Economic Rating and Spacing of LRT Traction Substations
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For propulsion modern light rail vehicles usually use DC power supplied by an external traction electrification system. The traction electrification system (TES) converts available medium-voltage AC power to low-voltage DC power that is then distributed to the trains. A typical TES consists of traction power substations connected to an existing utility grid and a DC distribution system. Two major types of DC distribution systems, differentiated by the final element on the power path from the substations to the vehicles, have evolved. They are the overhead contact system and the third rail, each with its own advantages and areas of application.

When a new light rail system is being designed, or an existing one extended, two major objectives become the center of attention:

1. To meet all performance, reliability, and safety criteria associated with transit operations and
2. To meet technical requirements at a minimum overall system cost.

In a broader sense, these two can be combined into one goal—to design an economically optimum TES under a set of constraints that represent performance, reliability, and safety requirements. This is a broad and far-reaching subject that is beyond the scope of this paper. Emphasis will be placed instead on the economic principles and relationships that can help in selecting TES parameters that result in economic design.

BASE COST FUNCTIONS

The acceptable ranges of the traction power substation rating and the line feeder size can be determined on the basis of technical feasibility, environmental or practical considerations, or a combination thereof. The unit costs of all feeder sizes and substation ratings can also be estimated and can be used to obtain corresponding curves, called base cost functions. They should include the total direct and indirect associated cost, materials, and labor.
For the substations (Figure 1), the base cost curve is defined as

\[ C_1 = q_1 \cdot P \]  

(1)

where

- \( P \) = nominal rating (kw)
- \( C_1 \) = substation cost ($1000/substation)

The substation cost will consist of equipment, site work (including land acquisition), and connection feeders.

For the DC distribution system, the base cost curve is defined as

\[ C_2 = q_2 \cdot A \]  

(2)

where

- \( A \) = overall cross-sectional area in thousands of circular mils (MCM)
- \( C_2 \) = line feeder unit cost ($/ft).

In case of overhead catenary systems (Figure 2), the line feeder cost will consist of overhead conductors, crossarm assemblies, and poles. In case of third rail systems, the line feeder cost will consist of the third rail with associated accessories.

The substation rating may be increased in increments of, say, 250 kw. The line feeder cross section increases with the standard conductor size increment and the number of conductors used. The base cost functions can be obtained analytically through the least squares curve fitting method. Polynomial approximations up to second degree would give satisfactory results.

PARAMETER OPTIMIZATION

As explained before, the TSB capital cost is a function of three interrelated parameters: substation rating, substation spacing, and line feeder size. For the established design criteria and permissible parameter ranges, there may exist many feasible solutions that consist of different combinations of substation ratings, spacings, and line feeder sizes. The economic solution that minimizes the overall system cost function may be obtained by the procedure outlined herein.

By selecting a certain substation rating \( P \) and varying the substation spacing, a series of line feeder sizes can be obtained starting from the minimum line feeder size. The greater the spacing, the larger the feeder line necessary to meet the voltage drop, ampacity, and short circuit current coordination requirements. The maximum spacing corresponding to a substation of rating \( P \) will be reached either when the substation short-term or long-term loading capabilities are exceeded or when the maximum line feeder size is reached, whichever comes first. Expressing the corresponding line feeder cost as a function of the substation spacing gives

\[ f_1 (L_s) = a_0 + a_1 L_s + a_2 L_s^2 \]  

(3)

where

- \( f_1 \) = line feeder unit cost ($/ft) and
- \( L_s \) = substation spacing in thousands of feet (MFT).

The unit substation cost (assuming the terminal substations are located approximately \( L_s/2 \) away from the end of the line) could be expressed as

\[ f_2 (L_s) = \frac{q_1 (P)}{L_s} \]  

(4)

Then the total system unit cost as a function of the substation spacing is given by

\[ C = f_1 (L_s) + f_2 (L_s) \]  

(5)

Assuming linear approximation for the function \( f_1 \) (which in most cases is accurate enough),
The minimum of this function, obtained through differentiation, is

\[ L_{opt} = \left( \frac{g_1(P)}{g_2} \right)^{1/2} \]  

(7)

Equation 7 reveals that the economical spacing of a substation of rating \( P \) is equal to the square root of the ratio of the total substation cost to the incremental change in the line feeder unit cost. This relationship will be more complex, resulting in a cubic equation, if the line feeder unit cost is not represented by a second degree polynomial.

The substation spacing \( L_{opt} \) obtained through Equation 7 can fall either within or outside the range of \( L_s \) in Equation 3. In the latter case, whichever substation spacing limit is closer to \( L_{sn} \) will be the most economical one.

After the set of economic substation spacings associated with each of the substation ratings has been derived, the system unit cost curves can be obtained. The first curve \( (X_1) \) represents the contribution of the substations to the overall unit cost; the second \( (X_2) \) represents a similar contribution from the DC distribution system.

Using the results from the parametric optimization obtained so far, the following relationships can be established:

\[ X_1 = X_1(P) \]  

(8)

and

\[ X_2 = X_2(P) \]  

(9)

where

\( X_1 \) = number of substations of the system as a function of the substation rating. Each point of the curve can be obtained by dividing the total line length by the established substation spacing corresponding to the rating \( P \).

\( X_2 \) = line feeder cross section as a function of the substation rating. Each point of the curve can be obtained by plotting the cross-sectional area corresponding to function \( f_1 \) from Equation 3. The spacing that corresponds to each substation size has already been obtained through Equation 7.

The substations unit cost function consequently can be expressed as

\[ y_1(P) = \left( \frac{X_1(P) \cdot g_1(P)}{L_s} \right) = d_0 + d_1 \cdot P + d_2 \cdot P^2 \]  

(10)

where

\[ y_1(P) = \text{substation unit cost curve ($/ft$)}, \]  
\[ L_s = \text{line length (ft)}, \]  
\[ d_0, d_1, d_2 = \text{coefficients of the second degree polynomial presentation}. \]

The DC distribution system cost function can be expressed as

\[ y_2(P) = g_2[X_2(P)] = e_0 + e_1 \cdot P + e_2 \cdot P^2 \]  

(11)

where

\[ y_2(P) = \text{feeder line unit cost curve ($/ft$)} \]  
and

\[ e_0, e_1, e_2 = \text{coefficients of the second degree polynomial presentation}. \]

Analysis of several \( y_1 \) and \( y_2 \) curves has indicated that both can be approximated to a good degree of satisfaction by a second degree polynomial (see Figure 3) using the least squares curve fitting method. Although the value of \( y_1(P) \) normally decreases with the increase of the rating \( P \), the unit cost increase due to the larger substation spacing may cause an increase in the substation unit cost.
line cost $y_2(P)$ on the other side exhibits an upward trend. The total TES unit cost function then can be expressed as

$$y = y_1(P) + y_2(P)$$

or

$$y = d_0 + d_1 \cdot P^2 + d_2 \cdot P + e_0 + e_1 \cdot P + e_2 \cdot P^2$$

The minimum of this function with regard to $P$ renders the economic substation rating ($P_e$). This minimum, obtained easily through differentiation, is

$$P_e = [(d_1 + e_1)/(2d_2 + 2e_2)]$$

The corresponding number of substations and the line feeder size can be found by substituting $P_e$ in Equations 8 and 9, respectively.

SAMPLE CASE

The procedure discussed in this paper was applied to the design of the Gauadalupe Corridor LRT system in San Jose, California. The system consists of approximately 21 mi of double track and was designed in accordance with the following basic concepts:

- Substation type: transportable, preassembled, walk-in, installed on concrete pads alongside the track;
- DC distribution system type: predominantly overhead catenary system with messenger wires serving as positive feeders; and
- Catenary systems of the two tracks paralleled electrically and supported by center poles with back-to-back crossarm assemblies.

The minimum substation rating was established as 1000 kw. The equivalent line feeder size included the catenary systems of both tracks, due to their electrical connection.

Table 1 gives the estimates that were used to establish base cost curves for substations. Table 2 gives the estimates that were used to establish base cost curves for equivalent line feeder (materials and labor).

The technical aspects of the analysis, such as establishing the maximum spacing for each substation and the corresponding minimum line feeder size, were performed with the help of computer simulations. Some of the relevant results from these studies are summarized in Table 3.

The maximum incremental cost change of the line feeder was roughly estimated to be in the neighborhood of $\Delta S = $3-4/ft-MFT. Substituting this value in Equation 7 results in economically optimal but
unconstrained substation spacing. It happens to be higher than the maximum permissible spacing obtained on the basis of technical requirements such as RMS and peak loads, ampacity, and voltage level constraints. For the 1000-kw substation, for example,

\[ L_{x} = 1/2 \left( \left[ \frac{g_{2}(P)}{a_{1}} \right] \right) \]

\[ = 1/2 \left( \left[ \frac{425 (\$1000)}{1/4 (\$/ft-MFT)} \right] \right) \]

\[ = 10.3 \text{ MFT} \]

In view of the load pattern, the 1000-kw substations cannot be spaced at such a distance, almost 2 mi, without overloading. Therefore the technically permissible spacing takes precedence over the economically ideal one.

The curves represented by Equations 8 and 9 need not be in analytical form. A table of discrete values related to the substation ratings (P) would be sufficient. Using the substation spacings and equivalent line feeder sizes obtained previously, these two functions can be expressed in tabular form:

<table>
<thead>
<tr>
<th>P (kw)</th>
<th>X1 (no.)</th>
<th>X2 (MCM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>20</td>
<td>1500</td>
</tr>
<tr>
<td>1500</td>
<td>14</td>
<td>2000</td>
</tr>
<tr>
<td>2000</td>
<td>10</td>
<td>2800</td>
</tr>
</tbody>
</table>

Finally, the TES unit cost curves \( y_{1} \) and \( y_{2} \) can be obtained through Equations 10 and 11. In tabular form these are:

<table>
<thead>
<tr>
<th>P (kw)</th>
<th>( y_{1} ) ($/ft)</th>
<th>( y_{2} ) ($/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>72.73</td>
<td>26.32</td>
</tr>
<tr>
<td>1500</td>
<td>61.72</td>
<td>34.14</td>
</tr>
<tr>
<td>2000</td>
<td>54.10</td>
<td>41.48</td>
</tr>
</tbody>
</table>

The total line length is approximately \( L_{L} = 110 \text{ MFT} \), including a 1 1/2-mi branch off the main route.

The analytical expressions of these two functions, obtained through the least squares approximation method, are:

\[ y_{1} = 104.922 - 0.03897P + 6.87 \times 10^{-4}P^{2} \]

\[ \text{dollars per linear foot} \]

and

\[ y_{2} = 33.24 - 0.01796P + 11.04 \times 10^{-4}P^{2} \]

\[ \text{dollars per linear foot} \]

Equation 13 will lend the economic substation rating. Substituting, the following is obtained:

\[ F_{e} = -\frac{(-0.03897 - 0.01796)/(2[6.871 \times 10^{-4} + 11.04 \times 10^{-4}])}{1} = 1589 \text{ kw} \]

The closest substation rating, using 250-kw increments is 1500 kw. To assess the sensitivity of the solution, the system unit cost function (Equation 12) is calculated for all substation ratings of the 1000- to 2000-kw range. The results are as follows:

<table>
<thead>
<tr>
<th>P (kw)</th>
<th>( y ) ($/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>95.95</td>
</tr>
<tr>
<td>1250</td>
<td>94.98</td>
</tr>
<tr>
<td>1500</td>
<td>92.85</td>
</tr>
<tr>
<td>1750</td>
<td>91.38</td>
</tr>
<tr>
<td>2000</td>
<td>95.58</td>
</tr>
</tbody>
</table>

**CONCLUSIONS**

Conflicting views have been expressed with regard to the selection of traction power substation rating and spacing. On one side there is the view that the substations should be frequently spaced and as small as possible, each substation just large enough to withstand its share of the current of two accelerating trains in the vicinity. There are also proponents of the opposing view, that traction power substations should be as large and spaced as far apart as allowed by the line feeder size, technical feasibility, or practicality or by some other considerations of a technical nature such as excessive track potentials. However, neither of these approaches ensures minimum overall system cost.

The method presented herein is an attempt to develop a systematic and analytical procedure for finding a combination of TES parameters that results in the least expensive technically acceptable system. It requires somewhat greater engineering effort in the design stage, but the reward can be a significant reduction of the traction electrification system capital cost. In the sample case, there is $6.2/ft differential between the maximum and the minimum values of the unit cost function. This is equivalent to $682,000 or approximately 8 percent of the actual procurement cost.

More experience with TESs that have various load patterns and with different components of cost structure is needed, however, before generalized assessments of the magnitude of potential savings can be made.