A GENERALIZED STREET NETWORK SIMULATION MODEL

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This paper describes a microscopic simulation model that reproduces traffic flow on a signalized street network under laboratory conditions. The input format and structure of the program facilitate application to any moderate-sized network. The program is economical to use, achieving a 6.5-to-1 real-time to simulation-time ratio for an 85-link network. The model has undergone a testing and validation procedure in which simulated vehicular travel times have been compared with actual travel times recorded on the test network to evaluate overall model accuracy. An analysis of individual model segments has been conducted to test their sensitivity to changes in key parameters. Results of these tests indicate that the model accurately and realistically simulates traffic flow through the street network.

•COMPUTER simulation is a tool that has been used effectively in business and scientific fields to gain more understanding of complex processes and to facilitate decision-making. Its application to traffic studies is especially appropriate because it allows a stochastic process, traffic flow, to be studied under controlled laboratory conditions.

To be useful, traffic simulation must satisfy three basic considerations:

1. The results of the simulation must fit the facts. Observations obtained as a result of simulation must agree with similar results obtained from observations of actual traffic flow.
2. The time required to simulate a problem must be reasonable. The ratio of simulated time to real time must be such that computer simulation of a street network is economically feasible. Before embarking on a study, its objectives should be thoroughly reviewed and alternative techniques for meeting them compared. The technique that does the job effectively at the least cost should then be chosen. When viewed from this standpoint, the use of simulation as a study tool can become much more attractive economically.
3. The results of simulation must be accessible in a format that is meaningful to those using them. The actual simulation takes place within the computer and is, of course, unobservable to the user (in the absence of some type of on-line visual display device). Thus it is necessary to devise some means of displaying simulation results in a form convenient to the user.

This paper describes a computer simulation program that was developed for use in the analysis of a signalized street network. The model is general so that it can be applied to any moderate-sized network, and the inputs and outputs can be understood by a traffic engineer not oriented toward computers.

MATHEMATICS OF VEHICLE BEHAVIOR

There are various methods that may be used to represent the flow of traffic within the computer. Early traffic simulations employed a physical notation (1, 2). Binary "1's" were used to represent vehicles and "0's" were used to indicate the spaces between vehicles. Groups of memory cells were figuratively placed end to end to represent the roadway. Algebraic manipulations caused the "1's" to change position, thereby simulating the flow of traffic. With this mode of representation the vehicles could occupy only certain specified locations (bit positions) along the roadway and individual vehicles had no identity as such.

The memorandum notation utilizes an entire word to represent a vehicle. Various parts of the word are used for such individual characteristics as its time of entry into the system and its desired velocity. These parts may be extracted and interpreted as desired. This method is more versatile in that each vehicle's characteristics are identifiable as it moves through the network, making it possible to compute delays associated with the individual vehicle.

A third method of representation has been called a mathematical notation (1). This form of representation is similar to the memorandum notation except that, in addition to its other characteristics, each vehicle is associated with its own position indicator. Its position is therefore continuous within the accuracy of the computer. A vehicle's new position can at any time be computed as a function of its last position, its velocity, its acceleration, and the time increment. Spacings between vehicles are available from their respective coordinates and the vehicle length.

A fully mathematical notation requires more complicated program logic. Maneuvers such as turns, which must be accomplished at a specified location, are more difficult when the vehicle can occupy any position at the start of the maneuver. Furthermore, mathematical processing of vehicles is more complex, thereby increasing execution time required. On the other hand, elimination of limitations on the position increment allows some increase in the size of the time increment for the same model accuracy and provides increased versatility.

The SIGNET model (SIGNET is the name of the simulation model developed in this project) employs a fully mathematical notation. The advantages to be realized from a virtually continuous position vector outweigh the additional execution time required. Furthermore, the high execution speed of the CDC 6500 computer somewhat offsets this loss.

Each vehicle in the SIGNET model is completely represented by the information contained in four computer words. Current position, current velocity, and current acceleration are each individual words (POSN, VEL, and ACCEL respectively). The fourth word, ICAR, contains information on ten variables, as shown in Figure 1. Each variable is easily accessed via an unpacking function.

The philosophy on which the SIGNET model is based is relatively simple in principle but involves complex programming for its implementation. The basic premise is that all drivers have a target velocity at which they would prefer to travel if conditions meet certain minimum requirements. Acting to limit the driver in the pursuit of his target velocity are limitations generated by interactions with other vehicles and the physical environment, including leading vehicles moving at a slower speed, red signal indications, turning movements, obstructions to lane changes, and conflicts with vehicles from other links at intersections.

The mathematical relationships describing vehicle behavior can be divided into nine separate areas: vehicle generation, car-following, free behavior, vehicle updating, amber acceptance, stopping performance, queue discharge, lane-changing, and turning performance.

Vehicle Generation

Vehicles are generated at the zero coordinate of each input link on a per-lane basis using a translated negative-exponential distribution. Traditionally the unmodified negative-exponential distribution has been used to obtain intervehicle headways. It is of the form

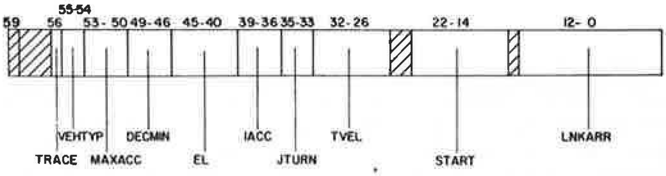
$$P\ (h \geq t) = \exp\ (-\beta_1 t)$$

where

$P\ (h \geq t)$ = probability of headway being greater than or equal to t; and
β_1 = vehicle flow rate in vehicles per second.

However, being distributed in $(0, \infty)$, the negative-exponential does not compensate for a minimum headway that of course exists for every vehicle. Therefore, as proposed by Gerlough (3), a better approximation is a negative-exponential with a translated axis:

Figure 1. Content of ICAR word.



LNKARR = link arrival time

START = initial coordinate at arrival on link

TVEL = target velocity

JTURN = turning movement

IACC = acceleration type

- 1 = stopped
- 2 = stopping
- 3 = stopped, waiting for turn
- 4 = stopping for turn
- 5 = stopping for lane change
- 6 = stopped for lane change
- 7 = free-behavior
- 8 = car following
- 9 = going in delayed left turn
- 10 = going in normal turn
- 11 = slowing for turn (no conflict)
- 12 = stopping for red signal

EL = effective length

DECMIN = minimum desired deceleration

MAXACC = maximum desired deceleration

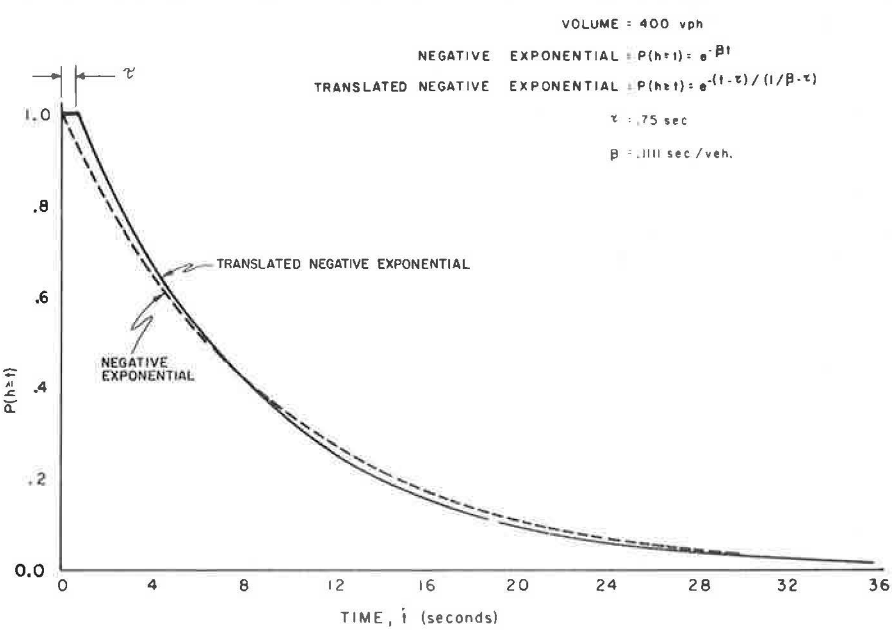
VEHTYP = vehicle type

- 1 = car
- 2 = truck

TRACE = vehicle being traced on travel-time route

- 0 = no
- 1 = yes

Figure 2. Comparison of negative-exponential and translated negative-exponential distributions.



$$P(h \geq t) = \exp\left(\frac{-(t - \tau)}{(1/\beta_1 - \tau)}\right)$$

where

τ = amount of translation; equivalently, in a physical sense, the minimum headway (Figure 2).

Work by Dawson and Chimini (4) in fitting the hyperlang probability distribution to intervehicular headways indicates a good value of τ (δ_1 in their study) to be 0.75 second under nonsignalized conditions. Input links to the SIGNET network are assumed not to be within the influence of upstream signals. Presence of a signal would necessitate reduction in this value of τ to reflect the lower headways of discharged vehicles.

In the SIGNET model the generation headway is independent of the simulation scan cycle. For each input lane in the network a variable is maintained to indicate the next arrival time of a vehicle in that lane. This tally is updated by the translated negative-exponential distribution at generation time to indicate the exact arrival time of the next vehicle in that lane.

Several descriptors of the vehicle's behavior are set at generation time, some stochastically and others deterministically. The vehicle's target velocity (TVEL), minimum desired deceleration (DECMIN), maximum desired acceleration (ACCMAX), and effective length (EL) are generated probabilistically according to their respective distributions. In addition, the vehicle type (IVEHTYP) is randomly determined from the link truck percentage input with the link traffic volume. The turning movement (JTURN) to be pursued at the link head is also determined randomly from the link turning probabilities. Finally, the link arrival time (LINKARR) is set to the current time, and the acceleration type (ACCTYP) is set to free behavior. All of the above are then packed into the ICAR word (Figure 1).

Car Following

At the heart of the simulation model are the free-behavior and car-following relationships. Much of the full spectrum of behavior at an intersection involves a tracking or following process, as seen in the case of queue discharge. Therefore the stimulus-response equations of car-following theory developed by Herman and associates (5, 6) are used to describe certain patterns of intersection performance.

Herman's works pertaining to car-following theory apply directly to the problem of processing vehicles in a digital simulation. With two basic exceptions (turning movements and stopping), one of the alternative vehicle behavior equations can be used to describe the behavior of individual vehicles within an intersection system.

Herman's equations have the general form:

response = sensitivity × stimulus

The best specific equation of this form Herman found was (in the notation of this study):

$$\text{ACCEL}(J+1, I+T) = a_0 \frac{\text{VEL}(J, I) - \text{VEL}(J+1, I)}{\text{POSN}(J, I) - \text{POSN}(J+1, I)}$$

where

ACCEL(J+1, I+T) = acceleration of car J+1, the follower, initiated at time I+T;
T = the car/driver lag;
VEL(J, I) and VEL(J+1, I) = the velocities of the leader and follower, initiated at time I;
POSN(J, I) and POSN(J+1, I) = the positions of the leader and follower initiated at time I; and
a_0 = the characteristic speed.

This equation is termed the reciprocal spacing model.

Free Behavior

Not all vehicles in a real system act as followers, however. An example of such a vehicle is the leader of a queue being discharged from a signal. In such a case behavior can be described by

$$ACCEL(J, I+T) = K\ (TVEL(J) - VEL(J, I))$$

where

ACCEL(J, I+T) = acceleration of car J initiated at time I+T;
K = proportionality coefficient;
TVEL(J) = target velocity of car J; and
VEL(J, I) = velocity of car J at time I.

This equation is termed the free-behavior model.

Stopping Performance

Two types of stops occur in the model: (a) stopping first in line at the intersection and (b) stopping behind another stopped vehicle. Empirical work at Ohio State University (7) indicated that use of a constant deceleration stopping model was realistic. It was found that, when given the choice, drivers tended to decelerate at an approximately constant rate throughout the duration of their stop.

In the SIGNET stopping model, the parameters of a minimum desired deceleration rate are supplied and a value of DECMIN(J) is randomly selected for each vehicle J at its generation time. During each scanning cycle the required stopping rate for the first vehicle on the approach is computed. When this rate is less than the minimum desired acceleration (implying a more severe stop), the vehicle begins stopping at the computed rate and continues to do so until zero velocity is reached. The stochastic nature of DECMIN thus accounts for different deceleration rates produced by each driver-vehicle combination.

A similar model is used for stopping behind another vehicle, with the principal difference being in the computation of the target stopped position. Vehicles stop at the position of the effective rear of the previously stopped vehicle, as determined by its effective length.

The effective length of a vehicle is equivalent to the average stopped spacing of vehicles stopped in queue, measured from the front bumper of the leading vehicle to the front bumper of the following vehicle, and therefore including the vehicle length and a clear space. Field studies have shown that it has a value of approximately 22 ft (6.7 m) for cars (8, 9). In SIGNET the vehicle's type (car or truck) is determined probabilistically at generation time. Then its effective length is determined stochastically from the parameters input for each of the vehicle types.

Turning Performance

Vehicles that desire to turn left or right at an intersection must at some point cease operating under the stimulus-response model and undertake an independent fixed turning schedule. The principal requirement is that vehicles must not exceed a given maximum speed during the turn. Maximum turning velocity is related to turning radius and side friction by the equation

$$VTURN = \sqrt{fgr}$$

where

VTURN = maximum turning velocity, feet per second;
f = coefficient of friction;
r = turning radius; and
g = acceleration of gravity.

The AASHO Policy on Geometric Design of Rural Highways (10) indicates that the 95-percentile turning speed is associated with side friction f = 0.3 for medium- to low-speed turns. Therefore,

$$\text{VTURN} = \sqrt{0.3\ (32.2)\ r} = \sqrt{9.66\ r}$$

where VTURN is in feet per second and r is in feet. Thus turning radii are supplied as input data and the maximum speeds associated with them are computed.

As a turning car approaches the intersection it is scanned at each simulation cycle. If the current velocity is greater than maximum turning velocity, the deceleration rate required to reach maximum turning velocity exactly at the start-turn point is computed. If this rate is less than the vehicle's minimum desired deceleration rate, the vehicle begins slowing to a maximum turning velocity, as in the stopping model. It continues to do so until reaching the start-turn position, unless affected by more stringent conditions. Maximum turning speed is maintained through the turn, whereupon the vehicle resumes behavior under one of the stimulus-response models, car-following or free-behavior, whichever is more stringent.

The foregoing does not imply that all vehicles will make their turns at maximum speed. Some will be affected by other vehicles in queue or vehicles on the receiving link so that their turns will be made at considerably lower speeds. However, none will exceed maximum turning speed.

Vehicle Updating

The various behavior relationships yield a negative or positive acceleration rate that begins after some reaction lag and continues for the rest of the scan cycle. In SIGNET both the reaction lag (REACT) and the scan cycle (CYCLE) are specified with the input data.

A theoretical analysis of driver reactions and highway events indicates the importance of driver reaction in safety and highway design. Therefore the inclusion of this parameter helps achieve realism within the model. In practice a wide range of values are used for reaction time; the Traffic Engineering Manual (11), however, recommends a time between 0.75 and 1.0 second for design purposes in urban traffic. To maintain continuity with the Carstens data (12) used in validating the queue-discharge model, a time of 0.75 second is recommended for use in SIGNET.

Movements of vehicles between scans are computed by adaptations of the equations of motion:

$$\text{V2} = \text{VEL(J, I)} + \text{ACCEL(J, I)} \cdot \text{REACT}$$

$$\begin{aligned}\text{POSN(J, I+CYCLE)} = {} & \text{POSN(J, I)} + \text{VEL(J, I)} \cdot \text{REACT} + 0.5 \cdot \text{ACCEL(J, I)} \\ & \cdot \text{REACT}^2 + \text{V2} \cdot (\text{CYCLE} - \text{REACT}) + 0.5 \\ & \cdot \text{ACCEL(J, I+REACT)} \cdot (\text{CYCLE} - \text{REACT})^2\end{aligned}$$

$$\text{VEL(J, I+CYCLE)} = \text{V2} + \text{ACCEL(J, I+REACT)} \cdot (\text{CYCLE} - \text{REACT})$$

where

V2 = the velocity of vehicle J after the reaction period;
REACT = the reaction time;
CYCLE = scan cycle;
POSN(J, I) and POSN(J, I+CYCLE) = the position of vehicle J at time I and I+CYCLE respectively;
VEL(J, I) and VEL(J, I+CYCLE) = the velocity of vehicle J at time I and I+CYCLE respectively; and
ACCEL(J, I) and ACCEL(J, I+REACT) = acceleration of vehicle J at time I and I+REACT respectively.

Geometric Configuration

The roadway is not represented physically as such, in the computer. However, it is necessary to specify certain roadway references in order to make meaningful the positions of vehicles contained in the POSN word. Thus there is a need for a coordinate system in which vehicles operate. The zero coordinate of each link is taken to be the point where all contributing turning movements are complete. In turn, the end of the link, or discharge boundary, is the point where all other contributing movements to the receiving link are complete. In reality there could be several contributing movements at the tail (i.e., zero coordinate) of the link, each ending at a different point. It is necessary to figuratively add a tangent section to each except the longest, thereby making all end at the same point.

Individually these computations are quite simple. When performed for an entire network, however, they become tedious and awkward. Therefore, the program PRESIG was developed as a companion to SIGNET. Its function is to compute a complete set of geometric data ready for input to SIGNET, based on easily acquired measurements from the network.

The logic of PRESIG may be divided into three areas as shown in Figure 3. Region I contains the primary input and initialization functions. Included in the input block is the verification of the input data, primarily achieved through checking card types and link numbers. Additional means for verification are obtained by printing out all input data.

Region II is concerned with the computation of a tentative set of discharge boundaries and begin-turn points for each link.

In Region III this tentative set is compared to revised discharge boundaries and begin-turn points, which are read from the input data. Any differences are changed to agree with the revised values. This option is provided to enable the user to easily correct any discrepancies that may arise between the PRESIG output and actual conditions. The final function of PRESIG is to output, both on the line printer and card punch, all the geometric data needed for input to SIGNET.

SIGNET PROGRAM STRUCTURE

The SIGNET program consists of a set of 33 nested, closed subroutines. The program is modular in construction, making it possible to insert new routines or modify existing ones. Twenty-eight routines occupy a fixed place in the program structure, while five routines serve auxiliary functions that are called on at various places in the program. All routines are written in FORTRAN IV for operation under the MACE operating system of the CDC 6500 computer.

Tasks performed by the main SIGNET program include

1. Input of all program data, program instructions, and link data, followed by formatting and writing these data.
2. Primary initializing tasks, which include initializing of all variables not connected with the statistical summary.
3. Activation of the traffic data input and initialization routines (INITRN and INIVOL) at the proper times as expressed on the program instruction cards.
4. Maintenance of the time loop, which includes control over the simulation time cycle and proper calling of the vehicle generation, vehicle update, and interlink transfer routines.
5. Program termination, which entails calling the final summary routine and writing final counts.

The main program chain is shown in Figure 4.

SIMULATION OUTPUT

The simulation program output is a detailed statistical tabulation of traffic characteristics in the network. The first section of output is a listing of input data supplied to the simulation program. Included are program parameters, link parameters, link geometry descriptors, traffic signal settings, turning probabilities, and traffic volumes.

Figure 3. PRESIG program chain.

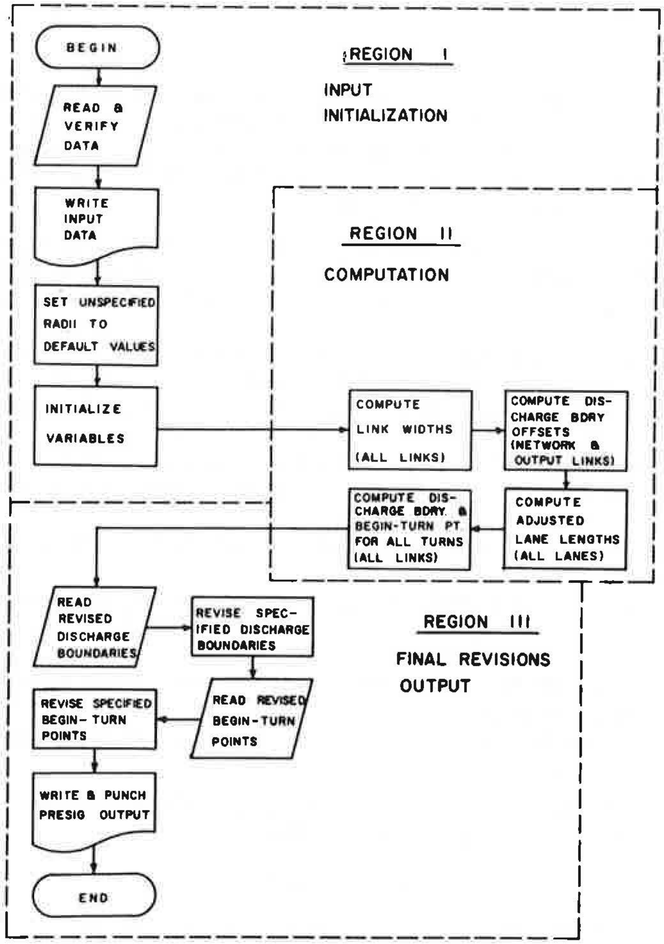
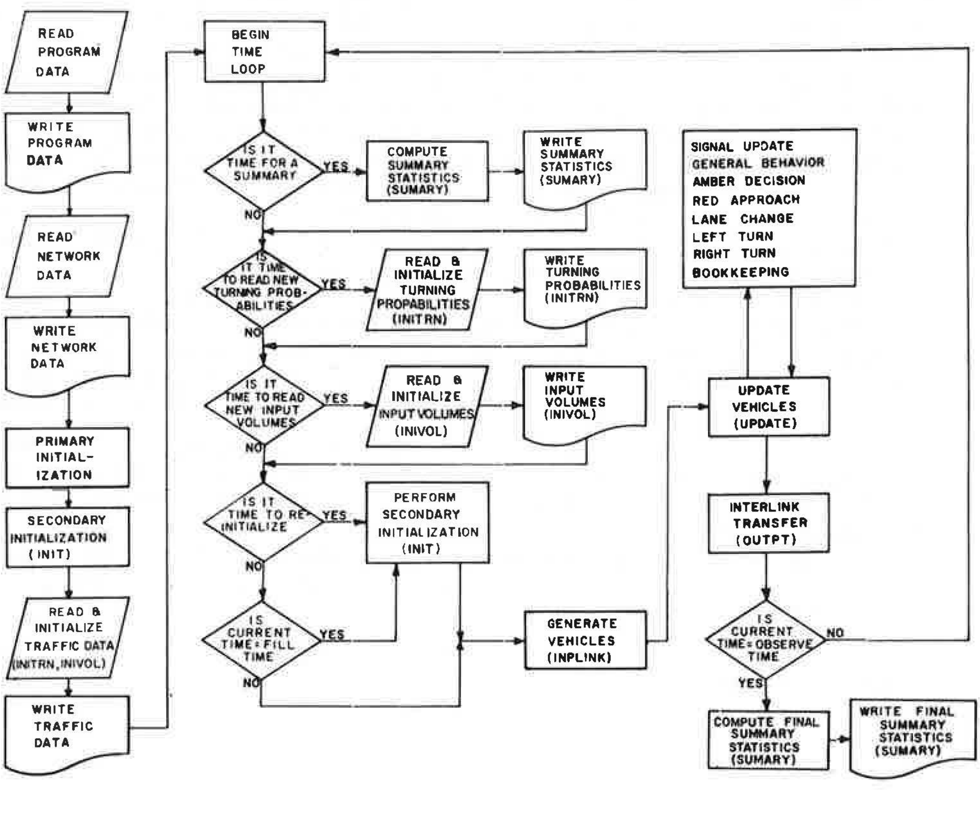


Figure 4. SIGNET main program chain.



The second section of output presents traffic operation data. Traffic magnitude data related to generated volumes, link exit volumes, and volumes traced along the specified route are reported, followed by statistics computed from the individual vehicles' performance. Values are presented for the overall system and for individual links. The following performance measurements are reported:

1. Total vehicle-miles—The total distance traveled on the link by all vehicles during the study period. Bookkeeping occurs when a vehicle leaves its link; the distance traveled by vehicles still on the link at the time of the summary, therefore, is not included in this tally.
2. Total delay (in seconds)—The sum of all vehicles' delays on a link. A vehicle's undelayed travel time is based on its target velocity and distance traveled. The delay encountered by the vehicle, then, is the difference between its actual and undelayed travel time.
3. Average delay (in seconds)—The total delay divided by the number of vehicles leaving the link. In this study average system delay is taken to be the primary measurement of system performance.
4. Delay standard deviation—The standard deviation associated with the above average delay.
5. Average delay (in seconds per vehicle-mile)—An extended form of the average delay. Its purpose is to facilitate the comparison of average delays among links.
6. Total travel time (in seconds)—The travel time for all vehicles.
7. Average travel time (in seconds per vehicle)—The average time required to traverse the link under consideration.
8. Average speed (in miles per hour)—The average speed achieved by all vehicles over the entire link or network.

The final sections of the output report contain frequency tables of queue lengths on the specified links and travel times on the route links.

TESTING AND VALIDATION

The SIGNET model was tested and validated to determine its accuracy in representing real-world conditions and its sensitivity to changes in input parameters. The first phase of the sensitivity analysis compared travel times produced by the simulation model with equivalent travel times obtained in the field.

Four simulation runs were made using different random-number generator seeds and network traffic volumes, turn percentages, and auto-truck ratios determined during the travel-time studies. Histograms were made for each link showing the number of vehicles having various simulated travel times. Such a histogram is shown in Figure 5 for a typical link during the test period. As indicated, the simulated travel time distribution was closely correlated to the actual travel times, with the mean simulated time being approximately equal to the actual travel times. This analysis indicated that the model working as a whole produced acceptable results.

The objective of the second phase of the testing was to determine the sensitivity of the model output to changes in various input parameters. An inherent characteristic of computer simulation, the ability to control all input parameters precisely, enables the investigator to change only one parameter and thereby to determine its influence on the output. This phase of the sensitivity analysis was useful for two reasons:

1. It provided additional data that could be used in confirming the reasonableness of the model. The direction of change in the output caused by modifying a parameter could be examined to see if it was logical, and the magnitude of the change could be checked for reasonableness.
2. It indicated whether particular portions of the model logic were indeed operating. If a parameter modification produced no change where one was expected, there was good evidence of logic flaws in the model.

Sensitivity tests were conducted by evaluating the effect of variation in seven different parameters:

Figure 5. Comparison of typical link travel times.

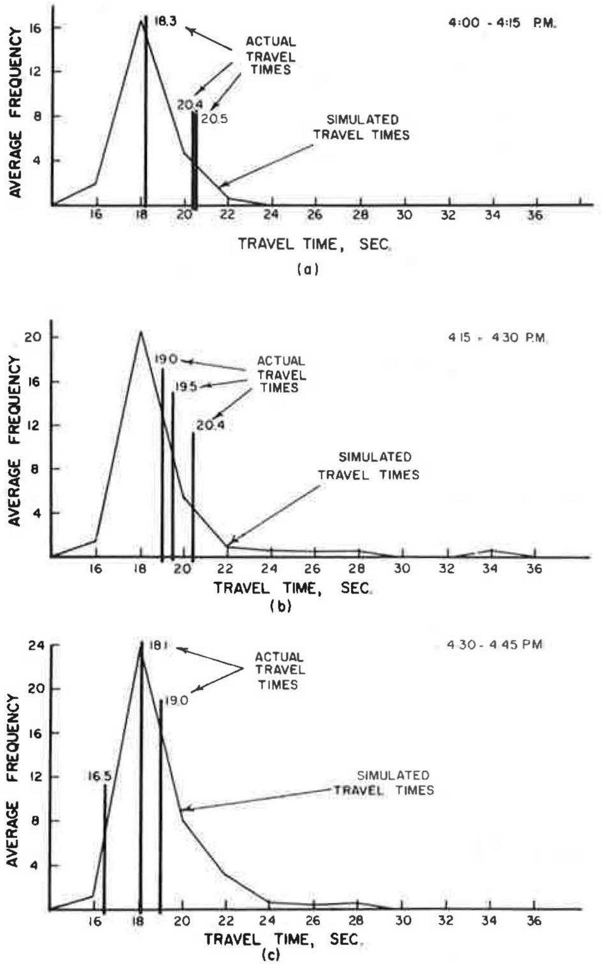


Figure 6. Effect of traffic volume on average vehicle delay.

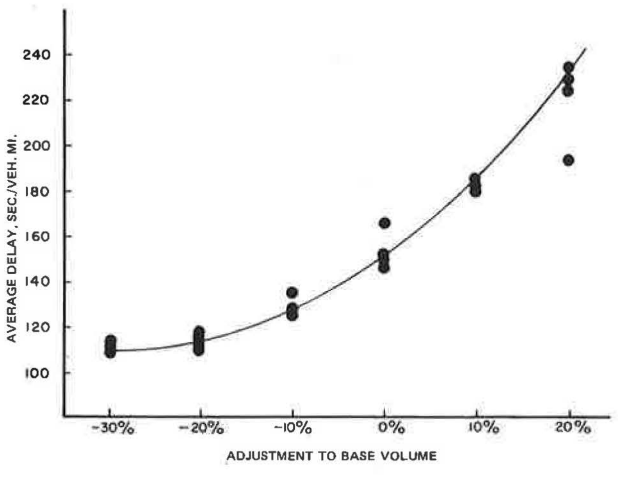


Figure 7. Effect of trucks on average vehicle speed.

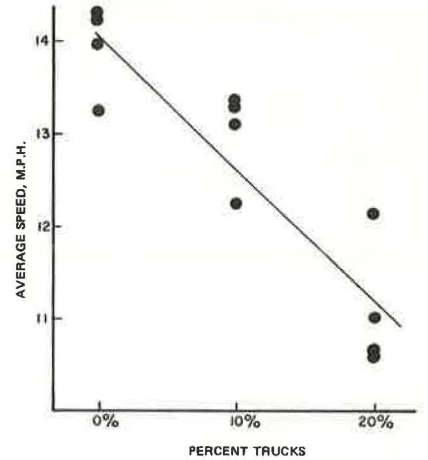


Table 1. Analysis of variance: effect of traffic volume on average vehicle delay.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F Ratio
Among treatments:	43,631.71	3	14,543.90	79.0[a]
Linear effect	39,856.58	1	39,856.58	216.0[a]
Quadratic effect	3,747.26	1	3,747.26	20.4[a]
Cubic effect	27.86	1	27.86	0.16
Experimental error	3,683.79	20	184.19	
Total	47,315.50	23		

[a]Effect is significant at $\alpha = 0.10$. Critical region: $F > 2.97$.

Table 2. Analysis of variance: effect of trucks on average vehicle speed.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F Ratio
Among treatments:	16.815	2	8.408	24.5[a]
Linear effect	16.188	1	16.188	47.2[a]
Quadratic effect	0.627	1	0.627	1.83
Experimental error	3.084	9		
Total	19.899	11		

[a]Effect is significant at $\alpha = 0.10$. Critical region: $F > 3.36$.

1. Input traffic volume;
2. Target velocity, which affects the free-behavior model;
3. Effective length of vehicles, which affects queue discharge;
4. Truck percentage, which affects overall traffic throughput;
5. Maximum desired acceleration, which affects the acceleration rate and therefore queue discharge;
6. Characteristic speed (a_0), which affects the car-following model; and
7. Proportionality coefficient (K), which affects the free-behavior model.

Four simulation runs were made for each value of the input parameters, providing data for an analysis of variance for each effect.

It was desirable to know not only that the simulation output varied with changes in the input parameters but also what form the variation took (i.e., if the response curve was linear, quadratic, or cubic in nature). If the output variation leveled out at some point, this response could be compared with expectations, providing added evidence of the model's acceptability. For this reason the treatment sum of squares was broken down into sums of squares associated with linear, quadratic, and cubic effects using the orthogonal polynomials method.

The effects of variation in two typical variables, input traffic volume and truck percentage, are shown in Figures 6 and 7. As shown, decreased traffic volumes resulted in less delay. The effect was reduced, however, as volumes became lower and signal delays became critical. The analysis of variance in Table 1 confirms this quadratic effect with 90 percent confidence.

Increased percentages of trucks in the traffic stream caused a reduction in average speed. As indicated in the analysis of variance in Table 2, this effect was linear within the range of percentages studied.

Adherence of these sensitivity test results to anticipated patterns indicated that the various individual portions of the model were functioning as intended. When viewed in combination with favorable travel time combinations, it was concluded that the model provided a realistic and accurate simulation of actual conditions.

SUMMARY

SIGNET is a microscopic simulation model that reproduces traffic flow on a signalized street network under laboratory conditions. Its input format and structure enables it to be readily applied to any moderate-sized signal street network.

The program is economical to run, achieving a 6.5-to-1 real-time to simulation-time ratio for an 85-link network. This ratio is of course variable, depending on network size and configuration and on input traffic volumes.

The model underwent a testing and validation procedure that included comparison of travel times with actual field data and a comprehensive sensitivity analysis of individual model segments. These tests indicate that the model realistically and accurately simulates traffic flow through the street network.

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