Crack Propagation in Plain Concrete

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A hypothesis for the fracture of concrete is presented. Initial cracks exist in the cement paste matrix and at the matrix-aggregate interface. The propagation of existing cracks in plain concrete is influenced by the properties of the cement paste and the aggregates.

The fracture toughness or resistance to crack propagation of a homogeneous material can be expressed as the stress intensity factor for the region near the tip of an existing crack at the onset of rapid crack propagation. A pseudo-fracture toughness is obtained for concrete when it is analyzed as a homogeneous material. The pseudo-fracture toughness of concrete is the summation of the fracture toughness of the cement paste and an arresting action of the aggregates on crack growth.

An experimental investigation formed part of the study and was designed to provide an evaluation of the hypothesis for fracture of concrete. Results indicate that concretes with typical coarse aggregate contents have approximately twenty percent greater resistance to crack growth than mortars with similar water-cement and sand-cement ratios.

•IN THE PAST the primary emphasis in the failure of plain concrete has been placed on the static strength rather than on the failure mechanism, crack propagation. Recently the technique of fracture mechanics has been applied to concrete to determine the necessary conditions for the rapid propagation of an existing flaw.

A hypothesis for the fracture of concrete is presented. The propagation of existing cracks in plain concrete under static loading is influenced by the properties of the cement paste matrix and the aggregates. The initial cracks exist in the matrix and at the matrix-aggregate interface. The cracks propagate through the matrix in two stages, slow and rapid. It is the onset of rapid crack propagation that leads to final rupture.

The fracture toughness or resistance to crack propagation of a homogeneous material can be expressed as the stress intensity factor for the region near the crack tip at the onset of rapid crack propagation. A pseudo-fracture toughness is obtained for concrete when it is analyzed as a homogeneous material. The pseudo-fracture toughness of concrete is the summation of the fracture toughness of the cement paste and an arresting action developed by the aggregates.

FRACTURE MECHANICS

Fracture mechanics deals with the forces associated with the rupture of a solid body. The fracturing process is a progressive failure that usually consists of three stages: crack initiation, slow crack growth and rapid crack propagation. The onset of rapid crack propagation leads to the final rupture of a solid.

Rapid crack propagation occurs when the energy released by a virtual crack extension is sufficient to supply the energy requirements of surfaces created by the virtual crack extension. There are two sources of energy available for crack extension: energy supplied by the work of the external forces, and the change of the stored strain energy. There are two energy requirements for the formation of the new crack surface: surface energy and energy dissipated in plastic deformations near the crack tip.

Spontaneous rapid crack propagation will occur when

$$\Delta E \geq \Delta T + \Delta W_P \tag{1}$$

where ΔE is the change of energy associated with a vertical crack extension ΔC, ΔT is the increase in surface energy and ΔW_P is the energy associated with plastic deformation. The instant of instability occurs when

$$\Delta E = \Delta T + \Delta W_P \tag{2}$$

for a virtual crack extension ΔC.

The energy increments of Eqs. 1 and 2 are functions of the crack extension ΔC. It is convenient to express the energies as rates of energy change with respect to the crack extension ΔC as ΔC approaches zero. The rate of release of energy is denoted by G, and it is

$$G = \lim_{\Delta C \to 0} \frac{\Delta E}{\Delta C} = \frac{\partial E}{\partial C} \tag{3}$$

The energy release rate assumes a critical value, G_c, at the instant of instability, and it is

$$G_c = \frac{\partial T}{\partial C} + \frac{\partial W_P}{\partial C} \tag{4}$$

G_c is assumed to be a material property that is independent of body geometry and loads. It is a measure of the resistance of a material to the propagation of an existing crack and is sometimes called the fracture toughness.

The energy release rate G may be related to the elastic stress and displacement fields near the tip of a crack in a homogeneous material.

Consider the homogeneous elastic plate Y of Figure 1 subjected to loads that are symmetrical with respect to the X_1 axis. An initial crack of length 2C exists along the X_1 axis. The plane strain solutions (1) for the stress σ_{22} and the displacement u_2 near the crack tip have the general form

$$\sigma_{22} = \left(\frac{EG}{(1-\mu^2)\,2\pi}\right)^{1/2} \left(X_1 - C\right)^{1/2} + \ldots \tag{5}$$

where $X_1 > C$, $X_2 = 0$, and

$$u_2 = \frac{4(1-\mu^2)}{E}\left(\frac{EG}{(1-\mu^2)\,2\pi}\right)^{1/2} \left(C - X_1\right)^{1/2} + \ldots \tag{6}$$

where $X_1 < C$, $X_2 = 0$, E is Young's modulus, and μ is Poisson's ratio.

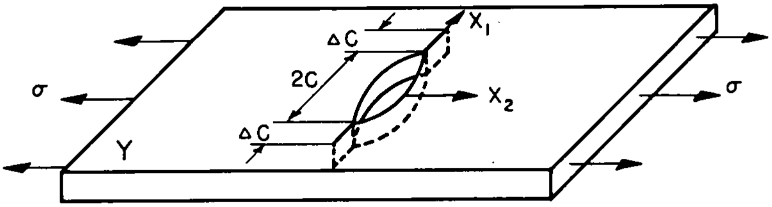

Figure 1. Infinite plate Y with an initial crack 2C.

The term $\left(\frac{EG}{1-\mu^2}\right)^{1/2}$ is denoted by the stress-intensity factor K for plane strain

$$K = \left(\frac{EG}{1-\mu^2}\right)^{1/2} \tag{7}$$

A critical stress intensity factor K_c corresponds to the critical energy release rate and is also a measure of fracture toughness

$$K_c = \left(\frac{EG_c}{1-\mu^2}\right)^{1/2} \tag{8}$$

HYPOTHESIS FOR CONCRETE FRACTURE

Several investigators (2, 3, 4, 5) have applied the energy concept of fracture mechanics to mortars and concretes. Kaplan (4) concluded that the concept could be applied, at least qualitatively, to concrete.

Concrete is a polyphase material that consists of a cement paste matrix and aggregates. The fracture mechanics concept for homogeneous materials must be modified for concrete. In this section the fracture of cement paste and concrete is described, and a model for crack development in plain concrete is formulated.

Paste Fracture

Kaplan (4) found the energy requirement for crack propagation in cement paste to be an order of magnitude greater than the surface energy of the new crack surface. Thus, the crack did not propagate as one single crack in the paste.

Since concrete does not exhibit ductile properties, Glucklich (2, 3) has suggested that the increased energy requirement for crack propagation in cement paste is caused by the formation of a microcracking region near the crack tip. Most of the microcracks are not incorporated into the main crack propagation. The total crack surface exceeds the area of the main crack surface, and the total surface energy is greater than the energy required to form only the main crack. The microcracks form as the stresses near the crack tip reach some limiting stress level. The distance from the crack tip to the edge of the microcracking region increases as the crack propagates. Thus the surface area of the microcracks and the total energy requirement increase with crack growth.

Consider an infinite plate of cement paste with an initial crack and a uniform tensile stress applied normal to the crack at infinity. As the applied stress is increased, the microcracks begin to form near the crack tip; the condition is reached when the energy required to form a virtual crack extension and the new region of microcracks is balanced by the energy released by the virtual extension, and the main crack begins to propagate. As the crack propagates, the size of the microcrack region and the energy required to form it increase. Thus it is possible for the energy system to remain balanced as the crack propagates. The stress intensity factor at the initiation of this controlled or slow growth is denoted by K_i. The slow growth continues until the zone of microcracking reaches a limiting size. There is no further increase in the energy requirement for crack growth, and rapid crack propagation occurs. The stress intensity factor at the onset of rapid crack propagation is denoted by K_c. It is a function of the limiting size of the region of microcracks and of the surface energy of the cement paste. If these values are properties of the cement paste, the critical stress intensity K_c is also a material property.

Concrete Fracture

The fracture of concrete is more complex than that of cement paste. Concrete consists of a cement paste matrix that surrounds fine and coarse aggregates. Initial cracks are present in the matrix and at the matrix-aggregate interface. These cracks propagate through the matrix. The fracture toughness of concrete depends on the energy requirement for crack propagation in the matrix and also on the heterogeneity of the concrete.

The onset of rapid crack growth in the cement paste matrix occurs at a maximum stress intensity factor K_c that is a material property of the cement paste. Slow crack growth may be initiated at some lower stress intensity factor K_i.

The elastic stress fields of a concrete body are complex, and it is convenient to consider the analogy of an infinite plate containing a bonded disc to show the crack development in concrete.

The infinite plate Z of Figure 2 has the material properties of a cement paste. The plate contains an initial crack of length C along the x_1 axis. There is a hole of diameter D with its center located on the x_1 axis, and a disc with the material properties of a concrete aggregate and with a diameter D is bonded into the hole. A uniform tensile stress σ is applied at a constant rate of loading normal to the x_1 axis at infinity. The initial crack is assumed to propagate along the x_1 axis toward the disc.

Crack development will depend on the relative magnitudes of the stress intensity factors associated with the energy requirements for crack growth in the plate and in the

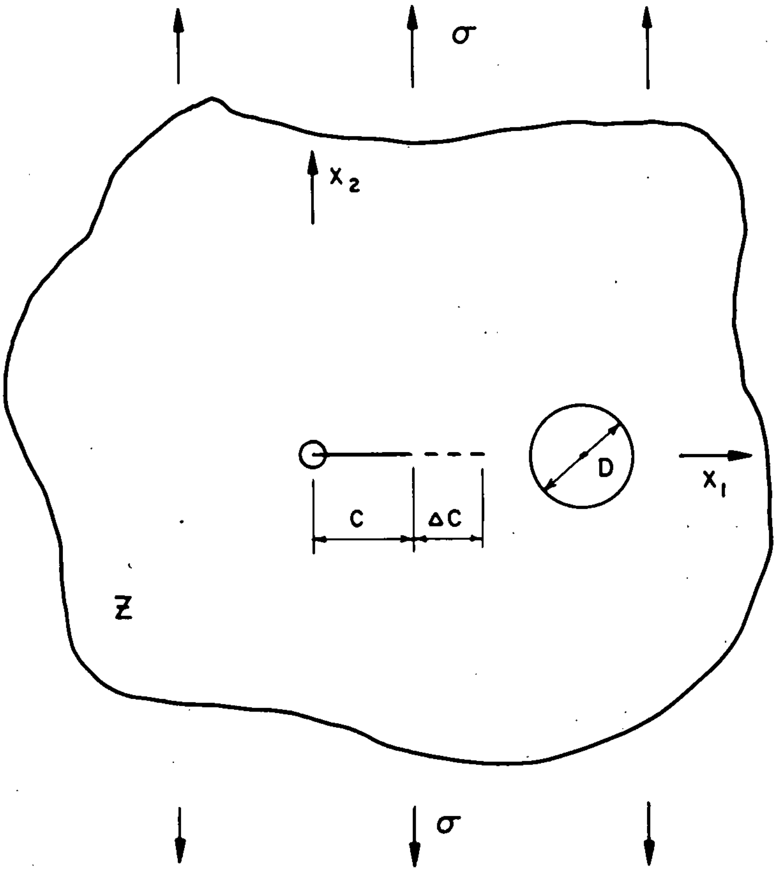

Figure 2. Infinite plate Z with bonded disc and initial crack C.

disc and of the stress intensity factors for the elastic fields in the plate and in the disc.

Stress intensity factors that are functions of the energy requirements of the materials are denoted by subscripts. They are K_{Pi} and K_{Di}, which are the stress intensity factors at the initiation of slow crack growth in the plate and in the disc, respectively, and K_{Pc} and K_{Dc}, which are the stress intensity factors at the onset of rapid crack growth in the plate and in the disc, respectively.

The stress intensity factors for the elastic fields are denoted by superscripts. K^P is associated with the elastic fields near a crack tip in the plate, and K^D is associated with the elastic fields near a crack tip in the disc.

The elastic fields in the plate are equal to the summation of the elastic fields in the plate subjected to the applied stresses for the special case of plate and disc having the same elastic properties and of the elastic fields in the plate for the case of the plate subjected only to boundary conditions at the hole that are the difference between the actual case and the special case of equal elastic properties. The stress intensity factor K^P for the actual case is the summation of the stress intensity factors for the two cases:

$$K^P = K^{PL} + K^{PA} \tag{9}$$

where K^{PL} is the stress intensity factor for the special case of a plate and disc with equal elastic properties subjected to the applied loads, and K^{PA} is the stress intensity factor for the body subjected to the difference boundary conditions at the hole.

An initial crack propagates in a plate that contains a disc with a greater modulus of elasticity in any of the following methods.

Method I. —The distance between the disc and crack tip is large, and the disc does not modify the elastic fields near the crack tip in the plate until the crack grows toward the disc. In this case

$$K^{PA} = 0$$

when

$$K^P < K_{Pi}$$

and

$$K^{PA} \leq 0$$

when

$$K_{Pi} \leq K^P \leq K_{Pc}$$

K^{PA} is negative because the effect of the disc is to reduce the stresses near the crack tip.

No crack growth occurs as the applied stress increases until

$$K_{Pi} = K^P$$

then slow crack growth is initiated. As the crack propagates toward the disc, the disc begins to modify the elastic fields near the crack tip, and

$$K^{PA} < 0$$

Further crack growth depends on the relative changes in K^{PL} and K^{PA} and on the energy requirements of crack growth in the materials. The slow growth will continue in the plate if

$$\frac{\partial K^{PL}}{\partial C} + \frac{\partial K^{PA}}{\partial C} \geq 0$$

and

$$K_{Pi} \leq K^{P} < K_{Pc}$$

In this case the only effect of the disc is a reduction in the rate of crack growth.

The disc will arrest the slow growth if

$$\frac{\partial K^{PL}}{\partial C} + \frac{\partial K^{PA}}{\partial C} < 0$$

since the energy released does not meet the increasing energy requirement of crack growth in the plate.

Rapid crack propagation occurs in the plate when

$$K^{P} = K_{Pc}$$

The rapid growth will continue when

$$\frac{\partial K^{PL}}{\partial C} + \frac{\partial K^{PA}}{\partial C} \geq 0$$

and it will be arrested when

$$\frac{\partial K^{PL}}{\partial C} + \frac{\partial K^{PA}}{\partial C} < 0$$

if the modification of the elastic fields is such that the kinetic energy of the moving crack and the released energy do not satisfy the energy requirements of crack growth in the material.

Method II. —The disc is a great distance from the crack tip and does not modify the elastic fields near the crack tip before the onset of rapid crack propagation. In this case

$$K^{PA} = 0$$

when

$$K^{P} \leq K_{Pc}$$

and the crack development is similar to the fracture of cement paste.

Method III. —The disc is near the crack tip and modifies the elastic fields near the tip of the initial crack. In this case

$$K^{PA} < 0$$

when

$$K^{P} \leq K_{Pc}$$

Slow growth is initiated at

$$K_{Pi} = K^{P}$$

and the onset of rapid growth occurs at

$$K^P = K_{Pc}$$

In any of the crack developments the energy requirements of the disc rather than of the plate will control crack growth when the crack reaches the plate-disc interface.

Slow crack growth will be arrested if

$$K^D < K_{Di}$$

and will continue if

$$K_{Di} \leq K^D$$

Rapid crack propagation can be arrested at the interface only if the increased energy requirement for crack growth in the disc exceeds the kinetic energy and the released energy of the moving crack.

A crack that has been arrested at the interface will begin slow growth again at

$$K_{Di} = K^D$$

and rapid growth at

$$K^D = K_{Dc}$$

A premature failure of the disc may occur any time during crack development. The disc may rupture, or a bond failure may occur at the plate-disc interface. Arresting action of the disc vanishes, and the stress-intensity factor for the crack tip in the plate will increase.

The crack will develop as a fracture in cement paste if

$$K^P < K_{Pi}$$

after the premature disc failure. Slow growth will be initiated by the disc failure (or continue) if

$$K_{Pi} \leq K^P < K_{Pc}$$

after the disc failure, and rapid propagation will be initiated (or continue) if

$$K^P = K_{Pc}$$

after the disc failure.

When a moving crack reaches a disc that has failed prematurely, the crack will pass through the existing failure surface. A plate-disc bond failure will appear as a disc "pull-out," and a disc rupture will appear as a fracture surface through the disc.

Crack arrest develops only when the modulus of elasticity of the disc, E_D, is greater than the modulus of elasticity of the plate, E_P. Crack arrest is increased by an increased modular ratio E_D/E_P. Increasing the disc diameter, D, also increases the arresting tendency. Crack arrest increases as the spacing between the crack tip and the plate-disc interface is decreased.

This analogy may be extended to a concrete body in concept only since the elastic stress fields of the concrete body are complex. The plate-disc body is a two-dimensional stress problem. The bonded disc extends through the plate thickness, and the modification of the elastic fields near the crack tip is independent of location in the direction of the plate thickness. A concrete body is a three-dimensional stress problem. The effect of any one aggregate on the elastic stress fields near the crack tip is dependent on its location in the direction of thickness. Superposition of the elastic field modifications of all aggregates would give the total arresting action or effective arrest. The effective arrest of the aggregates is less than the arrest of the bonded disc. However, the surface area of the aggregates is greater than the surface area of the disc, and premature bond failures will be delayed in concrete.

Fracture Model

The propagation of an initial crack in a concrete body can be analyzed by the energy concept of fracture mechanics. Plane strain conditions can be assumed for structural concrete members, since the concrete members found in practice have cross sections that are relatively thick.

Cracks in plain concrete propagate through the cement paste matrix. The energy requirements for crack growth in the matrix control the crack development in concrete. Slow crack growth is initiated at some stress intensity factor K_{Pi} in the paste, and rapid crack growth occurs at the critical stress intensity factor K_{Pc} for the paste.

The stress intensity factor for the elastic fields near the crack tip in the paste is K^P, a function of the applied loads and body geometry

$$K^P = K^{PL} - f(ARR) \qquad (10)$$

where K^{PL} is the stress intensity factor for a homogeneous body and f(ARR) is an arresting function that represents the modification of the elastic fields near the crack tip caused by the concrete aggregates.

The onset of rapid crack propagation in concrete occurs when the stress intensity factor in the paste reaches the critical value K_{Pc}

$$K^P = K^{PL} - f(ARR) = K_{Pc} \qquad (11)$$

K^{PL} is a pseudo-fracture toughness for concrete, and it will be denoted by K_c'. K_c' is obtained experimentally when a concrete body is analyzed as a homogeneous elastic material. Eq. 11 may be written

$$K_c' = K^{PL} = K_{Pc} + f(ARR) \qquad (11a)$$

and the resistance of concrete to crack propagation, K_c', is a function of the fracture toughness of cement paste, K_{Pc}, and of the crack arrest developed by the concrete aggregates.

The fracture toughness of cement paste might be affected by the water-cement ratio and the curing of the paste. The factors that might affect the crack arrest are numerous and interrelated. The maximum size and the grading of the aggregate and the aggregate percentages influence the body geometry of the polyphase concrete. Material properties of the cement paste and aggregates affect the modular ratio and the premature failure of the aggregates.

EXPERIMENTAL INVESTIGATION

An experimental investigation was carried out to provide an evaluation of the proposed hypothesis. The fracture toughnesses of several mortars and concretes were determined through flexural tests of notched specimens.

Scope of Tests

Mix Proportions. —Five mortar and five concrete mixes, made from type I portland cement, river sand and river gravel, were used. The experimental program was designed to investigate the effects of varying the aggregate-cement ratios in mixes of given water-cement ratio. Table 1 gives details of the mix designs.

Fabrication and Curing. —The flexural specimens were cast on their side in plywood forms. Notches were formed by steel molds that were rigidly attached to the side of

TABLE 1

MORTAR AND CONCRETE MIX DESIGNS

Series	Relative Weights				f'_c (psi)
	Cement	Water	Sand	Gravel	
A1a	1	0.50	2.4	—	7500
A1b	1	0.50	2.9	—	6530
A1c	1	0.50	4.0	—	6370
A3b	1	0.50	2.9	3.0	6240
B1b	1	0.55	2.9	—	5800
B3b	1	0.55	2.9	3.0	5430
C1b	1	0.60	2.9	—	4760
C2b	1	0.60	2.9	1.6	5080
C3b	1	0.60	2.9	3.0	4940
C4b	1	0.60	2.9	4.4	4330

the plywood forms. All specimens were moist cured for 26 days and then stored in a 50 percent relative humidity environment until testing at 28-days age.

Flexural Specimens. —Nominal dimensions of the flexural specimens were 4 by 4 by 12 in. A 30-deg notch with a sharp edge was cast in the tensile face of each specimen at midspan. Three nominal notch depths were used, ½ in., 1 in. and 1½ in. Five specimens were cast for each mix design.

Flexural Tests. —A 60,000-lb capacity hydraulic testing machine was used. The test setup is shown in Figure 3. A two-point load was applied to the flexural specimens by a steel load plate that also acted as a dynamometer. A deformeter was used to measure the deformation of the tensile surface between the load points. An X-Y plotter was used to obtain a continuous record of load and deformation during the tests.

Test Results and Analysis. —The load-deformation curves obtained for the notched specimens are useful in illustrating the behavior of the specimens under load. Changes in the load-deformation curve indicate changes in the load-deflection curve. A qualitative load-deflection curve is shown in Figure 4.

The curve exhibits four stages of behavior. In the first stage (linear) the load-deflection curve goes through a series of one or more straight lines of decreasing slope. In the second stage (microcracking) the curve slowly deviates from a straight line. In the third stage (slow growth) the deflections increase rapidly with a small increase in load. In the fourth stage (fracture) the deflections increase with no further increase in load.

The maximum load, P_{max}, corresponded to the fracture stage. The load at the initiation of slow crack growth, P_i, corresponded to the beginning of the slow growth stage. P_i was estimated from the original test record.

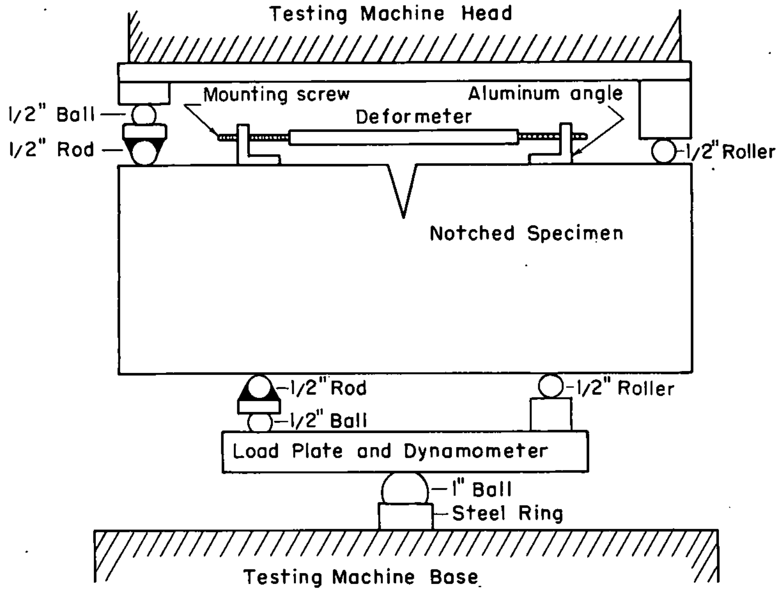

Figure 3. Test setup.

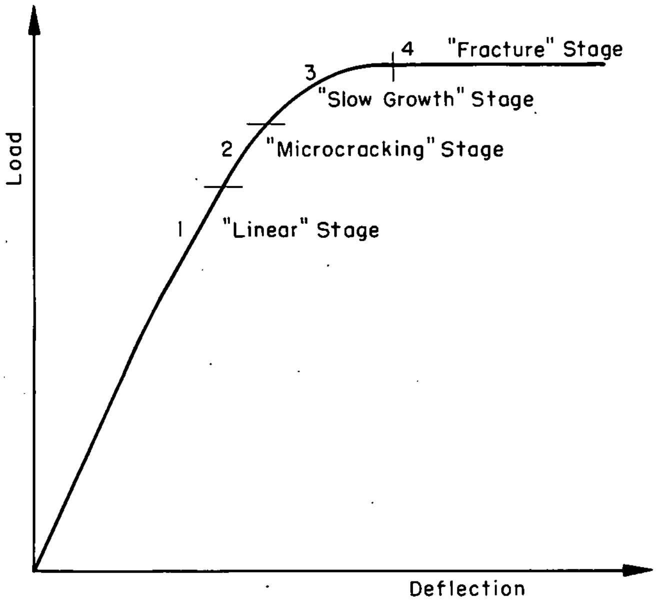

Figure 4. Qualitative load-deflection curve for test specimens.

Pseudo-Stress Intensity Factor. —Bueckner (6) has developed an expression for the stress intensity factor K for the homogeneous beam B of Figure 5 subjected to pure bending. The stress intensity factor is

$$K = \frac{6M}{wd^2} \cdot \left[\frac{2d}{\pi} h(C/d)\right]^{1/2} \tag{12}$$

where h(C/d) is a function of C and d,

$$h(C/d) = 10.08(C/d)^2 - 1.225(C/d) + 0.1917 \tag{13}$$

and C is the notch depth, w is the beam width, d is the beam depth, and M is the applied bending moment.

A pseudo-stress intensity factor K' is obtained for the concrete specimens by assuming concrete to be homogeneous. K' is a function of the instantaneous value of C. Slow crack growth increases the crack length to $C + \Delta C$. No accurate method was available to determine the slow growth, and the calculated values for K' are based on the original notch depth C.

The pseudo-fracture toughness K_c' is obtained by substituting M_{max} and the original notch depth C into Eq. 12. The pseudo-stress intensity factor at the initiation of slow crack growth K_i' is obtained by substituting M_i and C into Eq. 12. The mean values $\overline{K_i'}$ and $\overline{K_c'}$ and the coefficients of variation for each mix design are given in Table 2.

The mean pseudo-fracture toughness varied from 0.265 kips/in.$^{3/2}$ for one of the mortars to 0.356 kips/in.$^{3/2}$ for one of the concretes. The coefficient of variation ranged from 7.0 percent to 16.9 percent. $\overline{K_i'}$ varied from 0.210 kips/in.$^{3/2}$ for one mortar to 0.301 kips/in.$^{3/2}$ for one concrete, and the coefficient of variation ranged from 6.9 percent to 22.4 percent.

Effect of Concrete Parameters on $\overline{K_c'}$

$\overline{K_c'}$ vs water-cement ratio is shown in Figure 6 for three mortars with the same sand content and in Figure 7 for three concretes with the same sand and gravel contents.

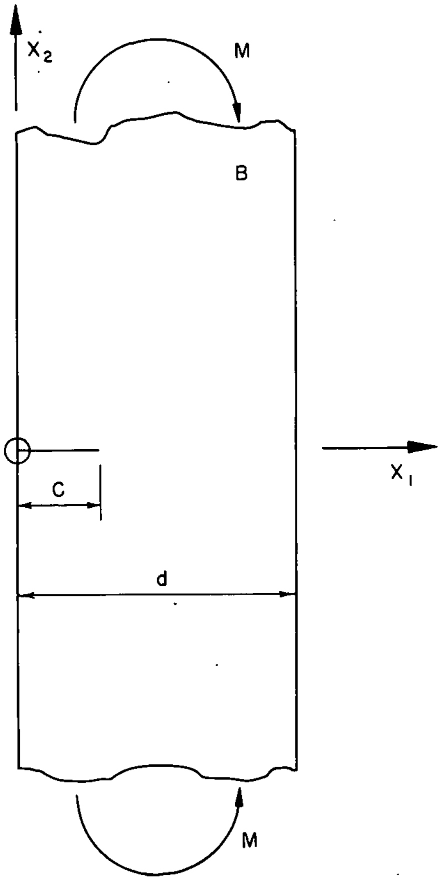

Figure 5. Beam B with an initial crack C.

TABLE 2

PSEUDO-STRESS INTENSITY FACTORS

Series	$\overline{K}_c'$ $\left(\frac{\text{kips}}{\text{in.}^{3/2}}\right)$	Coefficient of Variation (%)	$\overline{K}_i'$ $\left(\frac{\text{kips}}{\text{in.}^{3/2}}\right)$	Coefficient of Variation (%)
A1a	0.265	9.9	0.210	8.6
A1b	0.303	7.0	0.219	6.9
A1c	0.296	8.5	0.235	12.2
A3b	0.368	12.2	0.287	8.9
B1b	0.295	16.9	0.211	22.4
B3b	0.346	7.2	0.269	12.4
C1b	0.297	13.9	0.217	11.1
C2b	0.310	14.1	0.248	10.1
C3b	0.354	14.1	0.300	8.9
C4b	0.359	9.7	0.301	10.0

There is no significant variation in the pseudo-fracture toughness over the range of water-cement ratios investigated.

$\overline{K}_c'$ vs the percent of fine aggregate is shown in Figure 8 for three mortars with a 0.50 water-cement ratio. There is no significant trend in the variation of the pseudo-fracture toughness for the fine aggregate contents investigated.

$\overline{K}_c'$ vs the percent of coarse aggregate is shown in Figures 9 to 11. There is a significant increase in $\overline{K}_c'$ as the coarse aggregate content is increased from zero for mortar to typical values for concrete. $\overline{K}_c'$ for one mortar and one concrete with a 0.50 water-cement ratio is shown in Figure 9. $\overline{K}_c'$ for the concrete is 21.4 percent greater than $\overline{K}_c'$ for the mortar. $\overline{K}_c'$ for one mortar and one concrete with a

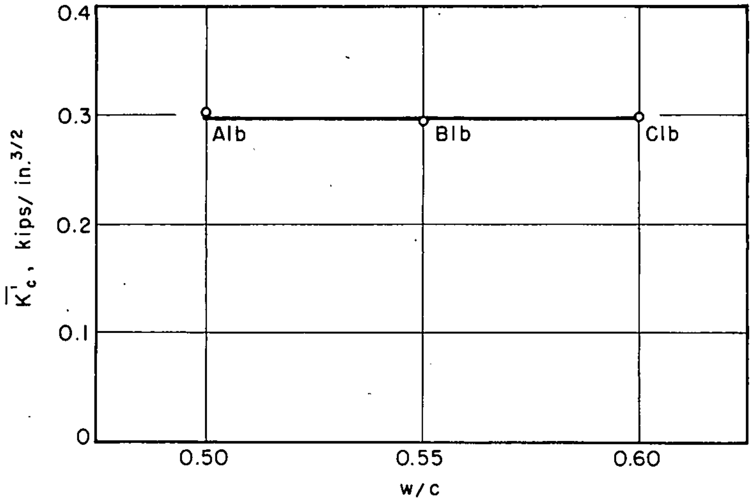

Figure 6. Effect of w/c ratio on $\overline{K}_c'$; three mortars.

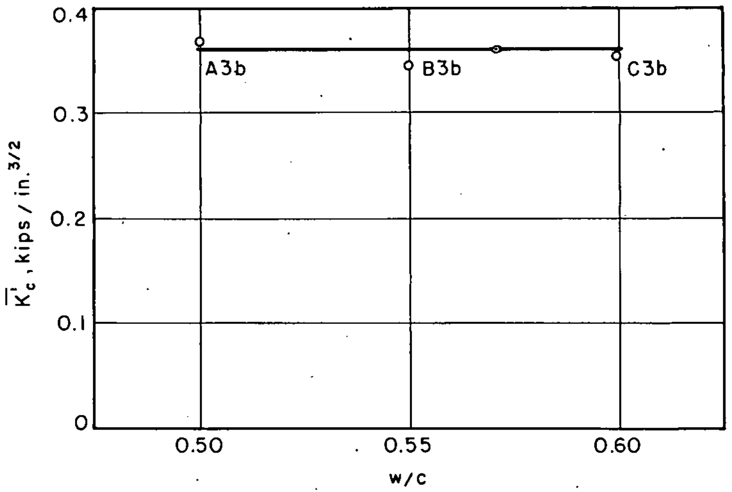

Figure 7. Effect of w/c ratio on $\overline{K}_c'$; three concretes.

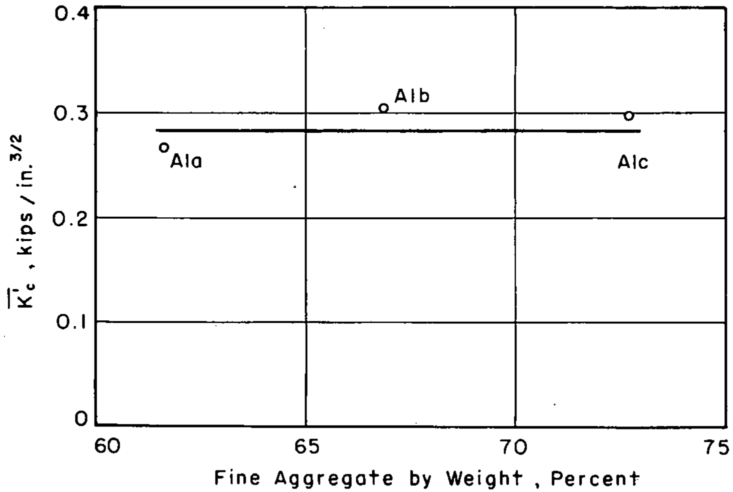

Figure 8. Effect of fine aggregate on $\overline{K}_c'$; three mortars, w/c = 0.50.

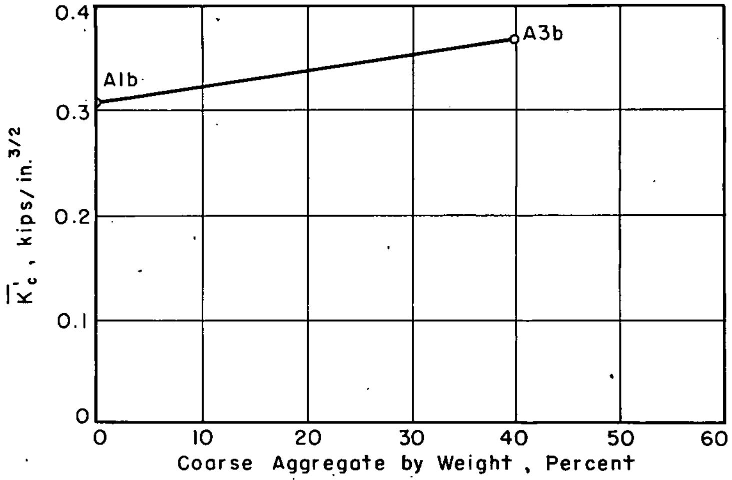

Figure 9. Effect of coarse aggregate on $\overline{K}_c'$; w/c = 0.50.

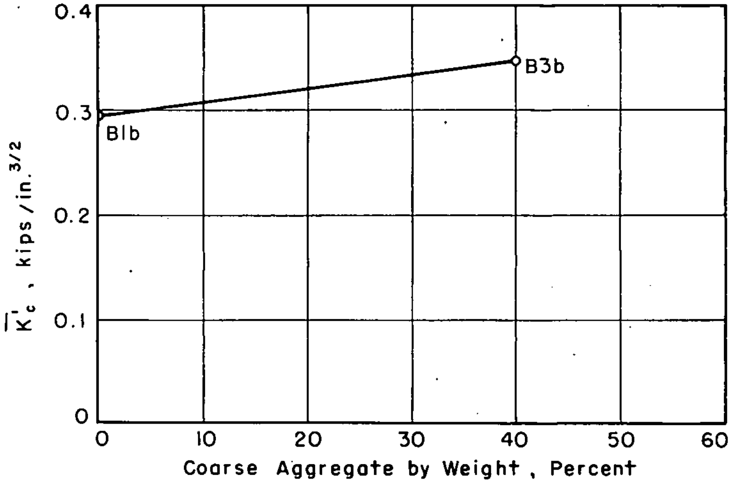

Figure 10. Effect of coarse aggregate on $\overline{K}_c'$; w/c = 0.55.

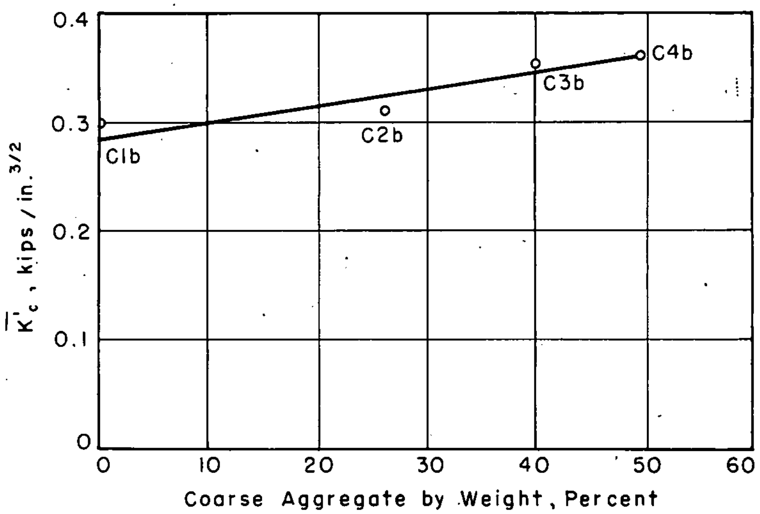

Figure 11. Effect of coarse aggregate on $\overline{K}_c'$; w/c = 0.60.

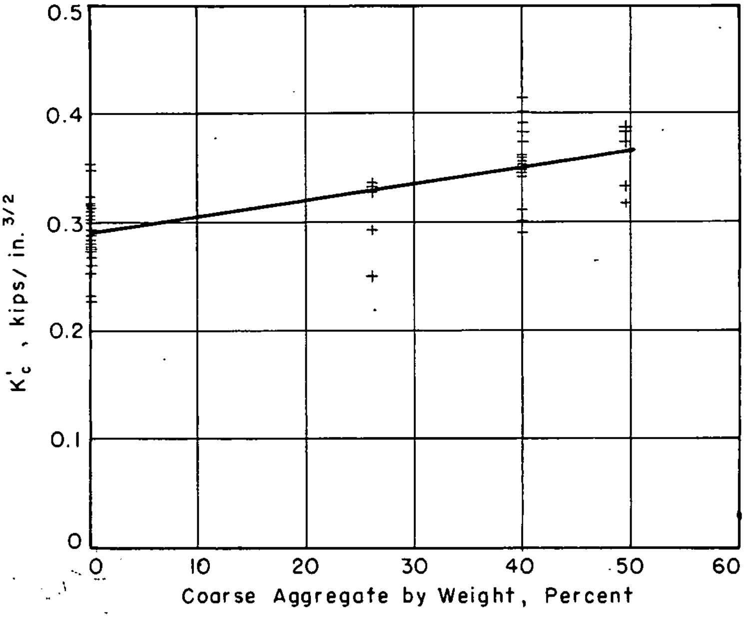

Figure 12. Effect of coarse aggregate on K_c'; all specimens.

0.55 water-cement ratio is shown in Figure 10; $\overline{K}_c'$ is 17.3 percent greater for the concrete than for the mortar. $\overline{K}_c'$ for one mortar and three concretes with a 0.60 water-cement ratio is shown in Figure 11. The least square straight line shows a significant increase in $\overline{K}_c'$ with respect to coarse aggregate content. K_c' for each specimen is plotted against the coarse aggregate content in Figure 12. Although these data include variation in water-cement ratio and sand-cement ratio, the least square straight line shows a significant increase in K_c' with respect to coarse aggregate content.

SUMMARY

Hypothesis for Fracture

The resistance of concrete to the propagation of an existing crack is a function of the cement paste matrix and also of the heterogeneity of the concrete. Initial cracks exist in the matrix and at the matrix-aggregate interface. These cracks will propagate through the matrix under certain conditions.

A fracture mechanics concept has been developed for homogeneous materials. An existing crack will propagate rapidly through a material if the energy released by a virtual crack extension supplies the total energy required for the formation of the new crack. The rate of energy released with respect to crack extension is related to a stress intensity factor, K, that is associated with the elastic fields near the crack tip. The critical stress intensity factor, K_c, corresponds to the onset of rapid crack propagation. It is a measure of the material resistance to crack propagation and is referred to as the fracture toughness of the material.

The fracture mechanics concept has been applied to cement paste. A zone of microcracks is assumed to form near the crack tip in cement paste. This microcracking increases the material resistance to crack propagation. The zone of microcracks grows with crack propagation until the zone reaches a limiting size. Slow crack growth occurs until the microcrack zone reaches this limiting size and then rapid crack propagation occurs. The critical stress intensity factor, K_{Pc}, is assumed to be a material property. The slow growth is initiated at a lower stress intensity factor, K_{Pi}, that is not necessarily a material property.

The application of fracture mechanics to concrete is complicated by the heterogeneity of the concrete. Cracks will propagate through the cement paste matrix when the stress intensity factor in the matrix reaches K_{Pi} (slow) and K_{Pc} (rapid). The concrete aggregates affect the elastic fields in the matrix. Crack development may be retarded when the effect of aggregates is a reduction in the stress intensity factor in the matrix.

A pseudo-stress-intensity factor, K', is obtained for concrete by analyzing it as a homogeneous material. The pseudo-fracture toughness, K'_c, is the critical value of K' at the onset of rapid crack propagation in the cement paste matrix. K'_c is related to the fracture toughness of the cement paste and to the arresting action of the aggregate.

$$K'_c = K_{Pc} + f(ARR) \tag{11a}$$

where K_{Pc} is the fracture toughness of the cement paste and f(ARR) is an arresting function that is dependent on the properties of the matrix and of the aggregate.

Experimental Results

Load-deformation curves of the specimens show four stages of behavior during loading. In the linear stage the deformation is proportional to the load; the curve may assume a lower slope as the specimen stiffness is reduced by rupture or unbonding of an aggregate. In the microcracking stage deformation starts to increase faster than the load as the assumed microcracks form near the crack tip. Deformations increase rapidly with a small increase in load in the slow growth stage. This corresponds to the slow crack growth that occurs while the assumed microcracking zone increases in size. In the fracture stage the deformations continue to increase without an increase in load. This corresponds to the rapid crack propagation associated with the final fracture.

The pseudo-fracture toughness was independent of the water-cement ratios of a series of three mortars. It was also independent of the water-cement ratios of a series of the concretes with the same aggregate percentages. The range of water-cement ratios investigated was from 0.50 to 0.60 for both the mortars and the concretes.

The pseudo-fracture toughness showed no apparent relation to the fine aggregate percentage of three mortars with a 0.50 water-cement ratio. The fine aggregate percentage ranged from 62 percent to 73 percent.

The pseudo-fracture toughness varied directly with the coarse aggregate content of mixes with constant water-cement ratio and fine aggregate content. The pseudo-fracture toughness of a concrete (30 to 40 percent coarse aggregate) was approximately 20 percent greater than the pseudo-fracture toughness of a mortar with the same water-cement ratio and fine aggregate content.

The experimental results were in general agreement with the hypothesis for fracture. The stages of crack development were reflected in the load-deformation curves of the specimens. The pseudo-fracture toughness increases with an increase in coarse aggregate content, indicating that an increase in crack arrest occurs.

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