ALGORITHM FOR A REAL-TIME ADVISORY SIGN CONTROL SYSTEM FOR URBAN HIGHWAYS

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To reduce acceleration noise and thereby accident probability, an on-highway traffic-responsive control system is proposed that transmits advance warning of impending slowdowns. In the proposed system, the control system output consists of command settings for advisory speed signs that are spaced along the highway at intervals of 0.1 mile (0.16 km). The system input consists of speed and vehicle-count information from roadbed detectors, also spaced at 0.1 mile intervals. This paper details the derivation of the sign control algorithm that culminates in a formula expressing the speed setting of an advisory sign in terms of the detected speed and vehicle-count data. The control algorithm is designed to influence the trajectory of a vehicle at times of impending slowdowns such that its contribution to the acceleration noise integral is minimized. Examples are presented that show the application of the sign setting algorithm to hypothetical traffic situations.

•ENTRANCE RAMP METERING is the only dynamic traffic-responsive method currently being employed to regulate the flow on heavily traveled expressways. However, entrance ramp metering is limited in that it exerts no control over vehicles already on the roadway. In dense traffic conditions it is well known that the unpredictable response of an inattentive driver can initiate a shock wave that causes upstream traffic to sharply decelerate or come to a prolonged standstill. Such an occurrence, which cannot be substantially mitigated by ramp control, produces severe strain and discomfort among drivers and increases the probability of an accident. A system that transmits advance warning of impending decelerations would be a stabilizing influence, serving to dampen the amplifying effect of car-following in dense traffic. Such a system would improve the driver's level of comfort and would substantially reduce the incidence of rear-end collisions. The reduction in accidents would mean less delay and higher average flows as well as a more reliable trip time.

The type of advance warning system that is presented is a real-time, computer-controlled advisory speed sign system. Such a system is intended for conditions of dense traffic where the demand restricted by ramp control is less than the capacity flow rate. The control system output consists of command settings for the advisory signs located along the highway at intervals of 0.1 mile (0.16 km). Roadbed detectors, also spaced at 0.1-mile intervals, continually monitor and transmit speed and density data to an on-line computer. Processing of this information in accordance with a control algorithm results in command settings for the advisory speed signs. These sign settings are considered to be valid for a 5-second period, after which traffic conditions are reevaluated to determine updated sign settings. For practical reasons, the sign settings are restricted to 5-mph (8-km/hour) increments, and the rate at which signs are changed is such that a driver is not presented with a rapidly varying sequence of sign settings.

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An advance warning advisory sign system is based on the anticipation of the propagation of any significant slowdown to upstream vehicles after the delay due to car-following. Such a system sets the upstream signs to values somewhat lower than the current speeds on the upstream sections. The intended effect is to induce a milder deceleration in advance of the natural car-following response.

Any driver will be predisposed to a certain degree of compliance with the message on the advisory sign, ranging from full compliance to merely a state of preparedness. Noncomplying drivers are forced to decelerate sharply because of either the deceleration of a complying driver further downstream or the propagation of the original disturbance by car-following. Thus, drivers are expected to gain confidence in the sign's message by reinforcement. The driver will adjust his response to the sign such that the ensuing response to the car ahead is not too uncomfortable. By controlling the signs in a consistent manner and by turning off all signs except those where a significant response is necessitated, the highest possible level of compliance will be elicited.

The sign control algorithm, as detailed in this paper, is derived on the basis of a single lane of traffic with no passing. It is likely that a control strategy formulated for a single-lane roadway may be applied to a multilane roadway by the introduction of appropriate averages over the several lanes. A justification of control on the basis of a lane composite is that in heavy traffic the flow on all lanes of a particular section of roadway exhibits a certain uniformity. Thus, it is assumed that the single lane represents one lane of a multilane expressway on which passing opportunities are rare because of dense traffic conditions.

Because it has been shown (1) that a quantitative measure of driver comfort and safety is the acceleration noise,

$$\sigma^2 = \frac{1}{T} \int_0^T [a(t)]^2 \, dt$$

the objective of advisory sign control is to minimize the acceleration noise summed over all vehicles. Thus, the sign control algorithm attempts to influence the trajectory of a typical vehicle at times of impending slowdowns such that the contribution of its deceleration to the integral $\int a^2(t)\,dt$ is minimized. Based on a calculus of variations solution to a simplified highway situation, a desired average deceleration is determined for vehicles on each 0.1-mile section of highway upstream of the slowdown. The desired deceleration is expressed in terms of measured section speeds and densities. An algorithm is presented in which sign settings are chosen so that the predicted average vehicle deceleration caused by the signs on each section equals the desired deceleration. The predicted average deceleration is derived from an assumed sign-following law and is a function of a car's speed when it enters the section and also the speed setting of the advisory sign located at the next section boundary.

A formula is derived for A_d, the desired average deceleration, and an expression for the predicted average acceleration A_s (of a vehicle due to a sign) is then determined. Equating the expressions for A_d and A_s yields the value of the advisory speed sign setting as a function of detector outputs.

CALCULATION OF DESIRED DECELERATION

To calculate deceleration, a simple highway situation is considered in which all vehicles are initially traveling at the same constant speed V_0 but at different spacings. The leading vehicle then decelerates to a new steady-state speed V_f. In this case it is assumed that the following vehicles eventually reach an equilibrium speed of V_f at a smaller spacing appropriate to the new speed. The optimum manner in which these vehicles should make the transition from V_0 to V_f, so that the integral $\int [a(t)]^2 dt$ is minimized for each vehicle, is considered.

Attention is directed to a particular following vehicle, and the problem is formally stated as follows: Given two vehicles having initial speeds of $v_1(0) = V_f$ and $v_2(0) = V_0 > V_f$

and initial spacing $x_1(0) - x_2(0) = S_0 > 0$ and given that $v_1(t) = V_f$ = constant, find the acceleration of the following vehicle $a_2(t)$ such that $I = \int_0^T [a_2(t)]^2\,dt$ is minimized subject to the final conditions $v_2(T) = V_f$ and $x_1(T) - x_2(T) = S_f < S_0$. The upper limit is given as T rather than infinity because it is expected that the optimum $a_2(t)$ is a function of the upper limit. Also, the following vehicle may be any of the vehicles upstream of the leading vehicle, not necessarily vehicle 2 in the line.

To simplify the calculation, the following normalized and equivalent problem is solved. Given a vehicle initially having $x(0) = 0$ and $v(0) = V_0$, find the acceleration $a(t)$ to minimize $I = \int_0^T [a(t)]^2\,dt$ and satisfy $x(T) = D$, $v(T) = 0$. This problem is of the form

$$\text{Minimize } I = \int_0^T f(t, x, \dot{x}, \ddot{x})\,dt = \int_0^T [\ddot{x}(t)]^2\,dt$$

with respect to $x(t)$, subject to given values of x and $\dot{x}$ at the upper and lower limits.

According to the calculus of variations (2, 3), the twice-differentiable function $x(t)$ that extremizes I satisfies the differential equation

$$\frac{\partial f}{\partial x} - \frac{d}{dt}\left(\frac{\partial f}{\partial \dot{x}}\right) + \frac{d^2}{dt^2}\left(\frac{\partial f}{\partial \ddot{x}}\right) = 0$$

Substituting $f(t, x, \dot{x}, \ddot{x}) = [\ddot{x}(t)]^2$ into this differential equation yields the solution

$$\ddot{x}(t) = A + Bt \tag{1}$$

The final solution is found by the following procedure: First, by integrating Eq. 1 and imposing the required values of x and v at the limits $t = 0$ and $t = T$, the values of A and B are determined as functions of T, D, and V_0. Second, the integral I is evaluated as a function of T, D, and V_0. Third, the value of T is found that minimizes I.

In the second step of this procedure, it is found that $I(T)$ is a monotonic nonincreasing function, as shown in Figure 1, approaching zero as T approaches infinity. (The derivative of $I(T)$ is zero at $T = 3D/V_0$ and $T = +\infty$.) It is tempting, because of the nature of $I(T)$, to choose T as large as possible so that $I(T)$ is minimized. However, it may be shown that for $T > 3D/V_0$ the acceleration waveform has the shape shown in Figure 2. Such an acceleration waveform is inadmissible because it implies a negative velocity for some values of $t < T$. Thus, the optimum acceleration is obtained by choosing $T = 3D/V_0$, which yields the motion given by

$$a(t) = \frac{2}{9}\frac{V_0^3}{D^2}\left(t - \frac{3D}{V_0}\right) \tag{2}$$

for $0 \le t \le 3D/V_0$ as shown in Figure 3. For $T = 3D/V_0$ the value of the integral I is

$$I = \frac{4}{9}\frac{V_0^3}{D} \tag{3}$$

Note that the minimizing function for the motion constrained by $v(t) \ge 0$ for $(0, T)$ has not been obtained; rather, the minimizing function for the unconstrained problem has been found, and the result has been modified to assure $v(t) \ge 0$ for $(0, T)$.

When the acceleration is assumed to be constant in (0, T) then the solution satisfying the initial and final conditions on spacing and speed is $a(t) = -V_0^2/2D$ for $0 \leq t \leq 2D/V_0$. For this case it is found that $I = V_0^3/2D$.

The constant deceleration waveform yields a value of I only 5.6 percent greater than the value obtained for the ramp function. For this reason and for reasons of mathematical tractability, the constant deceleration waveform is chosen as the desired deceleration waveform for the sign control algorithm.

For the first problem of a vehicle decelerating from V_0 to V_f while its spacing to a downstream car changes from S_0 to S_f, the solution is

$$A = \frac{1}{2} \frac{(\Delta V)^2}{\Delta S} \tag{4}$$

where $\Delta V = V_f - V_0$ and $\Delta S = S_f - S_0$.

PREDICTED AVERAGE DECELERATION

The average deceleration of a vehicle in response to a sign is determined from the sign-following model that is assumed to represent the dynamic response. Knowledge of car-following theory leads to a realistic sign-following model in the form of a stimulus-response equation. The sign-following model chosen is given by

$$\dot{v}(t) = \beta[V_s - v(t - \tau_s)] \tag{5}$$

The response is the acceleration at the current time, and the stimulus is the difference between V_s, the speed indication of the sign that the driver sees at time $t - \tau_s$, and $v(t - \tau_s)$, the vehicle's speed at $t - \tau_s$.

To facilitate further analysis of the model to obtain a sign setting algorithm, the model is modified to

$$\dot{v}(t) = \begin{cases} \beta[V_s - v(t - \tau_s)] & \text{for } t \geq \tau_s \\ 0 & \text{for } t < \tau_s \end{cases} \tag{6}$$

This modification has little effect on the dynamics of the response; in most cases the quantity $\dot{v}(t)$ is small for $0 \leq t \leq \tau_s$ because, near the end of the previous section of highway, the vehicle's speed will be nearly equal to the setting of the previous sign.

Values of β and τ_s are chosen to be equal to the corresponding parameters in the analogous car-following law for a very conservative driver. From data in Helly's paper (4), a reasonable choice is $\beta = 0.2 \text{ sec}^{-1}$ and $\tau_s = 2.0$ sec.

The average deceleration of a vehicle traversing a section under control of the sign on that section may be found from Eq. 6 if the initial condition on the vehicle's speed v(0) and the speed setting of the sign V_s are known.

However, for a given initial speed, it is desired to find the value of V_s that yields a specified average acceleration. By an iterative procedure, the value of V_s can be found that yields the specified average acceleration. Although this procedure may be readily implemented on a digital computer, a more direct solution for V_s would save substantial computation time. To this end an approximate solution is proposed. By trying several functions, it is found that

$$A_a = -0.00087[V_0^2 - V_s^2] \tag{7}$$

is a good approximation of the predicted average acceleration in response to a sign. This approximation yields an average absolute error of 5 percent for average section speeds between 40 and 55 mph (64 and 88 km/hour).

SIGN CONTROL ALGORITHM APPLIED TO A HIGHWAY SITUATION

Assumptions

It is assumed that the simple highway situation exists in which a line of vehicles is initially traveling at speed V_0 but at different spacings. The detectors spanned by these vehicles all register the speed V_0. It is assumed that the lead vehicle then decelerates to V_f and some time later passes a detector where the decrease in speed is registered.

At the next analysis time the settings for the advisory signs are computed on the basis of existing traffic conditions. These settings are valid for only 5 seconds, after which time conditions are reevaluated to determine new sign settings.

The basis for determining the sign settings is the temporary assumption that all vehicles upstream of the lead vehicle will eventually be forced to decelerate to V_f. The desired deceleration for these vehicles has previously been determined to be a constant given by $A_d = \frac{1}{2}(\Delta V)^2/(\Delta S)$ over an interval of time $T = 2(\Delta S)/(\Delta V)$. The approach used is to determine the sign settings such that the predicted average deceleration due to the sign equals the desired average deceleration for vehicles traversing an entire section under sign control.

Therefore, to determine the advisory speed setting for the i th sign, attention is focused on an imaginary vehicle located at the (i - 1) th detector that is 0.1 mile upstream of the i th sign. It is this vehicle that responds to the initial setting of the i th sign during the first 5 seconds of its journey on the i th section.

The desired deceleration of the imaginary vehicle A_d is expressed in terms of the measured section speeds and densities. This car's predicted average deceleration A_a is derived from the assumed sign-following law as a function of its speed when it enters the section and the speed setting of the advisory sign located at the next section boundary. Equating A_d to A_a yields the solution for the advisory sign setting in terms of detector outputs.

Computation of Desired Deceleration

A section of highway is shown in Figure 4. The delineators indicate the detector locations, spaced by 0.1 mile. The k th detector stores VD_k, the velocity of the vehicle most recently passing the detector, and ND_k, the number of vehicles currently on the k th section of roadway. The advisory signs are also spaced by 0.1 mile and the speed setting is represented as V_{s_i} for the i th sign.

In accordance with the simple highway situation previously described it is assumed that, at a particular sampling time, $VD_j = V_f$ and that all other upstream detectors register $VD_k = V_0$, where $V_f < V_0$.

To determine the speed setting for the i th sign requires that the desired deceleration of an imaginary vehicle located 0.1 mile upstream of sign i be expressed in terms of the detector outputs. The desired deceleration for this vehicle is

$$A_d = \tfrac{1}{2}(\Delta V)^2/(\Delta S) \tag{8}$$

for an interval of time $T = 2(\Delta S)/(\Delta V)$, where $\Delta V = V_f - V_0$ is the difference between the final speed after a slowdown and the initial speed and $\Delta S = S_f - S_0$ is the difference between final and initial spacings from the vehicle initiating the slowdown.

In this example the initial speed is $V_0 = VD_{i-1}$ and the final speed is $V_f = VD_j$, so that

$$\Delta V = V_f - V_0 = VD_j - VD_{i-1} \tag{9}$$

It is assumed that the vehicle that initiated the slowdown has just passed detector j. Thus, the initial spacing equals the distance in feet between detector i - 1 and detector j; i.e.,

$$S_0 = 528(j - i + 1) \tag{10}$$

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Page 68, Equation 10 should read: $S_0 = 528(j - i + 1)$

Figure 1. The integral I(T).

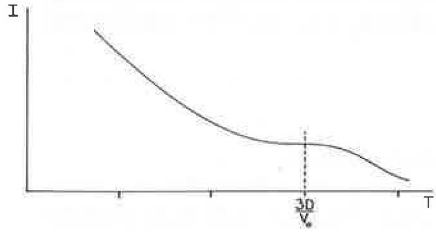

Figure 2. Acceleration waveform for $T > 3D/V_o$.

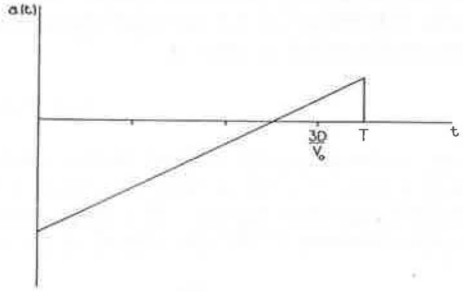

Figure 3. Optimal acceleration, $T = 3D/V_o$.

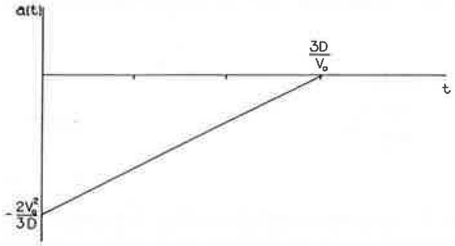

Figure 4. Representation of highway and detector locations.

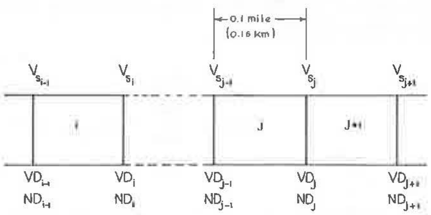

Table 1. Detector information available at t = 135 seconds.

I	NC(I) (vehicles)	VD(I) (mph)
11	4	50
12	5	50
13	5	50
14	5	50
15	3	48
16	4	46
17	4	35
18	6	34
19	2	—

Figure 5. Detector speed outputs for a typical case.

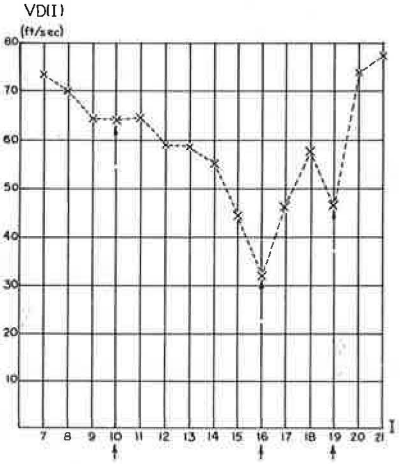

Table 2. Exact sign setting and rounded values.

Sign Number	Speed Setting fps	Speed Setting mph	Rounded Setting (mph)
17	44.0	30.0	30
16	60.0	40.9	40
15	66.1	45.1	45
14	68.3	46.5	45
13	69.4	47.3	45
12	70.2	47.9	Off

The final spacing between the imaginary vehicle currently at detector i - 1 and the vehicle assumed to be initiating the slowdown is determined from the number of vehicles between them and by the final speed. At the conclusion of the slowdown it is desired that each vehicle be at the spacing appropriate to its new speed. For the i th vehicle this desired spacing in feet is

$$DD_i = \alpha_i V_f + 20$$

where α_i is the desired headway factor for the i th vehicle-driver.

The ND_k vehicles on section k initially occupy 528 ft (161 m). After the slowdown they will occupy less space. For these vehicles the sum of the desired final spacings to the front bumper of the car ahead is expressed as

$$S_f(k) = \sum_{i=1}^{ND_k} (\alpha_i \times VD_j + 20) = 20\ ND_k + VD_j \sum_{i=1}^{ND_k} \alpha_i \tag{11}$$

where the sum is over the ND_k vehicles on the section. Because these vehicles initially occupied a 528-ft section,

$$528 = \sum_{i=1}^{ND_k} (\alpha_i v_i + 20) = 20\ ND_k + \sum_{i=1}^{ND_k} \alpha_i \times v_i \tag{12}$$

where v_i is the initial speed of the i th vehicle. The v_i are approximated by the estimated average section speed V_k, determined from the detectors at the section boundaries to be

$$V_k = (VD_{k-1} + VD_k)/2 \tag{13}$$

Replacing v_i in Eq. 12 by V_k from Eq. 13 gives

$$528 = 20\ ND_k + V_k \sum_{i=1}^{ND_k} \alpha_i \tag{14}$$

From Eq. 14,

$$\sum_{i=1}^{ND_k} \alpha_i = \frac{528 - 20\ ND_k}{V_k} \tag{15}$$

Substituting from Eq. 15 into Eq. 11 gives

$$S_f(k) = 20\ ND_k + \frac{VD_j}{V_k}(528 - 20\ ND_k) \tag{16}$$

or, using Eq. 13,

$$S_f(k) = 20\ ND_k + \frac{2\ VD_j}{VD_{k-1} + VD_k}(528 - 20\ ND_k) \tag{17}$$

Thus, the desired cumulative final spacing for the vehicles initially on sections i through j is

$$S_f = \sum_{k=i}^{j} S_f(k) \tag{18}$$

Substitution of the derived quantities into Eq. 18 gives, from Eq. 8,

$$A_d = \frac{1}{2} \frac{(VD_j - VD_{i-1})^2}{\sum_{k=i}^{j} \left[20\ ND_k + \frac{2\ VD_j}{VD_{k-1} + VD_k}(528 - 20\ ND_k)\right] - 528\ (j - i + 1)} \tag{19}$$

This is the desired deceleration for a vehicle traversing the i th section in response to a slowdown detected at the j th detector.

Solution for Sign Setting

Expressions for the desired average deceleration A_d and the predicted average deceleration A_a have been obtained. The value of the setting for the i th sign V_{s_i} is found by equating A_d from Eq. 19 to A_a from Eq. 7 with $V_s = V_{s_i}$. The solution for V_{s_i} is

$$V_{s_i} = \left\{VD_{i-1}^2 + \frac{575\ (VD_j - VD_{i-1})^2}{\sum_{k=i}^{j} \left[20\ ND_k + \frac{2\ VD_j\ (528 - 20\ ND_k)}{VD_{k-1} + VD_k}\right] - 528\ (j - i + 1)}\right\}^{1/2} \tag{20}$$

where VD_j is the speed indication of the downstream detector that is temporarily assumed to equal the final velocity of the upstream vehicles after the slowdown.

Rounding of Sign Settings

For practical reasons the sign settings are rounded to the standard 5-mph increments. In addition, a sign is not turned on if only a minor response to the calculated setting is anticipated.

After computing the exact sign settings via Eq. 20, each sign is examined and those that are off are skipped. If the i th sign is on, it is then ascertained whether a significant response is expected to the sign setting. If $VD_{i-1} - V_{s_i} \leq 2.5$ mph, the i th sign is turned off. If $VD_{i-1} - V_{s_i} > 2.5$ mph, the sign setting, V_{s_i}, is rounded to the nearest 5 mph. If, for the rounded sign setting, the quantity $VD_{i-1} - V_{s_i} \leq 2.5$ mph, then the sign is turned off. If $VD_{i-1} - V_{s_i} > 2.5$ mph for the rounded value, then the rounded value is retained as the current sign setting.

CALCULATION OF SIGN SETTINGS

An example from a simulation run with 80 vehicles is presented. [Details of the simulation are given elsewhere (6).] All vehicles enter the roadway at a speed of 50 mph and continue at this constant speed until t = 120 seconds. At this time the first vehicle decelerates to 35 mph at a rate of -3.0 fps^2.

At t = 125 seconds the slowdown is detected and one sign is set; at t = 130 seconds two signs are set; and at t = 135 seconds five signs are set. The situation at t = 135 seconds is examined in more detail.

The detector information available at t = 135 seconds is given in Table 1. The value of VD(I) is rounded to the nearest integer.

From several computer runs it is found that the acceleration noise is minimum when the constant 575 in Eq. 20 is replaced by a constant between 600 and 625. For the example under consideration, this constant is 625. Consequently, the appropriate equation for determining the setting of the i th sign due to a slowdown detected at VD_j is

$$V_{s_i} = \left\{ VD_{i-1}^2 + \frac{625\,(VD_j - VD_{i-1})^2}{\sum_{k=i}^{j} \left[20\,ND_k + \frac{2\,VD_j\,(528 - 20\,ND_k)}{VD_{k-1} + VD_k} \right] - 528\,(j - i + 1)} \right\}^{1/2} \qquad (21)$$

The data from several simulations indicate that it is preferable not to set the sign that is located directly at the point where the slowdown is detected. It is found that the setting of this sign often retards vehicles in recovering from a slowdown after the vehicle causing the slowdown increases its speed. Therefore, in this example, sign 18 is skipped.

In determining the exact setting (before rounding) for sign 17, the appropriate values to substitute into Eq. 21 are

$$j = 18$$
$$i = 17$$
$$VD_{16} = 66.9 \text{ fps } (46 \text{ mph})$$
$$VD_{17} = 50.9 \text{ fps } (35 \text{ mph})$$
$$VD_{18} = 50.9 \text{ fps } (35 \text{ mph})$$
$$ND_{17} = 4$$
$$ND_{18} = 6$$

Substituting these values into Eq. 21 gives $V_{s_{17}} = 44.0$ fps for the setting of sign 17.

To determine the exact setting for sign 14, for example, the values $j = 18$ and $i = 14$ should be used in Eq. 21. The value $j = 18$ should be used in finding V_{s_i} for all values of i less than j.

In the same manner, the exact settings of the other upstream signs are determined as given in Table 2. The results of the rounding algorithm are also tabulated. It is noted that sign 12 is turned off because $VD_{11} - V_{s_{12}} = 50.0 - 47.9 = 2.1$ mph is less than 2.5 mph. That is, the sign is not turned on because the vehicle 528 ft upstream of the sign is traveling at a speed only slightly greater than the computed sign setting.

APPLICATION OF SIGN CONTROL TO A GENERAL HIGHWAY SITUATION

In the simplified highway situation previously described, the speed indications of the detectors are monotonically decreasing in the downstream direction. This is intended to represent only an isolated component of the traffic pattern on an actual highway.

Sampling of the detector speed outputs at a random time generally yields a speed profile exhibiting peaks and valleys. A typical case is shown in Figure 5.

General Procedure for Sign Setting

The heuristic procedure for determining the sign settings is described for a typical case for which VD(I) are as shown in Figure 5. The detector speed outputs VD(I) are scanned, starting at VD(7), to locate the first "local minimum". A local minimum occurs at detector k if both

$$VD(k - 1) > VD(k)$$

and

$$VD(k + 1) \geq VD(k)$$

are satisfied. The three local minima are identified by arrows in Figure 5.

The first local minimum in Figure 5 is VD(10). Sign 10 is not set since it is located at the local minimum. Sign settings for signs 9 and 8 are determined from Eq. 21 using $j = 10$ and the appropriate values for VD and ND. That is, V_{s_9} and V_{s_8} are computed on the basis of the local minimum of VD(10). Sign 9 is set to 65.9 fps and sign 8 is set to 68.7 fps.

The scan is continued and the second local minimum is found to be VD(16). Sign 16 is not set because the local minimum occurs at the location of this sign. Signs 15 back through 8 are now considered on the basis of a slowdown assumed to originate from the second local minimum at VD(16). Thus, using the value j = 16 in Eq. 21, the computed settings (feet per second) for signs 15 back through 10 are

$$V_{s_{15}} = 38.5$$
$$V_{s_{14}} = 48.2$$
$$V_{s_{13}} = 51.9$$
$$V_{s_{12}} = 59.6$$
$$V_{s_{11}} = 60.8$$
$$V_{s_{10}} = 62.2$$

Sign 9 was considered previously because of the first local minimum of VD(10). However, because the second local minimum is of smaller value, a tentative setting is computed using j = 16. This computation yields 65.4 fps. Because 65.4 is less than the previously computed setting of 65.9 it is concluded that the second local minimum has the stronger influence on vehicles on section 10. Hence, sign 9 is reset to $V_{s_9} = 65.4$.

Similarly, a tentative setting for sign 8 is computed, using j = 16, and the result is 69.0. Sign 8 is not reset because the previously determined setting of 68.7 fps is less than 69.0.

The third local minimum is found at VD(19). Sign 19 is not considered and the setting for sign 18 is found to be $V_{s_{18}} = 47.6$ using j = 19 in Eq. 21.

Sign 17 is not considered because VD(16) is less than the speed at the third local minimum of VD(19). Thus, cars on section 17 will not be forced to decelerate because of the local minimum of VD(19). For the same reason sign 16 is not considered.

Sign 15 has been previously considered because of a local minimum further upstream of smaller value, i.e., VD(16). For this reason the setting of sign 15 is considered valid. Furthermore, additional scanning of signs further upstream is suppressed and the search for the next local minimum is initiated.

SUMMARY

Although a cost-benefit analysis has not been done, several factors indicate that the cost of a surveillance and control system with detectors spaced at 0.1-mile intervals may be justifiable. First, aside from the system benefit of reducing acceleration noise and thereby reducing the rate of occurrence of accidents, the surveillance part of the system would provide essentially instantaneous incident detection and location. Second, with the ever-decreasing cost of large-scale integration in circuit technology and the cost advantages obtained by large-scale use of identical chips, such as found in microprocessors, the system may well be built with all necessary computer capability at the site of each advisory sign, thereby eliminating the high cost of communicating with, owning, and operating a large central computer. Although field maintenance could be a drawback to such a distributed computing system, it is not likely to be expensive because it would be on a plug-in replacement basis.

Although this aspect has not been simulated, an increase of the sign spacing to 0.2 mile (0.3 km) could be tolerated, although an increase in the detectors' spacing would have a more detrimental effect because of the longer average delay before a slowdown would be detected. Of course, as pointed out by Altman (5), on an actual (inhomogeneous) highway, the detectors should be placed more densely in areas of greater intravehicular interference (e.g., near entrance ramps, physical bottlenecks, or permanent visual distractions). The optimum placement of detectors was not considered in this analysis, yet the derivation of the sign control algorithm is easily adapted to accommodate unequal detector or advisory sign spacings.

Compliance with the advisory sign messages is not expected to be a problem as long as the signs are used only when necessary and their messages are consistently reliable.

Each driver will learn to respond to the messages in a manner that maximizes his own comfort. An overall average level of response to the sign's message will be imposed on drivers due to the compliance of drivers further downstream.

The actual model used for a driver's response to a sign is not based on any experimental data but, rather, is patterned after the stimulus-response car-following models in the traffic flow literature. Although the constants in this model may be subject to question, the derivation of the sign control algorithm would be exactly parallel for different values of these constants.

The purpose of this paper has been to show that if a model for the response of a driver to a sign is assumed then an algorithm for controlling the signs along a highway may be readily found. An extensive simulation of the sign control system (6) using the algorithm derived in this paper indicates that even when driver compliance with the advisory signs is poor (i.e., 80 percent of the drivers ignore the sign's message completely) a significant reduction in acceleration noise is obtained. A detailed presentation of the simulation and its results will be made in a future paper.

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