FLEXIBLE PAVEMENT ANALYSIS SUBSYSTEM

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This paper presents an outline of the analysis techniques proposed for use in a pavement design check procedure that has been developed in the federally coordinated program of highway research on new methodology for flexible pavement design. The concepts and formulations used in the method have been developed in the National Cooperative Highway Research Program and by staffs of the Federal Highway Administration and the state transportation and highway departments. The techniques presented reflect current knowledge, and changes will be made as new information becomes available. The proposed structural analysis subsystem is based on the assumption that portions of the pavement act as viscoelastic elements, others as plastic elements, and others as elastic elements. The method also accommodates the important concept of element responses altered with temperature and loading rate fluctuations. The subsystem is incorporated in a procedure by which it is possible to check existing designs for structural adequacy to resist pavement damage due to cracking, rutting, and roughness. Eventually this subsystem will be integrated with a pavement management system that will allow consideration of optimum design concepts with respect to the planned used and life of the proposed roadway.

•THE FEDERAL Highway Administration's research program in flexible pavement design was planned and developed as part of the National Program of Research and Development in Highway Transportation (since replaced by the Federally Coordinated Program of Research and Development in Highway Transportation). With minor changes in emphasis, this program has been followed to the present time.

The major objective of the research program developed by the Pavement Systems Group of the Structures and Applied Mechanics Division, Office of Research, is a new structural subsystem for flexible pavements that will reliably predict in-service performance by a rational analysis of material properties, traffic loadings, and environmental conditions. This subsystem in the form of a pavement design check procedure will be available to the states on a trial basis. The procedure will be set forth in a users manual that will be supplemented by a completely documented computer program.

A great deal of effort by many research agencies has provided basic information that permits the presentation of this outline of the proposed design-analysis procedure. The outline relies heavily on concepts and work accomplished at the Massachusetts Institute of Technology (1, 2, 3, 4, 5, 6) and uses much of the work accomplished at the University of California (7, 8, 9, 10, 11, 12). Research efforts at Georgia Institute of Technology, Ohio State University, and Texas A&M University are being considered as refinements (13, 14, 15). Current work includes studies at the University of Utah, Materials Research and Development, Austin Research Engineering, Inc., and Pennsylvania State University.

As a long-range goal, the structural analysis subsystem is to become an integral part of an overall pavement design-management system, which will provide for total life planning. Pavement maintenance and economic factors will be integrated with the structural subsystem to provide a capability for optimizing the structural design. A schematic outline of the design-maintenance system is shown in Figure 1.

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CONCEPTUAL CONSIDERATIONS

A structural subsystem deals with the analysis of the structural response of the pavement system. It may be composed of one or more subsystem models and in general provides information about the primary and limiting responses of the pavement. Before entering on a discussion of the various models of the structural subsystem, the reader must understand the concepts behind the developments incorporated in this subsystem.

For more than 2 decades, pavement design engineers have been developing concepts and methodology by which pavement design can be transformed from an art to a science in which physical measurements of material properties, load applications, and environmental factors may be used to predict the performance of a pavement in place. This transformation requires that the measurements taken and the performance predicted be compatible with all rules of science and mathematics germane to the problem.

Therefore, the first phase of the research endeavor concentrated on the solution of boundary value problems and the development of constitutive equations. The researcher investigated size, shape, and makeup of the pavement layered system and developed formulations that attempted to predict its response when subjected to external influences. This research showed that the pavement response is manifested by both recoverable (elastic) and permanent (viscous and plastic) deformation, which eventually results in cracking and rutting. Portions of the response are time dependent (viscoelastic) and therefore partially nonrecoverable because of the time of the load applications on the pavement system and because of the effects of temperature on this response. In addition, laboratory tests were developed to investigate the behavior of the layer materials. Various configurations of material specimens were tested under different loading and environmental conditions. Procedures for characterizing the behavior of these materials led to the formalization of several types of laboratory tests that determine the material characteristics for use in predicting pavement response.

A second phase of the research was concerned with the development of formulations that allow a stochastic or probabilistic approach to the design problem. Variation is important in materials and in construction practices; therefore, the design-analysis system must take variability into account.

A third phase was concerned with the fatigue of flexible pavements. Currently the only available method for predicting fatigue life is an empirical on based on fatigue testing of sawed or formed beams and extrapolating those data to the fatigue of the pavement. The extrapolation procedure correlates the stress on the underside of the pavement, the expected temperature regime, and the fatigue test results. Fracture mechanics concepts have also been applied to the pavement-cracking problem. A predictive method is not yet available, but progress is being made. The concepts of viscoelastic fracture mechanics appear to have the best promise of a solution to fatigue cracking of flexible pavements.

The efforts of this work result in a rational analysis method for evaluating flexible pavement designs. This is a method in which all responses of the pavement can be stated in terms of the geometry of the pavement system, the physical properties of the materials, and the effect of climate and load on these properties.

STRUCTURAL SUBSYSTEM

The structural subsystem is composed of 3 separate sets of models: primary response, damage indicator, and performance. Each model depends on separate input variables and on interrelations of input and output among the models (1, 2, 3, 4, 5, 6, 18). For instance, the distress to the pavement incurred through the associated failure mechanisms is transformed into numerical values indicating the levels of serviceability of the pavement. A view of these interrelations is shown in Figure 2.

To account for the uncertainties and variabilities associated with the operations of a pavement system, computer programs allow inputs and outputs to be described in terms of probabilistic distributions instead of single-valued estimates. The methods of approach to the formulation of probabilistic models may be divided into simulation procedures and direct probabilistic procedures. The current version of the analysis incorporates Monte Carlo simulation techniques for the computation of primary response;

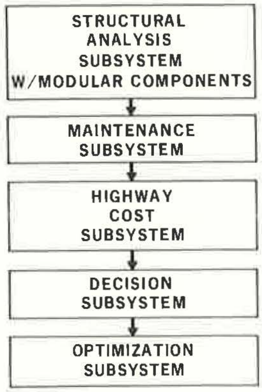


Figure 1. Design-management system.

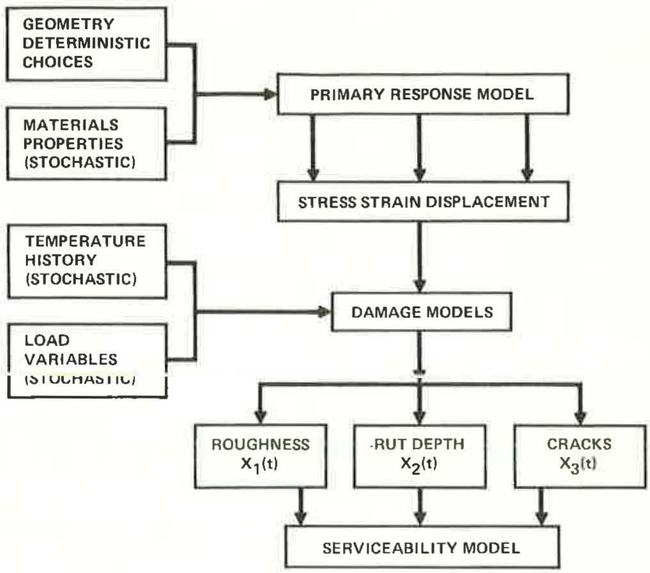


Figure 2. Structural subsystem.

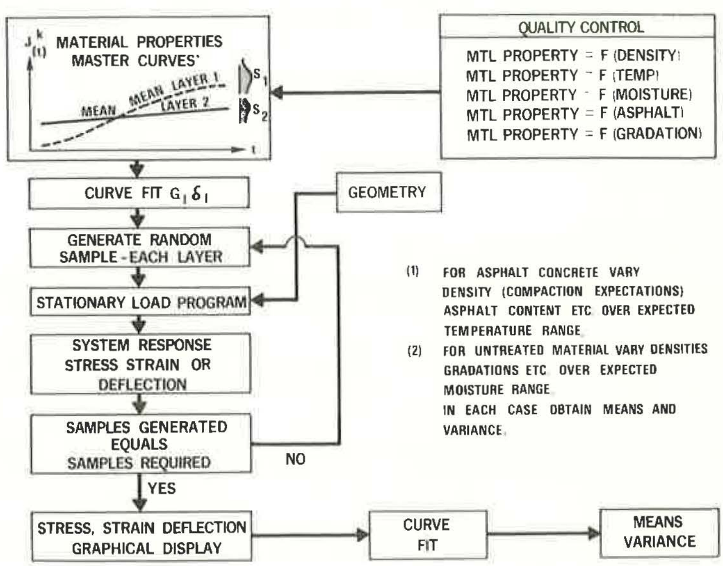


Figure 3. Primary response model.

direct, closed-form probabilistic procedures are used to compute the pavement's response to random loading. Variability in estimates of future traffic and in the properties of the layer material components can be accounted for, but the user must provide the computer programs with those data.

Primary Response Model

The primary response model is a mathematical model of the pavement structure in the form of computer program solutions to stationary (static) load conditions (1, 2). It now consists of a 3-layer linear viscoelastic boundary value problem that incorporates a probabilistic solution to account for the stochastic nature of input variables (3, 4). Output from this model consists of distributions of the mean value and variance of the resilient (elastic) and accumulative (time-dependent) stresses, strains, and deformations at any point in the pavement due to a stationary load applied at the pavement's surface. The components of this model are shown in Figure 3. The solution to the static load condition is similar to that described by Burmister except that the moduli of the material layers are allowed to behave as viscoelastic (rate-dependent) materials as well as elastic ones. In addition each material layer is assumed to be incompressible (Poisson's ratio 0.5). The computer program inputs provide for pavement geometry, magnitude and size of the statically applied load, and linear viscoelastic creep or elastic compliance function for each layer. The compliance function represents the material characterization of the layer materials to be used in the primary response model. (These properties are determined from the results of laboratory tests conducted on individual samples of each pavement layer.) It is expressed in terms of stress and strain as

$$D(t) = \frac{e_{zz}(t)}{\sigma_{zz} - 2\mu(t)\sigma_{rr}} \tag{1}$$

where

$D(t)$ = modular creep compliance function,
σ_{zz} = axial load in a tension or compression test with or without confinement,
σ_{rr} = confinement pressure,
e_{zz} = axial strain, and
$\mu(t)$ = Poisson's ratio.

For an elastic material, $D(t)$ is defined as the inverse of the elastic modulus or, as it is known today, the resilient modulus.

The modular creep compliance function is represented mathematically within the computer program by the exponential series

$$D(t) = \sum_{i=1}^{n} G_i \exp \delta_i t \tag{2}$$

where

G_i = constant coefficients determined by the series curve fit program, and
i = constants prescribed within the program.

Damage Indicator Models

A highway pavement is a structure built for use during a given period of time. During its design life, the structural integrity of the pavement may weaken and its inability to resist the imposed loadings and environment will give rise to accumulations of cracking and permanent deformation.

The factors that primarily influence these manifestations include properties of materials in each layer; magnitude, duration, and number of repetitions of load; and environmental factors such as moisture and temperature. Since each of these factors cannot be measured or specified in an exact form, their variations should be accounted for in the design-analysis procedure. For example, the quality of the material of each layer has certain variations that can be described statistically in terms of means and variances. The fluctuation of temperature and the randomness of traffic can also be described by means and variances. The structural subsystem has been uniquely formulated to account for these random parameters. By using stochastic procedures, the predictive capabilities of the system inherently include the interactions of these parameters. In addition the variation of the material properties along the roadway will give rise to longitudinal variations of the rut depth and cause longitudinal profile changes or roughness. In the computation of the distress indicators of the subsystem, 3 independent load-associated failure mechanisms are assumed (4, 5): fatigue failure, accumulative deformation in the wheel paths, and longitudinal roughness. The damage indicator models are shown in Figure 4.

Fatigue Failure Submodel—Cracking is a phenomenon associated with the brittle behavior of materials. A fatigue mechanism is assumed to cause progression of cracks in pavements. This distress mechanism is accounted for by a phenomenological approach, namely, a modified stochastic Miner's law for progression of damage within materials. Miner's law is given by the following equation:

$$C = \sum_{i=1}^{m} \frac{n_i}{N_i} \tag{3}$$

where

n_i = number of load applications at the strain state i, and
N_i = number of cycles to failure for that same strain state i.

When the amount of damage C reaches the value of 1, failure is said to have occurred. When C reaches any value less than 1, that value represents the percentage of pavement life used up. The number of cycles to failure N is related to the strain amplitude by the following relation:

$$N_i = K_1 \left(\frac{1}{\Delta\epsilon_i}\right)^{K_2} \tag{4}$$

where

$\Delta\epsilon_i$ = tensile strain amplitude at the underside of the asphalt concrete layer directly under the wheel load, and
K_1, K_2 = material characteristics of the fatigue model.

The values of K_1 and K_2 are usually determined in laboratory fatigue tests on beam specimens of the layer in which it is assumed fatigue cracking takes place (7, 8, 9). The deterioration of the pavement is computed through a probabilistic formulation of the fatigue equation and Miner's law. In the computer program the coefficients K_1 and K_2 may be statistically correlated; i.e., a coefficient of correlation of -1 means that an increase of K_1 corresponds to a decrease of K_2, and a coefficient of 0 means that K_1 and K_2 are statistically independent of each other. Values of K_1 and K_2 are to be prescribed by the user in terms of their mean value and variance. In general, the coefficients K_1 and K_2 are dependent on the configuration of the family of fatigue curves developed to represent the failure criteria. Recent analysis has shown that the variability of K_2 has a much greater influence on fatigue crack predictions than does the variability of the tensile strain (12). Therefore, for fatigue testing standard laboratory procedures must be developed that realistically reflect fatigue cracking in the pavement.

Rutting Submodel—Rutting distress results from the residual or permanent deformations occurring in the layers because of repeated load applications in the wheel paths. These accumulative deformations may occur in all layers; however, the mechanisms will be different for different materials. The rutting may be due to the viscous behavior of the materials or to compaction and reorientation of the individual particles

Figure 4. Damage indicator models.

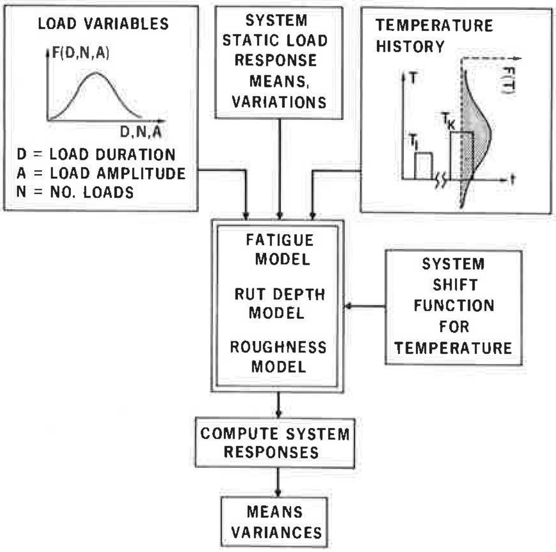
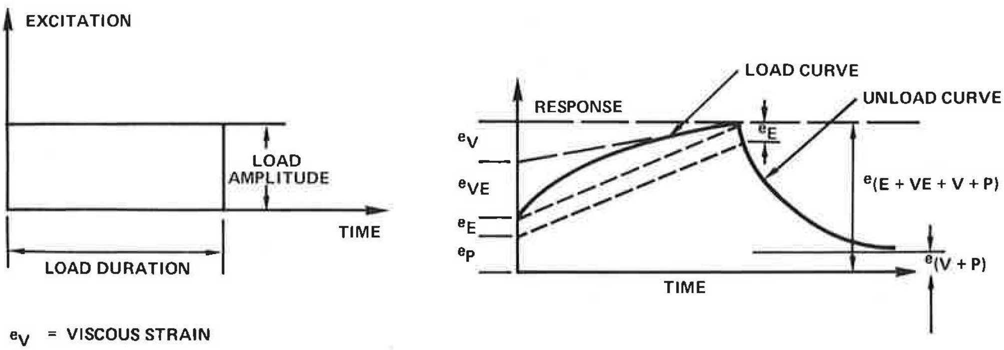


Figure 5. Material response.



e_V = VISCOUS STRAIN

e_{VE} = VISCOELASTIC STRAIN

e_E = ELASTIC STRAIN

e_P = "PLASTIC" STRAIN

upon application of wheel loads. Figure 5 shows the response of a material body and its rebound upon unload. Its true behavior could not have been known without a knowledge of the rebound curve. The permanent deformation is composed of a viscous part and a plastic part (the use of the word "plastic" denotes permanent deformations due to causes other than viscous flow), whereas the elastic and viscoelastic components are considered to be fully recoverable.

In general, the viscous component is of greater significance for asphalt-bound materials, and the plastic component has a greater effect on the development of permanent deformations in granular type materials. Fine-grained materials exhibit a predominantly elastic behavior when their moisture contents are below optimum values. As moisture contents are increased above optimum, viscous and plastic behavior becomes more predominant.

The rutting submodel of the program computes the amount of vertical deformation occurring in the wheelpath because of repeated traffic. The operational techniques used to predict pavement rutting have been programmed and are included in the random load program. The current version of this program computes rut depth by using the linear superposition integral. This operation is essentially a process where residual deformations are summed over many applications of a repeated load. The accumulative deformation, of course, will be a function of the duration of each load, the number of loads, the time between arrival of each load, the magnitude of each load, and the response behavior of the pavement system itself. A single load application is expressed mathematically as follows:

$$F(\tau) = A \sin^2 \omega\tau, \ 0 < \tau < \text{duration } F(\tau) \tag{5}$$

The function $F(\tau)$ is shown in Figure 6. The amplitude A and frequency take on random values associated with the traffic characteristics, which are prescribed by the user.

Roughness Submodel—This distress component defines the deformation along the longitudinal profile of the roadway. The rut depth along the wheelpath is assumed to vary in a random manner as a result of both quality control measures and construction techniques. For instance, if the materials along the roadway were placed during radical changes in environment or if a wide variety of construction practices were used or if different material sources were used, then one might expect the structural integrity to vary at different points along the roadway. In this submodel the roughness is expressed by the AASHTO definition for slope variance. It is computed both from a knowledge of the frequency distribution of rut depth and from an autocorrelation function that is a measure of the variation of material properties along the roadway. This function, however, must be determined from actual field measurements on existing roadways so that it reflects the in-place variations inherent in the pavement structure.

Performance Model

The performance of a pavement in a given environment is its ability to provide an acceptable level of serviceability with a specified degree of reliability at an assumed level of maintenance. Inability of the pavement to provide the necessary services in a given locale may then be considered as pavement failure. When viewed in this context, failure becomes a loss in performance; it is the extent to which the pavement is unable to render itself serviceable as a result of accumulation of damage during a given time period.

When a pavement constructed of known materials and geometry is subjected to an operational environment, the damage model predicts the distribution of each major distress component. One can use the expected values of these components to predict the expected value of the road serviceability after a given time period, provided one knows the relation between serviceability and damage components. The AASHTO serviceability model is assumed to be valid. Thus, the outputs of the damage indicator models are used in the following equation:

$$SI = a_0 + a_1(C) + a_2(RD) + a_3(SV) \tag{6}$$

Figure 6. Excitation and pavement response functions.

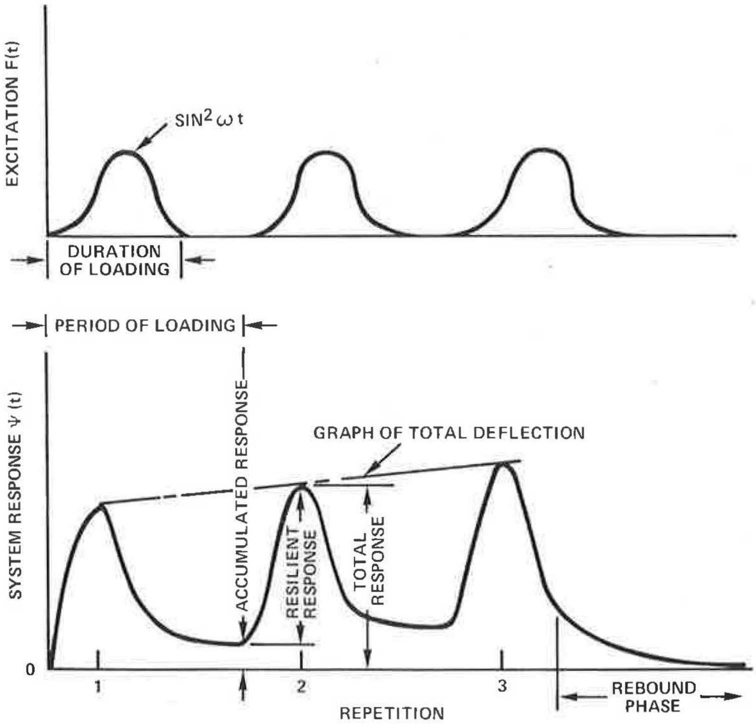
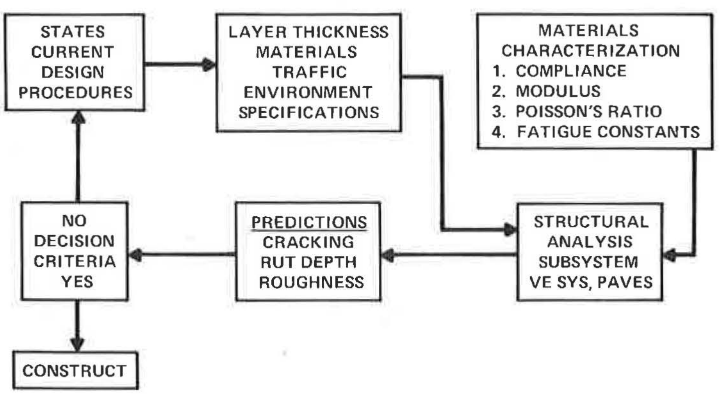


Figure 7. Design check procedures.



where

SI = present serviceability normalized with respect to its initial value (it is described by its distribution function, which can be used to determine the reliability of the system),
C = damage caused by pavement cracking,
RD = damage caused by change in the transverse profile, and
SV = damage caused by change in the longitudinal profile.

This estimates the change in serviceability due to accumulation of damage caused by the interaction between the pavement and the traffic in a given environment.

DESIGN CHECK PROCEDURE

The structural subsystem will be used initially as a check procedure for analysis of state pavement designs as shown in Figure 7. In general, the use of this package will involve the following:

1. Pavement sections are designed according to state's normal procedures;
2. The computer program (6) is used to analyze the design section;
3. If the structural analysis indicates that one or more of the failure mechanisms (cracking, roughness, or rutting) will occur at a more rapid rate than is thought to be tolerable, the original design is modified and evaluated by the computer program (step 2 above) until an acceptable design is obtained; and
4. After the pavement is constructed, performance measurements are taken and compared to the predicted values (as experience is gained, feedback information will indicate where the structural design subsystem may need modification).

COMPUTER PROGRAM

Input

Four input categories are used by the programs in predicting pavement performance: system geometry, material properties, traffic characteristics, and temperature history. In addition, spatial correlation coefficients must be prescribed for the roughness model. These coefficients range from 0 for a very rough pavement to 1 for a smooth pavement and are based on the history of material and construction control in the state. Until more precise data become available, coefficients based on information gained in quality assurance research have been incorporated in the program for initial trials.

System Geometry—In the current program, system geometry is expressed in terms of the thickness of the first and second layers.

Material Properties—The material properties are divided into 2 categories: those expressing the stress-strain relations of each layer of material and those describing a failure characteristic. The stress-strain relations require determination of the creep compliance function for rate-dependent materials and elastic moduli for rate-independent materials. The rate-dependent properties are obtained from creep tests. Values of the modular creep compliance, as described by Eq. 1, are plotted versus time on log-log paper. Care should be taken in testing materials to ensure that test results reflect the effects of stress state, temperature, moisture content, and conditions corresponding to those of the in situ pavement. When asphalt-bound samples are tested, a sufficient number of tests at different temperatures should be run to establish the master creep compliance curves and hence the time-temperature shift factor a_T.

Since the programs will also handle variations in the material properties, the user has the option of specifying those significant variations in the properties that he or she expects in the field. A very simplified method of estimating anticipated variations of material properties is presented by Kenis in another report (16). Figure 8, from that report, shows how the estimated standard deviations of creep compliance vary for different points in time. An average coefficient of variation can be obtained from these values for input to the programs. In practice, the user need only punch selected values from the mean compliance curve and the average coefficient of variation of this curve

Figure 8. Material variation.

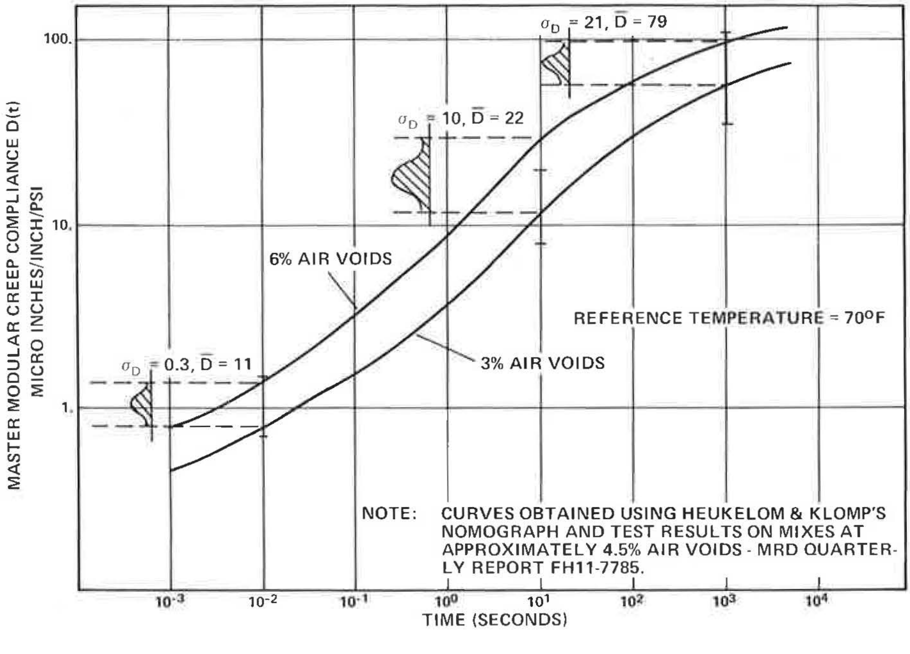
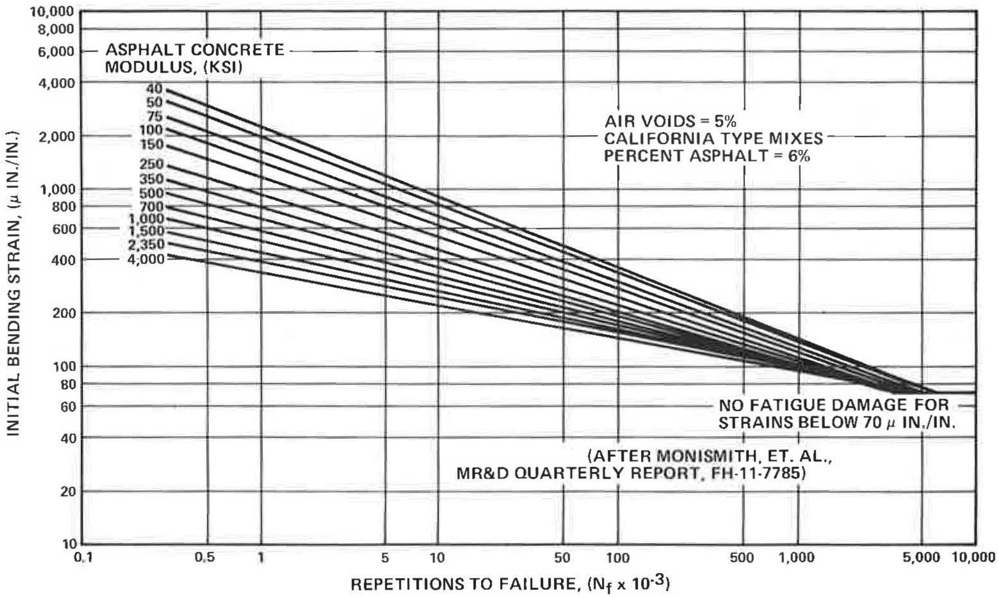


Figure 9. Fatigue failure criteria.



onto computer cards and insert them in the deck.

The elastic or resilient modulus can be obtained for granular or cement-treated materials from complex or dynamic modulus tests, or the instantaneous elastic response occurring in the creep test can be used. The use of an elastic modulus for any given layer would indicate that this layer exhibited an insignificant amount of creep under long-term loading. When the modulus depends on the state of stress, such as in a granular base course, it may be desirable to use the following relation:

$$M_R = A_1 \phi A_2 \tag{7}$$

where M_R is the resilient modulus, ϕ is the first stress invariant, and A_1 and A_2 are constants determined from laboratory tests (7, 10, 12, 13, 19, 20).

Certain fine-grained materials will also exhibit rate-dependent properties and may be characterized as such (21, 22). Tests to establish material properties are being further developed and will be standardized by ASTM. However, current methods have been adopted by many of the researchers and will be recommended for use in conjunction with the check procedure.

When fatigue failure properties are established, fatigue curves similar to those shown in Figure 9 are customarily developed. In computing fatigue life, the computer makes use of 2 constants, K_1 and K_2, which are developed from the curves shown in and are related to Eq. 4. Mathematically K_1 can be expressed as the intercept b raised to the -1/m power (b-1/m), and K_2 is the reciprocal of the slope of the curve (-1/m). These constants play a significant role in the computation of fatigue life; therefore, the variance of the values of K_1 and K_2 plays an important role in the reliability of the computations.

Traffic Characteristics—Figure 10 shows statistical characteristics that have been assumed to represent the loading conditions for a typical highway. The loading of a pavement system is assumed to be a process of independent random arrivals. Vehicles arrive at some point on the pavement in a random manner both in space and in time. The arrival process is modeled as a statistical distribution, a Poisson process, with a mean rate of arrival. It is assumed that a logarithmic-normal distribution is suitable to represent the scatter in load amplitudes. Means and variances of load amplitudes are also used to represent this scatter.

The load duration, a function of vehicle speed on the highway, is also a random variable. In a typical highway, for example, speeds may vary from 40 to 70 mph. Accordingly, the load duration is assumed to have a statistical scatter represented by its mean and variance from distributions obtained by traffic studies.

The load variables must be determined for specific conditions and are used as input to the computer program. The mean and the standard deviation of each variable are determined by the user. The lateral distribution of traffic must also be known. In this program it is assumed that 75 percent of the traffic is channelized. A summary of the loading variables is as follows:

1. Radius of the applied loads, in inches;
2. Intensity of loads, in pounds per square inch;
3. Duration of the loads, in seconds; and
4. Rate of load applications per month and the proportion of channelized loads.

Temperature History—The current version of the programs automatically accounts for annual temperature variation. The variations in pavement response during the year from one temperature period to another are determined through application of the time-temperature superposition principle. One can choose the temperature periods in such a way that averaging temperatures within these periods is justified. The present computer program allows for the study of hourly, daily, weekly, monthly, quarterly, or yearly intervals of time. Application of the time-temperature superposition principle has demonstrated that the relation

$$\log a_T = 0.09(To - T) \tag{8}$$

Figure 10. Distribution of load characteristics.

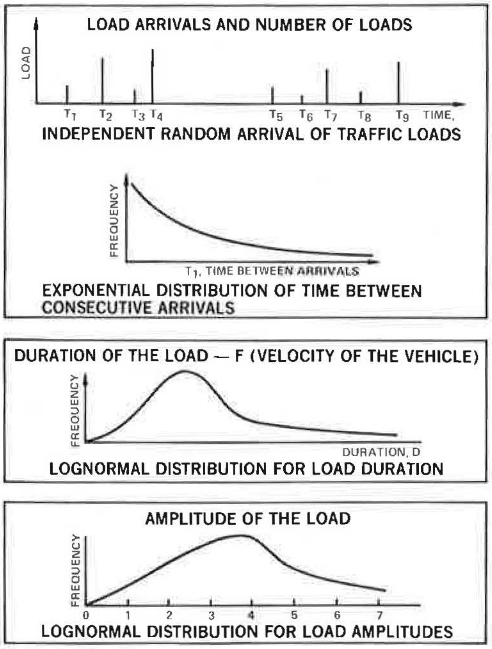


Figure 11. Temperature shift factor.

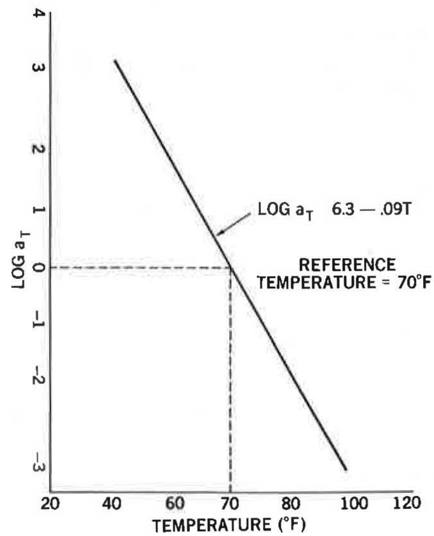
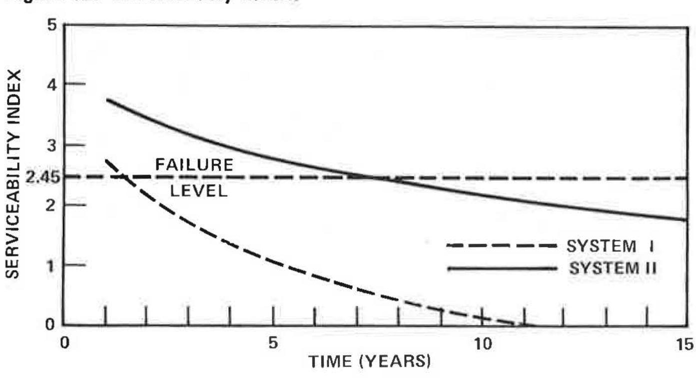


Table 1. Response history of 2 pavement systems.

	Rut Depth		Slope Variance		Cracks	
Years	Expected Value	Variance	Expected Value	Variance	Expected Value	Variance
System 1						
1	0.1058124 D-04	0.7016140 D-11	0.8957364 D-09	0.3189094 D-19	0.2545644 D-02	0.3626053 D-02
3	0.1363651 D-04	0.7362124 D-11	0.9353431 D-09	0.3330551 D-19	0.7636933 D-02	0.1087816 D-03
6	0.1567957 D-04	0.7507012 D-11	0.9468578 D-09	0.3371604 D-19	0.1527387 D-03	0.2175632 D-03
10	0.1740845 D-04	0.7628664 D-11	0.9530464 D-09	0.3393539 D-19	0.2545644 D-03	0.3625053 D-03
12	0.1805977 D-04	0.7679161 D-11	0.9548168 D-09	0.3399763 D-19	0.3054773 D-03	0.4351263 D-03
15	0.1884290 D-04	0.7747835 D-11	0.9565659 D-09	0.3405849 D-19	0.3818466 D-03	0.5439079 D-03
System 2						
1	0.7749276 D-05	0.3763186 D-11	0.4804291 D-09	0.9242749 D-20	0.3366507 D-02	0.6420968 D-02
3	0.1000001 D-04	0.3950931 D-11	0.5019319 D-09	0.9657749 D-20	0.1009952 D-03	0.1926290 D-03
6	0.1149962 D-04	0.4029092 D-11	0.5081370 D-09	0.9777295 D-20	0.2019904 D-03	0.3852581 D-03
10	0.1276699 D-04	0.4094688 D-11	0.5114631 D-09	0.9840995 D-20	0.3366507 D-03	0.6420968 D-03
12	0.1324438 D-04	0.4121933 D-11	0.5124144 D-09	0.9859065 D-20	0.4039809 D-03	0.7705161 D-03
15	0.1381838 D-04	0.4159002 D-11	0.5133544 D-09	0.9866735 D-20	0.5049761 D-03	0.9631451 D-03

Figure 12. Serviceability index.



is reasonably valid for a wide variety of asphalt mixes (11). In this expression a_T is the time-temperature shift factor, and To and T are the reference and prevailing temperatures respectively. This curve is shown in Figure 11 for a reference temperature of 70 F.

Output

The output of the computer programs provides information dealing with both pavement response and its relation to pavement performance. Values are presented as means and variances of the following:

1. Rutting in the wheelpath at the pavement surface in terms of accumulated deformation of the component layers;
2. Pavement roughness in terms of slope variance resulting from the variance of rut depth in the wheelpath;
3. Strain in the wheelpath on the underside of the asphalt layer;
4. Fatigue damage in terms of cracked surface area that is related to percentage of pavement life; and
5. Present serviceability index, as defined by AASHTO, at specified points in time.

Typical computer output for 2 pavement systems is given in Table 1. The material properties of the 2 systems were varied while all other inputs were held constant (4). System 1 has less cracking but more rutting and roughness. This comparison is intended to emphasize that, although a given system may reflect adequate structural integrity in one failure mode, it may not resist another. Serviceability index for the 2 systems is shown in Figure 12. This view indicates that the serviceability index as defined by AASHTO is less influenced by the amount of cracking than it is by roughness and rutting. These comparisons are only included to indicate the capabilities of the systems. Numerical values are dependent on realistic inputs from experimental field observations. As experimental and field data become available, the structural subsystem models will be adjusted accordingly.

SUMMARY

A brief overview of the research accomplishments emanating from the federally coordinated research project on new methodology for flexible pavement design has been presented. The use of a structural subsystem as a design check procedure was described. More research is under way not only to improve and refine the methods presented but also to develop and test a complete system that will incorporate maintenance, economics, and decision theory as integral parts of a design-management system.

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