BASIC PRINCIPLES OF HIGH-STRENGTH CONCRETE

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A hypothesis that concerns concrete strength is proposed based on testing under compression with elimination of surface friction. Three basic parameters are used: splitting strength of aggregate, tension strength of mortar or cement-stone, and adhesion between aggregate and mortar or cement-stone. The structural stresses from shrinkage of concrete are taken into account, and the degree of relaxation with time is evaluated. The hypothesis, which is confirmed by experimental results of testing concrete made with aggregates from several rocks, is used for producing high-strength concrete.

POSSIBLE WAYS of producing high-strength concrete with portland cement and other binders were considered at a meeting on development of high-strength concrete (1). It was noted that increasing concrete compressive strength can be achieved by increasing the tensile strength of the mortar using, in particular, dispersed reinforcement or by improving adhesion between the aggregate and cement-stone. Also indicated was the necessity for further in-depth investigation into internal stresses in concrete. The need for improving adhesion and tensile strength of cement-stone or mortar is also discussed by A. J. Harris, who referred to the method of dispersed reinforcement as one that needs further investigation, especially for clearing up the basic thesis by which concrete strength can be predicted. Use of high-strength aggregate with good adhesion to cement-stone was also discussed. Portland cement of higher strength will produce higher concrete strength when used correctly with traditional methods of concrete preparation. Naturally the question arises whether the traditional technology has the hidden reserves. One must consider the existing theory of concrete strength to answer this question.

Skramtaev (2) considers the macrostructure of concrete made with one size of coarse aggregates and focuses on shear. Cement-stone structure was discussed by Powers and Brownyard (3). Davin (4) discusses the possibility of application of Mohr's envelope curves to heterogeneous material like concrete. This idea was developed by Pelter (5).

There are macrostructural theories of concrete strength (6, 7). Baker (7) does not consider the concentration of stresses in the heterogeneous body and does not evaluate the stress state inside the aggregate. As in the hypothesis of Reinius, the internal stresses caused by the shrinkage of concrete are disregarded, although these play an important role in concrete strength. Local concentration of compression stresses equal to 4.37 P and tension stresses equal to P, according to Timoshenko's data, appear on the surfaces of aggregate particles included in cement-stone under compression P. Our tests confirm the existence of high stress concentrations near the aggregates. The tension stresses, equal to the average compression stress, check the strength of concrete.

These tests also show that splitting stresses, which can break the aggregate, arise in the aggregate under compression. According to l'Hermite (8), transverse deformations of the aggregate cause stresses in the cement-stone surrounding the aggregate. These transverse stresses cause separation of the cement-stone from the aggregate or cracking of the cement-stone. But even before loading concrete by the external force, there is a heterogeneous field of internal stresses in it caused by the shrinkage of the cement paste during hardening. Thus the concrete loaded without surface friction, which

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is the case in axially compressed elements such as columns and walls, is destroyed under compression because of (a) splitting of aggregates when the strength of cement-stone is great enough and the adhesion is good, (b) breaking of the bond between the mortar or cement-stone and coarse aggregate, and (c) rupture of cement-stone. The internal stresses play a significant role in the strength of concrete (according to l'Hermite, Basant, Pelter, and Desov). The concrete specimens loaded without surface friction are destroyed with the formation of vertical cracks in two perpendicular directions. Tests on crack stability (9) have shown that the destruction of concrete with more than 5 kgf/cm^2 takes place with almost the same relative strain; in other words the first hypothesis of concrete strength is the hypothesis of maximum stresses.

Three processes occur simultaneously in the concrete and in the mortar with time: the increase of compressive and tensile strengths; the increase of shrinkage and internal stresses; and the decrease of internal stresses under constant strain, i.e., the relaxation of stresses.

The total development and relaxation of internal stresses may be expressed by the following exponential equation:

$$R_{csp} = \frac{C_1 f\ (w/c,\ D,\ n)}{t^2\ (e^{C_o/t} - 1)} \tag{1}$$

where

R_{csp} = mean internal stress at age t,
w/c = water-cement ratio,
D = aggregate size,
n = aggregate-cement ratio,
t = age of the concrete, and
C_o and C_1 = experimental factors.

The R_{co} compressive strength of concrete can be expressed by the following formula:

$$R_{co} = AR_{sp} + FR_{bo} + BR_p + R_{csp} \tag{2}$$

where

$A = \frac{\pi}{4}\frac{D^2}{(D + d)^3}(a + b)K$;

$F = \frac{\pi}{2}\frac{D^2}{(D + d)^3}(a + b)(1 - K)$;

$B = \frac{(a + b)d}{(D + d)^3}$, where the height of the potential surface of rupture between aggregates in the perpendicular direction to the plain of section is assumed to be 1 cm;

R_{sp} = the splitting strength of aggregate;
R_{bo} = mean bond strength;
R_p = tensile strength of cement-stone or mortar;
d = thickness of cement-stone layer;
a, b = dimensions of specimen; and
K = coefficient.

For finding the four coefficients A, F, B, and R_{csp} use this system:

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 &= R_1\\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 &= R_2\\
a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 &= R_3\\
a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 &= R_4
\end{aligned} \tag{3}$$

The following values of coefficients were obtained from concrete tests with four rock aggregates in specimens 10 by 10 by 10 cm: $A = x_1 = -0.4$, $F = x_2 = +14.32$, $B = x_3 = +2.02$, and $R_{csp} = x_4 = -32.0$. The tests were done when surface friction was eliminated.

Substitution of these values into Eq. 2 gives values of the strength in the last column of Table 1. There is good agreement, except for the case of the concrete with granite aggregate II, which had a very low splitting strength (19 kgf/cm^2).

Several experimental and theoretical works have appeared in which attempts were made to create a theory of concrete strength. For instance, aside from the publications already cited, Alexander and Taplin (10) proposed a relationship between concrete compressive strength R_{oo} and bond strength by bending R_{bb} and the bending strength of cement-stone R_{cb}, as follows:

$$R_{co} = 480 + 2.08R_{cb} + 1.02R_{bb} \tag{4}$$

The units of all values are measured in psi. From Eq. 4 it is seen that a change in bending strength of cement-stone changes the concrete twice as much as a change in bond strength. The metric form of Eq. 4 is

$$R_{co} = 33.6 + 29.71R_{cb} + 14.57R_{bb}$$

It is hard to imagine how the bending strength of cement-stone can play a role in the fracture of concrete by uniaxial compression. It would be more logical and more clearly physical if, instead of bending strength, the direct tensile strength of cement-stone were used. Also, instead of bond strength by bending, it would be more correct to introduce adhesion by tension or by shear because this form of bond strength is influenced more by the diameter and shape of aggregate. It is well known that the ratio between tensile strength by bending and direct tensile strength is approximately 2. The average value of this ratio (11) also equals approximately 2. For the bond by bending and bond by tension or shear, this ratio may be taken as equal to 1.

As a result, the equation of concrete strength (10) can be written

$$R_{co} = 33.6 + 14.85R_p + 14.57R_{bo} \tag{5}$$

Concrete strength is determined by the usual method, that is, without elimination of surface friction (Eq. 5). Surface friction increases the measured concrete strength, depending on specimen size. The real strength equals nearly 0.6 of the concrete strength tested without elimination of surface friction on specimens 15 by 15 by 15 cm. It equals only 0.4 of the standard concrete strength measured on specimens 10 by 10 by 10 cm. The strength formula (Eq. 5) can be written as follows if one takes into consideration surface friction:

For specimens 15 by 15 by 15 cm [$K = 0.6$]

$$R_{co} = 34 + 8.9R_p + 8.75R_{bo} \tag{6}$$

For specimens 10 by 10 by 10 cm [$K = 0.4$]

$$R_{co} = 34 + 5.95R_p + 5.83R_{bo} \tag{7}$$

For the additive constant, the K correction factor is not used because it expresses internal stresses that do not depend on surface friction.

Equation 2 with the calculated values of coefficients A, F, B, and R_{csp} for the specimen 10 by 10 by 10 cm are

$$R_{co} = -0.4R_{sp} + 14.32R_{bo} + 2.02R_p - 32 \tag{8}$$

If the first member representing aggregate splitting is omitted, then the formula is like that of Alexander and Taplin, but with the coefficients for the five tested rocks it becomes

Table 1. Comparison of experimental and theoretical strength of concrete.

Rock Type	Splitting Strength of Aggregate (kgf/cm^2)	Mean Bond Strength With Cement-Stone (kgf/cm^2)	Tensile Strength of Cement-Stone (kgf/cm^2)	Compressive Strength	
				Experimental	Theoretical
Sandstone	a_{11} = 37.3	a_{12} = 15.0	a_{13} = 26.2	R_1 = 221	221
Liporite	a_{21} = 99.0	a_{22} = 20.0	a_{23} = 26.2	R_2 = 268	267
Granite I	a_{31} = 70.0	a_{32} = 15.0	a_{33} = 26.2	R_3 = 222	208
Granite II	a_{41} = 19.0	a_{42} = 11.0	a_{43} = 26.2	R_4 = 214	171

Figure 1. Influence of aggregate diameter on concrete strength as provided by coefficients A, F, and B.

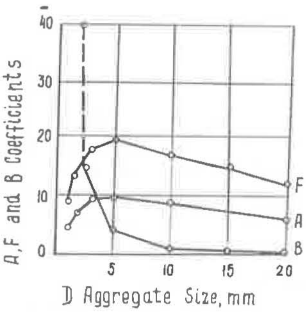


Figure 2. Influence of specimen size on concrete strength by uniaxial compression with (—) and without (- - -) friction.

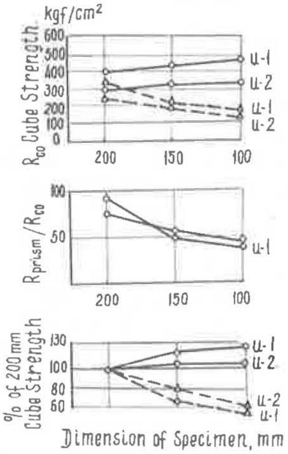
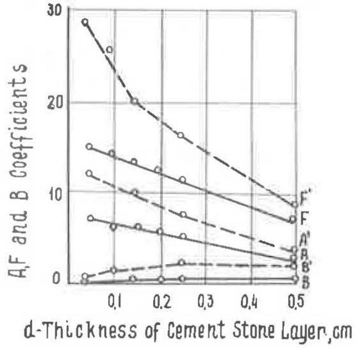


Figure 3. Influence of cement-stone layer thickness (composition of concrete mix) on compressive strength of concrete as provided by coefficients A, F, and B.



$$R_{oo} = 3.92R_{bo} + 6.29R_p - 32$$

The number of the destruction planes, as can be seen from Figures 1 and 2, depends to a great extent on the average diameter of the aggregate and on the composition of the concrete mix, i.e., on the thickness of cement-stone layer between particles. In addition, it is necessary to take into account that Eq. 2 was developed for possible fracture along two planes. This is confirmed by theoretical and experimental work (12).

The concrete is represented by a two-phase system (13). The strength is determined by the mean compression strength of mortar and that of the aggregate and the percent and modulus of elasticity of every phase. A comparison of this hypothesis with experimental data gives very great differences between calculated and experimental strengths.

CONCLUSIONS

The hypothesis about concrete strength in a limit state (Eq. 2) permits one to arrive at a number of important consequences and practical conclusions.

1. From the theoretical point of view, the strength of the concrete made with the same diameter of the aggregate and tested with the elimination of surface friction must decrease with a decrease in specimen size. It must not increase as it has until now because of the results of tests performed without the elimination of surface friction. Our tests confirm this theoretical conclusion (Fig. 2) both from qualitative and quantitative points of view. (In Fig. 2, calcium carbonate aggregate U-1 has a 5- to 10-mm gradation, w/c = 0.48 by weight, unit weight = 2.31 ton/m^3, and Vebe workability = 30 sec. U-2 has a 10- to 30-mm gradation, w/c = 0.44 by weight, unit weight = 2.35 ton/m^3, and Vebe workability = 50 sec.)
2. The concrete strength measured on 20-cm cubes without surface friction is theoretically 78.9 percent of the standard 20-cm cube strength that is close to the prismatic (20 by 20 by 80 cm) strength.
3. The role of aggregate is twofold. The number of planes of destruction in a limited state decreases with an increase of aggregate diameter in a specimen of the same size and, therefore, the strength of concrete must diminish. Also the internal stresses sharply increase with the increase of the aggregate diameter, i.e., both of these causes must decrease the strength. Tests confirm this up to a certain limit (Fig. 2).
4. The theoretical strength of concrete must increase with the decrease of the thickness of the cement mortar layer or with the decrease of cement content up to a certain limit (Fig. 3). Tests of many investigators confirm this thesis (15, 16).
5. The prismatic and cube strengths theoretically can be equal at 25 by 25 by 25 cm. Tests (17) confirm this thesis.
6. Dispersed fiber reinforcement (a) does not influence the splitting strength of aggregate; (b) does not improve, perhaps even reduces, the adhesion of cement-stone to the aggregate; and (c) can only improve the tensile strength of cement-stone. Dispersed reinforcement essentially postpones the occurrence of cracking in the concrete.
7. The strength of a cube specimen with biaxial stresses should be less than the strength obtained with uniaxial compression, when surface friction is eliminated. The result of tests of a number of investigators also confirms this conclusion (18, 19, 20).

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