DILATANCY OF GRANULAR MEDIA IN TRIAXIAL SHEAR

K. P. George, Department of Civil Engineering, University of Mississippi; and
N. S. Shah, Sargent and Lundy, Chicago

ABRIDGMENT

• COHESIONLESS (granular) aggregates depend on the friction and the interlocking action between the individual particles for strength and stability. Crushed rock and natural river gravel, both widely used in flexible pavements, are examples of this type of material.

The fundamental assumptions of the strength theories ignore the fact that granular soil consists of individual grains rather than a homogeneous mass with certain mechanical properties. A particulate approach is emphasized in this study; that is, granular soils consist of randomly arranged, irregularly shaped, discrete particles that are free to displace relative to each other.

Idealizing the granular media to a continuum, the criterion of failure under the triaxial stress system, where the effective principal stresses are $\bar{\sigma}_1$ and $\bar{\sigma}_2 = \bar{\sigma}_3$, is given by

$$\bar{\sigma}_1 = \bar{\sigma}_3 \tan^2 \frac{\pi}{4} + \frac{\phi_d}{2} \tag{1}$$

in which $\bar{\sigma}_1$ and $\bar{\sigma}_3$ = maximum and minimum principal effective stresses respectively and ϕ_d = drained angle of friction. A straight slip line is predicted at failure that subtends an angle of $(\pi/4 + \phi_d/2)$ to the major principal plane. Equation 1 suggests that the assembly slides at failure with no volume change, just as in the case of two blocks of material in a simple friction test. Nevertheless, Osbourne Reynolds as early as 1885 reported that dense sands expand at failure, a condition which he named dilatancy. Several other researchers (4, 7, 8) investigated this problem and concluded that, in general, all specimens weak at yield (very loose) must be compacting and that all specimens strong at yield (dense) must be dilating.

In this study equations for volume change are derived in accordance with the particulate model advanced by Rowe (10). Using the stress-deformation results obtained from triaxial compression tests, the volumetric strain of two granular materials (a natural gravel and a crushed limestone) during triaxial loading are predicted and compared with the observed data. We also discuss how dilatancy/contraction is influenced by (a) surface texture of particles, (b) elastic properties of individual grains, (c) geometrical factors such as grain size and shape factors, and (d) packing of the mix.

ANALYSIS OF A SIMPLIFIED MODEL OF COHESIONLESS SOIL

The behavior of granular masses subject to external forces and displacement is governed by the individual forces and displacements occurring at each particle contact. Thurston and Deresiewicz (12) used a regular array of perfectly uniform spheres, in which each sphere was in contact with 12 others (face-centered cubic array), and discussed the conditions under which sliding of adjacent layers occurs. They derived a ratio of shearing to normal forces on the potentially sliding layer of spheres, given as

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$$\frac{D \cos \beta}{2\sqrt{3}\, R^2 \sigma_3 + D \cos \gamma} = \frac{3 + 4\sqrt{2}\mu}{2(\sqrt{6} - \mu)} \tag{2}$$

in which D = the force tending to cause movement; R = radius of a single sphere; σ_3 = initial hydrostatic pressure to which the assembly is subjected; μ = coefficient of physical friction; and β and γ = respectively the angles made by D with the $\bar{y}$- and $\bar{z}$-axis. In the absence of friction, shearing force is required to do work in order to lift the sliding balls toward the crest against the hydrostatic stress, which can be obtained by setting $\mu = 0$ in Eq. 2. This latter component of shear strength is often designated as the dilatancy component.

PROPORTIONING THE MATERIALS

Two contrasting granular materials were selected for the laboratory testing: a natural gravel (specific gravity 2.82) and a crushed limestone (specific gravity 2.65). Each of these materials was separated into various fractions; for example, 25.4 mm to 19.0 mm, 19.0 mm to 12.7 mm, and so on. Various components stored separately were then combined to result in a graded material, as shown in Figure 1.

TRIAXIAL TEST DETAILS

The granular aggregates were tested in triaxial compression using samples measuring approximately 100 mm (4 in.) in diameter and 200 mm (8 in.) in height. Complete saturation was ensured before testing under a confining pressure of 100 kPa (15 psi). A constant rate of strain, 0.3 percent per minute, was maintained throughout the test.

The volume change of the sample was estimated by reasoning that in a fully saturated sample the volume of water expelled or drawn in is a direct measure of the volume change of the sample. As described by Bishop and Henkel (1), the volume of water expelled or drawn in is measured by a 100-ml burette.

The observed strength results were adjusted for membrane correction, as proposed by Henkel and Gilbert (3), and for the chamber piston friction by direct measurement. The nonuniformity in deformation resulting from possible end restraints was minimized by using lubricated end plates.

STRESS-DEFORMATION CHARACTERISTICS OF THE MATERIALS

Stress ratio and volumetric strain are plotted in Figure 2 as functions of axial strain for the two gravels compacted to respective maximum densities. During the initial stage the strains are small and nearly proportional to stress. Sliding between particles does not begin until the stress increment exceeds some critical stress ratio, approximately 3 and 4 for round gravel and limestone respectively. Due only to slippage, both materials exhibited nonlinear stress-strain relationships, the more so in the round gravel. In both gravels, as the shear motion continues, the degree of interlocking decreases, as evidenced by the decrease in shear resistance. Of the two mixes, round gravel experiences considerable shear motion accompanied by an increase in dilatancy (Fig. 2) to give rise to a somewhat sharp drop in shear strength beyond the peak.

As expected, these mixes decrease in volume under the initial increments of stress followed by an increase in volume with a net volume increase.

DILATANCY PREDICTION

Using a rigid plastic model, Yamaguchi (13) derived expressions for the Poisson's ratio and for unit expansion in the flow state during triaxial testing. The expression for unit expansion v is

$$v = \frac{\epsilon_1 \sin^2 \phi \left(\frac{\sigma_1}{\sigma_3} + 2\right)}{\frac{\sigma_1}{\sigma_1} - (1 + \sin \phi)^2} \tag{3}$$

in which ϕ = angle of shearing resistance and ϵ_1 and ϵ_3 = major and minor principal strains respectively. These results show that dilatancy increases with an increase in the angle of friction mobilized during loading.

Dilatancy of limestone is predicted in accordance with Eq. 3 and is plotted in Figure 3. The computed curve overpredicts the experimental volumetric strain shown by data points. The large difference can be attributed to, among other factors, the fact that in this analysis no allowances have been made for the elastic compression of the specimens.

To improve on the volumetric strain prediction, an alternate expression—in accordance with the stress-dilatancy theory—is derived here. According to this theory, soil particles in a dense mass move in such a way that a minimum of internal energy is absorbed. Rowe (11) computed an expression for the energy ratio (ratio of the instantaneous rate of work done on the sample by $\bar{\sigma}_1$ to that done by the sample against $\bar{\sigma}_3$) as

$$\dot{E} = \frac{1}{2}\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_3}\right)\frac{d\epsilon_1}{d\epsilon_3} = \tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right) \tag{4}$$

where ϕ_μ = angle of physical (solid) friction.

The stress-strain relation obtained from a drained triaxial test can be approximated by a series representation,

$$\epsilon_1 = A\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_3}\right)^n \tag{5}$$

in which A and n are empirical constants determined by a least-squares curve-fitting procedure. The differential $d\epsilon$ from Eq. 5 is substituted in Eq. 4. The resulting expression is integrated to give ϵ_3, which when divided by ϵ_1 gives the Poisson's ratio of the aggregate, $\bar{\nu}$:

$$\bar{\nu} = \frac{n}{2(n+1)\tan^2\left(\frac{\pi}{4} + \frac{\phi}{2}\right)}\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_3}\right) \tag{6}$$

Since the Poisson's ratio is known, the volumetric strain, v, is computed by

$$v = \epsilon_1(1 - 2\bar{\nu}) \tag{7}$$

Figure 3 shows a comparison of the experimental data points and the volumetric strain predicted by Eq. 7 for round gravel and limestone. For all six limestone mixes of various gradations, the predicted volumetric strain agrees with the experimental data. In round gravel, however, the volumetric strain values developed from theoretical calculation are significantly larger than observed in the laboratory test. In actual test, due to slippage, not all the grains will move in an expanding direction during shear; the measured behavior, therefore, is a result of the statistical averaging of the movements of many grains.

MATERIAL PROPERTIES RELATED TO DILATANCY

The various material characteristics that govern the strength and dilatancy of granular materials may be classified into two groups: intrinsic factors, such as surface texture, shape, size, and elastic properties of grains, and extrinsic factors, such as grading and porosity or packing of aggregations.

Dilatancy in Relation to Surface Texture

Surface texture is often expressed by the angle of surface friction, ϕ_μ. As in Eq. 1, the strength of a granular medium increases with increasing ϕ_d, which in turn is a function of ϕ_μ. Morris (9), for example, has documented that an increase in texture alone would increase the strength by about 37 percent.

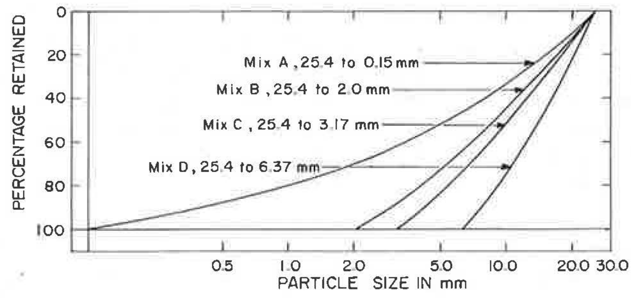


Figure 1. Particle size distribution curves for various mixes, material smaller than 25.4 mm.

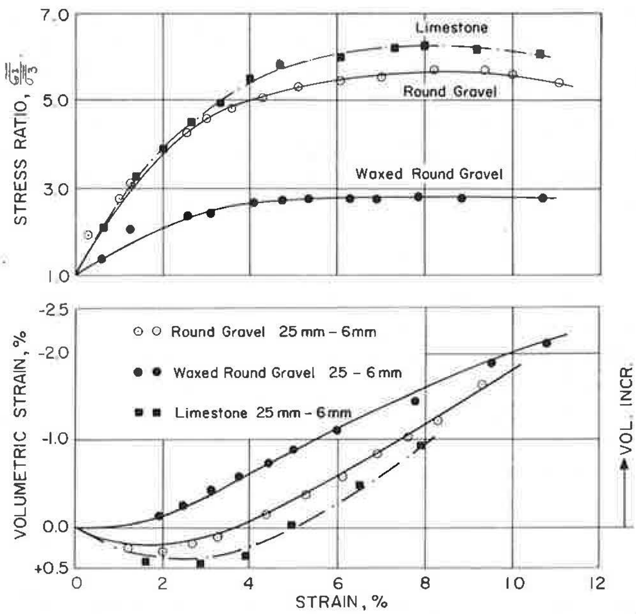


Figure 2. Drained triaxial test results: Stress ratio and volumetric strain related to axial strain, confining pressure 15 psi (100 kPa).

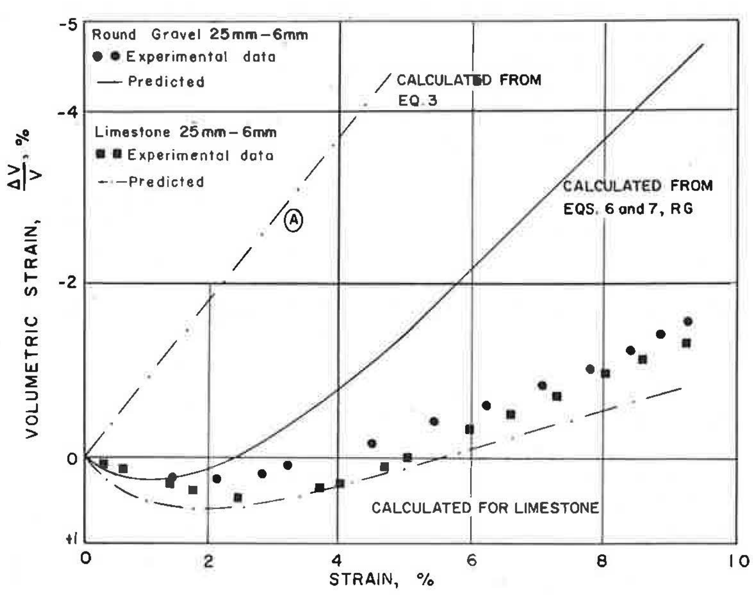


Figure 3. Volumetric strain of round gravel and limestone: Comparison of experimental and predicted values.

To clarify the effect of ϕ_μ on dilatancy, the stress-deformation-volumetric strain relations of round gravel with and without wax coating are predicted by Eqs. 6 and 7. In wax-coated gravel ϕ_μ is reduced from 28 deg to 22 deg, implying that the strength is substantially decreased, whereas the dilatancy is increased. Considering the energy dissipation during loading, both of these observations can be substantiated. The tentative conclusion, therefore, is that dilatancy of granular media increases with decreasing surface texture.

Strength and Volumetric Strain in Relation to Modulus of Particles

To study the stiffness characteristics of an aggregate theoretically, we may make use of the analysis of Deresiewicz (2). He derived the stress-strain relations for an aggregate of equal spheres (simple cubic lattice) subjected to triaxial loading. Because shear modulus for round gravel (29×10^6 kPa) is higher than that for limestone (15×10^6 kPa), the theory predicts that the round gravel is stiffer than the limestone (Fig. 4). The experimental curves in Figure 2, however, contradict the theoretical results. The fact that ϕ_m and the grain concentration of limestone are greater than for round gravel partially explains and supports the experimental findings. These results are in general agreement with those of Morris (9), who reports that so-called tough and hard materials possess little, if any, strength advantage over relatively soft and friable materials, unless they differ in roughness.

Effect of Geometric Factors

The geometrical factor in soils is reflected in the grain size, shape, grain distribution, and density of packing.

Grain Size—The effect of grain size is studied by testing various mixes labeled A, B, C, and D in Figure 1. The results, as shown in the right side of Figure 5, indicate that ϕ_d remains constant regardless of d_{10}; however, the dilation component ϕ_δ increases with an increase in d_{10}.

By setting $\mu = 0$, Eq. 2 becomes

$$D_0 = \frac{1}{2\sqrt{2}\cos\beta}(2\sqrt{3}\,\sigma_3 R^2 + D_0 \cos\gamma) \tag{8}$$

where D_0 is the value given to the inclined force required to initiate failure when the sphere-to-sphere friction is zero. Equation 8 clearly demonstrates that D_0, which is the force required to dilate the sample, increases with the square of the radius of the constituent particles.

Effect of Roughness—Morris (9) proposed the concept of roughness, which includes the effects of both shape and texture of particles. The shape of the particles governs the number of contacts that will occur between adjacent particles, and the texture at the point of contact determines the strength or resistance to particles sliding over each other.

The effect of shape of the aggregate is investigated by testing two mixes prepared from the parent crushed limestone. The first mix consisted predominantly of "platey" particles ($r_v = 0.63$, where r is the shape factor) and the second of "chunky" or nearly cubical particles ($r_v = 0.72$). A comparison of the strengths (or ϕ_d) and the failure strains of three samples reveals that the peak strength is not materially changed, whereas the slope of the stress-strain diagram decreases with decrease in shape factor. This suggests that an aggregate of chunky particles is superior to one composed of platey particles, which is in agreement with others' results (8). The volumetric strain data confirm some previous results (7) that crushed gravels of elongated, platey particles will undergo a decrease in volume during failure in contrast to chunky materials, which will dilate during failure.

The effect of texture was discussed earlier, where it was shown that the texture influenced the strength more than the dilatancy characteristics. It is concluded that between the two factors, shape and texture, the former primarily controls the dilatancy characteristics whereas the latter influences the undrained shear strength.

Figure 4. Stress-strain relation for a simple cubic array of like spheres (radius 25 mm).

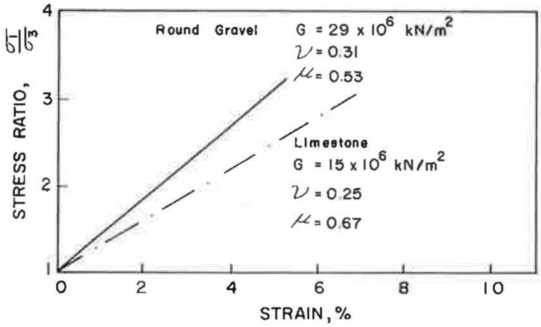
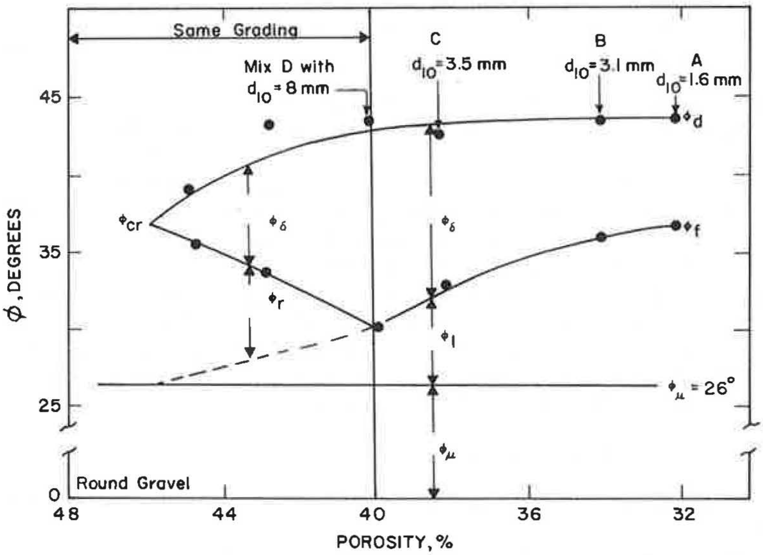


Figure 5. Components of ϕ for round gravel at various densities and different gradations: Three pairs of points on the left side are results from mix D compacted to three different porosities.



Porosity

The dilatancy component of shear strength, computed in accordance with the stress-dilatancy theory, is related to porosity. A minimum energy line is plotted for each test (10) and a value of ϕ_f is obtained from the best-fit curve for the plotted points by using

$$\bar{\sigma}_1 = \bar{\sigma}_3 (1 + K_F) \tan^2 \left(\frac{\pi}{4} + \frac{\phi_f}{2}\right) \tag{9}$$

in which $K_F = (d\dot{v}/v)/\dot{\epsilon}_1$ at failure, where $d\dot{v}/v$ = increment in volume change per unit volume, positive on expansion; $\dot{\epsilon}_1$ = maximum principal strain rate, positive on compression; and ϕ_f = that portion of ϕ_d with dilation removed.

The ϕ_f values on the left side of Figure 5 are obtained from mixes of different density having the same width of grading (mix D). In both materials ϕ_f increases with increase in porosity from a minimum value (which, according to Rowe, is equal to ϕ_μ) to ϕ_{cv} at the maximum porosity. We also note that $\phi_\delta (\phi_\delta = \phi_d - \phi_f)$ decreases with porosity; this relation reinforces the conjecture that, for a given mix of given gradation, looser packing lowers the energy spent in dilation.

Increasing the width of grading in a basic mixture (by adding more fines, mixes A, B, and C) results in decreased porosity. The results, as shown in Figure 5, show that, when the porosity is decreased, ϕ_d remains unchanged; however, ϕ_δ is somewhat reduced. For a graded sand Kirkpatrick (6) reported a decrease in the angle of shearing resistance with the decrease in porosity; he attributed this decrease, at least in part, to the reduction in dilatancy component.

The observation that ϕ_f increases with the decrease in porosity, however, contradicts Rowe's original hypothesis that, at the minimum porosity, ϕ_f tends to equal ϕ_μ. While clarifying certain aspects of stress-dilatancy theory, Horn (5) asserts that ϕ_μ should always be a few degrees smaller than ϕ_f. Accordingly, the statement may be advanced that the drained shear strength is constituted of the following four components rather than only three as proposed by Rowe (10): (a) strength arising from surface friction, ϕ_μ; (b) strength due to interlocking (which increases with the density of the mix), ϕ_i; (c) strength corresponding to the energy spent in remolding (which is zero in a dense mix), ϕ_r; and (d) strength equivalent to the energy spent in dilating the sample against the confining pressure, ϕ_δ. It may be noted that the present classification recognized an additional component, namely, the strength due to interlocking, which, according to the experimental results, increases with the decrease in porosity.

CONCLUSIONS

1. The main cause of strain in granular materials is relative movement (sliding and rolling) between particles.
2. The volumetric strain of crushed limestone, predicted according to the stress-dilatancy theory, is in agreement with the experimental data; in smooth gravel, however, the predicted value is larger than that of the observed data.
3. The dilatancy during failure increases with decreasing values of physical (solid) friction of the grains.
4. The strength and dilatancy of granular aggregates depend not on the stiffness of the constituent particles but on the shape and surface texture of the grains.
5. Crushed gravels of elongated (platey) particles undergo a decrease in volume in contrast to chunky subrounded aggregates, which tend to dilate at failure. A previous study (8) reports that all gravel materials contract at confining pressures of 50 psi and greater.
6. Dilatancy increases with increasing effective size (d_{10})—more so in the rounded natural gravel.
7. Increasing the coefficient of uniformity produces a negligible effect on ϕ_d; doing so, however, decreases the dilatancy component in both gravels.

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