure of pavement condition being extensively used at the AASHO Road Test is the present serviceability index, developed by the Road Test staff. This index has been described in detail elsewhere (1). For present purposes it is sufficient to report that its value depends primarily upon the relative smoothness of the pavement, and to a lesser extent upon the amount of visible damage — cracking, patching and rutting — present in the pavement. The index varies between a minimum value of zero for a very rough, badly damaged pavement to a maximum value of five for a perfectly smooth pavement with no visible damage.

The serviceability index, together with the design and load factors, will be the variables in the Road Test single-load equations now in the process of being derived. Some function of the serviceability index will be used as the measure of accumulated damages, g, required by the mixed-traffic theory, as illustrated in the following section.

A HYPOTHETICAL SINGLE-LOAD EQUATION

A hypothetical single-load equation applying to rigid pavement is given here, and its multiload version in the next section. These examples are given to illustrate, in one specific instance at least, the result of applying the mixed-traffic theory. The equations should not be construed as reflecting the results of the AASHO Road Test.

The single-load equation is

$$n = 10^{f} g \tag{2}$$

in which n is the number of applications of an axle load of a specified type (single or tandem) and specified weight required to reduce the serviceability index of a given pavement from an initial value, P_0, to the value P; the accumulated damage, g, is given by

$$g = \log_{10} (1 + P_0 - P) \tag{3}$$

and f represents a function of slab thickness and axle load, and has separate forms for single and tandem axles. For single axles,

$$f = 7.0 + 0.5D - 0.8\sqrt{L} \tag{4a}$$

and for tandem axles,

$$f = 7.0 + 0.5D - 0.7\sqrt{L} \tag{4b}$$

In Eqs. 4a and 4b, D is the slab thickness in inches and L is the axle load in kips.

CORRESPONDING MULTILOAD EQUATION

Before writing the multiload version of the foregoing single-load equation (Eq. 2), it is necessary to provide an appropriate mechanism for characterizing the traffic. Accordingly, the following rules applying to the counting of tandem axles and steering axles are observed.

Tandem axles: The two closely-spaced axles forming a tandem unit are counted as a unit, and a tandem axle load is understood to be the total load carried by the two axles making up the unit.

Steering axles: Steering axles are counted on passenger cars, pick-ups, panel-bodied trucks, buses and empty trucks, but not on loaded trucks.

Except for those steering axles excluded under the rules, all the axles applied daily to one lane of a highway are divided into a specified number of categories, with each category containing a specified number of axles all of the same type (single or tandem) and approximately of the same weight.

Table 1 is an abbreviated example of such a subdivision of traffic. Here each category is given a sequential number, i, and it is presumed that every axle in the category can be represented by a specified load, L_i. The symbol, C_i, represents the proportion of all axles occurring in the ith category. The total number of categories is represented by k and the number of single-axle categories by k'.

TABLE 1

EXAMPLE OF THE TRAFFIC PARAMETERS, L_i, C_i, k' and k

(Figures apply to one lane)

Axle Type	Category No., i	Load, L_i (kips)	No. Axles per Day	C_i
Single	1	6	300	0.30
	2	12	180	0.18
	3=k'	18	110	0.11
Tandem	4	24	230	0.23
	5=k	32	180	0.18
Total			1,000	1.00

With these definitions established and by application of the theory described in Part II, the multiload equation may be written as follows:

$$N = \frac{10^h g}{A + B} \tag{5}$$

in which g is defined by Eq. 3

$$h = 7.0 + 0.5D \tag{6}$$

$$A = \sum_{i=1}^{k'} C_i 10^{0.8\sqrt{L_i}} \tag{7}$$

$$B = \sum_{i=k'+1}^{k} C_i 10^{0.7\sqrt{L_i}} \tag{8}$$

and N represents the total number of applications of axles of all types and weights (excepting those steering axles excluded under the rules) required to reduce the present serviceability index from P_0 to P.

APPLICATION TO DESIGN OF PAVEMENTS

An illustration of the use of a multiload equation in pavement design is as follows:

Suppose that it is desired to compute, from Eq. 5, the thickness of concrete pavement required to carry the traffic given in Table 1 for a period of 20 years, the serviceability index meanwhile being permitted to drop from an initial value of 4.5 to a value of 2.5 (It being beyond the scope of this paper to discuss the effect of changes in environment or materials on pavement behavior, it is assumed that this pavement is to be constructed in the same environment and of the same materials as those existing at the Road Test.)

The computation is made as follows:

$$g = \log(1 + P_0 - P) = \log(1 + 4.5 - 2.5) = 0.47712$$

$$N = 20 \text{ years} \times 365 \text{ days} \times 1{,}000 = 7.3 \times 10^6 \text{ appl.}$$

$$A = 0.30 \times 10^{0.8\sqrt{6}} + 0.18 \times 10^{0.8\sqrt{12}} + 0.11 \times 10^{0.8\sqrt{18}} = 406$$

$$B = 0.23 \times 10^{0.7\sqrt{24}} + 0.18 \times 10^{0.7\sqrt{32}} = 2{,}259$$

$$7.3 \times 10^6 = \left(\frac{0.47712}{406 + 2259}\right) \times 10^h$$

Solving for h, and then for D (Eq. 6), one obtains D = 7.2 in.

LIMITATIONS ON APPLICATION

The general limitations on the application of AASHO Road Test single-load equations to pavement design have been discussed elsewhere (2). It need be added here only that these limitations apply also to the corresponding multiload equations. An additional limitation on the latter stems from the fact that they cannot be obtained directly from AASHO Road Test data, but arise instead from the indirect use of these data in the mathematical theory described in Part II. Thus predictions from the multiload equations can be checked directly only by comparing them with the observed behavior of highways subjected to mixed traffic of known composition.

Nevertheless, it is well known that practicing engineers find ways of overcoming the limitations both of theories and experiments and are able to turn the findings from both to practical advantage. Perhaps such will be the case in the present instance.

II. Mixed Traffic Theory

GENERAL MULTILOAD EQUATION

Consider a highway of design D, where D is some function of the pavement design parameters such as surface thickness, base thickness, etc. The design, materials and environment are similar to those at the AASHO Road Test. The highway is subjected to mixed traffic.

The daily traffic in a selected lane of the highway is characterized by the parameters k, k', L_i and C_i, defined in Part I under "Corresponding Multiload Equation." It is assumed that these parameters do not change with time; i.e., the traffic is well mixed.

Let N be the number of axles of all weights and types which have passed a fixed point in the lane from time $t = O$ (taken as the time the highway was first opened to traffic) to time $t = t_1$. Let P be the present serviceability index at time $t = t_1$.

Let $g(P)$ be any function, g, of P, continuous in the interval $O < P < P_o$, where P_o is the initial value of P (i.e., $P = P_o$ when $N = O$). As before, it is assumed that g is a measure of damage accumulated under traffic and therefore can be expressed as a function of the traffic passing over the pavement in the interval, $t = O$ to $t = t_1$, and of the pavement design D, as follows:

$$\begin{aligned} g &= \text{a function of the variables } N_1, N_2, \ldots, N_i, \ldots, N_k, \\ &\quad \text{and the constants } L_1, L_2, \ldots, L_i \ldots, L_k, \text{ and } D, \end{aligned} \tag{9}$$

where $N_1 = C_1N$, $N_2 = C_2N$, ..., $N_i = C_iN$, ..., $N_k = C_kN$, and N_i is the number of axles in in the ith category which have passed a fixed point in the lane from time $t = O$ to $t = t_1$.

In the interval $t = t_1$ to $t = t_1 + dt$, there is a change, dg, in g, given by

$$dg = \frac{\partial g}{\partial N_1} dN_1 + \frac{\partial g}{\partial N_2} dN_2 + \ldots + \frac{\partial g}{\partial N_1} dN_i + \ldots + \frac{\partial g}{\partial N_k} dN_k \tag{10}$$

Because $N_i = C_iN$, it follows that $dN_i = C_idN$. Substitution of C_idN for dN_i in the ith term of Eq. 10, gives

$$dg = \sum_1^k C_i \frac{\partial g}{\partial N_i} dN \tag{11}$$

According to the differential equation given in Part I (Eq. 1), each partial derivative, $\partial g/\partial N_i$, appearing in Eq. 11 can be expressed as a function of L_i, D and g. Let this function be Y_i. Then

$$\frac{\partial g}{\partial N_i} = Y_i = \text{a function of the constants } L_i \text{ and } D, \text{ and the variable } g \tag{12}$$

It follows from Eq. 12 that the variables g and N of Eq. 11 can be separated. Consequently,

$$N = \int \frac{dg}{\sum_1^k C_i Y_i} + \text{constant of integration} \tag{13}$$

in which each Y_i is to be derived from the appropriate Road Test single-load equation, and the constant of integration is to be found from the initial condition that $P = P_o$ when $N = O$.

DERIVATION OF THE RATE FUNCTION, Y_i

Either of the Road Test single-load equations now being derived can be expressed in the general form:

$$n_i = X(L_i, D, g) \tag{14}$$

in which $X(L_i, D, g)$ is a function, X, of L_i, D and g and n_i is the number of applications of the axle load, L_i, required to reduce the serviceability index of a pavement of design, D, from P_o to P. The function Y of L_i, D and g is defined as

$$Y(L_i, D, g) = \frac{\partial g}{\partial n_i} \tag{15}$$

in which L_i and D are regarded as constants. Then

$$Y(L_i, D, g) = \frac{1}{\frac{\partial X(L_i, D, g)}{\partial g}} \tag{16a}$$

or, in a briefer notation,

$$Y_i = 1 / \frac{\partial X_i}{\partial g} \tag{16b}$$

in which $Y_i = Y(L_i, D, g)$ and $X_i = X(L_i, D, g)$.

Thus the rate function, Y_i, appearing in the multiload equation (Eq. 13) may be readily obtained by differentiating the appropriate single-load equation with respect to g, and inverting the result, as indicated in Eq. 16b.

MULTILOAD EQUATION FOR SINGLE AND TANDEM AXLES

For a given pavement type (flexible or rigid) there will be two forms of the function X, say X' for single and X'' for tandem axles. Derived from these there will also be two forms of the function Y, say Y' for single and Y'' for tandem axles. In terms of Y' and Y'', Eq. 13 becomes

$$N = \int \frac{dg}{\sum_{1}^{k'} C_i Y'_i + \sum_{k'+1}^{k} C_i Y''_i} \tag{17}$$

in which the functions g, Y'_i and Y''_i are to be taken from the appropriate Road Test single-load equation.

AN APPLICATION

As an illustration the transformation procedure is applied to the hypothetical single-load equation for portland cement concrete pavement given in Part I and repeated here in a slightly different notation.

$$n_i = 10^{f_i} g \tag{18}$$

in which g is defined by Eq. 3 and n_i is the number of applications of an axle load, L_i, required to reduce the serviceability index of a pavement of thickness D from an initial value P_0 to the value P. For single axles,

$$f_i = 7.0 + 0.5D - 0.8\sqrt{L_i} \tag{19a}$$

and for tandem axles,

$$f_i = 7.0 + 0.5D - 0.7\sqrt{L_i} \tag{19b}$$

L_i is expressed in kips, and D in inches. According to Eq. 16b and 18,

$$Y_i = 10^{-f_i} \tag{20}$$

It follows from the definition of Y' and Y'', and from the two forms of the function f, that

$$Y'_i = 10^{-(7.0 + 0.5D - 0.8\sqrt{L_i})} \tag{21}$$

and

$$Y''_i = 10^{-(7.0 + 0.5D - 0.7\sqrt{L_i})}. \tag{22}$$

Noting that Y'_i and Y''_i are here independent of the variable g, one arrives at

$$N = \int \frac{dg}{\sum_{1}^{k'} C_i Y_i' + \sum_{k'+1}^{k} C_i Y_i''} = \frac{g}{\sum_{1}^{k'} C_i Y_i' + \sum_{k'+1}^{k} C_i Y_i''} \tag{23}$$

by performing the integration indicated by the general multiload equation for single and tandem axles.

That the constant of integration is zero follows from the condition that N and g are both zero when $P = P_O$; for, when $P = P_O$, N is zero by definition and $g = \log_{10}(1 + P_O - P_O) = 0$.

When the expressions for Y_i' and Y_i'' given in Eqs. 21 and 22 are substituted in Eq. 23, the result is the multiload equation (Eq. 5) already given in Part I.

The task of finding the multiload transform of Eq. 18 was made easier by the fact that n_i was linear in the damage function, g; thus, the integration of the general multiload equation offered no difficulty. On the other hand, it seems possible at the present writing that the final form of the Road Test single-load equations may not be linear in the damage function. In this event it may be necessary to employ some numerical technique in the integration of the general multiload equation.

REFERENCES

1. Carey, W.N., Jr., and Irick, P.E., "The Pavement Serviceability-Performance Concept." HRB Bull. 250 (1960).
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