MOISTURE VARIATION IN HIGHWAY SUBGRADES AND THE ASSOCIATED CHANGE IN SURFACE DEFLECTIONS

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The objectives of this research are to determine the influence of subgrade moisture conditions on surface deflections and to develop a means of predicting relative changes in surface deflections from variations in pertinent pavement system properties. Five field test sites were established and pertinent data were collected for 3 years. A multilinear regression analysis technique was used to determine the relationship between the dependent variable (percent changes in surface deflection) and selected independent variables. Surface deflections are significantly influenced by changes in pavement surface temperature and subgrade moisture. A predicting equation for relative changes in surface deflections from pertinent pavement system properties is developed.

•THE seasonal change in moisture content of subgrade soils with its resultant effects on structural pavement performance is an area of interest to many highway engineers. It is well documented in the literature that the shearing strength of a subgrade soil can be greatly reduced by the influx of moisture during spring thaw or long periods of heavy rainfall (1, 2, 3, 4). This reduction in strength is generally attributed to an increase in moisture content of the subgrade soil resulting in high excess pore pressures between the soil particles and sometimes an associated decrease in soil density. Consequently, the bearing capacity of the subgrade will also be reduced significantly, and extensive deflection of the pavement may result (4, 5, 6, 7).

It is generally accepted that, in fine-grained soils, the shearing strength and bearing capacity of a subgrade reach their lowest value at the beginning of the thawing period in the spring when the excess pore pressures reach a maximum (4, 7, 8, 9). During this time frost boils, pumping, and pavement breakup may occur under a moving load (9). The excess pore pressures will gradually decrease with time, and consequently the subgrade will gradually gain in strength (4). An equation has been developed for the purpose of predicting seasonal changes in pavement surface deflections.

PROCEDURE OF THE INVESTIGATION

Field Sites and Data Collection

For this study, field test sites were selected throughout Pennsylvania. Five of the sites were flexible and three were rigid. Pits 3 ft (0.91 m) wide by 4 ft (1.21 m) long by 5 ft (1.52 m) deep were opened in the center of one of the travel lanes for the purpose of installing thermocouples and moisture cells. Moisture contents, densities, liquid limits, plastic limits, and gradations of all pavement materials were determined by the appropriate ASTM or AASHO standard test. In addition, the thicknesses of the pavement layers were measured.

Type T thermocouples (copper-constantan) from Omega Engineering, Inc., with an error tolerance of ± 0.75 F (0.41 C) were employed in this study to monitor temperature fluctuations. Continuous readout was supplied by Esterline Angus Corporation Model

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E1124E Multipoint Recorders, which cycle every 15-20 minutes, depending on the number of thermocouples in the pavement profile. The surface thermocouple was placed 0.25 in. (0.63 cm) from the surface of the wearing course to protect it from traffic damage. In addition, thermocouples were placed at the interface and midpoint of the surface, base, and subbase layers and every 6 in. (15.24 cm) into the subgrade to a depth of 5 ft (1.52 m).

The moisture-sensing devices consisted of Soiltest Model MC-310A fiberglass soilmoisture cells constructed of two Monel screen electrodes separated by and encased in a wrapping of fiberglass fabric. The units also contained thermistors, which were employed as a check against thermocouple readings, for monitoring temperature. A Soiltest Model MC-300A moisture meter was used to monitor the changes in electrical resistances with associated moisture changes. Calibration of the cells was done essentially by the method proposed by Turner and Jumikis (9). The cells were placed at the same elevations in the pavement profile as the thermocouples, except none were placed in the pavement surfaces.

Deflections were taken on the pavement surface using a Model RR-400 Road Rater manufactured by Foundation Mechanics, Inc. Attached to the front end of a vehicle, the Road Rater consists of a steel mass, hydraulic vibrator, and deflection sensors. The 160-lb (72.6-kg) mass was oscillated at 25 Hz. Pavement displacement, measured by deflection sensors, is clearly displayed on instrumentation located inside the supporting vehicle, adjacent to the operator. The methods of calibration and data collection are explained in detail elsewhere (5).

Data collection at all of the sites consisted of obtaining the moisture and temperature readings in the pavement profile and the associated surface deflection readings at points (2,5), (2,7), and (4,6) as shown in Figure 1. In addition, the accumulated monthly precipitation data were acquired from adjacent Weather Bureau stations. These data, plus the engineering index properties and thicknesses of the pavement layers, were then used in the analysis of the changes in deflections with time at the experimental field sites.

Variables Considered in This Study

A number of authors (3, 6, 10, 11, 12) have recognized the following variables to be contributing factors in the percentage change in subgrade support during the spring-thaw period: moisture content, temperature, density, liquid limit, plastic limit, plasticity index, gradation of the subgrade soil, and thickness of pavement section.

The primary factors of interest in determining the percent change in surface deflection (which can be related to subgrade support), as shown in the literature (4), are the changes in moisture content (Δ MC) of the subgrade and the temperature (Δ T) of the pavement surface. As indicated in the literature (12), the thickness of the various pavement layers (D_p) determines the relative stiffness of the pavement acting as a whole. Logically, therefore, these terms must be included as independent terms in predicting changes in surface deflection. Moreover, the nature of the subgrade soils as reflected by their engineering index properties—particle size distribution, liquid limit (LL), plastic limit (PL), plasticity index (PI), and density (γ d)—significantly influence the overall pavement deflection. Consequently, a statistical mathematical model for establishing the prediction of percent change in deflection must compensate for different soil types.

In this investigation all the previous parameters were studied. The percentage change in surface deflection is expressed as $\frac{1}{2}$

$$\Delta \delta = \frac{\delta - \delta_{min}}{\delta_{min}} \times 100 \tag{1}$$

where $\Delta \delta$ = percent change in deflection from the minimum deflection (δ_{min}) and the deflection (δ) at any date. Likewise,

$$\Delta MC = \frac{MC_{\delta} - MC_{\delta_{min}}}{MC_{\delta_{min}}}$$
 (2)

where ΔMC = percent change in moisture content from the moisture content that corresponds with the minimum deflection ($MC_{\delta_{min}}$) and the moisture content associated with the deflection at any date (MC_{δ}). The particle size distribution or gradation of the subgrade soil is represented by the percent passing the No. 4 and No. 200 sieves.

RESULTS AND ANALYSES

Subgrade Moisture

Figure 2 shows the average change in actual subgrade moisture content as indicated by moisture data from 8 different sites and for the years 1970 through 1973. In addition to the 5 flexible pavement sites used in the deflection analyses, 3 rigid pavement sites are also included to arrive at these subgrade moisture change curves. It is presumed that the type of pavement surface has no significant effect on subgrade moisture variation as long as an adequate seal exists against vertical moisture movement through the surface. The sand curve is comprised of data from the Lairdsville, Wellsboro, and Wilkes-Barre sites, which have an A-2-4 subgrade soil classification. Data from the Clarion, Lantz Corners, and Meadville sites, which have an A-4 subgrade soil classification, make up the silt curve. State College and Washington sites, which have an A-7 and A-6 subgrade soil respectively, provide data for the clay curve.

It can be readily seen that all three subgrade soil types experience the greatest increase in moisture content in March and April. The sand soils show the greatest increase in moisture (3 to 4 percent) as compared to increases of 2 to 3 percent for silt soils and only 1 percent for clay soils. As anticipated, the increased moisture content in the sands drops back to a base level at a faster rate than do the silts. All subgrade soil types perennially reach a base level or minimum moisture content by September and October. The subgrade moisture variation, which occurs throughout the entire instrumented depth of 5 ft (1.52 m), can be associated with lateral movement from the shoulders, fluctuation in the water table and capillary zone, and, to a somewhat lesser degree, infiltration through surface cracks.

Influence of Temperature on Pavement Deflections

Among the variables that affect the deflection of a pavement is the temperature of its surface course. This variable could be included in the multiple linear regression program; however, it can be isolated from the other variables principally for two reasons:

- 1. A related study (5) by the authors has already established the relationship between the temperature of a pavement's surface course and its surface deflection; and
 - 2. A better insight into the influence of all other variables can be attained.

To eliminate the influence of temperature on surface deflections, all readings must be corrected to some standard value. In this study a base temperature of 60 F (15.56 C) is used. Hence, the actual surface deflection values shown in the third column of Table 1 are corrected to values based on pavement surface temperature of 60 F using the appropriate temperature adjustment factor from Figure 3. This linear equation is developed by regression analysis of data recorded at two sites with similar subgrade soil during a period when the moisture remained unchanged.

Seasonal Relationship of Deflection, Moisture, and Precipitation

The effect of subgrade moisture variation on the change in corrected surface deflection is shown in Figures 4 through 8. A definite relationship between moisture and deflection is apparent. A corresponding increase in surface deflection generally occurs with an increase in subgrade moisture, except during winter months when deflections are constant because of the frozen conditions of both the pavement and the subgrade.

A comparison of monthly precipitation with moisture variation indicates erratic peaks and no definite increases in moisture due to periods of heavy rainfall. In some instances, however, points of maximum subgrade moisture are preceded by a few months by periods of high precipitation. It is also noted that after October the subgrade moisture content tends to increase each month during the winter up to a maximum in March or April. This trend was also found by Yao and Broms (4).

Figure 1. Plan view of test site.

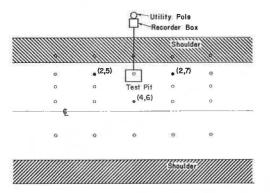


Figure 2. Seasonal change in subgrade moisture content.

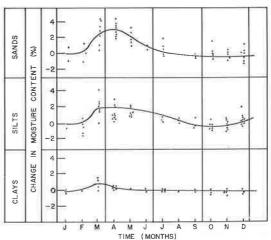


Table 1. Actual and corrected surface deflection data.

Site	Month-Year	Actual Deflection ^a (mils)	Pavement Temperature ^b (deg F)	Temperature Correction Factor ^o (60 F base)	Temperature Corrected Deflection (mils)
Lairdsville	April-72	0.50	55	1.0365	0.52
	May-72	0.47	89	0.7747	0.36
	July-72	0.45	85	0.8055	0.36
	Sept72	0.46	91	0.7593	0.35
	March-73	0.49	51	1.0673	0.52
	April-73	0.47	64	0.9672	0.46
		0.47	85	0.8055	0.38
	May-73	0.46	86	0.7978	0.36
	July-73	0.45	110	0.6130	0.27
	August-73 Sept73	0.36	.93	0.7439	0.26
Meadville	April-72	1.34	79	0.8517	1.13
	May-72	1.08	95	0.7285	0.78
	July-72	0.87	96	0.7208	0.62
	Sept72	0.85	79	0.8517	0.72
	March-73	0.97	46	1.1058	1.07
	April-73	1.05	83	0.8209	0.85
	May-73	1.01	69	0.9287	0.63
	July-73	0.94	100	0.6900	0.64
	August-73	0.98	82	0.8286	0.80
	Sept73	0.83	80	0.8440	0.70
State College	April-72	1.17	64	0.9672	1.12
-	May-72	0.76	73	0.8979	0.68
	July-72	0.94	95	0.7285	0.68
	March-73	0.75	56	1.0288	0.77
	April-73	0.75	73	0.8579	0.67
	May-73	0.72	84	0.8132	0.58
	July-73	0.72	79	0.8517	0.60
	August-73	0.76	88	0.7824	0.59
	Sept73	0.81	74	0.8902	0.71
Washington	April-72	1.34	86	0.7978	1.06
	May-72	0.84	98	0.7054	0.59
	July-72	0.80	95	0.7285	0.58
	Sept72	0.80	94	0.7362	0.58
	March-73	0.90	60	0.9980	0.90
	April-73	0.86	71	0.9133	0.70
	May-73	0.84	64	0.9672	0.77
	July-73	0.83	91	0.7593	0.62
	August-73	0.71	82	0.8286	0.58
	Sept73	0.61	76	0.8748	0.53
Wilkes-Barre	April-72	0.95	49	1.0827	1.03
TALKES DAILE	May-72	0.63	83	0.8209	0,51
	July-72	0.58	83	0.8209	0.47
	Sept72	0.67	95	0.7285	0.48
	March-73	0.75	57	1.0211	0.77
	April-73	0.84	54	1.0442	0.87
	May-73	1.02	81	0.8363	0.85
	July-73	0.85	89	0.7747	0.65
	August-73	0.83	110	0.6130	0.50

^aAverage of points (2,5), (2,7), and (4,6), which are nearest instrument pit in site grid system, ^bAverage of thermocouples inserted in surface course, ^cSee Figure 3,

Figure 3. Deflection adjustment factor for temperature.

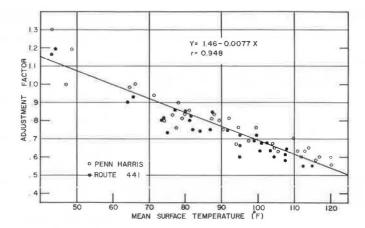


Figure 4. Seasonal relationship of precipitation, subgrade moisture, and corrected surface deflection for Lantz Corners.

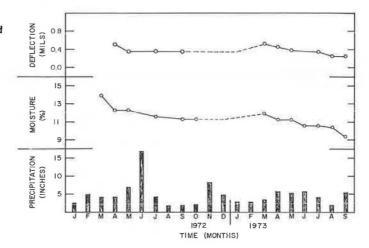


Figure 5. Seasonal relationship of precipitation, subgrade moisture, and corrected surface deflection for Meadville.

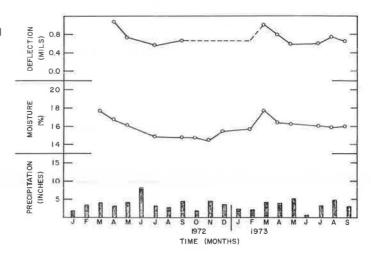


Figure 6. Seasonal relationship of precipitation, subgrade moisture, and corrected surface deflection for State College.

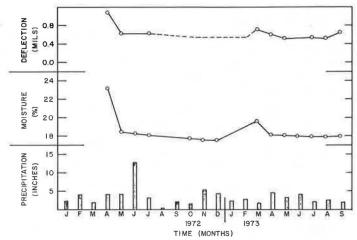


Figure 7. Seasonal relationship of precipitation, subgrade moisture, and corrected surface deflection for Washington.

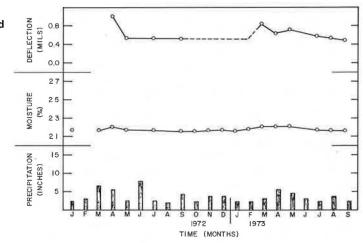
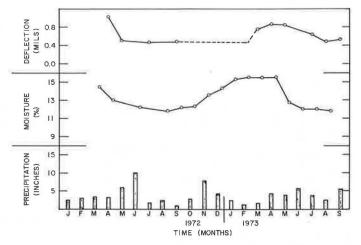


Figure 8. Seasonal relationship of precipitation, subgrade moisture, and corrected surface deflection for Wilkes-Barre.



Statistical Analysis

A stepwise multilinear regression analysis was employed to establish the degree of relationship between the dependent variable (percent change in deflection) and the independent variables—the percent change in moisture content (Δ MC), thickness of entire pavement system (D_p), density (γ_d), liquid limit (LL), and percent by weight passing the No. 200 sieve (%200). The following statistical parameters were evaluated:

- 1. Coefficient of multiple correlation (R);
- 2. Coefficient of determination (R2);
- 3. Analysis of variance ratio (F-ratio);
- Standard error of the estimate (S_v); and
- 5. Regression coefficients for the associated variables.

Tables 2 and 3 show the field data that were analyzed in the development of the regression equation.

Multiple linearity was assumed to exist between the percent change in deflection and the associated variables. A high R-value would substantiate this assumption whereas a low R-value would negate linearity, indicating that possibly a polynomial relationship exists or that no relationship exists at all.

Initially, the percent passing the No. 4 sieve (\$4), the plastic limit (PL), and the plasticity index (PI) of the subgrade soils were also included as independent variables. However, the correlation matrix for investigating correlation between two or more independent variables indicated the following variables were intercorrelated:

- 1. Liquid limit (LL) and plastic limit (PL);
- 2. Liquid limit (LL) and plasticity index (PI); and
- 3. Percent material passing No. 4 and No. 200 sieves.

If these independent variables are intercorrelated, then one of the variables may be removed without affecting the functional relationship. In selecting the independent parameters, the one contributing most significantly to the coefficient of determination value (R^2) must be considered. The logic for this is dictated by the fact that the R^2 -value is defined as the ratio of the explained variation of the regression analysis to the total variation, which in turn indicates the percentage of the total variation that is attributable to the independent variables. With this point in mind, the percent passing the No. 4 sieve, the plastic limit, and the plasticity index were eliminated from the final relationship with no significant reduction in the R^2 -value.

The multilinear regression analysis performed with field data from the 5 bituminous pavements produces the following equation for predicting percent changes in surface deflections:

$$\Delta \delta_{\text{BOE}} = 2.8496 \; (\Delta \text{MC}) + 0.7000 \; (\$200) + 2.4484 \; (D_p) - 0.8342 \; (LL) - 0.3979 \; (\gamma_d)$$
 (3)

This equation gives computed values that produce relatively high multiple R- and R^2 -values and reasonably low standard errors of estimate when compared with the actual values.

The "F" to remove, listed along with other pertinent statistical data in Table 4, is an indicator of the importance of the corresponding independent variable in predicting deflection changes. A higher F-value indicates a more significant variable. Therefore, the change in subgrade moisture content is certainly the most significant of the five variables, and the percent passing the No. 200 sieve is the second most significant variable. The remaining three variables are considerably less significant, and their introduction into the equation produces relatively small increases in the multiple R-value.

The multilinear regression analysis forces a linear equation that produces the best fit using combinations of all variables. As a result, a regression coefficient of any corresponding independent variable reflects not only the variation that is directly due to that independent variable but also the variation that is due to the other independent variables correlated with it $(\underline{13})$. Therefore, it is not possible to relate any one independent variable to the dependent variable.

Table 2. Change in corrected surface deflection with associated change in subgrade moisture.

Site	Month-Year	Corrected Deflection (mils)	Moisture Content (percent)	Percent Change in Surface Deflection	Percent Change in Moisture Content ^b
Lairdsville	April-72	0.52	12.3	100	30.9
	May-72	0.36	12.3	38	30.9
	July-72	0.36	11.6	38	23.4
	Sept72	0.35	11.3	35	20.2
	March-73	0.52	11.9	100	26.6
	April-73	0.46	11.3	77	20.2
	May-73	0.38	11.3	46	20.2
	July-73	0.36	10.6	38	12.8
	August-73	0.27	10.4	4	10.6
	Sept73	0.26°	9.4	_	-
Meadville	April-72	1.13	16.9	82	12.7
	May-72	0.78	16.3	26	8.7
	July-72	0.62°	15.0	-	-
	Sept72	0.72	14.9	16	-0.7
	March-73	1.07	17.8	73	18.7
	April-73	0.85	16.1	37	7.3
	May-73	0.63	16.4	2	9.3
	July-73	0.64	16.2	3	8.0
	August-73	0.80	16.0	29	6.7
	Sept73	0.70	16.1	13	7.3
State College	April-72	1.12	23.3	90	28.7
	May-72	0.68	18.5	15	2.2
	July-72	0.68	18.2	15	0.6
	Sept72 March-73	0.77	19.8	31	9.4
	April-73	0.67	18.3	14	1.1
	May-73	0.59	18.2	-2	0.6
	July-73	0.60	18.1	2	0.0
	August-73	0.59°	18.1	_	-
	Sept73	0.71	18.1	20	0.0
Washington	April-72	1.06	22.2	100	2.3
	May - 72	0.59	21.9	11	0.9
	July-72	0.58	21.8	9	0.5
	Sept72	0.58	21.7	9	0.0
	March-73	0.90	22.2	70	2.3
	April-73	0.70	22.2	32	2.3
	May-73	0.77	22.2	45	2.3
	July-73	0.62	21.8	17	0.5
	August-73	0.58	21.7	9	0.0
	Sept73	0.53°	21.7	-	-
Wilkes-Barre	April-72	1.03	14.5	119	18.9
	May-72	0.51	13.0	9	6.6
	July-72	0.47°	12.2	-	-
	Sept72	0.48	11.8	2	-3.3
	March-73	0.77	15.5	64	27.0
	April-73	0.87	15.5	85	27.0
	May-73	0.85	15.5	81	27.0
	July-73	0.65	12.7	38	4.1
	August -73	0.50	12.0	6	-1.6
	Sept73	0.55	11.8	17	-3.3

Table 3. Pertinent engineering characteristics of field test sites.

Site	Corrected a Minimum M Deflection D	Moisture Content at Minimum Deflection (percent)	Depth of Pavement Section (in.)	Engineering Properties of Subgrade Soils				
				Density (pcf)	Liquid Limit	Plasticity Index	Percent Passing No. 200 Sieve	Subgrade Classification
Clariona	=	= .	24.5	117	31	8 -	61.5	A-4(3)
Lairdsville	0.26	9.4	16.0	124	24	3	26.0	A-2-4(0)
Lantz Corners*	-	-	17.0	112	25	2	65.1	A-4(0)
Meadville	0.62	15.0	12.0	110	24	3	66.2	A-4(0)
State College	0.59	18.1	18.0	100	55	20	74.9	A-7-5(17)
Washington	0.53	21.7	15.0	105	37	11	85.9	A-6(10)
Wellsboro	-	-	19.0	116	21	1	29.4	A-2-4(0)
Wilkes-Barre	0.47	12.2	24.0	127	16	NP	29.2	A-2-4(0)

^aConcrete pavement,

 $^{^{}a}Computed as (\delta - \delta_{min})/\delta_{min}, \\ ^{b}Computed as (MC - MC @ \delta_{min})/(MC @ \delta_{min}), \\ ^{c}Corrected minimum surface deflection (\delta_{min}), \\$

Table 4. Multilinear regression analysis data.

Independent	Confident	"F"	Multiple	Increase
Variable*	Coefficient	to Remove	R	R
ΔMC	2.8496	40.79 ^b	0.8423	0.8423
½200	0.7000	7.29°	0.8824	0.0692
D_p	2.4484	4.13	0.8859	0.0035
LL	-0.8342	3.98	0.8898	0.0039
γa	-0.3979	2.96	0.8980	0.0082

^aDependent variable is percent change in surface deflection (△δ) corrected to 60 F temperature base.

Figure 9. Actual versus computed percent change in deflections (adjusted to 60 F temperature base).

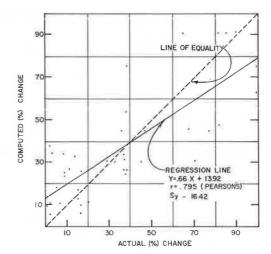


Figure 9 shows a scatter diagram and the associated statistical parameters of the percent change in deflection computed with the multilinear regression equation versus the actual percent change measured in the field. When the regression line coincides with the line of equality, perfect correlation is established. The moderately high r-value and low standard error of estimate indicate a good correlation between predicted and actual change in deflection values. Thus, the prediction of the change in deflection appears to be suited for bituminous pavements placed on plastic subgrade soils. Furthermore, the validity of the equation predicting deflection change, Eq. 3, is established for the five investigated sites.

SUMMARY

- 1. Surface deflections of flexible pavement systems are significantly affected by temperature variation in the surface and moisture variation in the subgrade.
 - 2. Seasonal variations in subgrade moisture occur as anticipated.
- 3. Percent change in surface deflection of flexible pavements can be predicted with reasonable confidence by the following equation:

$$\Delta \delta_{60 \text{ F}} = 2.8496 \ (\Delta MC) + 0.7000 \ (200) + 2.4484 \ (D_p) - 0.8342 \ (LL) - 0.3979 \ (\gamma_d)$$

RECOMMENDATIONS FOR APPLICATION

- 1. Along with field testing, spring load restrictions can be established.
- 2. Maximum and minimum surface deflection values can be predicted from field conditions.
- 3. Fatigue life of flexible pavement surfaces can be predicted from change in surface deflection.

bSignificant at 1 percent level.

Significant at 5 percent level.

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