Use of Road Test Equations in Pavement Cost Allocation

H. C. DUZAN, J. C. OEHMANN, and J. M. DAVIS
National Highway Planning Division, Bureau of Public Roads

Highway pavements are designed to accommodate the magnitude and frequency of expected loads with due consideration for environmental conditions. Thus, the most suitable method of determining pavement increments is to start with the design of the pavement that will actually be built for mixed traffic and, holding all factors except axle load constant, to ascertain by means of the modified Road Test equations the lower design requirements of successively lighter axles. To hold the total axle applications constant the heaviest group of axles is considered to be replaced by a like number of the next to the heaviest axles. This process of transferring axles to the next lower weight group is continued until finally the total number of axles in the traffic stream is considered to be in the lightest weight group.

Considerations other than the numbers and weights of axles in each group do not enter into the determination of the size of the pavement increment required and there is no reason to look upon the first increment as a pavement that might be built. The first increment is simply the portion of pavement structure that remains when the added structural requirements occasioned by the heaviest axles because of their weight, but not their number, are removed.

The procedure described in this paper evades the problem of modifying the terms of the Road Test equations to make them applicable to other soils and climates by equating the designs actually used for mixed traffic to the axle-load composition of the traffic for which they are built. The procedure can be used with any method of modifying the single-load equations to make them applicable to mixed traffic.

• The incremental method of allocating responsibility for highway costs to the vehicles which occasion them is based on the concept that each element of highway design and cost can be broken down into the increments occasioned by vehicles of different types, sizes, and weights. This concept, as it applies to the structural design of highway pavement, is illustrated in Figure 1. For simplicity, the number of increments is held to four and only one component of pavement is shown although all pavement courses (surface, base and subbase) must be taken into account.

The heaviest group of axles, here represented by the rear axle of the fully loaded truck at the right, is shown as requiring the total thickness of pavement; that is, all four increments of pavement thickness. The front axle of the fully loaded truck and the diminished rear-axle load produced when the truck is not quite full or is carrying a lighter commodity are accommodated by a thinner pavement made up of three increments. The difference in pavement thickness is the increment occasioned by the heaviest group of axles—those represented by the rear axle of the fully loaded truck. The successively lighter axle loads that result from further reductions in payload (or are produced by vehicles of a lighter type) are accommodated by progressively thinner pavements until, at the left, the general group of light vehicles, including passenger cars and small trucks of the pickup type, requires only the first increment of pavement—the upper layer (Fig. 1). This initial increment of pavement thickness is also shown as adequate for a larger vehicle when it is empty.

It is not significant that the back axle is shown as invariably requiring one more increment than the front axle (except for the empty truck); nor is it to be inferred that all empty trucks can be accommodated by the first increment and all fully loaded trucks require all increments. The intent is merely to illustrate that a vehicle or vehicle combination does not necessarily require the same number of increments for all axles and for all loadings.

Inasmuch as the first increment is required

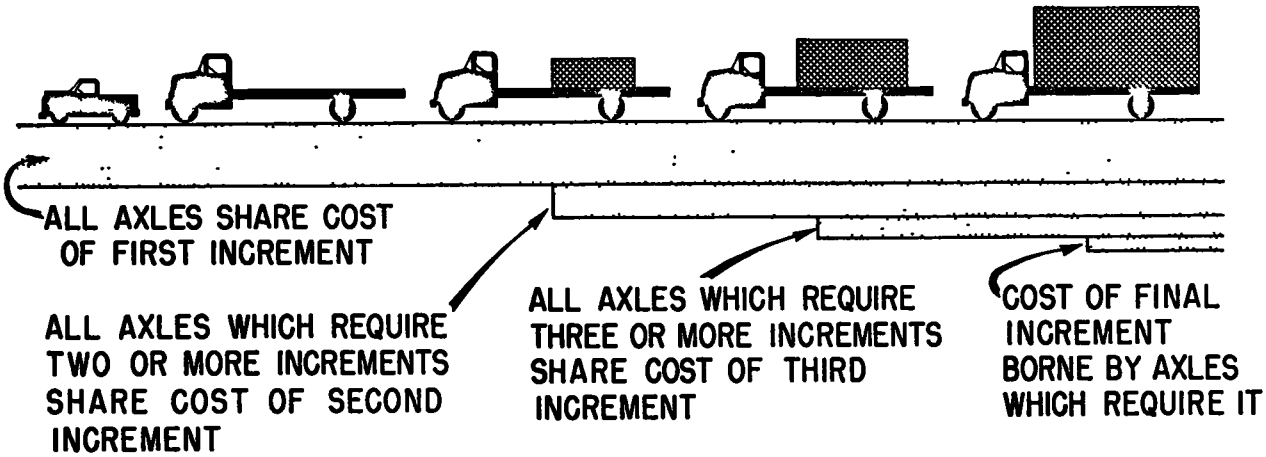

Figure 1. Incremental concept of highway cost allocation.

by even the lightest axle loads and is an integral part of the total pavement thickness required by heavier axle loads, the cost of the first increment is distributed at a single rate among all axles, regardless of load. Similarly, the second increment is divided among all axles except those that require only the first increment. The cost of each successive increment is distributed among the progressively smaller number of axles that require it.

This paper describes the use of the Road Test equations to determine pavement increments for use in the incremental solution made as a part of the Highway Cost Allocation Study.

ASSUMPTIONS

The Road Test equations relate pavement design to the variables (axle load and number of axle applications) that determine the relative pavement design requirements of different vehicles. In this respect they are well suited to the calculation of pavement increments, but the Road Test equations are directly applicable only to the environmental and controlled traffic conditions of the Road Test. Their use in the Highway Cost Allocation Study with mixed traffic and for other environments is based on three assumptions, as follows:

1. The single-load equation can be modified to account for the combined effect of the variety of axle loads that occurs in mixed highway traffic.
2. The variation of design requirements with axle load given by the equations, as modified for mixed traffic, holds for other environments.
3. The pavement designs used by the States for mixed traffic are the proper ones for the traffic volume, traffic composition, and environment for which they are used.

Two possible methods of working the combination of axle loads found in mixed traffic into the equation are given in the preceding paper by Scrivner and Duzan. At this time there are no experimental data with which to test either the validity of the theories on which the two methods are based or the accuracy of the results, but it is encouraging that the two approaches give much the same answer. The fact of importance is that this assumption must be made and a method of applying the equations to mixed traffic must be adopted, at least for the present, if the equations are to have practical application to highway problems.

The second assumption is no more susceptible of absolute proof at this time than the first. The proposed satellite tests and other experimental work will undoubtedly lead to means of translating the Road Test findings to other climates and other soils, but this is all in the future. In the meantime, it is reasonable to adopt the neutral position that changes in environment will not materially affect the variations of pavement design requirements with axle load that were found at the Road Test site.

The three assumptions can be combined into a statement of the role played by the equations in pavement cost allocation, as follows: the pavement design used by the States properly reflects the requirements of environment and traffic and can be broken down into increments by means of the Road Test equations, modified for application to mixed traffic.

This use of the equations avoids coming to grips with the problem of modifying the equations to adapt them to different environments. This approach has another attractive feature. If the pavement built for mixed traffic is not precisely tailored to the volume and composition of traffic to which it is subjected the maladjustment is distributed among all the increments in accordance with the traffic composition. This is preferable to concentrating it in some one increment.

USE OF THE EQUATIONS

To use the equations it is necessary to know the pavement design and to have an estimate of the traffic composition by observed axle-load intervals. Separate rigid and flexible pavement designs representative of the practice of the States in each Census Division were obtained for each traffic volume group on each Federal-aid system by means of a questionnaire, "Incremental Design Standards and Cost Factors for Road Construction." The traffic volume and weight studies made by the highway planning organizations in each State highway department produced estimates of the axle-load distribution of travel on each road type and system.

Tandem Axles

The increment of pavement thickness required by any given range of single-axle loads is also required by some range of tandem-axle loads and the cost of the increment must be divided among single and tandem axles. It is, therefore, necessary to group tandem axles with the single axles that have the same design requirements.

Because of the form of the Road Test equation, the equivalent tandem-axle load can not be obtained directly except for the special case when the terminal serviceability index is 1.5. For values of terminal serviceability greater than 1.5, the equivalent tandem-axle load is obtained by successive trial calculations.

Figure 2 shows the single-tandem axle load equivalence obtained for flexible and rigid pavements. For each, the bottom edge of the band indicates the equivalence when the terminal serviceability index is 1.5. All other equivalence values for terminal serviceability indexes up to 3.0 will lie within the area of each band. As is evident from the width of the bands, the effect of the terminal serviceability index on the equivalence is quite small. It is also apparent from the separation of the two bands that separate equivalent ratios between single and tandem axles must be used for flexible and rigid pavements. For a tandem-axle load of 32 kips, the single-axle load equivalence is about 19.8 and 17.2 kips, respectively, for rigid and flexible pavements. The single-axle load intervals and the corresponding tandem-axle load intervals used in the computations for the two pavement types are given in Table 1.

These load intervals give approximate ratios between tandem-axle loads and equivalent single-axle loads of 0.54 and 0.62 for flexible and rigid pavements, respectively.

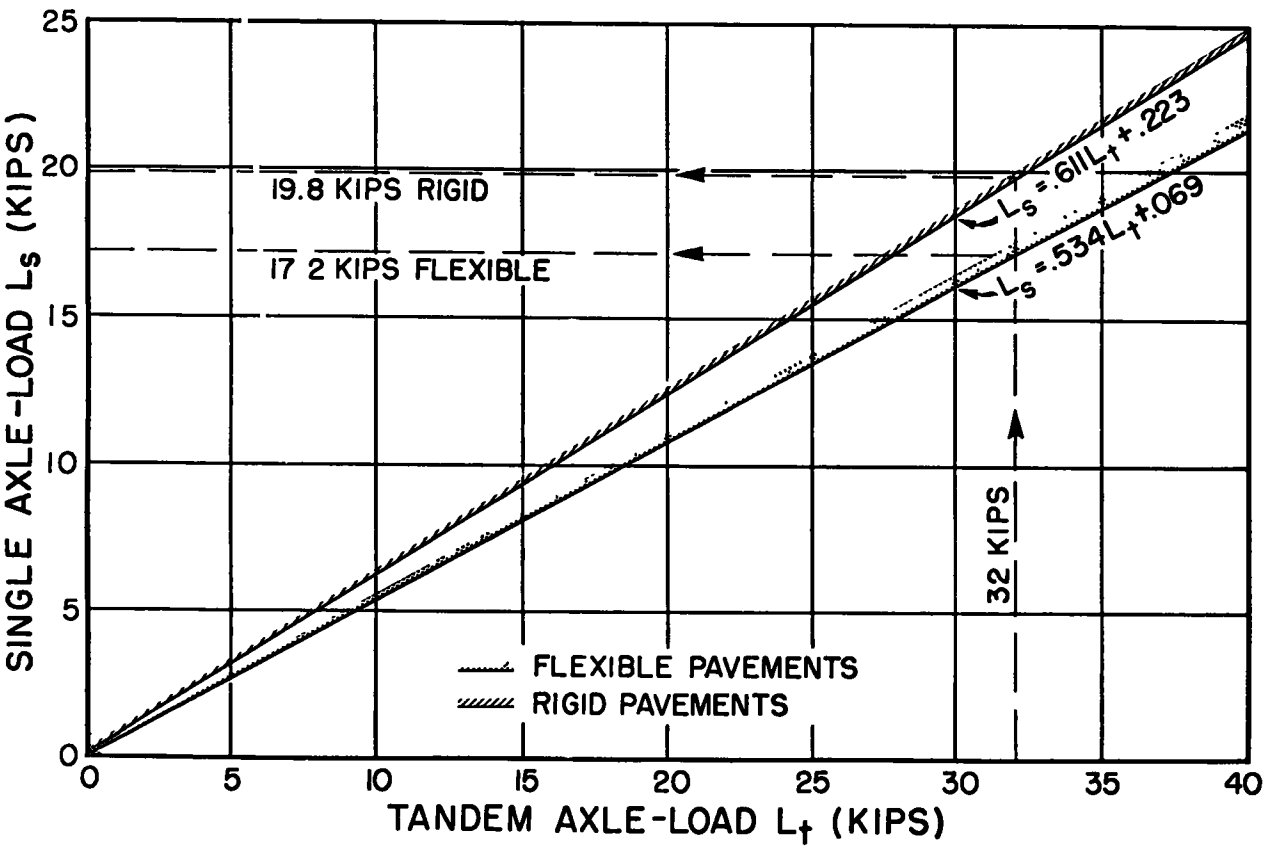

Figure 2. Single/tandem axle-load equivalence.

Computation of Pavement Increments

The modified Road Test equation derived for mixed traffic by the equivalent applications approach is used in describing the method of computing the increments of pavement design occasioned by axles of different weights. This equation is used in preference to the differential equations derived by the mixed traffic theory. The basic principles illustrated are the same no matter which equation is used, but it is easier to demonstrate them algebraically with the equivalent applications equation than with the mixed traffic equation.

The general form of the Road Test equations is

$$\log W = \log \rho + \frac{G_t}{\beta} \qquad (1)$$

or

$$W = \rho 10^{G_t/\beta} \qquad (2)$$

in which ρ is the number of applications required to reduce the present serviceability index, P_t, to a value of 1.5, the value at which pavement sections were removed from the test. For $P_t = 1.5$ the equation is simply

$$W = \rho \qquad (3)$$

It is convenient to start with this special form of the equation in explaining the calculation of incremental pavement designs.

The equivalent applications version of the modified Road Test equation applicable to mixed traffic for the special case of $P_t = 1.5$ is

$$W \Sigma C_i R_i = A_0' (D + 1)^{A_1} \qquad (4)$$

in which

W = Number of applications of all axle loads in mixed traffic, without regard to weight;

$$\Sigma C_i R_i = C_a R_a + C_b R_b + \ldots + C_i R_i \qquad (5)$$

$C_a, C_b, \ldots C_i$ = portion of total axle loads that are of weight $a, b, \ldots i$, respectively, in ascending order (by definition, then $\Sigma C_i = 1.0$); and

$R_a, R_b, \ldots R_i$ = number of applications of axle load a that is equivalent to one application of axle load $a, b, \ldots i$ (by definition, then $R_a = 1.0$).

The expression $A_0' (D + 1)^{A_1}$ represents the Road Test expression for ρ when a single-axle load of weight a is substituted in the load terms. In this expression D represents pavement design; that is, D_2 for rigid pavement and $a_1D_1 + a_2D_2 + a_3D_3$ for flexible pavement.

Each term in $\Sigma C_i R_i$ is the multiplying factor by which the number and weight of the axles in that class convert W to equivalent applications of load a. The left-hand member of Eq. 4 (that is, $W \Sigma C_i R_i$) is the equivalent number of applications of axle load a in mixed traffic and the equation is treated as a single-load equation for axle load a. The equation may be written in terms of any desired axle load. As will become apparent later, it is convenient to write the equation in terms of the lightest axle load so that R for the lightest axle load (in this case, R_a) will equal 1.0.

TABLE 1

SINGLE-AXLE LOAD INTERVALS AND CORRESPONDING TANDEM-AXLE LOAD INTERVALS

Single Axles (kips)	Tandem Axles: Rigid Pavement (kips)	Tandem Axles: Flexible Pavement (kips)
0 - 2.9	0 - 4.4	0 - 5.4
3.0 - 6.9	4.5 - 11.0	5.5 - 12.9
7.0 - 11.9	11.1 - 19.2	13.0 - 22.2
12.0 - 15.9	19.3 - 25.7	22.3 - 29.6
16.0 - 19.9	25.8 - 32.3	29.7 - 37.0
20.0 - over	32.4 - over	37.1 - over

The use of the modified Road Test equations to compute incremental pavement designs is shown in Figure 3. For simplicity, only four axle-load groups are used. The four weight groups are designated, in ascending order of weight, by a, b, c, and d.

The pavement designed and built to accommodate mixed traffic is represented by the combined thickness of all four increments. This design is designated D_m, the D being the design term (D_2 in the case of rigid pavement) for the Road Test equations and the subscript m indicating that the design is for mixed traffic. The value of D_m and the axle-load composition of mixed traffic can be substituted in the equation shown at the lower left in Figure 3 and the equation solved for W. The same equation, except that it is written in the form to solve for D, is shown at the right. This form of the equation is used to calculate the pavement design that will hold W constant as axle loads d, c, and b are successively replaced by lighter loads. It is obvious that if the value of W obtained by solving the equation at the left is substituted in the equation at the right one gets again the design for mixed traffic, D_m.

The equation used to calculate the pavement design D_c, required when the axles of weight d are replaced by an equal number of axles of weight c, is shown in the portion of the chart that represents three increments of pavement. The only difference between this equation and the one at the lower right is that the number of axles in the heaviest weight group, d, is now multiplied by R_c, the equivalent application factor for weight group c; that is, the number

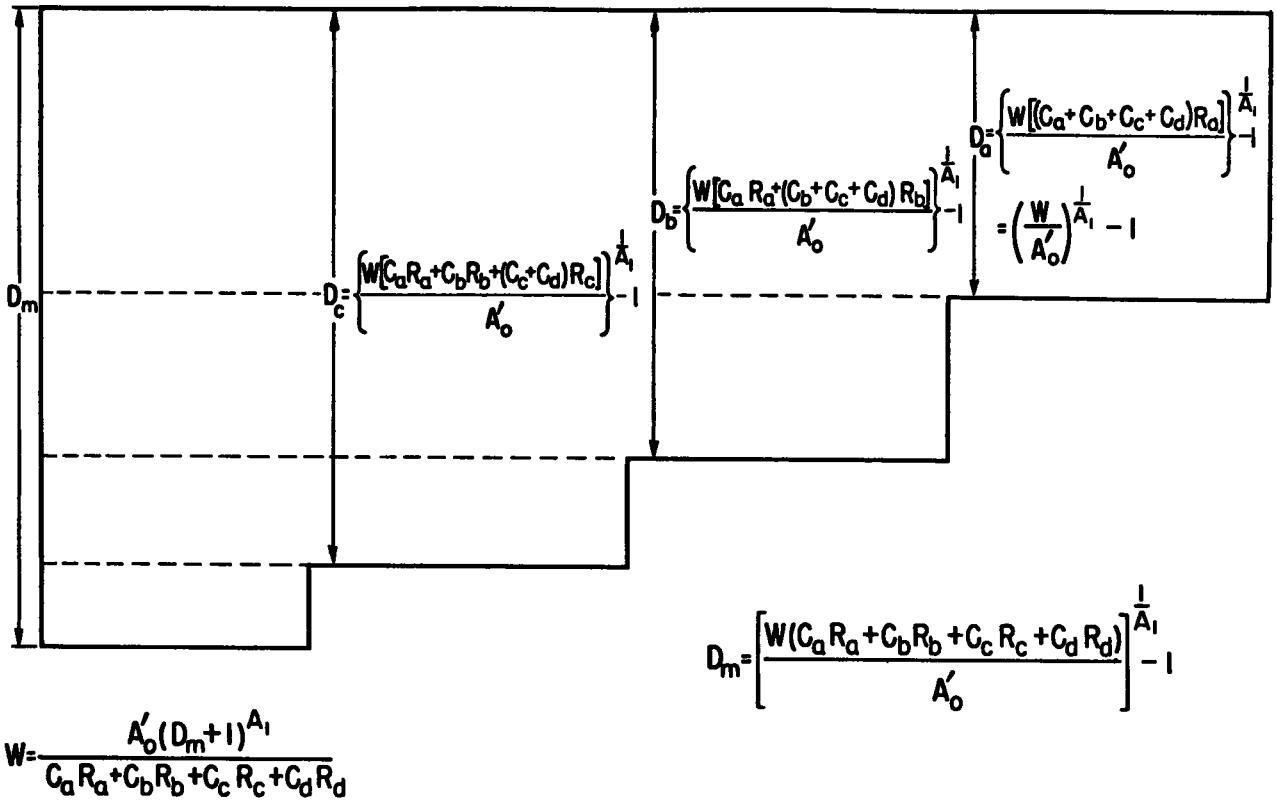

Figure 3. Illustrative calculation of pavement increments.

of axles is not changed but axles of weight d are treated as if they were of weight c.

The equation for D_b differs from the one for D_c in that axles of weight d and c are now considered to be of weight b. Finally, all axles are considered to be of weight a in computing D_a, the first increment of pavement design. Because $C_a + C_b + C_c + C_d = 1$, the equation for D_a becomes

$$D_a = (W/A_o')^{1/A_1} - 1 \qquad (6)$$

if $R = 1$. It is for this reason that it is convenient to base the equivalent application ratios (R_a, R_b, etc.) on the lightest axle load.

Values of D obtained through these calculations must be transposed into the structural elements (surface, base and subbase) comprising the pavement structure.

In the case of rigid pavements, the value of D consists of one design variable only—thickness, in inches, of the portland cement concrete slab. Therefore, the changes in D for the rigid pavements are made entirely in the thickness of the concrete pavement and the value of subbase thickness, if any, reported for the pavement designed to accommodate mixed traffic remains unaffected.

Change in the structural elements comprising a flexible pavement, due to the incremental decrease in the design term D, is not as easy. For flexible pavements, the design term is the summation of three factors. Each factor is an expression of the strength contributed to the pavement structure by one of the three elements —surface, base and subbase. The magnitude of the factor depends on the thickness and type of material used in the element. For the incremental design requirements outlined here, the pavement composition reported for mixed traffic was used as a guide and the thickness of each element was reduced to satisfy the required D.

It can be seen that considerations other than the numbers and weights of axles in each weight group do not enter into the determination of the size of the increment occasioned by the group and there is no reason to look upon the first increment as a pavement that might be built. The first increment is simply the portion of pavement structure that remains when the added structural requirements occasioned by the heavier axles because of their weight, but not their number, are removed.

The equation used for illustrative purposes in Figure 3 is the simple form assumed by the modified Road Test equations at a serviceability index value of 1.5. The principle of successively replacing the heaviest axle loads by a like number of the next heaviest loads is applicable to other serviceability index values but the calculations are not as simple. For values of

Equivalent application theory:

$$W = \frac{A_0'(D_m+1)^{A_1}}{C_a R_a' + C_b R_b' + C_c R_c' + C_d R_d'} 10^{\left(b_0 + \frac{G}{\frac{B_0'}{(D_m+1)^{B_1}}}\right)}$$

$$W = \frac{1}{\frac{C_a}{\rho_a 10^{\frac{G}{\beta_a}}} + \frac{C_b}{\rho_b 10^{\frac{G}{\beta_b}}} + \frac{C_c}{\rho_c 10^{\frac{G}{\beta_c}}} + \frac{C_d}{\rho_d 10^{\frac{G}{\beta_d}}}}$$

Mixed traffic theory: $G = \log g$

$$W = \int_0^{g_1} \frac{dg}{C_a\left[\frac{\beta_a}{\rho_a} g^{\left(1-\frac{1}{\beta_a}\right)}\right] + C_b\left[\frac{\beta_b}{\rho_b} g^{\left(1-\frac{1}{\beta_b}\right)}\right] + C_c\left[\frac{\beta_c}{\rho_c} g^{\left(1-\frac{1}{\beta_c}\right)}\right] + C_d\left[\frac{\beta_d}{\rho_d} g^{\left(1-\frac{1}{\beta_d}\right)}\right]}$$

Figure 4. Mixed traffic equations for all values of *p*.

serviceability index other than 1.5 the equation of Figure 3 takes the form shown in Figure 4. Part of this equation will be recognized as the equation of Figure 3, with one important exception. The equivalent application terms R_a, R_b, etc., are now shown as R_a', R_b', etc. This is because, for any serviceability index value other than 1.5, the equivalent application ratios vary with D. The expression $b_0 + \frac{B_0'}{(D+1)^{B_1}}$ signifies the Road Test expression for β when a single axle of weight a is substituted in the load terms.

As shown in Figure 3, the first step is to substitute the value of D_m and solve for W. This can be done because for any selected value of serviceability index G is a constant and the equivalent ratios R_a', R_b', etc., are determined by the known value of D_m.

It will be seen that the equation cannot be solved directly for D. To find the value of D_c that will hold W constant when the axles of weight d are reduced to weight c it is necessary to assume a value for D_c, substitute this value and the corresponding values of R_a', R_b', and R_c' in the equation and solve for W. This process is repeated until a value of D_c is found that will give the required value of W.

The second equation of Figure 4 is simply the first equation written in another form.

Also shown in Figure 4 is the differential equation for mixed traffic derived by the mixed traffic theory as explained in the preceding paper by Scrivner and Duzan. The symbols C_a, C_b, C_c, and C_d are the same in all three equations. To conserve space the expressions for ρ and β are not written out in the second and third equations. Subscripts are used to indicate the axle load for which ρ and β are evaluated. As indicated $G = \log g$.

In the differential equation at the bottom of Figure 4, the process of replacing axle loads by the same number of lighter axle loads is carried out by successively dropping the expression in brackets for the heaviest axle load and combining the value of C for that axle load with the value of C for the next lighter load.

CONCLUSION

The approach described in this paper is believed to be applicable to any method of adapting the single-load equations to mixed traffic. It is based on the hypothesis that the mode of variation of pavement design requirements with axle load given by the Road Test equations can be used to break down the pavement designs for mixed traffic into the increments occasioned by the axles in each weight group. It is offered as a reasonable and prudent application of the Road Test equations until such time as further research may either verify it or provide means of refining it.