SENSITIVITY ANALYSIS OF THREE RIGID PAVEMENT DESIGN TECHNIQUES

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Three rigid pavement design techniques were studied to determine the relative effects of various design parameters on pavement thickness. The analysis phase of this investigation consists of formulating each technique into a comprehensive mathematical or graphical thickness model. An evaluation of the influences on thickness of the major design factors was accomplished by a sensitivity analysis with 1 theoretical and 2 practical measures of parameter importance. The theoretical measure reveals considerable differences among techniques as to the process of resolving design thicknesses and as to the relative theoretical influence of various parameters that estimate the same major design factors. Practical measures of parameter importance, which account for variations in parameter values as well as the manner in which the design variables are formulated in the thickness model, show a greater consistency in the importance of generic factors among design methods. Although the relative importance of the traffic load differs with the 3 design methods investigated, this parameter and the flexural strength of the pavement are the influential factors in the design of rigid pavement thicknesses.

•THE OBJECTIVE of rigid pavement design is the provision of an acceptable riding surface that can withstand the deteriorating effects of traffic and environment for the service life of the facility. This goal is considered an integral part of the total highway transportation program and is constantly sought in quantitative measures. Various measurable parameters are used to quantify the physical demands imposed on the pavement structure and the subjective desires of road users for a good riding surface. As a result, several design techniques have been developed to combine in a logical manner these design parameters to determine the required pavement thickness.

Pavements designed for the same traffic load, soil support, and environmental conditions, built of the same quality of materials and workmanship, and expected to exhibit similar performance characteristics should be equal in thickness regardless of the agency responsible for the design and construction. Such agreement is not the case because, in addition to the differences of the various design procedures available, much engineering experience and subjective judgment enter into the decisions required for resolving the design of pavement thicknesses.

Determining a satisfactory design thickness is inherently difficult, and the optimum design cannot be ascertained even though the major factors affecting pavement thickness have been identified. Part of this problem is due to the uniqueness of the road structure and the conditions under which the facility must serve. A pavement is a thin narrow structure relative to its length and is built at or near the ground surface. Thus, a single design, which is seldom changed during the contracted length of the roadway, must satisfy a variety of subgrades and environmental influences. The heterogeneous nature of pavement-building materials and their changing behavior with time and ambient conditions also contribute to the uncertainty of the pavement design

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process. Although design is logically influenced by the performance expected of the pavement, methods of pavement evaluation and definitions of failure conditions have not always been clearly established for economical engineering purposes.

The purpose of this research was to identify and examine the relative effects on design thickness of various design parameters that measure subgrade support, traffic load, pavement material properties, environmental factors, and performance criteria (2). Three rigid pavement design techniques, which are representative of present design practices, were analyzed to define the important set of pavement design variables considered in each design method. The relative importance of these design factors was determined in a sensitivity analysis that was developed to investigate the impact of changes in parameter values on the rigid pavement structure.

PROCEDURE

The procedure used in the analysis of selected rigid pavement design techniques is subdivided into 3 phases: selection of pavement design methods, modeling of the design techniques, and sensitivity analysis.

Selection of Pavement Design Methods

Because numerous techniques exist for the design of rigid pavements, selection of several design methods was necessary to carry out this research investigation. Common usage of the technique, a rational approach to resolving the design thickness, and the availability of literature pertaining to the design method were the main criteria used in selecting the 3 design methods for analysis (5). The rigid pavement design methods that best satisfy these criteria and that are representative of present design practices are those issued by AASHO (1), the Corps of Engineers (4), and the Portland Cement Association (3).

Modeling of the Design Techniques

After a thorough review of the literature pertaining to the selected pavement design methods, models of each design method were formulated to mathematically or graphically systematize the design-variable relations for the purpose of executing the sensitivity analysis. Design charts to facilitate the determination of the required pavement thickness and mathematical equations that are basic to the various design processes are available for each method chosen for this research investigation.

Sensitivity Analysis

The effects on thickness of the factors considered in pavement design were quantitatively evaluated in a sensitivity analysis. Applied to each design technique investigated, the sensitivity analysis basically examined the change in design thickness produced by changes in the various design parameters.

The underlying premise of the sensitivity analysis is that, as the change in required pavement thickness produced by a design parameter becomes larger, the more important that parameter becomes in the design method. On this basis, the rate of change of thickness with respect to a particular parameter (quantified by the first partial derivative of the thickness with respect to that parameter) defines the theoretical measure of parameter importance. On the other hand, the magnitude of the thickness change produced by comparable parameter variations specifies the practical measure of parameter importance.

Theoretical Measure of Design Parameter Importance—The relative theoretical importance is defined as the positive ratio of the partial derivative of a thickness with respect to a particular parameter to the sum of the absolute values of the partial derivatives for all parameters. This determination is symbolically written as

$$RPP = 100 \times \left| \frac{\partial T}{\partial P_i} \right| \div \sum_{i=1}^{n} \left| \frac{\partial T}{\partial P_i} \right| \qquad (1)$$

where

RPP = relative percentage of a partial,

$\left| \frac{\partial T}{\partial P_i} \right|$ = absolute value of the first partial derivative of the thickness function T with respect to the design parameter P_i, and

$\sum_{i=1}^{n} \left| \frac{\partial T}{\partial P_i} \right|$ = summation of the absolute values of all partial derivatives considered in the sensitivity analysis of a design method.

The above relative percentage adequately determined the importance of each parameter as formulated in the thickness function, but this technique implicitly assumes that parameter variations are numerically "small" and equal.

Measures of Practical Importance of Design Parameters—The sizes of parameter variations encountered in actual pavement design practice differ considerably among design parameters, and these increments of parameter change influence the range of design thicknesses required by each parameter. Therefore, the sensitivity analysis was expanded to examine the combined effects of both the manner in which parameters were included in the thickness function and the actual parameter variations that occur. This combination resulted in 2 measures of practical importance. One relative percentage measure of importance is mathematically shown as follows:

$$RPTC = 100 \times \left| \frac{\partial T}{\partial P_i} dP_i \right| \div \sum_{i=1}^{n} \left| \frac{\partial T}{\partial P_i} dP_i \right| \quad (2)$$

where

RPTC = relative percentage of the thickness change,

$\left| \frac{\partial T}{\partial P_i} dP_i \right|$ = absolute value of the product of the first partial derivative of the thickness function T with respect to the design parameter P_i and the incremental parameter change dP_i, and

$\sum_{i=1}^{n} \left| \frac{\partial T}{\partial P_i} dP_i \right|$ = summation of the absolute values of all parameter products (partial derivative × increment) considered in the sensitivity analysis of a design method.

If appropriate dP_i increments are chosen as representative of actual parameter variations, then the absolute value of the term $\frac{\partial T}{\partial P_i} dP_i$ becomes a realistic measure of the change in thickness caused by a pavement design parameter. The portion of the total thickness change produced by a parameter and calculated as a percentage of the total change provides a relative measure of design parameter importance with respect to the total design process rather than the formulated thickness function alone.

Because the validity of the relative percentage of the thickness change may be questionable as deviations from the mean parameter values become large, a second approach was devised as a corroborating measure of the practical importance of design parameters. If 2 different values are selected for the same parameter, then 2 corresponding thicknesses are determined in the design of a rigid pavement. The difference between these 2 thicknesses is interpreted as the actual change in the design thickness produced by the parameter and its variation if the 2 parameter values are indicative of realistic parameter variations normally encountered. Based on this uncomplicated concept, a thickness change caused by a variation in one parameter can be expressed as a percentage of the sum of all changes that are similarly calculated for each design parameter. This second measure of practical importance is summarized by the following equation:

$$RPATC = 100 \times \left| \Delta T_{P_i} \right| \div \sum_{i=1}^{n} \left| \Delta T_{P_i} \right| \quad (3)$$

where

$RPATC$ = relative percentage of the actual thickness change,

$|\Delta T_{P_i}|$ = absolute change in thickness due to the P_i parameter, and

$\sum_{i=1}^{n} |\Delta T_{P_i}|$ = sum of all the absolute thickness changes produced by all parameters in the design method under consideration.

Because very little difference was observed between the 2 measures of practical importance, only evaluations for the partial differential method are presented in this report of the sensitivity analysis of 3 rigid pavement design methods. The increment of parameter change was always selected to approximate 1 standard deviation of the "population" for each design parameter.

RESULTS

The techniques of sensitivity analysis that were developed in the preceding section were applied to the design methods of AASHO, Corps of Engineers, and PCA. Both theoretical and practical importance measures were quantified to demonstrate the significance of each design parameter in determining the resultant rigid pavement thickness.

AASHO

The sensitivity analysis of the AASHO rigid design method used the following equation (1):

$$\log W = 7.35 \log (D + 1) - 0.06 + \frac{\log \frac{c_o - p}{c_o - 1.5}}{1 + \frac{1.624 \times 10^7}{(D + 1)^{8.46}}} + (4.22 - 0.32p) \log \left[\frac{f_t (D^{0.75} - 1.132)}{690 \left(D^{0.75} - \frac{18.416k^{0.25}}{E^{0.25}} \right)} \right] \quad (4)$$

where

W = total number of 18-kip single-axle load applications;
D = concrete pavement thickness, in.;
c_o = initial serviceability index;
p = terminal serviceability index;
f_t = 0.75 × modulus of rupture of concrete = working flexural strength, lb/in.^{2};
k = modulus of subgrade reaction, lb/in.^{3}; and
E = modulus of elasticity, lb/in.^{2}.

Each of the variables in the above equation was investigated in the sensitivity analysis to evaluate its relative influence on the design thickness D. Although importance measures were calculated for many combinations of design parameter values, a summary of the sensitivity analysis is presented only for likely combinations in this report.

Summary plots of theoretical parameter importance are shown in Figure 1 for terminal serviceability indexes of 2.0 and 2.5. The terminal and initial serviceability indexes are theoretically the most important design parameters in the formulated thickness function. In addition, design conditions that are reflected by subgrades and concretes of different strengths and traffic loads of different magnitudes have little influence on the theoretical importance of the AASHO parameters.

If a subgrade modulus of 100 lb/in.^{2}, a concrete working flexural strength of 450 lb/in.^{2}, a terminal serviceability index of 2.0, an initial serviceability index of 4.0, and a 4-lane highway operating at a volume level of 1,200 passenger cars per hour per lane with 20 percent trucks are regarded as a typical design situation, then the approximate corresponding relative theoretical parameter importances are 77 percent for the terminal serviceability index, 21 percent for the initial serviceability index, and 2 percent for the concrete, the subgrade, and the traffic parameters. If the terminal serviceability index is increased from 2.0 to 2.5 and no changes are made in the other design parameters, then 65, 33, and 2 percent are respectively the relative theoretical parameter importances for the terminal serviceability index, the initial serviceability index, and the combination of the concrete, the subgrade, and the traffic design parameters. Thus, approximately 98 percent of the pavement thickness is based on the 2 measures of pavement serviceability when the sensitivity analysis of the AASHO rigid design method is performed to ascertain the relative theoretical importance of the various design parameters.

Practical measures of relative importance, which provide a more realistic determination of parameter importance in the design process by accounting for the actual amount of variation that occurs in each factor, are shown in Figure 2 for the AASHO rigid design method. Ranges of practical importance that represent various percentages of trucks and traffic volumes are shown by short horizontal lines and extended curves respectively. For parameters of lesser importance, the short lines or extended curves are not distinguishable, and a single line adequately represents the range of these practical measures. The 5 short lines identify parameter importance for a traffic stream composed of 10, 20, 30, 40, and 70 percent trucks, and the 3 extended lines describe practical importance for a highway that carries the equivalent of 1,200, 1,500, and 2,000 passenger cars per hour per lane. Because greater traffic factor values imply higher volumes of traffic or a larger percentage of trucks, the conditions represented by each line can easily be ascertained.

Because each plot demonstrates the controlling importance of the AASHO traffic parameter, the total equivalent 18-kip single-axle load was concluded to be the most important parameter influencing the design objective. The concrete modulus of rupture ranked second in practical importance. However, as the weight and frequency of vehicular traffic increase, the traffic parameter becomes less important, and the concrete flexural strength and the terminal serviceability index assume an increasing importance in the determination of rigid pavement thicknesses. If the same typical design situation as appraised for the theoretical parameter importance is again assumed for a pragmatic evaluation of the AASHO design factors, then the relative practical parameter importances are 60 percent for the traffic parameter, 20 percent for the flexural strength parameter, 9 percent for the modulus of subgrade reaction, 9 percent for the terminal serviceability index, and 2 percent for the initial serviceability index. Although the AASHO traffic parameter shows the most significant increase in practical importance as compared to the theoretical measure in which it had no influence, the serviceability indexes exhibit the most prominent decrease. In both sensitivity analyses according to the theoretical and the practical importance of each design parameter, the measure of subgrade support is relatively insignificant in the thickness determination of rigid pavements.

Corps of Engineers

Because the rigid design method of the Corps of Engineers was based substantially on the plotted empirical relations (4), the design chart shown in Figure 3 was selected as the best available model for the sensitivity analysis. The few mathematical equations pertaining to this design technique did not express the relations used to prepare the design chart and, hence, could not be used to yield the necessary information that could be obtained by graphical interpretation of the design chart.

In the evaluation of the theoretical importance of the various parameters, a graphically determined first-order partial derivative of thickness with respect to each design parameter was expressed as a percentage of the sum of all partial derivatives for a wide range of parameter-value combinations. The theoretical importance measures are shown in Figure 4 for 2 common design conditions. An overwhelming theoretical

Figure 1. Relative theoretical importance of AASHO rigid design parameters.

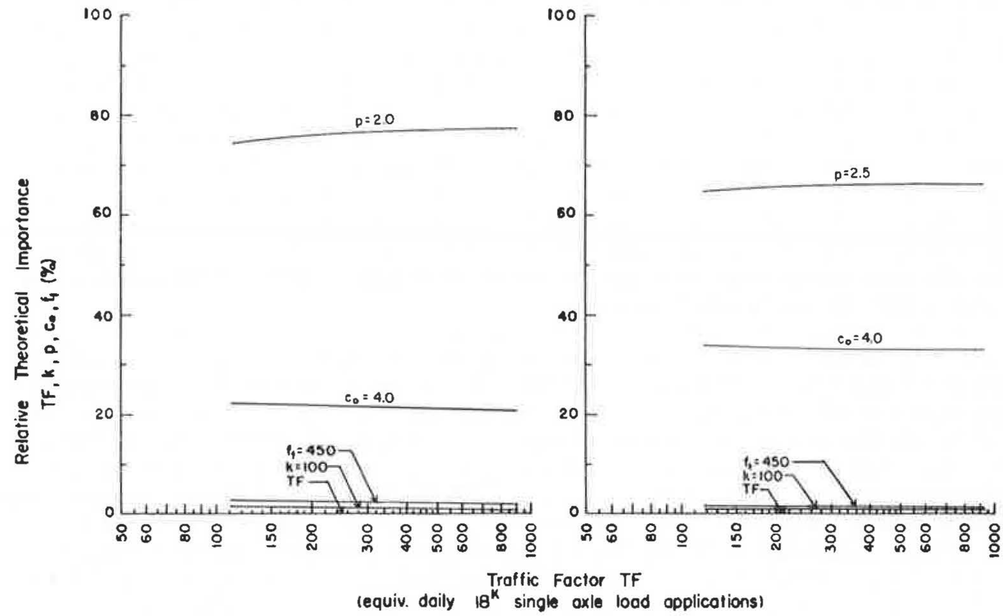


Figure 2. Relative practical importance of AASHO rigid design parameters for total differential approach.

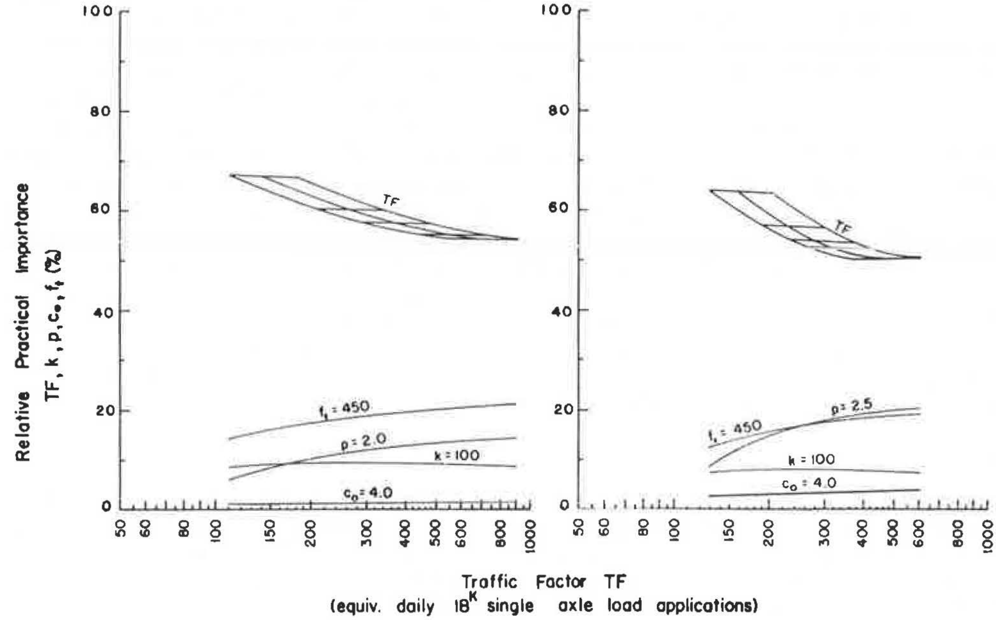


Figure 3. Rigid design chart of Corps of Engineers.

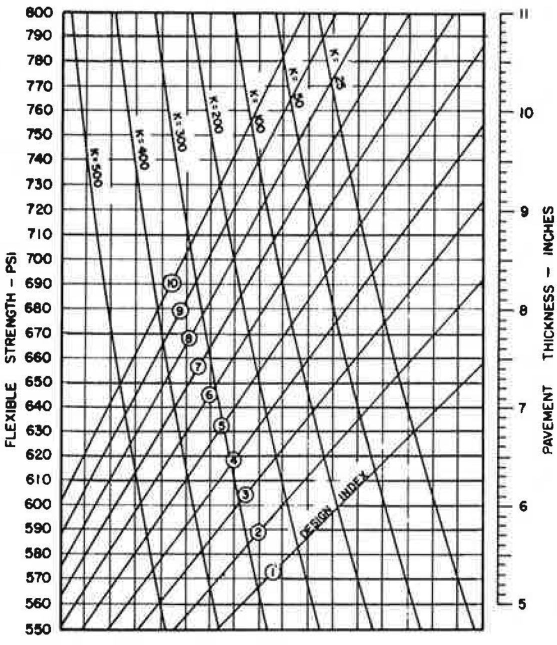
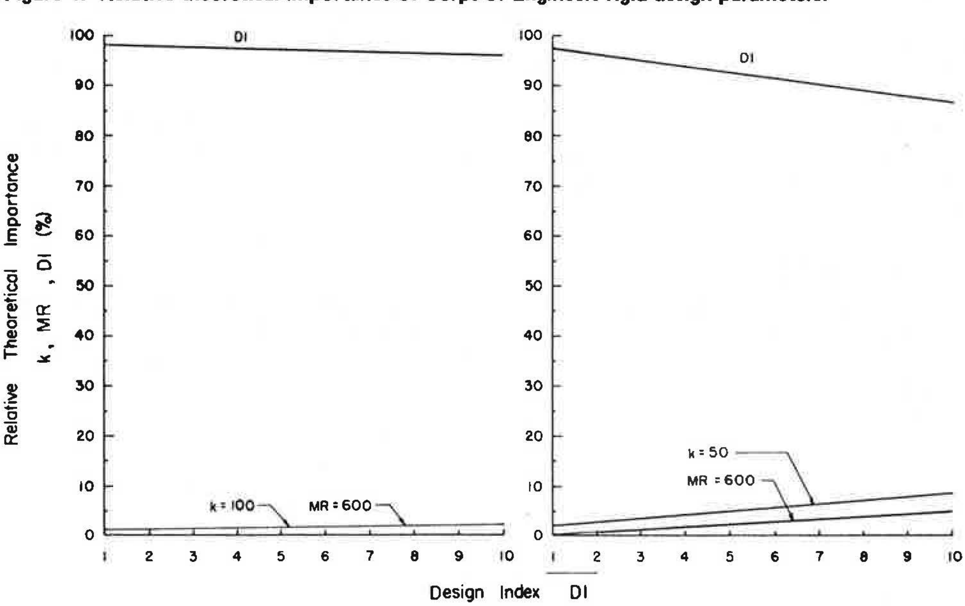


Figure 4. Relative theoretical importance of Corps of Engineers rigid design parameters.



importance is evidenced for the design index (traffic) parameter for most design situations. Either the subgrade modulus or the modulus of rupture accounts for no more than 10 percent of the relative rate of thickness change, and the design index accounts for no less than 85 percent of that measure of importance. If actual parameter variations are neglected, the Corps of Engineers rigid thickness requirements could essentially be quantified by a traffic load estimation alone.

Parameter variations, however, are an integral part of the design process and were considered in developing the relative practical measures of parameter importance. To predict representative variations in the Corps of Engineers rigid traffic parameter, we assumed that the design index was directly and linearly proportional to the traffic parameter used in the California Division of Highways stabilometer design method for flexible pavements. Thus, readily determined variations in the California stabilometer traffic index were translated into similar approximate variations of the Corps of Engineers rigid design index parameter. This method of estimating Corps of Engineers rigid design index variations does not enable average daily traffic or percentage of trucks to influence directly the increments of change in this design parameter. Design index values greater than 5 do not apply to the usual pneumatic-tired vehicular traffic and were neglected in the practical importance measures.

Because the practical importances of the design parameters were nearly identical for both the "portions-of-the-total-differential" and the "relative-thickness-changes" approaches, only values calculated by the former method are shown in Figure 5 for subgrade reaction values of 100 and 50. For the average conditions described by a design index of 4.0 (k = 100), the relative practical parameter importance is approximately 38 percent for the modulus of rupture, 42 percent for the subgrade modulus, and 20 percent for the design index. As the design index value decreases, there is a significant decrease in its relative importance. A decrease in the modulus of subgrade reaction, which implies weaker supporting soils, indicates a slight increase in the importance of that parameter with a corresponding decrease in the importance of the design index. Stronger concretes as reflected by increases in the value of the modulus of rupture produce a slight decrease in the importance of both the design index and the modulus of rupture and an increase in the importance of the subgrade reaction.

For the Corps of Engineers rigid design method, the modulus of subgrade reaction and the modulus of rupture are the important design variables. The traffic parameter is ranked third for the assumptions used in developing the practical importance measures.

Portland Cement Association

Although further development of thickness models for the sensitivity analysis of the AASHO and the Corps of Engineers rigid processes was not necessary, the following mathematical model of the total PCA rigid design process (3) was formulated in this research investigation:

$$\log\,[\mathrm{ADT}\ (\mathrm{PTT})\ \mathrm{AT}_i] = \frac{20.24P_i}{(\mathrm{MR})\ D^2}\left[1.0 - \frac{0.2034(a_i)^{0.500}(k/D^3)^{0.125}}{0.925 + 0.0091(A_i)(k/D^3)^{0.250}}\right] - 12.0 \qquad (5)$$

where

ADT = average daily traffic in both directions, vehicles per day;
PTT = percentage of total traffic that is trucks;
AT_i = axles per 1,000 trucks for axle load class i;
P_i = design load equal to the midpoint of axle load class i, lb;
MR = modulus of rupture, lb/in.^{2};
D = pavement thickness, in.;
a_i = radius of contact, in.; and
k = modulus of subgrade reaction, lb/in.^{2}

This design model relates the modulus of subgrade reaction, the modulus of rupture, and the 3 factors characterizing traffic loads to the required thickness for a 125 percent fatigue resistance as the failure criterion. In the development of this equation for the sensitivity analysis, reasonable assumptions were made in regard to the lateral distribution of traffic, the axle load allocations among vehicles in the traffic stream, and the design life of the facility.

The relative theoretical importance of each design parameter as indicated by the first partial derivative of the thickness function is shown in Figure 6 for modulus of rupture values of 600 and 500 $lb/in.^2$. To facilitate the presentation of the relative importance measures, appropriate California stabilometer traffic index values were selected to summarize the traffic parameter for the 3 PCA factors that describe traffic conditions. For "average" conditions (MR = 600 $lb/in.^2$, k = 100 $lb/in.^2$, and TI = 12), the relative theoretical importance measures are 66 percent for the modulus of rupture, 31 percent for the subgrade modulus, 2 percent for the axles per 1,000 trucks parameter, 1 percent for the percentage of trucks, and negligible importance for the average daily traffic.

The same combinations of design parameter values were also used in the evaluation of practical parameter importance as shown in Figure 7 (MR = 600). For the same average conditions, the relative practical importances are 60 percent for the modulus of rupture, 25 percent for the subgrade modulus, 15 percent for the average daily traffic, and a negligible amount for the other 2 traffic factors. This practical measure of importance indicates a remarkable increase in the importance of the average daily traffic as a design parameter, but little change is noted from the theoretical measures for the other parameters. In this comparison between the theoretical and the practical measures of relative importance, the average daily traffic parameter increased from 0 to 15 percent, and the relative importances of the modulus of rupture for concrete and the subgrade modulus were each reduced on the average by 6 percent. The remaining 3 percent is accounted for by the loss of importance in the parameters of the axles per 1,000 trucks and the percentage of trucks. Therefore, the modulus of subgrade reaction and the modulus of rupture are the more important parameters in the PCA rigid design process.

SUMMARY OF RESULTS AND CONCLUSIONS

The AASHO, Corps of Engineers, and Portland Cement Association design methods were analyzed to identify and examine the relative effects on thickness of the various factors considered in these pavement design processes. A modeling of the design technique, a measure of relative theoretical parameter importance, and 2 measures of relative practical parameter importance were employed in the investigation of these rigid pavement design methods. Although the design objective of an adequate highway pavement to serve the imposed physical and subjective demands is common to all pavement design techniques, the factors considered and their manner of employment are particular to each design technique. Subgrade characteristics, traffic loads, concrete flexural strength, and performance criteria are generally regarded as the primary factors affecting rigid pavement design.

After a model of each technique that was adequate for the sensitivity analysis was formulated, a quantitative evaluation of the theoretical influences of major factors on thickness delimited the inconsistencies among rigid design techniques. Although the Corps of Engineers rigid traffic parameter is of considerable theoretical importance, the AASHO and the PCA rigid traffic parameters have a negligible influence in the formulated thickness functions. In a similar manner, the design parameters of soil support and pavement material are theoretically important elements in the design of a rigid pavement by the PCA rigid process but unimportant in the AASHO and the Corps of Engineers rigid methods. A numerical summarization of the theoretical importance of the various design parameters is given in Table 1 for each design method.

Dissimilarities were also evident among the 3 rigid pavement design methods as a result of the sensitivity analysis for practical importance. The relative values of practical importance are given in Table 1 for the design parameters that are appropriate

Figure 5. Relative practical importance of Corps of Engineers rigid design parameters.

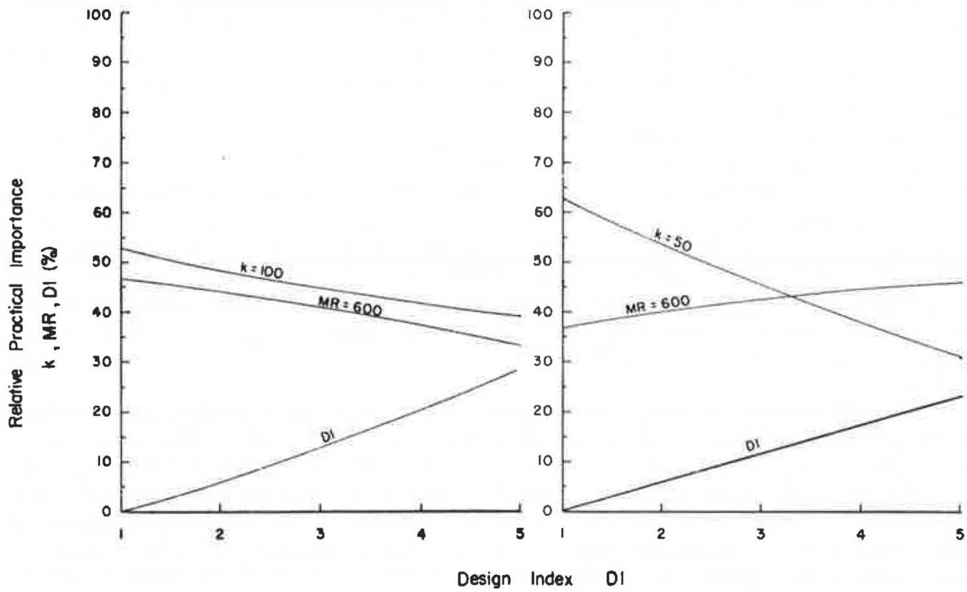


Figure 6. Relative theoretical importance of PCA rigid design parameters.

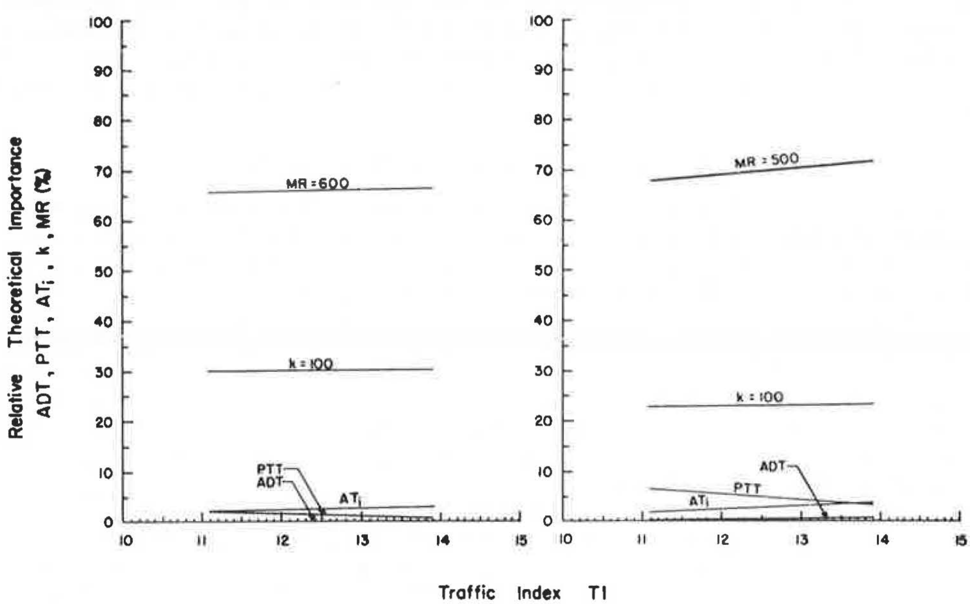


Figure 7. Relative practical importance of PCA rigid design parameters.

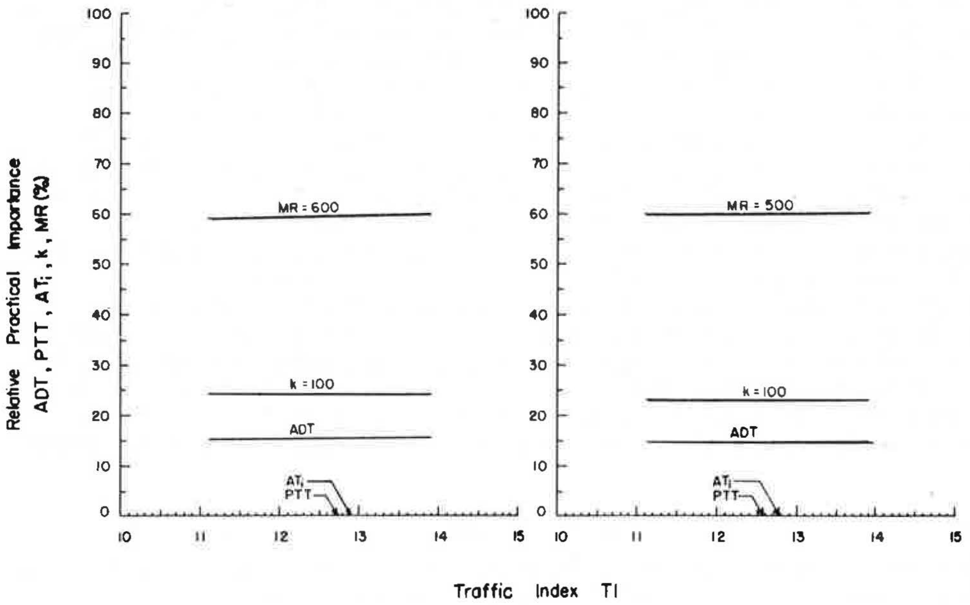


Table 1. Parameter importance in percent for average design conditions.

Design Method	Importance Measures	Traffic[a]	Supporting Soil k	Pavement Material[b]	Initial Performance c_0	Terminal Performance p
AASHO	Theoretical	0	1	1	21	77
	Practical	60	9	20	2	9
Corps of Engineers	Theoretical	98	1	1	—	—
	Practical	20	42	38	—	—
PCA	Theoretical	0, 1, 2	31	66	—	—
	Practical	15, 0, 0	25	60	—	—

[a]TF for AASHO; DI for Corps of Engineers; and ADT, PTT, and AT_i respectively for PCA.
[b]f_t for AASHO and MR for Corps of Engineers and PCA.

to each pavement design procedure. Although the traffic parameter is highly important in the AASHO rigid design method, the input of traffic data provides only minor practical importance in the determination of rigid pavement thicknesses by the Corps of Engineers and the PCA design processes. On the other hand, soil support is the most important design parameter in the Corps of Engineers rigid design method, although the strength of the pavement material occupies a slightly less degree of practical importance. The strength characteristics of concrete are highly important in the PCA rigid design procedure. However, the pavement material provides less input in the thickness determination for both the Corps of Engineers rigid and the AASHO rigid design methods. Soil support does account for some practical importance in the PCA rigid design procedure, but this design parameter has little quantitative impact on pavement thickness when the AASHO rigid design method is selected.

This sensitivity analysis of the physical variables and the subjective factors that affect thickness and expected pavement performance provides the design engineer with greater insight into the decision-making process of accomplishing the structural design of rigid pavements. The effect of actual deviations associated with these parameter design values identifies those phases of the design process that require closer attention and study and indicates those areas where design information is exceedingly precise.

REFERENCES

1. AASHO Interim Guide for the Design of Rigid Pavement Structures. Committee on Design, American Association of State Highway Officials, 1962.
2. Buick, T. R. Analysis and Synthesis of Highway Pavement Design. Joint Highway Research Project, Purdue University, Rept. 13, July 1968.
3. Thickness Design for Concrete Pavements. Portland Cement Association, Chicago, 1966.
4. Rice, J. L. An Evaluation of the Corps of Engineers Pavement Design Criteria Utilizing the Results Obtained From the AASHO Road Tests. Ohio River Division Laboratories, U.S. Corps of Engineers, Tech. Rept. 4-41, 1966.
5. Yoder, E. J. Principles of Pavement Design. John Wiley and Sons, New York, 1959.