TOWARD A UNIFIED DISTRIBUTION SYSTEM-WAREHOUSE LOCATION THEORY

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Many of the problems of urban goods movements may be related to the present methods of analyzing freight distribution systems in urban areas. This paper considers the problems associated with the physical distribution of freight, as well as the problems of distribution center location. The authors suggest that these two areas are very much interrelated and should be considered together as components of a single system. An extensive literature review of freight distribution and terminal location research is presented with special reference to the feasibility of designing a unified distribution system-terminal location theory. Also an attempt is made to match theoretical approaches and insights with the practical requirements and concerns of freight distribution in urban areas. The authors conclude that much of the purely theoretical work has little, if any, relevance to the solution of real-world distribution and location problems. It can be concluded that serious faults underlie component-by-component analysis of the distribution system. This approach is questionable because the performance of the whole system is the decisive element in the functioning of a distribution system, rather than the individual operation of its components.

•A SUBSTANTIAL PORTION of the ongoing research on urban goods movement is concerned with identifying the interrelationships of freight transportation and the urban area. These relationships address problems such as the joint use of transportation facilities for freight and passenger service; space requirements for shipping and receiving operations; and legal, regulatory, labor, and financial constraints associated with the distribution of freight in urban areas.

Many of the problems encountered with urban goods movement can be identified by investigating the theoretical and practical methods of analyzing freight distribution systems in urban areas. The inadequacies of these techniques, both methodological and structural, may provide insights into the appropriateness of these models and techniques in analyzing processes of urban goods movements. Many of the suboptimalities experienced in urban goods movements may be related to the incomplete and perhaps erroneous analysis of a firm's distribution system. We may define the physical elements of a distribution system as one or more terminals (warehouses, depots), a set of routes between these terminals and the consignees serviced by the system, and vehicles that routinely transport the freight within this system.

Terminals are included in the distribution system because their functions are directly related to the objectives of the actual physical distribution of freight. Terminals break down line-haul shipments into smaller lots for distribution to individual consignees, act as intermediate storage points (between the primary producers and the consignees) to provide "production smoothing" of the flow of goods to the consignees, and provide for the transferral and reassembly of freight (break-bulk operations) from the incoming method of transportation to that of the outgoing method. This latter point does not necessarily indicate that there must be a change of mode.

This paper is concerned both with the problems associated with a physical distribution system as a whole and with the problem of distribution center location. The two problem areas are so intertwined that they must be considered simultaneously as interrelated components of a single system. The logical basis for developing, operating, and financing a unified freight distribution system in an urban area must be the consideration of warehouse location, which takes into account the other elements of the total distribution system.

For ease and clarity of exposition, the authors have chosen to discuss the literature on terminal location and distribution system problems separately. As will be shown later, research in the two areas tends to make different assumptions about the given and the unknown elements of the overall system.

The three major objectives of this paper are to analyze the state of the art of freight distribution and terminal location research and to investigate the feasibility of designing a unified distribution system-terminal location theory based on a critical analysis of the literature. A third objective is to attempt to match theoretical approaches and insights with the practical requirements and concerns of freight distribution in urban areas. We concluded very early in the research effort that much of the purely theoretical work had little, if any, relevance to the solution of real-world distribution and location problems.

WHY A UNIFIED THEORY OF WAREHOUSE LOCATION AND DISTRIBUTION SYSTEM ANALYSIS?

A large number of contributions to the literature on warehouse location and physical distribution analysis have been made by operations researchers, management scientists, and management consultants. These groups tend to have highly specialized interest in specific components of the distribution system. This orientation has resulted in many models and heuristic methods for accomplishing warehouse location, design, and operation; fleet scheduling, routing, and size; inventory analysis and control; and service area definition for a warehouse. A particular characteristic of the mathematical models is that they depend on information inputs from other components of the distribution system. Many of these data requirements, however, cannot be met for a practical application of the model. Also, mathematical modelers are often forced to make simplifying assumptions about the behavior of other components of the distribution system to facilitate the application of their models. The result is a patchwork of optimization methods that do not realistically describe the components of the distribution system. These methods, in almost all of the studies examined, did not produce useful information on operation of the particular component being investigated, which could be used as input for the analysis of another component in the distribution system (1).

We know of two simulation approaches developed to overcome the problem of piecemeal analysis of the total distribution system: the IBM software distribution system simulator (2) and Michigan State University's long-range environmental planning simulator (1). These simulators link the elements of production, warehousing, and customer demand; however, they do not provide a unified theory that quantifies the interrelationships of all the components of the distribution system.

A management consultant approaches the problem of warehouse location with the interest of minimizing the total cost or maximizing the total profit of the firm. With this in mind, he cannot afford to consider only certain components of the total distribution system, but he must encompass the entire scope of the problem and proceed to find a feasible solution (3).

A study of facility location errors by a loading consulting firm (4) identified the 10 most common faults in location, many of which occurred because of the failure to consider the interactions of the distribution system components.

LOCATION MODELS FOR DISTRIBUTION CENTERS

The following paragraphs will discuss and criticize the current theoretical and nontheoretical methodologies of location and distribution system analysis. The logical basis for developing, operating, and financing a unified freight distribution system in an urban area must be the consideration of warehouse location, which takes into account the other elements of the total distribution system. Eilon and Watson-Gandy (5) stressed the multifaceted character of terminal location problems, but their identification of the four fundamental components of a distribution system—number of terminals, location of each terminal, allocation of customers to each terminal, size of each terminal—is still quite incomplete, as will be shown later.

When theorists talk about the problem of locating distribution or collection centers, they implicitly assume an optimization objective. This objective is obviously only meaningful in relation to a measure or set of measures (criteria) to be optimized (e.g., minimum number of distribution vehicles, minimum total route distance, storage requirements, and manpower), subject to a number of constraints (e.g., access, land-rent, labor force, and legal and regulatory considerations).

Cooper (6) has stated the general problem in the following form: Given the location of each destination, the demands at each destination, and a set of shipping costs for the relevant area of distribution, determine (a) the number of distribution centers, (b) the location of each center, and (c) the capacity of each center.

The solution to this problem proved to be very difficult from both theoretical and computational perspectives. Consequently, other researchers have redefined the problem, changed the basic assumptions, and experimented with exact as well as heuristic solution techniques.

Mathematical approaches to this problem date back as far as 1647 when Cavalieri found that determining the point whose sum of distances from three given points is a minimum required that each side have an angle of less than 120 deg with the given minimum point (7, p. 332). Many of the recent approaches to the optimal location problems are based on or related to the generalization of the problem of determining the location of a point, in two-dimensional Euclidean space that represents the minimum distance or cost for a number of weighted destinations, as formulated by Weber (8). In mathematical terms the problem can be stated as follows:

$$\text{Min } \Phi = \sum_{j=1}^{n} \beta_j \left[(X_{Dj} - X)^2 + (Y_{Dj} - Y)^2\right]^{1/2}$$

where

X_{Dj}, Y_{Dj} = coordinates of known destination in two-dimensional Euclidean space, $j = 1, \ldots, n$,

X, Y = coordinates of unknown distribution point, and

β_j = weights relating to amounts to be shipped or any other weights.

The time required to solve all possible combinations of the generalized Weber problem is excessive except in cases where the problem involves very small numbers of terminals and destinations. Some authors have redefined the problem into a single source problem by subdividing the area of concern into several subareas each with its own source or terminal (9). For problems of industrial importance, heuristic solution methods, which incorporate a consideration of the customers to be served by each terminal, seem to provide the only answer to the multiple-source location problem (6, 10).

The solution approaches to the optimal location problem can appropriately be grouped into two categories (5): the infinite set approach and the feasible set approach. The first approach is based directly on the Weber model. Generally, these models are developed under the assumption that transportation costs are a linear function of distance. The objective function minimizes the sum of the weighted distances between sources (terminals) and destinations weighted by their demands. Solution of the optimal number of terminals is arrived at by establishing the optimal solution for 1, 2, 3 . . . terminals with respect to transportation costs (11).

The feasible set approach attempts to improve on the infinite set approach by (a) digressing from the assumption of linearity of transportation costs with respect to distance; (b) taking into account the overhead costs of a terminal that might be strongly affected by its specific location; and (c) considering the potential economies of scale in the operation of a distribution center.

Rather than consider all locations within a geographical space, the feasible set approach selects those locations that fulfill the following requirements:

1. The locations are feasible with respect to land availability, rental costs, and so on;
2. The operating costs for a terminal in such a location can be determined;
3. The absolute optimum set need not be in the solution set; and
4. Transportation costs need not be related to distance.

The objective of this approach is to determine the set of locations with minimum total cost from a preselected set of feasible locations (5).

The major shortcomings or difficulties with this latter approach are that subjective evaluation criteria will have to enter the selection process, that the problems of data gathering on all cost items can be tremendous, and that feasible set problems tend to become very large because of the greater number of constraints.

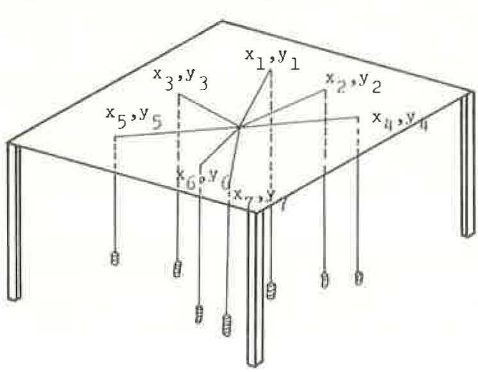
After reviewing and testing a number of solution strategies in each of these two approach categories, Eilon and Watson-Gandy (5) concluded that their "total cost model" of the feasible set type represents the most promising and efficient heuristic solution technique. Their model takes into account three cost items:

1. Transportation costs from production site to distribution terminal,
2. Local distribution costs, and
3. Warehousing costs.

Through an iterative procedure, the solution technique arrives at the lowest cost alternative for the depot locations. By using a so-called "drop-routine," the authors also determined the optimal number of terminals in the system. It should be pointed out that this approach does not pinpoint exact locations for the distribution center, but merely indicates the general area where suitable sites should be considered. A detailed benefit-cost analysis of any of these selected areas would be advisable to determine the exact optimal location of each distribution center. Naturally, this analysis would not look at optimality in a systemwide context but would concentrate on each terminal location in isolation from the others originally located in an optimal manner.

The majority of the models reviewed are static rather than dynamic. Vergin and Rogers (12) proposed that optimal locations for centers of economic activities could be determined by identifying the spot where the sum of the costs of transporting goods between existing source and destination points and the new terminal location is a minimum. This center-of-gravity concept is frequently used in attempts to determine the optimal location of distribution centers. Vergin and Rogers suggested several methods of solving the above problem. One method is the mechanical analogue (Fig. 1), which operates as follows: A map is secured to a table, and holes are drilled through the table at the customers' locations. Strings are passed through these holes with one end carrying a weight proportional to β_i (analogous to β_i in the generalized Weber model) and the

Figure 1. Mechanical analogue.



other tied to a small ring or washer. The ring will locate itself at a point of minimal potential energy, which is the position at which the transport costs are at a minimum (11)

There are several disadvantages to this method, one of which is that the mechanical analogue of the generalized Weber model is only able to locate one terminal at a time, which in the case of a multiterminal location problem requires a subdivision of the entire region into areas each being served by a single depot. This clearly appears to be a feasible solution. There is, however, no established method of dividing the region exactly, and this can produce suboptimal location. A second disadvantage is that there is no indication of transportation and facility costs at the location involved, and such costing must be calculated by some other means. A third fault is that it is very difficult to construct a large model that will have the accuracy required for an optimal solution.

The weighted arithmetic mean is a second approach used by Vergin and Rogers (12). In this procedure, the optimal location can be determined at the intersection of the weighted arithmetic mean of the points of demand along two orthogonal axes, where the demand points are the destination-origin of the tonnage of materials flowing to and from the facility to be located. It was pointed out that finding the weighted arithmetic mean coordinates is analogous to the center-of-gravity approach of a two-dimensional object in mechanics. An illustration using rectangular movement and straight-line movement (Pythagorean theorem application) was given to solve a single-facility location problem.

The rectangular movement was designed to minimize the sum of transportation costs, which can be expressed as

$$C = \sum_{i=1}^{n} W_i D_i$$

where

C = total cost of transporting goods,
D_i = distance between location point and the n known destinations,
W_i = $T_i V_i$,
V_i = weight or volume of goods, and
T_i = charge/unit of distance/unit of weight or volume.

Given a location designated as (X, Y), the distance to point $i(x_i, y_i)$ can be expressed as $|x - x_i| + |y - y_i|$. It can be shown that the median of a discrete set of points X_i is such that the sum of absolute directors from it $\sum |x - x_i|$ is a minimum, and $\sum |y - y_i|$ is also a minimum. Therefore C is a minimum when the coordinates of the location point are the median value above.

In the straight-line approach the movement in the x direction depends on the movement in the y direction. Distance between the location point and the demand point i can now be expressed as:

$$D_i = \left[(x - x_i)^2 + (y - y_i)^2\right]^{1/2}$$

It can be shown that this equation is convex, indicating that there exists a single global minimum for C.

The arithmetic mean method, when used to find the optimum coordinates (x, y), produces answers that may be quite removed from the optimal. This is due to the difference in the amount of tonnage at each destination. When the tonnage delivered to each destination is similar, the weighted arithmetic mean produces results close to the optimal location. However, as large changes in tonnage occur at destination points, the error in the process increases rapidly. Therefore, it can be concluded that not only are weighted mean coordinates seldom optimal when weighting factors for destination differ greatly, but they are not necessarily optimal when all delivery points are equally weighted (12).

Goldstone (13) suggested an iterative solution procedure to solve the straight-line problem. His method requires the selection of an efficient initial estimator (x_0, y_0). Goldstone proposed that, because Vergin and Rogers' square weighted start point gave costs closer to optimal, the possibility of higher powers might mean a quicker solution to the same result. An example of 24 warehouses serving two to 10 shops was tested, with results of higher powers yielding quicker optimal solutions.

A number of other solution strategies and techniques have been proposed that do not fit into the general framework of the two-category classification presented. Three examples of such approaches follow.

Eilon and Deziel (14) propose the use of a general-purpose analogue computer to solve the straight-line distance, single-facility location problem. If more than one distribution center is to be optimally located, the final solution depends on the initial choice of center locations and the subsequent allocation of delivery points (customers) to these centers. Using the examples of one center and 10 customers and two centers and four customers, they show that their results in terms of the value of the objective function are very close to those achieved by Miehle (15) obtained by an iterative analytical technique.

Eilon and Deziel assume linear transportation costs. If nonlinear transportation costs are assumed, a larger number of amplifiers would be required in the analogue computer. The method they describe can also be used to determine the optimal number of distribution centers for a given network.

Another approach to the terminal location problem was developed by Griffiths (16). His regression analysis approach deals with a distribution system that involves three distribution centers and a network of transit depots. He considers the delivery costs from transit depots to customers, which are divided into two categories: number of vehicles and drivers and mileage traveled by the vehicles. Griffiths' objective was to develop an estimate of the time required by a delivery vehicle to satisfy a certain demand.

To determine depot locations, he developed a route-independent measure of mileage. The run of the straight-line distances from each town to the depot produced the best results for such a measure.

Many studies of location have been performed in the context of optimal layout of manufacturing plants, large service operations, and department stores. An example of an approach that is applicable to the solution of both terminal location and layout problems was proposed by Curry and Skeith (19). They developed a dynamic programming formula to solve the problem of minimizing total cost when k facilities are allocated in m facility locations and n demands are assigned.

The typical context would be the most efficient way of arranging supply facilities with respect to fixed demand points in a manufacturing plant so as to minimize the travel distance between them. Both conceptually and computationally the approach presented in their work appears to be suitable also for use in determining warehouse locations if a number of specific sites are known as suitable alternative locations.

The authors formulate the problem as a nonlinear minimization problem that can be transformed into a dynamic programming formulation. Because the problem involves two decision variables in one of the constraint equations (namely two 0-1 variables indicating the allocation of a facility to a location and allocation of a demand location to a facility location), it is separated into a multistage optimization problem in which the stages, representing facility locations, are optimized sequentially. This problem is overcome by using the Lagrange multiplier technique and by including one of the constraint equations in the objective function.

Computationally this dynamic programming approach has the property that adding facilities or facility locations to the problem will only have an additive effect on the solution time as compared to an exponential (or fractional) effect for an exhaustive search method. The state space for each stage of the problem, however, is equal to the product of the number of fixed demand points and the number of possible facility locations. This latter aspect makes this approach impracticable for multiple-terminal location problems involving large numbers of demand points.

DISTRIBUTION OF FREIGHT

Analyses of the physical distribution of freight, from terminal to demand point, have produced algorithms that determine vehicle fleet routing under a variety of constraints and the set of demand points that are to be most economically served from a particular terminal. Also, the costing of physical distribution has received a great deal of attention recently in response to the precise data requirements of new cost models of the distribution process. The following sections will discuss relevant models and algorithms developed to analyze the distribution of freight.

The first mathematical fleet routing algorithm was developed by Dantzig and Ramser (20). The problem was to route a vehicle fleet from a single depot to a set of customers, which had individual, constant demands for a homogeneous commodity supplied by the depot, such that the total mileage of all routes is minimized. All customer demands must be satisfied and all fleet vehicles are assumed to have the same capacity C.

The authors noted that, if the capacity of a vehicle was greater than the sum of customer demands to be fulfilled from the depot, the problem reduces to the traveling salesman problem. Within this formulation, it is assumed that the vehicle can visit every demand point. Dantzig and Ramser, however, confined their attention to the case in which a vehicle cannot make all of the deliveries in one journey from the depot because of the capacity restriction of the vehicle.

The procedure of their algorithm is based on a series of N stages of aggregation in which suboptimizations are carried out. In the initial stage of aggregation, only those points whose combined demand does not exceed $(1/2^{N-1})C$ and whose interpair distances satisfy the criterion described below are allowed to pair up to begin a route. In the next stage any groups from the suboptimal solution of the initial stage may pair up provided that the combined demand of such pairings does not exceed $(1/2^{N-2})C$. This procedure is continued until N stages of aggregation have been examined.

In each stage minimum interpair distances are determined. These pairings of points or groups of points are the suboptimal solutions achieved in each stage. In the final stage the sum of route lengths is near minimum for all routes. This stage also links the ends of the chains formed to the depot. Every point is connected to no more than two points and this series of connections must form a "circular" chain with one of its links being the depot.

The authors noted that the algorithm could consider multiproduct demands, provided that, from the carrier's point of view, the goods are similar to each other (weight, volume) so that, regardless of the product mix, the vehicle could still accommodate the same number of units. Also, Danzig and Ramser suggested that, if a vehicle fleet of varying capacity was to be considered, the optimization function should be redefined as a total cost minimization rather than a total mileage minimization. The cost function would be composed of charges based on unused unit volume and unit mileage. The implicit assumption of the equal-capacity vehicle fleet is that slack capacity incurs no cost. With a variable-capacity fleet, effort should be made to use the differential capacity most efficiently by minimizing the unused space in each vehicle. This would imply, according to Dantzig and Ramser's methodology, that the vehicles should be loaded as fully as possible when they leave the depot, a philosophy practiced by many fleet managers.

Clarke and Wright (21) investigated the same problem except that they specifically considered a vehicle fleet of varying capacity. They observed that the Dantzig-Ramser algorithm caused delivery points that had been aggregated in a stage to remain aggregated in later stages. This produced a method that emphasized filling trucks to their capacity and only partially minimized total route distance.

In addition to their algorithm, Clarke and Wright produced a criterion for including a customer on a route (which evolved from Dantzig and Ramser's minimum interpoint distance)—the "route savings" criterion. Route savings is a measure of the priority, in terms of linear distance, of linking customers A and B to each other on a single delivery route, instead of having two out-and-back trips to serve A and B from the depot.

Distant customers are given priority in the route search technique because it is more economical to incorporate these outlying customers on one route than to serve them by more than the number of vehicles specified by this procedure. Isosavings curves are used to partition the set of customers according to sets of successively greater savings to be achieved by incorporating them on a multiple-delivery route (22). Computer routing would progress from the sets of greatest savings to those of least savings.

In their algorithm, pairs of points that would experience similar levels of savings are linked on a route if the following conditions are met:

1. The points are linked to the depot by a route,
2. The points are not already allocated on the same truck route, and
3. The additional demand requirement, which results in the removal of trucks allocated to serve the points in question and the allocation of a truck to serve the new augmented route, is not larger than the greatest capacity vehicle that has not yet been allocated to a route.

This approach, when subjected to the data considered by Dantzig and Ramser (20), produced routes with a total distance of 290 units. The algorithm of Dantzig and Ramser produced different routes that yielded a total distance of 294 units. Also the Clarke-Wright algorithm is far less involved computationally than the earlier method. The method of allocating trucks to routes does not ensure maximum capacity utilization; however, practical constraints on vehicle requirements may easily be incorporated into the formulation.

As Clarke and Wright point out, although the solution to their algorithm produces a sequence of customers to be serviced on a route, the traveling salesman problem should be solved for each final route to determine the true optimum order of visitation.

A flaw in this algorithm is that, once a link is established, its contribution to route minimization is never reevaluated; the link remains a part of the route even if a series of future links would have rendered the choice of this particular link inappropriate. This argues for a dynamic programming formulation of the vehicle routing problem.

Gaskell (22) also considered the problem as stated by Dantzig and Ramser with the additional constraint that total mileage of any route may not exceed a specified limit. Essentially, Gaskell sought to determine a simple, near optimal method for fleet vehicle routing. He compared five methods for determining whether a customer should be included on a route. The first method was a manual search of all possible routes within the artificially developed cases. This search was extensive and was considered optimal. The remaining four methods were variations of the route savings criterion. The mathematical property of these variations is that they altered the order or priority in which demand points are considered for linking on a common route. Gaskell determined that no particular method was superior in all cases and that the method suggested by Clarke and Wright for vehicle routing was reasonable.

Christofides and Eilon (23) proposed an "r-optimal tour" method to route vehicles where the problem is the same as considered by Gaskell. The origin of the r-optimal tour method lies in two properties of the minimal traveling salesman tour: Such a tour does not intersect itself, and the tour that does not intersect itself is one that cannot be reduced in length by replacing any two links by any other set of two links. This latter property is known as a 2-optimal tour. The same principle is extended to form a 3-optimal tour.

Although the 3-optimal tour could be extended to an r-optimal tour, as r approaches n (the number of points to be served on a route), checking sets of links becomes a complete enumeration of all possible tours to that group of customers. The authors determined that a 3-optimal tour produces good results.

The algorithm proceeds as follows. An arbitrary random tour that is feasible (i.e., it satisfies all constraints) is developed. A 2-optimal tour is generated, and the improved tour is then used as the basis for forming a 3-optimal tour. The procedure is repeated several times (i.e., a different set of customers is incorporated into the initial tour, given the constraints) and the most improved (least cost) tour is selected. This procedure does not necessarily minimize the number of vehicles required to supply the customers.

Christofides and Eilon noted that, although the general routing problem can be formulated in a dynamic programming structure (24) or as an assignment problem in integer programming (25), the computation time and storage demands become excessive for large problems.

O'Neil and Whybark (26) compared the efficiency of five routing heuristics that were computationally simple. Only two of the formulations proved uniformly near optimal: the route savings method and the clustering and travel time saved heuristic. The latter method was developed from the statistical concept of clustering analysis. Customer groups are formed on the basis of customers' proximity to one another in terms of travel time. A cluster is chosen so that the demands of the customers cannot be served by a single visit to that cluster by a vehicle. This is done to ensure that the first vehicle will not be assigned to a route that underutilizes its capacity. The route savings method is then applied to members of this cluster. This procedure iterates and redefines new clusters as routes are established. The heuristic works well when customers are clustered naturally. The clustering of demand points is an observed phenomenon in many real-world situations.

Vehicle routing, as Higgins (27) suggested, can be accomplished by simple, nonalgorithmic models when the fixed and variable costs of freight distribution can be identified. These costs can be used to determine variable costs that are then applied to a series of Monte Carlo simulations of deliveries of a particular commodity to demand points or clusters from a depot. The route simulation that yields the least total cost routing policy is adopted as the routing strategy.

Many depot locations have been determined by the analogue machine approach to solving the generalized Weber problem. This approach does not assume that there will be out-and-back delivery routes, but rather that the cost of the eventual routes will vary linearly with distance of the customers from the depot. There is a discrepancy between the minimized radial distances generated by solving the generalized Weber problem and the actual distances traveled by vehicles dispatched from the depot to serve customers. This difference is accentuated because, in practice, deliveries to more than one customer on a single trip from the depot are common. Christofides and Eilon (23) computed from a series of 42 randomly generated problems a regression equation of the actual distance traveled as a function of the radial distances. They noted that the relationship indicated a high degree of correlation and that the position of the depot with respect to the customers, which was varied in several problems, did not significantly alter the strength of the relationships.

Most distribution studies consider the location of the terminal to be fixed. If there is a cost trade-off between serving a portion of the customers' demands by the corporate fleet and making deliveries by using alternate distribution services, geographical boundary areas within which it is most economical for a firm to use its own vehicle fleet for distribution should be established for the terminal. Buxton and Quayle (28) suggest a method for determining boundary areas, for a fixed warehouse location, that delineates the economical regions for the firm's fleet and common carrier distribution. Two constraint boundary lines are developed. The time constraint boundary line determines the geographical area a vehicle may service in a day. The available number of driving hours per day, average miles per hour per zone, ratio of the furthest peripheral point in miles (i.e., greatest diameter of the route) on a route, and the total mileage of the route are the informational requirements necessary to compute this boundary. The cost equalization boundary is the set of points at which the cost difference between fleet and common carrier distribution is zero. This boundary equation requires measures of fleet distribution, costs per mile and per ton, current average route mileage, and market demand in tons per square mile in the study zones. The innermost boundary curve (with respect to the depot) defines the delivery area for the depot.

Certain distribution situations require the fleet to make deliveries to demand points, which vary from day to day. In this situation, Christofides (29) suggested that fixed areas for distribution instead of fixed routes should be established. The fixed area for distribution is the nearest approximation to the fixed route for the situation of variable demand. An algorithm similar to that used to generate fixed vehicle routes is used to successively build up delivery areas from the basic areal units defined by customer locations.

The determination of depot delivery areas constitutes an extension of vehicle fleet routing methods where an extra degree of freedom, in terms of vehicle fleet operation, is incorporated.

EVALUATION AND PRACTICAL APPLICATION OF DISTRIBUTION MODELS

Throughout this paper both location and distribution models have been discussed to determine the feasibility and applicability of these models in describing and analyzing the operation of the entire distribution system. Shortcomings of the particular examples used were cited. We will now take a closer look at these shortcomings and comment on their influence on the analysis of the complete system.

Distribution system analysis is characterized by cost functions that fail to develop accurately the total costs of distribution processes. Specifically, in both the theoretical and practical approaches to distribution system analysis, the costs that are generated by physical distribution are not well understood or fully identified. This situation has resulted in the use of oversimplified, noncomprehensive cost functions, which can produce serious errors (30) if they are incorporated in a location or distribution analysis. The following important cost factors are commonly misunderstood.

1. The ton-mile statistic is used as a transport cost statistic when it is intended to measure only transportation work. A ton-mile statistic forces equal weighting of the ton and the mile. Many location models, especially gravity formulations, are based on statistics of this type.
2. Transportation costs vary nonlinearly with distance, except when carriers are constrained by regulation to operate under fixed rates.
3. In practice multiple deliveries occur, and a customer may be served on that particular trip if sufficient vehicle capacity exists. Otherwise the demand point must wait to be serviced by another vehicle. This "combinatorial" element of cost (30) indicates that inclusion of shipment size and demand point location only would result in an incomplete cost function.

An early case study of a distribution system (10) did not give adequate consideration to many cost factors. These misunderstood factors, including the three mentioned previously, were used in siting a pair of factories.

The use of a simple function to describe distribution costs is characteristic of the approach to the problems of warehouse location and distribution analysis taken by operations research people. Managers of firms that accomplish their own physical distribution are vitally interested in the complex of costs ascribable to the distribution system. Costs associated with warehousing and inventory are allocated and recorded far more comprehensively and accurately than those costs associated with the actual physical distribution of freight (31).

Also, most operations research approaches to distribution system analysis tend to ignore the complex interactions of the firm's various organizational units that effect physical distribution. A particular example is the assumption of many operations research location models that customer demand is fixed. Notably, these models consider the short-run situation that deals with a fixed set of demands in locating the warehouse, which is a long-term facility (32).

According to Christopher and Wills (33), the total cost concept or the "total logistics" concept is probably the most important concept in physical distribution management. The total cost is as follows:

$$\text{Total cost} = F + I + T_1 + T_2$$

where

F = fixed costs of warehousing,
I = stock holding cost,
T_1 = cost of trucking, and
T_2 = cost of local deliveries.

This principle is well indoctrinated into most firms, but there is little effort to apply this idea in practice (34). Even though distribution is the link between the production and marketing of a product, decisions made without consideration of distribution (e.g., all deliveries made within 48 hours upon receiving the order) will cause an unbalanced cost condition.

Figure 2 (33, p. 214) shows the general relationship of the components of the total cost equation. The curves are not strictly continuous or accurate in shape. They do, however, indicate how the costs behave in relation to an increase in the number of warehouses within the system.

Hoch (4) indicates that, in retrospect, many executives admit that the original decision to locate was made primarily on the basis of freight cost comparisons with no detailed study of rentals, payrolls, taxes, insurance, or inventory carrying costs, where these cost factors actually complete the fixed cost of warehousing.

With this background, changes in the components imply that T_2 would be minimized by establishing a warehouse at every customer's location. Similarly I can be minimized by having a zero inventory, which would result in disastrous serviceability. Therefore, given the possible factors of cost reduction and their implications, it becomes clear that the true minimum cost is the one that minimizes the sum of all costs. This has particular impact as Bowersox commented (1).

Minimizing cost has been the prevailing goal of the models reviewed. However, it must be pointed out that all the authors reviewed had a different concept of what cost should be minimized. As previously discussed, these models can be categorized as either the infinite set approach or the feasible set approach. The main disadvantage of the former is that the method requires that the transport costs be directly related to distance, which is not a valid assumption in all cases. The main disadvantage of the feasible set approach is that not only is a considerable amount of effort and expense involved in building a list of sites and their costs, but also in a changing situation available sites may not be known (33). The models presented consider only cost items relating to weights, destinations, positions, and transportation costs, the sum of which is not the total cost of a distribution system.

The factors determining the location of distribution centers vary substantially from place to place. One is well advised to keep in mind those locational factors that are most important in practical locational decisions. Table 1 gives the decisive factors used in locating truck terminals in the Hall Street area of St. Louis, Missouri. The outstanding feature of this table is the fact that land availability clearly outweighed the

Figure 2. Total cost and component curves.

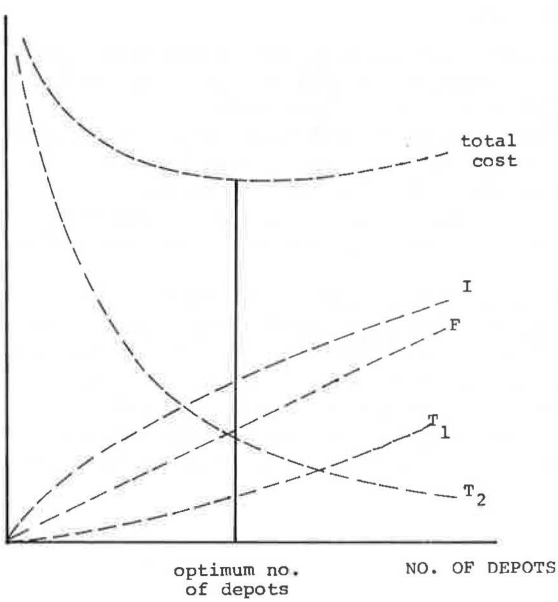


Table 1. Truck terminal location factors (35).

Factor	Percentage of Respondents Considering Factor Most Important
Land availability	81
Proximity to other carriers	35
Proximity to shippers and consignees	27
Cost of land	23
Access to major highways	23

other factors by a substantial margin. The fact that the location of truck terminals has moved outward from the central city supports the importance of land availability, in addition to other locational decision factors. Schwar (36) has documented this latter development in the Chicago metropolitan area for the period 1950 to 1960.

Hoch (4) analyzed the typical mistakes made in warehouse location. His study was based on observations and reports from more than 1,000 U.S. manufacturing corporations. The main point of his analysis is the fact that, when asked "Has your warehouse location been completely successful and, if not, what have been the most important problems?," the following were the most frequent answers given (in descending order):

1. Failure to consider total costs,
2. Carelessness in checking site,
3. Failure to anticipate growth,
4. Underestimating the importance of taxes,
5. Miscalculating labor costs,
6. Inadequate labor reservoir,
7. Lack of supporting facilities,
8. Lack of distribution know-how,
9. Location by imitation or compromise, and
10. Incorrect cost relationship.

Although there was no indication of the percentage of companies dissatisfied with their operation, it is clear that the problems cited above are not part of the data considered throughout the literature on location and distribution.

The elaboration of each of the above indicates that there is a great fallacy in the premise that location and distribution can, indeed, be thought of as a linear operation.

Demczynski (37) believes that the most important single factor that results in such mistakes, as listed above, is the lack of communication between the mathematicians and the line executive. He indicates that mathematicians fail to explain their work in terms that the nonspecialist can understand. The line executives are often suspicious of the methods proposed because they do not fully understand what is proposed and because they carry the responsibility for implementation and results. The magnitudes of the proposals are startling to the executives, and they are not willing to place their reputation on something they do not fully understand. As previously mentioned, heuristic methods appear to be the most reliable techniques to employ in solving the many-faceted location problem.

CONCLUSIONS

This paper has attempted to provide the informational base for evaluating the present capability and applicability of techniques, methods, and models for analyzing distribution system structure. It was determined that no universally accepted and suitable technique or theory exists at the present time. This resulted in the necessity for presenting and evaluating a variety of research efforts documented in the literature. It can be concluded that serious faults underlie component-by-component analysis of the distribution system. This approach is questionable because the performance of the whole system is the decisive element in the functioning of a distribution system and not the individual operation of its components.

Real-world problems are of such a nature that many components must interact. Piecewise analysis is characterized by analytical methods that deal with the components only within their own structure. Optimality is reached within the operation of the individual components but rarely in the context of the whole system-suboptimization.

Clearly, research should be initiated to develop techniques and methods that specifically express or quantify the component interactions typically encountered in real-world problems. Emphasis should be placed on developing a manageable distribution system-warehouse location theory.

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