# EVALUATION OF BUS MAINTENANCE PROCEDURES THROUGH SYSTEMS ANALYSIS: A CASE STUDY

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The purpose of this study was to apply different systems analysis techniques, especially the queuing theory, to evaluate the bus maintenance problems of a large transit company. The case study centers around the Denver Metro Transit Company. The maintenance facility of the company is analyzed, in terms of storage capacity, service rates for the various types of repairs, and other pertinent data, to arrive at a statistical service distribution. The statistical distribution of bus arrival for maintenance and channel configuration of the repair shop are established. The results indicate various effects on waiting time, the broken-down rate, arrival rate by changing facility capacities, and maintenance policy. At this stage of study, most efforts were concentrated on the facility aspect of the problem. The study established the theoretical basis for the maintenance procedure.

•EVERY transit company is faced with maintenance operations of buses. Each bus that is in the garage is a loss of revenue. For a transit system to operate in an efficient manner, the buses must receive proper maintenance and repairs with a minimum loss of time. This study is an attempt to gain an understanding of the bus maintenance procedures of a large transit company and to apply the systems analysis techniques, specifically the queuing theory techniques, so that the bus maintenance problem can be evaluated. The Denver Metro Transit Company (DMT), which is owned and operated by the City and County of Denver, is used as a case study.

## GENERALIZED MODEL OF VEHICLE MAINTENANCE PROCEDURES

### Model Construction

It is hypothesized that a generalized model of any maintenance shop can be developed through analysis of the maintenance procedures of DMT. A complexity arose because of the numerous types of vehicles in use at DMT and their requirement for specific parts, which are not necessarily interchangeable among the various vehicles or even needed on all of the vehicles. With this in mind, a simplified model of vehicle maintenance procedures is developed based on the following assumptions:

1. The company operates with one type of vehicle, totaling V.
2. Each vehicle contains P number of major parts per vehicle.
3. Preventive maintenance procedures (referred to as inspections) of vehicles are performed every $m_0$ miles. There are K different types of inspections; Kth is the most complete. These K inspections are carried out on a cyclic basis. The maintenance mileage at the i th inspection, $m_i$, is $im_0$. After the Kth inspection, the mileage on the vehicle is recorded as zero, and the sequence of inspections is repeated.
4. A maintenance period is assigned to each major part. Because all parts get maintained only when vehicles get inspected, the maintenance period for part i, $I_i$, will be $nm_0$, where n is a positive integer.

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Publication of this paper sponsored by Committee on Maintenance and Operations Systems.

5. The probability density function of the failure of part i at mileage m, $f_i(m)$, is Erlang-distributed. The lifetime distribution function $F_i(m)$ or the probability that part i breaks down before mileage m is expressed as

$$F_i(m) = \int_0^m f_i(x)dx \tag{1}$$

6. For the convenience of vehicle dispatching, a predetermined number of vehicles $V_k$ is assigned for the k th type of inspection each day according to time allowances for that particular inspection. If the number of vehicles requiring the k th type of inspection exceeds the capacity of the maintenance shop, those vehicles with the highest mileages are assigned, and the remainder continue in operation.

7. There are J maintenance channels and an equal number of crew members in the inspection shop. Each channel can handle all types of inspection at a service rate $\eta_k$ with k = 1 to K.

8. Vehicles that break down on their routes are pulled into the repair shop. There are R repair channels. Each repair channel specializes in the repair of one major part. The number of spare units of the i th part are equal to $S_i$ with i = 1 to P.

9. Vehicles break down in a random fashion (Poisson distribution), the time to repair part i is Erlang-distributed with mean $\mu_i$, and the time to remove a worn part and replace it with a new part is relatively short and negligible.

In this study, the model is constructed with special emphasis on the facility aspects. Two other related aspects, manpower and cost, should be taken into consideration in the future to make up a complete model.

## Inspection Queue

The concept of the inspection queue is somewhat different from what one might think of first. Service rate of this queuing system is the assignment rate, rather than the actual inspection rate. Consequently, the service channel is referred to the assigning process, rather than the actual inspection channel. From this viewpoint, the system of the inspection shop can be thought of as K single-channel queues. The input for the k th queue, or the vehicles that reach the maintenance mileage of the k th inspection, is Poisson-distributed with mean $\lambda_k$, where k = 1 to K.

$$\lambda_k = \frac{m_t}{m_k} \tag{2}$$

where

$m_t$ = total daily operating mileage, and
$m_k$ = total operating mileage from k th inspection to the next k th inspection = $Km_o$.

The service rate of the k th queue, or the assignment rate for the k th inspection, is constant and equal to $A_k$. Then the average additional operating mileage, $M_k$, is

$$M_k = \frac{T_k m_t}{V} \tag{3}$$

where $T_k$ = average additional daily operating time per day before vehicles can get inspected.

## Effect of the Input Distribution

The values of $T_k$ depend on the input rate $\lambda_k$, the assignment rate $A_k$, and the types of input distribution. For the fixed $\lambda_k$ and $A_k$, $T_k$ is determined by the distribution function of the input.

1. If the input is Poisson-distributed, then

$$T_k = \frac{\rho_k}{2A_k(1 - \rho_k)} \quad (4)$$

where $\rho_k = \frac{\lambda_k}{A_k}$.

2. If the input is uniformly distributed ($\lambda_k < A_k$) and a steady-state system exists, then $T_k = 0$.

Thus, a uniform or regular dispatching would be preferred to a random dispatching. Also, if $M_k$ is greater than zero, the possibility of failure of parts would increase from $F_i(I_i)$ to $F_i(I_i + M_k)$. Therefore, the best dispatching rule is that the input rate of inspection can be kept uniform.

Broken-Down Rate

Because broken-down rates are involved with several calculations in this study, they are examined here in detail. If part i with expected life mileage $E_i(m)$ undergoes maintenance at mileage $(I_i + M_k)$, then the average operating mileage of part i before it breaks down or goes in for maintenance is

$$D_i = \int_0^{I_i+M_k} m f_i(m)dm + \int_{I_i+M_k}^{\infty} (I_i + M_k)f_i(m)dm \quad (5)$$

$$D_i = E_i(m) - \int_{I_i+M_k}^{\infty} (m - I_i - M_k)f_i(m)dm \quad (6)$$

$$D_i = (I_i + M_k)\,[1 - F_i(I_i + M_k)] + \int_0^{I_i+M_k} m f_i(m)dm \quad (7)$$

Thus, the daily number of part i to come to the maintenance shop for either inspection or repair is

$$VM_i = \frac{m_t}{D_i} \quad (8)$$

The broken-down rate $B_i$ and the number coming for inspection $VI_i$ are respectively

$$B_i = VM_i F_i(I_i + M_k) \quad (9)$$

and

$$VI_i = VM_i\,[1 - F_i(I_i + M_k)] \quad (10)$$

Effect of the Assignment Rate

Although the assignment rate is constant, service time of inspection is not. Realistically the rates are assumed to be exponentially distributed with mean $\eta_k$ for k = 1 to K. The time to serve the assignment rate $A_k$, $t_k$ is Erlang-distributed with

$$f(t_k) = \frac{t_k^{(A_k-1)}}{(A_k - 1)} \exp\left[-\frac{A_k t_k}{k}\right] \quad (11)$$

Let TN be the total working time for all the channels, then

$$P\left(\sum_{k=1}^{K} t_k \le TN\right) = \int_0^{TN}\int_0^{TN-t_1}\cdots\int_0^{TN-t_1-\ldots t_{k-1}} \sum_{k=1}^{K} P(t_k)dt_1\ldots dt_k \quad (12)$$

The probability that the inspection crew cannot finish the assigned vehicles and have to work overtime is

$$P\left(\sum_{k=1}^{K} t_k > TN\right) = 1 - P\left(\sum_{k=1}^{K} t_k \le TN\right) \quad (13)$$

The average overtime length TM is

$$TM = E\left[\left(\sum_{k=1}^{K} t_k - TN\right)\middle|\left(\sum_{k=1}^{K} t_k > TN\right)\right] \quad (14)$$

The expectation is taken over the summation of $t_k$.

By increasing $A_k$, $M_k$ will be reduced as will the broken-down rate, while the same time TM will be increased. From the viewpoint of minimizing cost, the optimal $A_k$ will be as follows:

$$\text{Opt. COST} = \min_{A_k} [(\text{average overtime labor cost}) \times TM + (\text{average cost per repair for part i}) \times B_i] \quad (15)$$

Inspection Crew Size and Number of Inspection Channels

It was assumed in the model that the number in the inspection crew was equal to the number of inspection channels J. If the number of channels is increased, then the number of the crew size must also be increased; therefore, there will be an increase in total working time for all inspection channels and a decrease in overtime. The optimal number of inspection channels will depend on the availability of a night shift, the wages of mechanics, and the cost of increasing channel capacity. One can also increase the number of mechanics in each channel to reduce the service time, but the marginal savings gained on the service time by increasing the crew will eventually decrease. The optimal number of members in each channel would be reached when (marginal savings on overtime work)×(labor rate of overtime) = (wages of increased number of mechanics).

Determination of Maintenance Mileage for Individual Parts

In the case when $m_o$ is given, the optimal maintenance mileage for part i, $I_i$, would be such that

$$\hat{I}_i = \hat{n} m_o$$

$$\begin{aligned}\text{Opt. COST} &= \min_{n} [(\text{avg. cost/repair}) \times B_i + (\text{avg. cost/inspection}) \times VI_i] \\ &= \min_{n} \{(\text{Avg. cost/repair}) \times VM_i \times F_i(nm_o) + (\text{avg. cost/inspection}) \times VM_i \times [1 - F_i(nm_o)]\}\end{aligned}$$

where n is positive integer. If the average additional operating mileage $M_k$ is known,

$B_i$ in the above equation becomes $VM_i \times F_i(\hat{n}m_o + M_k)$ instead of $VM_i \times F_i(\hat{n}m_o)$. $VI_i$ becomes $VM_i \times [1 - F_i(\hat{n}m_o + M_k)]$.

## Repair Queue

If the time required to remove a worn part and replace it with a spare unit is comparatively short and negligible, the repair shop system can be described as having P single-channel queues, where P is the number of types of major parts. The input of each queue is Poisson-distributed with mean $B_i$. The service time or the time needed to repair part i is assumed to be Erlang-distributed with mean $\mu_i$ and Erlang constant $\ell_i$. The average number of vehicles idle in the repair shop E(n) is equal to

$$E(n) = \sum_{i=1}^{P} E_i(n) = \sum_{i=1}^{P}\left(\frac{\ell_i + 1}{2\ell_i} \times \frac{B_i}{\ell_i\mu_i(\ell_i\mu_i - B_i)} + \frac{B_i}{\ell_i\mu_i}\right) \qquad (16)$$

and if no spare parts are available, the average expected waiting time at the repair shop $E_i(w)$ would be equal to

$$E_i(w) = \frac{\ell_i + 1}{2\ell_i} \times \frac{B_i}{\mu_i(\mu_i - B_i)} + \frac{B_i}{\mu_i} \qquad (17)$$

## Effect of Providing Spare Units

If $S_i$ spare units are provided for part i, then the probability that a vehicle arrives and finds a spare unit available is equal to the probability that a vehicle arrives and finds the total number of failed parts i in the system less than $S_i$.

$$P(s_i < S_i) = \sum_{s_i=0}^{S_i-1} P(s_i)$$

$$= P(s_i = 0) + \sum_{s_i=1}^{S_i-1} P(s_i)$$

where

$$P(s_i = 0) = 1 - \ell_i\alpha_i$$

$$\alpha_i = \frac{B_i}{\ell_i\mu_i}$$

$$P(s_i) = (1 - \ell_i\alpha_i) \sum \alpha_i^{\beta}(-1)^{i}\binom{\beta}{i}\binom{\beta+j-1}{j} \quad \text{for } s_i > 0 \qquad (18)$$

where the summation is taken over the partitions of $s_i$ such that $s_i = \beta + i\ell_i + j$.

The average time saved from waiting by providing $S_i$ spare units $TS(S_i)$ is

$$TS(S_i) = \frac{P(s_i < S_i)}{\mu_i} \qquad (19)$$

and the average waiting time is reduced to $[E_i(w) - TS(S_i)]$. The average number of vehicles at the repair shop idle because part i failed can be reduced to

$$E_i(n) = \sum_{s_i=1}^{S_i-1} s_i P(s_i) - S_i[1 - P(s_i < S_i)] \tag{20}$$

The benefits gained from providing an additional spare unit decrease with the increasing number of spare units already on hand. If we have $(s_i - 1)$ spare units for part i, then the time that can be saved by providing an additional unit is

$$\begin{aligned} \Delta TS(s_i) &= TS(s_i) - TS(s_i - 1) \\ &= s_i \mu_i P(s_i - 1) \end{aligned} \tag{21}$$

The time decreases with an increase in $s_i$.

If the capital cost for one spare unit of the i th part is $C_i$ and the expected life mileage is $E_i(m)$, then the optimal $\hat{S}_i$ is such that

$$\text{Opt. } C_i = \underset{S_i}{\text{Min}} \left[E_i(m) \times B_i \times \Delta TS(S_i)\right] \tag{22}$$

The flow chart of the model is shown in Figure 1.

## PROBLEMS CONCERNING RELAXATION OF ASSUMPTIONS

### Operation With Many Vehicle Types

It is assumed in the model that bus companies operate with only one type of vehicle, but in the real world a bus company would operate with a variety of vehicles. This fact affects almost every aspect of maintenance procedure.

First of all, daily operating mileages are different for each type of vehicle. One of the reasons is that some models are more suitable to serve some specific area or route to meet various passenger capacities. Table 1 gives the mileage variation on different models.

Next, one might argue that there may be different optimal inspection mileages for the various models, but this causes some complications from the viewpoint of management. Except for a few extreme cases, all models are inspected at the same mileage.

From the nature of the Poisson distribution, the distribution of the sum over several Poisson-distributed random variables is still Poisson. If the number of vehicles due for inspection for each model is Poisson-distributed, then the total number of vehicles due for each type of inspection is also Poisson-distributed. The average time needed to get through each inspection is almost the same, without significant differences for each model. Therefore the analysis discussed in a previous section still holds.

More complications arise, however, in the repair shop. It is not unusual to find that parts (e.g., the engine) are not interchangeable among the various models. Time to replace and time to repair are also different for some models. For most parts the time required to replace or to repair varies from case to case, but it is independent of the model.

Another problem that arises is the provision of spare units. The inability to change parts from one model to another requires the provision of spare units for each model. To solve this problem, parts from different models can be treated as different parts. The total number of parts in the system would therefore increase sharply, although the methodology would remain the same.

### Assignment Discipline

The assignment discipline might be different from one company to another, and depends on the maintenance capacity, the maintenance system, and the management viewpoint of each company. For instance, a company might not run overtime for inspection at night. Those buses that were assigned for inspection but could not get through are left to be finished the next day. The assigned number for the next day is consequently

**Figure 1. Vehicle flow of model.**

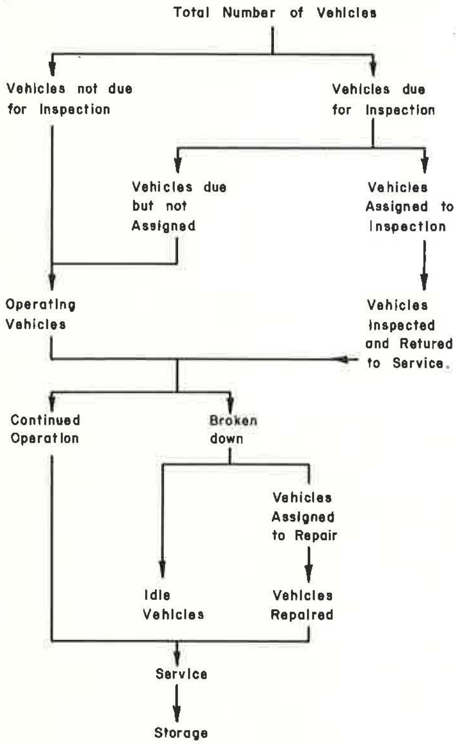



**Table 1. Average monthly mileage for DMT (July 1972).**

| Vehicle Model | Mileage per Month (miles) |
|---|---|
| Stickshift, GMC 47 | 1,560 |
| GMC 51 | 1,704 |
| MACK 45 | 1,758 |
| GMC 45 | 2,810 |
| GMC 53 | 2,988 |
| FLEX 53 | 2,894 |

**Table 2. Proposed daily inspection assignment rate.**

| Type of Inspection | Assigned Vehicles per Day | | | | | Total per Week |
|---|---|---|---|---|---|---|
| | Mon. | Tues. | Wed. | Thurs. | Fri. | |
| A | 5 | 5 | 6 | 6 | 4 | 26 |
| B | 6 | 6 | 5 | 5 | 4 | 26 |
| C and D | 9 | 9 | 11 | 11 | 12 | 52 |

reduced. If the distribution of the service rate P(n) is Poisson-distributed then the assignment rate becomes a truncated Poisson distribution with

$$P'(n) = P(n) \quad n = 1, 2, \ldots, N - 1$$
$$P'(N) = 1 - P(i) \tag{23}$$

where

N = predetermined assignment rate,
P(n) = distribution of service rate, and
P'(n) = distribution of actual assignment rate.

There is no analytical solution for such a problem thus far, but by the use of a random number generator, the problem can be analyzed through computer simulation.

## Significant Removal and Installation Time for Vehicle Parts

In most cases, the time required to replace the part was less than the time required to repair it. This is the assumption made in the analysis. Logically, to what extent can one argue (a) that the time needed to replace the part is negligible and (b) what to do if it is significant? It was found that if the time needed to replace parts is negligible, then the analysis is independent of the number of service channels. If, however, time is significant, then the number of service channels plays a central role in the analysis.

Usually the service channels in the repair shop can be classified as hoists, pits, and stalls, which are suitable to serve some specific parts. If the number of one of these channels is greater than or equal to the number of parts that needed this type of channel to replace it, then the analysis described previously is still applicable. If the number of channels is less than the number of parts (after the parts are removed), the vehicles can be withdrawn from the channels and thus leave channels ready to serve other vehicles; the analysis is still applicable. The problem occurs when the number of channels is less than the number of vehicles needing service, and the vehicle being served has to remain in the channel until a repaired unit is installed. The analysis then would not hold.

## Preventive Maintenance

In the previous model, the time required for preventive maintenance of all parts is included in the inspection time. It is noted, however, that maintenance of some parts takes longer time periods and sometimes needs special facilities. Therefore, the maintenance of these parts is not performed at the inspection shop but at the repair shop.

Suppose for part i, the time required for maintenance is Erlang-distributed with mean $q_i$ and Erlang constant $\ell_i'$. If the maintenance mileage is $I_i$, the arrival rate $VI_i$ for preventive maintenance is

$$VI_i = VM_i[1 - F_i(I_i + M_k)] \tag{10}$$

The distribution of arrivals can be tested to determine whether it is Poisson if the number of vehicles is large.

The repair rate $\mu_i$ and the preventive maintenance rate $q_i$ can be the same or different depending on the nature of the part and on the Erlang constants $\ell_i$ and $\ell_i'$. If $u_i = q_i$ and $\ell_i = \ell_i'$, the arrival rate for repair $B_i$ and preventive maintenance rate $VI_i$ can be combined into

$$B_i + VI_i = VM_i \tag{24}$$

This new arrival is still Poisson-distributed. If $\mu_i \neq q_i$ or $\ell_i' \neq \ell_i'$, then one should treat them as two different sources for the repair queue system but use the same spare units.

# CASE STUDY—DENVER METRO TRANSIT COMPANY

## General Description

The general philosophy of DMT is to provide a second car for a family, to improve service, and to increase ridership. The facility was originally designed for motor coaches by the Tramway Company in 1956, which was privately owned and operated. It was purchased in April of 1971 by the City and County of Denver and renamed the Denver Metro Transit Company.

DMT operates with 9 different types of models for a total of 250 vehicles. The peak-hour morning run requires 234 vehicles, and the peak-hour afternoon run requires 235 vehicles. This leaves 15 vehicles in reserve for inspection, repair, and overhaul.

## Routine Maintenance

DMT requires four types of inspections (A, B, C, and D) for its vehicles. The A inspection requires 30 to 45 min and four crew members. It consists of a brake adjustment and visual inspection of parts. The B inspection requires 1 to $1\frac{1}{2}$ hours and two crew members. It consists of a brake adjustment, lubrication, visual inspection of parts, oil and filter change, battery hydrometer check, stall test on the engine and transmission, voltage regulator volt test, and a check of the oil-cooled alternators. The C and D inspections require 10 min each and consist of a brake adjustment and rapid visual inspection. Inspections are required at 1,500-mile intervals: C at 1,500 miles, A at 3,000 miles, D at 4,500 miles, and B at 6,000 miles. Before inspection, the engine is washed a day or two in advance so that oil leaks may be checked for on inspection. Table 2 shows the proposed inspection assignment rate.

Overhauls are assigned according to compression ratings and oil consumption. If the inspection area falls short of work, the inspectors report oil consumption and highest mileages, and these vehicles are inspected. If work due to breakdowns and accidents offsets inspections, those vehicles scheduled for inspection are scheduled for split shifts and the shortest runs, so that they can be pulled in during the middle of the day for inspection and be returned to service by the afternoon run. The inspections are made so that most of the vehicles can be on the road; shortest repairs are done first, and these vehicles are back on the road first.

DMT provides three maintenance channels for inspections. There are two short pits and one long pit at these channel locations. These channels will permit four vehicles to be inspected at one time.

The inspection vehicles are selected on the basis of mileage. They are scheduled two days in advance. The inspection rate for July, August, and September of 1972 gives a general feel for system operation (Table 3). The actual arrival rate for inspection for July is given in Table 4 as a comparison.

## Analysis of Inspection Queue

The model is constructed so that all rates, such as arrival rate and service rate, are measured with the unit of number of vehicles per day. It is further assumed that vehicles assigned can get through inspection by the next morning because there is a night shift crew provided in the DMT maintenance shop. (The service discipline of concern is how vehicles get assigned rather than how vehicles get inspected.)

In principle there exists a fixed and predetermined assignment rate. If the number of vehicles ready for inspection on a specific day exceeds this assignment rate, only a number equal to the assignment rate will be inspected on that day. The rest are left for the next day. On the other hand, if the number ready for inspection is less than the assignment rate, the inspected number on that specific day will be equal to the arriving number. In practice, however, the assignment rate is more flexible. From the experiences of DMT, one can see that the inspected number sometimes exceeds the assignment rate. In other words, the assignment is somewhat random, and no explicit decision rule is followed. In Figure 2, the number of daily inspected vehicles at DMT is fit to the inverse Erlang distribution. Several simulations were run on the computer so that differences between fixed and random assignment rates could be compared.

**Table 3. Inspection rate.**

| Type of Inspection | July | | | | | August | | | | | September | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Mon. | Tues. | Wed. | Thurs. | Fri. | Mon. | Tues. | Wed. | Thurs. | Fri. | Mon. | Tues. | Wed. | Thurs. | Fri. |
| A | 5 | — | 5 | 5 | 5 | | 5 | 5 | 6 | 4 | | | | | 4 |
| B | 5 | | 5 | 4 | 5 | | 6 | 4 | 5 | 4 | | | | | 4 |
| C and D | 10 | | 8 | 11 | 9 | | 13 | 10 | 11 | 9 | | | | | 8 |
| A | 5 | 5 | 5 | 5 | 5 | 5 | 5 | 6 | 6 | 4 | — | 5 | 6 | 7 | 5 |
| B | 5 | 5 | 5 | 5 | 5 | .6 | 6 | 5 | 5 | 4 | | 6 | 5 | 4 | 4 |
| C and D | 10 | 11 | 10 | 9 | 8 | 11 | 12 | 10 | 11 | 8 | | 11 | 9 | 10 | 10 |
| A | 5 | 5 | 6 | 5 | 5 | 4 | 6 | 6 | 6 | 4 | 5 | 5 | 5 | 4 | 5 |
| B | 5 | 5 | 5 | 6 | 5 | 6 | 6 | 5 | 5 | 4 | 5 | 7 | 5 | 5 | 4 |
| C and D | 12 | 12 | 11 | 11 | 9 | 11 | 11 | 11 | 11 | 8 | 9 | 4 | 9 | 11 | 10 |
| A | 5 | 5 | 6 | 6 | 5 | 5 | 6 | 5 | 6 | 4 | 5 | 5 | 6 | 6 | 4 |
| B | 4 | 6 | 5 | 5 | 5 | 5 | 6 | 5 | 5 | 4 | 6 | 6 | 5 | 5 | 4 |
| C and D | 9 | 9 | 9 | 10 | 8 | 11 | 11 | 11 | 11 | 7 | 11 | 10 | 11 | 10 | 8 |
| A | 3 | | | | | 5 | 4 | 7 | 5 | | 4 | 7 | 6 | 6 | 4 |
| B | 6 | | | | | 6 | 6 | 5 | 5 | | 6 | 6 | 5 | 5 | 4 |
| C and D | 12 | | | | | 12 | 11 | 10 | 9 | | 11 | 11 | 9 | 10 | 8 |

[a]Independence Day. [b]Labor Day.

**Table 4. Actual arrival rate.**

| Type of Inspection | Sun. | Mon. | Tues. | Wed. | Thurs. | Fri. | Sat. | Sun. | Mon. | Tues. | Wed. | Thurs. | Fri. | Sat. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| A | 0 | 2 | 7 | 1 | 9 | 1 | 2 | 1 | 2 | 2 | 4 | 0 | 7 | 7 |
| B | 1 | 0 | 4 | 5 | 4 | 3 | 4 | 2 | 3 | 2 | 5 | 5 | 2 | 4 |
| C and D | 2 | 2 | 8 | 3 | 9 | 2 | 14 | 2 | 1 | 9 | 8 | 7 | 11 | 10 |
| A | 1 | 0 | 8 | 11 | 4 | 6 | 2 | 2 | 0 | 3 | 5 | 2 | 4 | 5 |
| B | 2 | 1 | 2 | 7 | 6 | 5 | 2 | 0 | 2 | 4 | 5 | 2 | 4 | 4 |
| C and D | 5 | 3 | 11 | 7 | 11 | 9 | 8 | 4 | 1 | 9 | 4 | 7 | 9 | 10 |
| A | 1 | 2 | 1 | | | | | | | | | | | |
| B | 2 | 1 | 4 | | | | | | | | | | | |
| C and D | 3 | 1 | 7 | | | | | | | | | | | |

Note: From July 1972 operations of DMT.

**Figure 2. Distribution of inspection rate.**

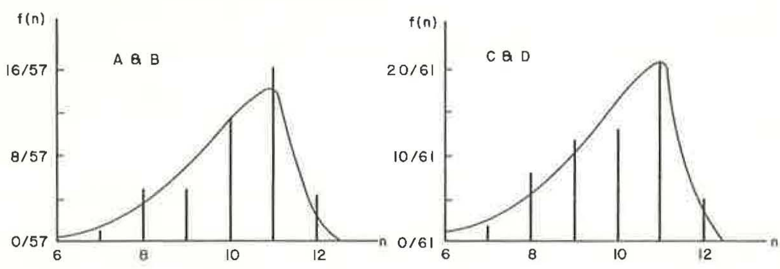


Figure 3 shows the observed data for inspections C and D at DMT from July to September 1972. The curves are from simulation based on the assumptions of Poisson arrivals and constant assignment rate. Because the arrival rate data fits Poisson distribution well (Fig. 4) and the assignment rate is basically constant, they are used in the following analysis. Input data for the inspection queue are

1. Number of vehicles—250;
2. Average daily operating mileage—89.9 miles per day per vehicle;
3. Inspection period—every 6,000 miles for all types of inspection;
4. Assignment—constant with rate 5.20 per day;
5. Assignment discipline—first arrive, first assigned; and
6. Inspection rate—exponentially distributed with the following means: $\eta_A = 40$ min, $\eta_B = 75$ min, $\eta_C = \eta_D = 10$ min.

By applying these data to the model, the following results:

1. Arrival rate—4.25 per day;
2. Average waiting time—0.25 days; and
3. Average additional operating mileage—19.33 miles per day per vehicle.

Both the number of buses and average daily operating mileage are beyond control of the maintenance shop, and the rest of the factors are determined either by the facility capacity or by the maintenance policies. The resulting average additional operating mileage is what we are most concerned with, because it has a direct effect on the broken-down rate. We have to increase the assignment rate, i.e., to speed up the actual inspection rate to reduce the additional operating mileage. Another alternative to reduce this mileage is to prolong the inspection period. However, this will increase the broken-down rate sharply. These relationships are shown in Figures 5, 6, and 7.

## Repair Shop

The input for the repair shop is determined by the pull-ins and road calls. Road calls are received, and it is determined at this time whether or not it is necessary to send a replacement vehicle or a repair vehicle to the scene. DMT has three pick-ups and one tow truck to answer these road calls; three of the four vehicles are radio equipped for easier dispersion to disabled buses. Not more than 5 min in route time is lost before a repair vehicle meets the bus on route for a road call. The repair vehicles meet the buses along their scheduled runs so that service is never disrupted. A completely stopped bus requires a replacement; therefore within 20 min after the call, a replacement is at the scene to complete the run of the disabled vehicle. A bus will continue in service as long as it is operating properly with no danger to the passengers or operator.

A pull-in is taken to the repair shop and checked. There are six lanes with no hoists or pits. If a major repair is indicated, the bus is taken to the overhaul or body shop for repair. The repair shop has a crew that is taken from the inspection area as needed.

## Repair Shop Analysis

Currently, DMT operates nine different types of buses; this fact makes the problem of repair and maintenance much more complex and the analysis more difficult. Differences between the various types of buses are ignored here so that more insight into the mechanism of the repair shop may be obtained. A computer simulation should be used to accommodate the problem of operating with many vehicle types.

There are 29 major parts listed in the file of the DMT repair shop. Although a maintenance mileage is suggested for each part, no preventive maintenance is carried out at the present time. From the DMT repair records, a life-mileage curve, according to the vehicle type, is fitted for each part without further breakdown. Some examples are shown in Figure 8. The detailed data, including the life-mileage curve, time required for replacement and repair, number of square units, and maintenance mileages suggested by DMT, are given in Table 5. Among these 29 parts, four parts that occupy the repair shop for the longest period of time were chosen for detailed analysis. These four parts are the engine, the transmission, the transmission governor, and the cylinder head.

Figure 3. Observed and theoretical inspection rate.

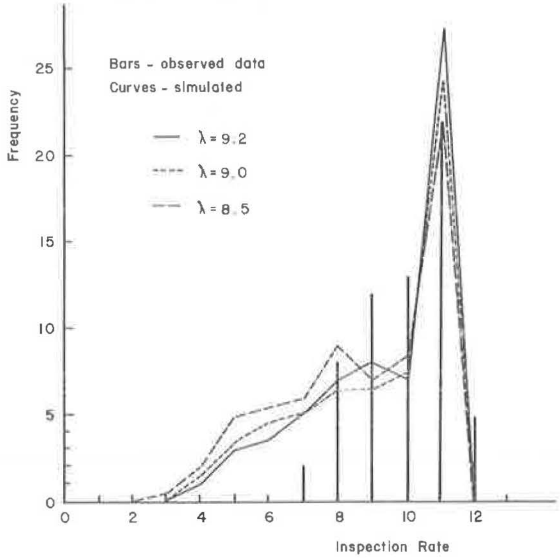



Figure 4. Distribution of arrival rate.

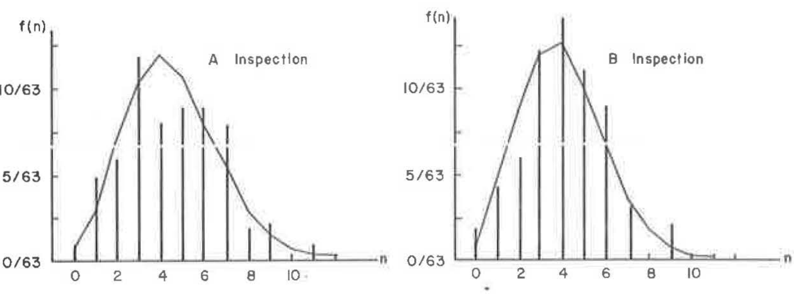



Figure 5. Effect of assignment rate on additional operating mileage.

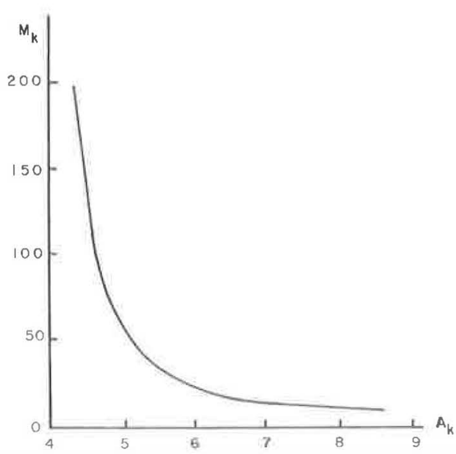


Figure 6. Effect of inspection period on broken-down rate.

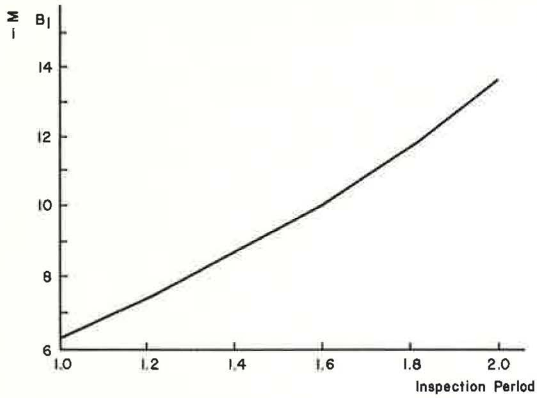



Figure 7. Effect of inspection period on additional operating time.

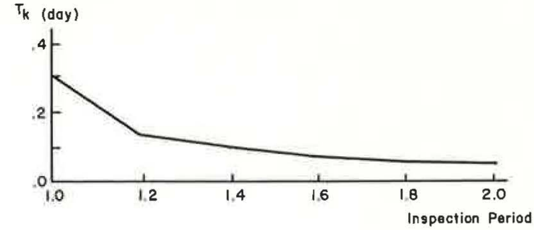



Figure 8. Cumulative distribution of life mileage ($10^4$ miles for transmission, engine, and cylinder head; $10^3$ miles for transmission governor).

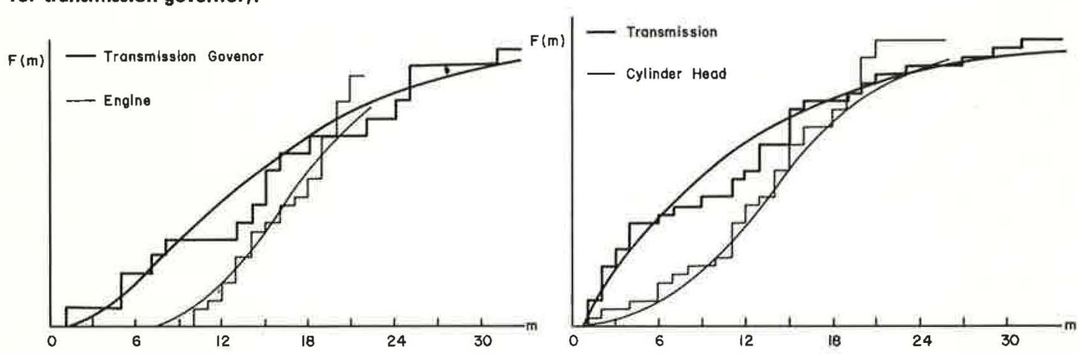



Table 5. Major part characteristics.

| Parts | Life Mileage | | Proposed Maintenance Period | Time (hours) | |
|---|---|---|---|---|---|
| | Erlang Constant | Expected | | Remove | Repair |
| Engine | 10 | 180,000 | —[b] | 8 | 192.5 |
| Transmission | 1 | 105,000 | 200,000 | 8 | 32 |
| Trans. gov. | 2 | 16,000 | 36,000 | 12.5 | 32 |
| Starter | 1 | 60,000 | 150,000 | 2.4 | 1 |
| Generator | 6 | 140,000 | 125,000 | 1 | 1.5 |
| N/S solonoid | —[a] | —[a] | —[a] | 1 | 1 |
| Compressor | 2 | 70,000 | 150,000 | 1.5 | 5 |
| Comp. gov. | 3 | 75,000 | 125,000 | 0.5 | 0.5 |
| Comp. lub. valve | —[a] | —[a] | —[a] | 0.5 | 1 |
| Shutter stat. | 1 | 40,000 | 78,000 | 0.5 | 1 |
| Shutter cylinder | 3 | 90,000 | 125,000 | 2 | 1 |
| Injectors | 1 | 60,000 | —[b] | —[a] | —[a] |
| Clutch cylinder | 2 | 36,000 | 155,000 | 0.5 | 0.25 |
| Throttle cylinder | —[a] | —[a] | —[a] | 0.5 | 0.5 |
| Water pump | 3 | 45,000 | 125,000 | 0.5 | 2.5 |
| Alarmstat. | 6 | 32,000 | 78,000 | 0.5 | —[c] |
| Fuel pump | 6 | 140,000 | —[b] | 1 | 0.5 |
| Cylinder head | 6 | 140,000 | —[b] | 7 | 4.5 |
| Blower | 6 | 140,000 | —[b] | 3 | 11 |
| Eng. gov. | 2 | 120,000 | —[b] | 2 | 1.5 |
| Clutch mag. valve | —[a] | —[a] | —[a] | 0.5 | 1 |
| Eng. thermo. | —[a] | —[a] | —[a] | 1 | 0.5 |
| Throttle lip | —[a] | —[a] | —[a] | 0.1 | 0.5 |
| Radiator | 1 | 90,000 | —[a] | 4 | 8 |

[a]Not available. [b]Overhaul. [c]Disposable.

At the DMT repair shop, including the overhaul shop, there are four pits, four hoists, and numerous stalls. Because the number of channels of each type is greater than the number of parts needing this specific type channel and also because each of these four parts goes to four different mechanics for repair, the analysis from the previous section can be applied here.

Another interesting aspect is that the time required for repair is the same for either the broken-down vehicle or the vehicle that comes in for preventive maintenance. Therefore, the actual arrival rate at the repair shop is the sum of these two cases. $VM_i$, the average arrival number at the repair shop, and $B_i$, the average number of broken-down vehicles due to failure of part i, were shown previously in Eqs. 9 and 10. Both of these rates are the function of the maintenance mileage, $I_i$. The broken-down rate is a monotonic increasing function of $I_i$, and the arrival rate, including the broken-down vehicle and vehicles for preventive maintenance, is a convex function of $I_i$. They are shown in Figures 9 and 10. It was observed that the optimal maintenance mileage for the lowest arrival rate occurs at from 70 to 110 percent of the expected life mileage. However, a minimum arrival rate could probably mean a high broken-down rate because the broken-down rate curves are monotonically increasing. For example, the arrival rate due to failure of the cylinder head reaches minimum at 70 percent of the expected life mileage, and the broken-down rate at this mileage is 0.35, which is much higher if compared to 0.08 of the transmission. Therefore the optimal maintenance mileage should be located at some time when the broken-down rate is thought to be tolerable.

The optimal number of spare units that should be provided for each part is also a management decision. Five parts are chosen to test the effects of the spare number provided. It is observed that the decreasing rate of the number of idle vehicles depends on the value of $\alpha$, the ratio of arriving rate to the service rate of $\ell\mu$. For the engine, the radiator, and the compressor (they all have small values of $\alpha$), the effect of providing one spare unit is significant. Both the transmission and the transmission governor have high values of $\alpha$; the effect of the provision of one spare unit is not as dramatic. The efficiency of the provision of n spare units can be defined as one minus the ratio of the number of idle buses if n spare units are provided to that if no spare units are provided at all. The results are shown in Figure 11.

With further analysis, one can extract more information from Figure 12. If the service rate is increased, which reduces the value of $\alpha$, greater benefits can be achieved by providing the same number of spare units.

## SUMMARY AND CONCLUSIONS

The main purpose of this study was to provide an analytical basis for a bus maintenance shop. The bus maintenance procedure is primarily based on the mileages of the buses and the life mileages of major parts on each bus. This procedure becomes complicated as the variety of bus types increases and when the number of parts taken into consideration grows. To make the formal analysis possible, a generalized model was constructed. This model consisted of submodels of the inspection shop and the repair shop and a cost minimization submodel. The cost optimization model was not presented because no data were available to validate and demonstrate it. The validity of others was established through the comparison between observed data and that produced by the model. Information made available by DMT was used as the observed data base.

Queuing theory plays a key role in the analysis of the inspection shop and the repair shop. The inspection shop is treated as many single queues with a Poisson arrival rate and constant service, or assignment, rate. The inspection period had the most sensitive effect on the breakdown rate of parts that are inspected on a routine basis. The repair shop had many single channel queues with Poisson-distributed input and Erlang-distributed service time. There is more variety to the input into the repair shop than in the inspection shop, and it contains different channel types. Some spare units for each part are also provided in the repair shop. The relationship among these various factors is examined, and it was found that most of the characteristics of each part were determined by curve fitting to the life-mileage curve, which can be determined by curve fitting to the actual data.

**Figure 9. Maintenance mileage related to broken-down rate.**

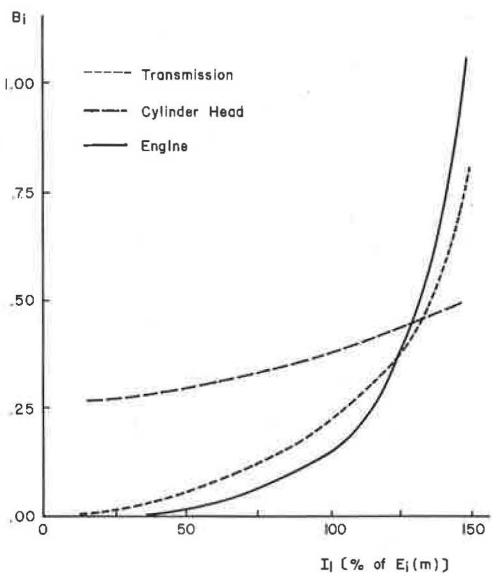



**Figure 10. Maintenance mileage related to arrival rate.**

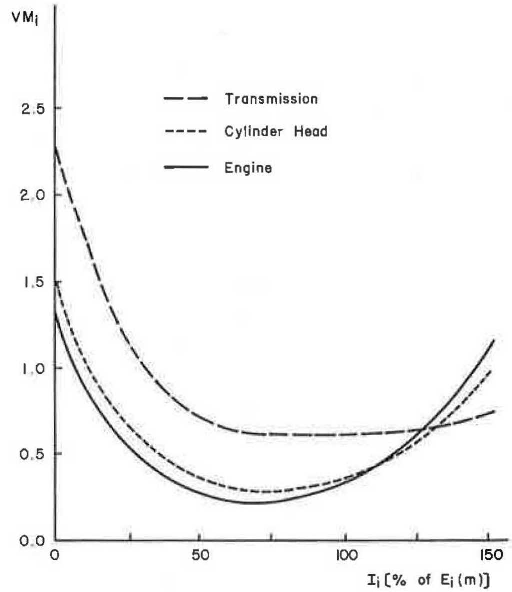



**Figure 11. Efficiency of providing spare units (TG = transmission governor, T = transmission, C = compressor, R = radiator, and E = engine).**

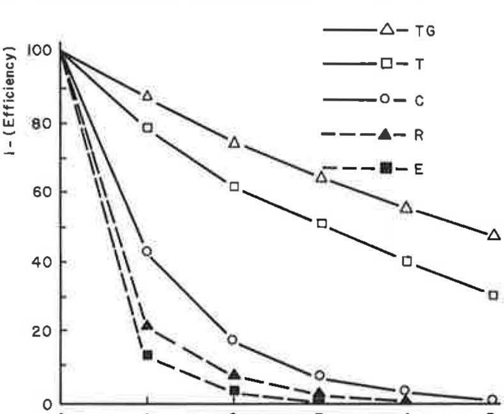
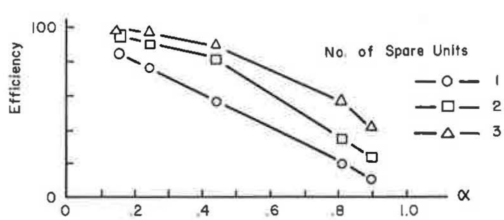



**Figure 12. Effect of value of $\alpha$ on efficiency (compressor).**



Although the model is quite simple, it provides much insight into the problems and complexities of a maintenance shop. If a computer simulation were applied, the model could be modified to become even more realistic.

## ACKNOWLEDGMENT

The research presented in this paper is part of a project sponsored by the Urban Mass Transportation Administration. The results and views expressed are those of the authors and not necessarily those of the sponsoring agency. The assistance of Sharon Tomich in the data collection and preliminary analysis is gratefully acknowledged.

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