

# METHODS FOR PREDICTING MODULI AND FATIGUE LAWS OF BITUMINOUS ROAD MIXES UNDER REPEATED BENDING

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The use of modern pavement design methods to prevent cracking necessitates, among other things, determining the moduli and fatigue laws in bending of the bituminous road mixes (base courses and wearing courses). These determinations are now possible, but they involve sophisticated experiments that are not well adapted to practical purposes. The objective of this paper is to present an original contribution to the prediction of moduli and fatigue laws in sinusoidal bending (controlled stress tests) of bituminous mixes. The results obtained with a large variety of mixes having different bitumen, composition, and size distribution of the aggregates show that this prediction requires only the knowledge of the volumetric composition of the mix (aggregate volume content, bitumen volume content, and void content) and the knowledge of some characteristics of the bitumen used (asphaltene content and penetration). On the basis of the experimental results obtained, general formulas are presented to permit the prediction of the moduli and the fatigue law of a mix. Practical criteria for the selection of the bitumen are also proposed.

•WORK on predicting moduli and fatigue laws of bituminous road mixes (under repeated bending) is part of a more general research project on pavement design in progress at Belgium's Centre de Recherches Routières. This project covers design criteria, vehicles and traffic, multilayer theory, mechanical properties of road materials, influence of climatic effects, and experimental roads (1).

## MIXES AND BITUMEN

The particulars of the mixes investigated are given in Table 1. Mixes cut out of experimental roads are designated with an "r" (2, 3). The other mixes were manufactured in the laboratory by static compaction. The maximum size of the coarse aggregates varied from 12 mm to 32 mm. The sand, which varied from 74  $\mu$ m to 2 mm, was a mixture of natural sands except for the mixes P1r to P6r, P20 to P23, and S12 to S19r, where it was a mixture of  $\frac{2}{3}$  natural sand and  $\frac{1}{3}$  crushed sand. The filler (<74  $\mu$ m) was a limestone filler except for mixes P15, P16, and 017 where it was a mixture of limestone and clay; limestone and chalk; and limestone, clay, and chalk respectively. The letter symbol under weight percentage of a mix refers to the kind of stones in the mix (P = porphyry, G = rounded gravel, S = hard sandstone, L = limestone, and PV = Voutre porphyry).

The characteristics of the bitumen used are given in Table 2. The meanings of the symbols are as follows: penetration at 25 C/100 g/5 s = Pen; softening point ring and ball =  $T_{AB}$ ; penetration index = IP; asphaltene content defined as the percentage of the insoluble phase of the bitumen in normal heptane at the ambient temperature =  $\alpha$ ; specific gravity =  $\gamma_L$ ; (d log pen)/(d log t) = derivative B; viscosity at 25 C for low shear stresses =  $\eta_0$ ; and the ratio between viscosities at 25 C for low and high shear stresses =  $\eta_0/\eta_\infty$ .

Table 1. Particulars of the mixes.

Mix	Weight Percentages				Volume Percentages			
	Stones (P)	Sand (S)	Filler (F)	Bitumen (L)	Voids	Aggregates	Bitumen	Bitumen <sup>a</sup>
P0	58	35	7	5.8	4.8	82.3	12.9	DU 60
P1r	53.8	36.2	10	6.1	3.5	83.2	13.3	T3 50-70
P2r	53.8	36.2	10	5.9	3.5	83.5	13.0	T2 50-70
P3r	53.8	36.2	10	6	3.5	83.5	13.0	T1 50-70
P4r	53.8	36.2	10	5.8	3.5	83.7	12.8	T3 80-100
P5r	53.8	36.2	10	5.7	3.5	83.9	12.6	T2 80-100
P6r	53.8	36.2	10	5.9	3.5	83.6	12.9	T1 80-100
P7	58	35	7	5.8	4.9	82.5	12.6	DT 40
P8	80	18	2	4.9	12.5	77.4	10.1	DT 180
P9	58	35	7	5.8	4.7	82.6	12.7	DT 180
P10	80	18	2	4.4	14.2	76.8	9	DT 80
P11	50	40	10	6.7	3.9	81.7	14.3	DU 80
P12	55	37.7	7.3	6.2	6.2	80.3	13.5	L 80
P13	55	37.7	7.3	6.2	6.1	80.9	13.1	D 80
P14	55	36	9	6	4.7	82.3	13	DU 60
P15	55	36	7+2	6	3.5	83.3	13.2	DU 60
P16	55	36	8.5+0.5	6	4.9	82.1	13	DU 60
P17	55	36	6.5+2+0.5	6	3.7	83.2	13.1	DU 60
P18	35	52.8	12.2	8	6.9	76.9	16.2	DU 40
P19	55	36	9	5.8	4.8	82.6	12.6	DU 60
P20	55	38	7	6.1	5.3	81.5	13.2	DU 60
P21	65	35	—	6.4	6.3	80.2	13.5	DU 40
P22	80	18	2	6.25	10.2	76.9	12.9	DU 60
P23	55	37	8	6.3	4.4	82.0	13.7	HN 80
P24	55	36.4	8.6	5	8.3	81.0	10.7	DU 60
P25	53	38.4	8.6	4.75	8.7	81.2	10.1	DU 60
P26	55	36.4	8.6	5.4	6.9	81.5	11.6	DU 60
P27	53	38.4	8.6	5.2	8.1	80.8	11.1	DU 60
P28	55	37	8	5.6	5	82.5	12.5	DT 80
L1	55	37.7	7.3	6.2	6.1	80.8	13.1	DU 40
L2	55	37.7	7.3	6.2	6.2	80.7	13.1	DU 80
L3	55	37.7	7.3	6.2	6.2	80.8	13.0	D 80
G1	55	37	8	5.8	5.1	82.3	12.6	DT 80
S1	55	37.7	7.3	6.2	5.1	81.3	13.6	L 80
S2	55	37.7	7.3	6.2	5.5	81.4	13.1	D 80
S3	55	37.7	7.3	6.2	5.7	81.3	13.0	H 50
S4	55	37.7	7.3	6.2	6.6	80.4	13.0	B 80
S5	55	37.7	7.3	6.2	6.6	80.4	13.0	E 50
S6	55	37.7	7.3	6.2	6	80.8	13.2	SG 80
S7	55	37.7	7.3	6.2	6.4	80.5	13.1	SK 80
S8	55	37.7	7.3	6.2	6.8	80.1	13.1	E 80
S9	55	37.7	7.3	6.2	5.1	81.6	13.3	SK 50
S10	35	52.8	12.2	8	7.3	76.7	16.0	DU 40
S11	55	37.7	7.3	6.2	6.1	80.8	13.1	LA 80
S12r	55	37.7	7.3	6.2	2.7	83.9	13.4	SK 80
S13r	55	37.7	7.3	6.2	1.9	84.5	13.6	SK 50
S14r	55	37.7	7.3	6.2	1.7	84.9	13.4	LA 80
S15r	55	37.7	7.3	6.2	1.8	84.9	13.3	H 50
S16r	55	37.7	7.3	6.2	1.9	84.7	13.4	E 50
S17r	55	37.7	7.3	6.2	2	84.4	13.6	SG 80
S18r	55	37.7	7.3	6.2	2.4	84.4	13.2	B 80
S19r	55	37.7	7.3	6.2	2.1	84.4	13.5	E 80

<sup>a</sup>From Table 2.

Table 2. Characteristics of the bitumen.

Type of Bitumen	Pen	T <sub>AS</sub>	IP	$\alpha$	$\gamma_v$	B	$\eta_o \times 10^{-5} \text{ N/m}^2$	$\eta_o/\eta_{100}$
L 80	107	45.6	-2.04	0.9	0.995	0.495	0.56	1.33
D 80	83	46.6	-1.00	9.7	1.031	0.471	0.79	1.36
H 50	39	60.1	+0.89	29.3	1.046	0.312	30.0	7.50
B 80	81	49.1	-0.31	22.4	1.033	0.404	1.57	2.28
E 50	42	55.1	-0.02	24.1	1.028	0.345	14.40	4.74
SG 80	83	46.4	-0.23	16.4	1.020	0.440	1.10	1.72
SK 80	81	48.4	-0.18	19.8	1.022	0.384	1.45	1.71
E 80	79	49.7	-0.61	23.4	1.021	0.429	1.86	2.70
SK 50	49	55.7	+0.75	23.4	1.025	0.332	2.32	4.48
LA 80	70	49.5	-0.02	25.4	1.056	0.386	2.20	2.00
H 50 <sup>a</sup>	33	—	—	32.9	1.056	—	70.40	8.66
B 80 <sup>a</sup>	74	—	—	26.7	1.054	—	3.41	2.34
E 50 <sup>a</sup>	29	—	—	29.2	1.044	—	54.10	4.92
SG 80 <sup>a</sup>	114	—	—	18.6	1.025	—	1.37	2.40
SK 80 <sup>a</sup>	76	—	—	23.1	1.033	—	2.79	2.07
E 80 <sup>a</sup>	39	—	—	27.4	1.040	—	25.30	4.36
SK 50 <sup>a</sup>	43	—	—	27.9	1.040	—	21.00	4.43
LA 80 <sup>a</sup>	76	—	—	27.2	1.056	—	2.42	1.79
T1 50-70	60	50.0	-0.49	14.7	1.038	0.438	2.56	1.20
T2 50-70	64	53.5	+0.31	25.8	1.028	0.360	6.76	3.30
T3 50-70	57	55.5	+1.01	27.9	1.030	0.336	5.46	4.70
T1 80-100	84	47.0	-1.07	15.3	1.032	0.458	2.31	2.40
T2 80-100	80	49.0	-0.23	22.8	1.027	0.400	2.91	1.80
T3 80-100	92	49.0	+0.27	26.4	1.027	0.407	2.99	3.40
DT 40	46	57.9	+0.40	24.3	1.028	0.317	5.65	2.60
DT 80	93	48.5	+0.03	18.0	1.024	—	0.73	2.00
DT 180	208	37.3	-1.12	12.7	1.019	0.500	0.21	1.30
DU 40	42	59.0	+2.01	24.7	1.028	0.321	22.30	7.20
DU 60	57	54.5	+0.42	22.9	1.023	0.349	4.46	2.90
DU 80	97	47.5	-0.13	19.6	1.029	0.429	0.93	1.80
HN 80	73	48.5	-0.48	18.2	1.032	0.400	—	—

<sup>a</sup>Bitumen recovered from cores cut out of wearing course.

## EXPERIMENTAL CONDITIONS

The tests were performed on specimens having trapezoidal shapes (bases were 9 cm and 3 cm; height was 35 cm; thickness was 3 cm) that were fixed at their larger base and detached at the other base in sinusoidal bending (controlled stress tests).

Frequencies ranged from 3 to 100 Hz; temperatures ranged from -20 to +30 C. Moduli were determined for a stress level such that fatigue failure would not occur for fewer than  $10^7$  repetitions. In fatigue tests, the number of cycles (N) before failure varied between  $5 \cdot 10^3$  and  $10^6$ .

Manufacture of the specimens and the description of the equipment are presented in a detailed research report (4).

## METHOD FOR PREDICTING STIFFNESS MODULI

Experimental Results

Table 3 identifies the moduli that have been determined on 52 of the mixes given in Table 1. Each mix is characterized by the stiffness modulus,  $|E^*|$ , and the loss tangent,  $\text{tg } \varphi$ , both of which are functions of temperature, T, and frequency, f:

$$|E^*| = \sqrt{E_1^2 + E_2^2}$$

and

$$\text{tg } \varphi = E_2/E_1$$

where

$E_1$  = storage modulus and  
 $E_2$  = loss modulus.

The values of  $E_1$ ,  $E_2$ ,  $|E^*|$ , and  $\text{tg } \varphi$  were determined for 5 frequencies (3, 10, 30, 54, and 97 Hz) and 4 temperatures (-20, -5, +15, and +30 C). The numerical values of  $|E^*|$  and  $\text{tg } \varphi$  obtained for the 52 mixes given in Table 3 are for 3 frequencies (3, 30, and 97 Hz) and the 4 temperatures mentioned. The interpretation of these values confirmed previously obtained results:

1. The  $|E^*|$  moduli of bituminous mixes obey the frequency-temperature superposition principle—for every temperature, T, a factor  $\alpha_T$  can be found, such that the curve ( $|E^*|$  versus  $\alpha_T \times f$ ) at temperature T coincides with the curve ( $|E^*|$  versus f) at a chosen reference temperature  $T_s$ . The shifting factor  $\alpha_T$  is given by an equation of the Arrhenius form

$$\alpha_T = \exp \frac{\Delta H}{R} \left( \frac{1}{T} - \frac{1}{T_s} \right) \quad (1)$$

T and  $T_s$  are in kelvins, R is the universal gas constant, and  $\Delta H$  (apparent activation energy) is practically a constant (5). For all the mixes investigated here,  $\Delta H \approx 2.09 \times 10^5$  J/mol and  $T_s = 288$  K (15 C).

2. The complex moduli  $E^*(i\omega) = E_1 + iE_2$  can be written as the analytical formulation of a rheological model made of springs with 2 so-called parabolic cells (i.e., cells with a power-law creep function).

$$E^*(i\omega)/E_\infty = 1/[1 + \delta(i\omega\tau)^{-k} + (i\omega\tau)^{-h}] \quad (2)$$

where

$E_\infty$  = purely elastic modulus characterizing the mix at very low temperatures or at very high frequencies or both,

$\omega$  = angular frequency,

$\tau$  = parameter with the dimensions of a time, and

h, k, and  $\delta$  = empirical factors (6).

The values of h, k,  $\delta$ , and  $E_\infty$  obtained for the 52 mixes are given in Table 4.

Table 3. Numerical values of  $|E^*|$  and  $\text{tg } \varphi$ .

Mix	3 Hz								30 Hz								97 Hz							
	30 C		15 C		-5 C		-20 C		30 C		15 C		-5 C		-20 C		30 C		15 C		-5 C		-20 C	
	$ E^* ^a$	$\text{tg } \varphi^b$	$ E^* ^a$	$\text{tg } \varphi^b$	$ E^* ^a$	$\text{tg } \varphi^b$	$ E^* ^a$	$\text{tg } \varphi^b$	$ E^* ^a$	$\text{tg } \varphi^b$	$ E^* ^a$	$\text{tg } \varphi^b$	$ E^* ^a$	$\text{tg } \varphi^b$	$ E^* ^a$	$\text{tg } \varphi^b$	$ E^* ^a$	$\text{tg } \varphi^b$	$ E^* ^a$	$\text{tg } \varphi^b$	$ E^* ^a$	$\text{tg } \varphi^b$	$ E^* ^a$	$\text{tg } \varphi^b$
P0	13	55	53	39	181	12	255	4	33	51	92	29	221	9	283	3	50	44	119	24	246	6	298	<1
P1r	11	78	50	42	165	13	246	6	27	66	87	32	202	9	273	4	42	57	109	26	222	8	287	3
P2r	8.3	83	45	51	174	15	262	7	23	71	83	36	215	10	289	3	38	63	110	30	238	9	303	1
P3r	10	128	82	56	245	9	311	3	40	93	144	32	280	5	331	1	66	69	177	25	300	—	343	—
P4r	5.8	82	36	55	162	19	259	7	17	78	72	40	206	11	291	5	31	65	98	36	232	10	307	3
P5r	6.8	88	46	57	180	15	266	5	21	82	87	37	221	10	291	2	37	74	114	31	243	8	305	2
P6r	9.6	128	70	48	217	9	283	3	36	84	125	28	250	5	303	—	61	74	153	22	266	4	312	<1
P7	12	78	56	45	190	13	261	6	32	64	96	31	227	9	287	4	48	56	119	27	245	8	301	2
P8	3.8 <sup>c</sup>	111 <sup>c</sup>	17	81	127	20	202	6	14 <sup>c</sup>	101 <sup>c</sup>	46	60	163	13	223	3	26 <sup>c</sup>	87 <sup>c</sup>	65	48	181	10	233	2
P9	4.3 <sup>c</sup>	106 <sup>c</sup>	20	86	137	22	240	7	16 <sup>c</sup>	99 <sup>c</sup>	52	59	181	13	264	3	32 <sup>c</sup>	71 <sup>c</sup>	76	48	203	10	277	2
P10	5.2	80	28	53	118	15	183	5	15	73	55	39	147	11	199	3	26	60	72	33	162	9	208	2
P11	4.8	88	29	62	149	19	236	6	15	81	63	45	190	13	265	4	28	61	84	37	220	9	278	3
P12	7	110	71	46	203	6	252	2	28	98	121	27	226	6	264	1	53	58	146	19	238	2	270	1
P13	6.5	104	49	53	182	8	236	2	24	90	93	33	205	5	248	1	45	54	117	22	219	3	255	<1
P14	10	71	48	45	179	14	248	5	26	70	88	37	210	9	274	3	42	53	112	28	233	8	282	3
P15	19	109	66	56	187	18	269	8	35	80	105	43	228	13	295	5	53	63	133	30	251	8	315	2
P16	9	68	43	52	172	17	255	8	24	71	81	37	208	11	277	4	39	52	105	28	227	8	290	2
P17	14	73	56	45	185	16	263	6	31	77	98	36	224	11	287	5	48	58	125	27	245	8	300	3
P18	8	59	35	41	126	16	204	10	19	65	61	35	156	12	223	7	30	45	76	25	175	8	224	2
P19	22	51	76	23	175	9	224	4	44	44	108	20	198	8	239	3	59	35	124	16	209	5	239	<1
P20	11	69	44	45	151	13	226	4	27	62	79	32	185	9	249	3	42	51	103	26	209	6	267	1
P21	14	60	48	38	151	12	222	5	31	53	80	30	184	9	247	2	45	44	101	24	201	7	267	2
P22	8.1	75	33	51	127	15	187	7	21	62	61	37	160	10	212	5	32	61	78	30	176	9	225	4
P23	10	97	49	56	190	14	265	4	32	78	95	35	225	8	286	2	47	68	120	30	245	7	298	0
P24	11	78	42	48	140	14	203	7	27	62	76	33	172	9.5	227	5	41	61	94	30	187	9	228	4
P25	10	72	42	48	153	16	207	7	27	65	76	33	186	12	230	4	41	58	97	29	204	9.6	242	4
P26	10	69	43	47	151	14	221	6	27	63	78	33	184	10	246	3	41	62	98	28	203	8	253	2
P27	10	71	41	50	144	14	217	6	27	63	75	34	177	9	239	3	41	58	97	28	195	8	249	2
P28	10	77	45	50	183	14	248	5	28	69	85	35	226	10	275	4	45	55	110	26	253	6	295	1
L1	15	67	56	38	156	12	214	5	32	56	87	27	184	9	230	3	46	42	107	21	203	5	238	1
L2	4	70	26	56	130	19	209	6	11	93	55	49	168	13	229	4	28	42	74	33	188	8	237	2
L3	9.2	100	64	51	194	9	219	2	31	85	109	32	220	5	252	2	53	55	135	21	231	3	260	<1
G1	5.7	85	33	57	142	17	234	7	17	80	66	41	176	11	250	3	29	72	89	34	197	9	259	2
S1	6.6	141	76	57	221	8	300	3	29	117	128	30	248	5	315	2	53	87	156	22	264	3	317	3
S2	8.6	111	70	51	206	8	258	2	33	97	119	30	230	5	271	<1	53	75	144	24	245	3	280	<1
S3	19	69	73	33	187	11	252	5	40	52	107	25	212	9	268	3	56	48	138	22	231	6	276	<1
S4	10	82	56	44	170	10	238	3	28	73	93	32	200	9	256	3	44	53	115	23	216	5	262	<1
S5	22	51	76	23	175	9	224	4	44	46	107	20	197	8	238	4	59	35	124	16	209	5	239	<1
S6	7.7	91	47	53	166	12	243	6	24	78	87	37	196	9	261	4	41	52	109	24	211	5	261	1
S7	7.1	86	37	46	141	12	210	4	18	73	70	36	172	10	230	3	32	45	87	31	189	7	239	<1
S8	13	63	58	32	164	9	229	4	29	56	91	26	192	7	245	2	43	41	110	20	205	4	253	<1
S9	14	61	54	34	160	11	227	5	32	56	86	27	188	9	245	3	47	43	106	20	202	4	250	<1
S10	8.3	84	38	37	125	15	190	6	20	61	66	31	155	9	211	4	30	45	81	24	168	9	217	5
S11	8	53	45	54	157	15	231	6	24	81	81	40	190	11	250	4	36	63	100	30	208	7	255	2
S12r	7	89	43	50	168	14	248	6	21	77	80	37	203	10	277	4	36	66	106	31	226	8	292	3
S13r	21	62	82	30	196	10	267	5	45	48	121	23	233	7	296	3	65	41	147	19	255	6	313	4
S14r	12	74	63	42	192	13	275	5	33	65	107	29	230	8	300	4	51	56	135	25	254	7	314	2
S15r	30	54	97	26	219	8	285	4	60	42	138	19	253	6	308	2	61	36	162	17	275	5	326	<1
S16r	36	51	115	22	236	9	298	4	66	37	153	16	265	5	318	3	67	33	177	14	285	5	335	<1
S17r	9	102	58	57	206	9	295	<1	30	85	111	36	250	8	322	2	49	72	140	29	274	6	336	2
S18r	11	92	61	46	199	13	277	5	30	77	106	31	236	8	300	2	49	62	134	25	258	7	315	1
S19r	20	62	91	29	222	10	290	6	46	51	133	22	254	7	314	3	65	43	159	18	275	5	329	<1

<sup>a</sup>  $|E^*| \times 10^8$  in  $\text{N/m}^2$ .

<sup>b</sup>  $\text{tg } \varphi \times 10^2$  in  $\text{N/m}^2$ .

<sup>c</sup> Temperature = 25 C.

In view of these 2 conclusions,

$$|E^*| = E_\infty \times |R^*(f_r)| \quad (3)$$

where

$$\begin{aligned} |R^*(f_r)| &= \text{a reduced modulus varying between 0 and 1 and} \\ f_r &= \text{a reduced frequency equal to } \alpha_f \times f. \end{aligned}$$

#### Relation Between $E_\infty$ and Mix Composition

On the basis of the numerical values of  $E_\infty$  as given in Table 4 for the 52 mixes and for 20 other mixes investigated by Van der Poel (7), Bazin and Saunier (8), Harlin and Uge (personal communication), and Huet (6), it was possible to establish that  $E_\infty$  depends only on the composition of the mix and is given by

$$E_\infty(\text{N/m}^2) = 1.436 \times 10^{10} \times R^{0.55} \times \exp(-5.84 \times 10^{-2} \times v) \quad (4)$$

where

$$\begin{aligned} R &= V_A/V_L, \text{ the ratio between the aggregates percentage volume } (V_A) \text{ and the bitumen} \\ &\text{percentage volume } (V_L) \text{ (range of variation for the 72 mixes: 0.12 to 12) and} \\ v &= \text{void contents (range of variation for the 72 mixes: 1.5 to 32 percent).} \end{aligned}$$

Figure 1 shows the comparison between the observed values of  $E_\infty$  and the values calculated on the basis of Eq. 4.

#### Relation Between $|R^*(f_r)|$ and Bitumen Characteristics

The analysis of the numerical values of  $|R^*(f_r)| = |E^*|/E_\infty$  gives rise to the conclusion that  $|R^*(f_r)|$  is dependent only on the bitumen characteristics Pen, B, or  $\alpha$ .

Fitting a satisfactory formula on the basis of the empirical correlation to predict  $R^*$  from  $f_r$  could only be achieved piecemeal. The best formulas derived for 3 ranges of values of  $f_r$  are shown in Figure 2. A complete derivation of these formulas is presented in a detailed research report (9).

#### Predicting $|E^*|$

On the basis of the results presented by the Federal Highway Administration (1), Huet (3), and Verstraeten (4) and those in Figure 2,  $|E^*|$ , for different temperatures and frequencies, can be predicted solely by knowledge of the volumetric composition of the mix ( $V_A$ ,  $V_L$ , and  $v$ ) and the characteristics of the bitumen (Pen, B, or  $\alpha$ ). A computer program has been written to facilitate use of the formulas. Figure 3 shows the result of the comparison between the measured values and the calculated values of  $|E^*|$  for mix P17.

Knowledge of  $|E^*|$  is sufficient to apply pavement design methods based on the theory of elasticity (multilayer theory). For methods based on the theory of viscoelasticity, knowledge of the storage moduli ( $E_1$ ) and the loss moduli ( $E_2$ ) would be necessary. A method for predicting these moduli based on Eq. 2 is presented in detail in a research report (9).

### METHOD FOR PREDICTING FATIGUE LAWS

#### Experimental Results

Table 5 identifies fatigue laws for 42 of the mixes that were given in Table 1. The results presented here complement the precise results presented by Verstraeten (5).

For the mixes investigated, the fatigue law in sinusoidal bending may be written

$$\sigma/|E^*| = \epsilon_r(N) = K \times N^{-a} \quad (5)$$

where

Table 4. Numerical values of k, h, σ, and E<sub>∞</sub>.

Mix	k	h	δ	E <sub>∞</sub> × 10 <sup>-8</sup> N/m <sup>2</sup>
P0	0.182	0.480	1.91	303.1
P1r	0.146	0.493	1.74	325.6
P2r	0.186	0.511	1.75	320.2
P3r	0.184	0.675	1.58	351.8
P4r	0.180	0.501	1.69	338.9
P5r	0.193	0.576	1.76	319.3
P6r	0.183	0.665	1.81	309.6
P7	0.182	0.512	1.88	325.5
P8	0.223	0.621	1.95	246.3
P9	0.205	0.598	1.78	294.9
P10	0.200	0.510	1.72	219.2
P11	0.197	0.532	2.11	303.7
P12	0.238	0.666	2.18	264.7
P13	0.167	0.587	1.34	253.8
P14	0.186	0.495	1.59	300.4
P15	0.212	0.633	1.61	334.7
P16	0.185	0.495	2.07	314.1
P17	0.193	0.546	1.88	331.5
P18	0.141	0.446	2.16	290.0
P19	0.205	0.525	2.65	298.1
P20	0.165	0.463	1.54	272.5
P21	0.152	0.419	1.42	269.7
P22	0.161	0.471	1.69	253.9
P23	0.200	0.580	1.60	298.7
P24	0.148	0.480	1.31	275.6
P25	0.225	0.545	2.41	261.4
P26	0.184	0.477	1.57	267.6
P27	0.151	0.467	1.38	277.8
P28	0.203	0.536	2.23	303.4
L1	0.178	0.485	1.84	252.3
L2	0.215	0.516	2.55	250.0
L3	0.208	0.612	1.37	258.1
G1	0.177	0.513	1.55	284.0
S1	0.103	0.681	1.78	335.6
S2	0.191	0.669	1.52	275.5
S3	0.180	0.475	1.50	294.4
S4	0.154	0.524	1.42	273.6
S5	0.193	0.497	2.86	251.6
S6	0.116	0.519	1.36	309.2
S7	0.178	0.528	1.81	247.1
S8	0.145	0.440	1.43	261.0
S9	0.157	0.461	1.72	262.2
S10	0.133	0.410	1.62	259.7
S11	0.179	0.538	1.52	289.9
S12r	0.164	0.525	1.65	322.6
S13r	0.107	0.415	1.46	365.5
S14r	0.159	0.487	1.46	342.9
S15r	0.144	0.414	1.78	343.0
S16r	0.130	0.383	1.39	358.6
S17r	0.172	0.582	1.35	336.5
S18r	0.195	0.571	1.93	325.0
S19r	0.147	0.444	1.62	350.1

Figure 1. Comparison of observed and calculated values of E<sub>∞</sub>.

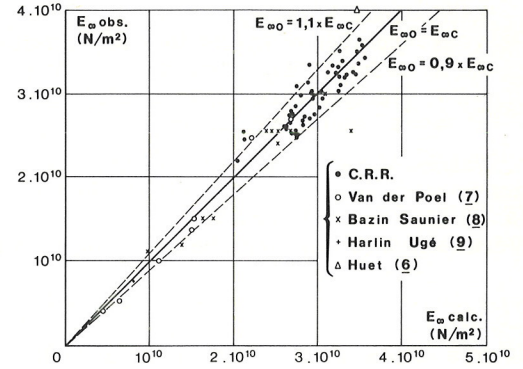


Figure 2. Formulas for predicting |R\*(f<sub>R</sub>)|.

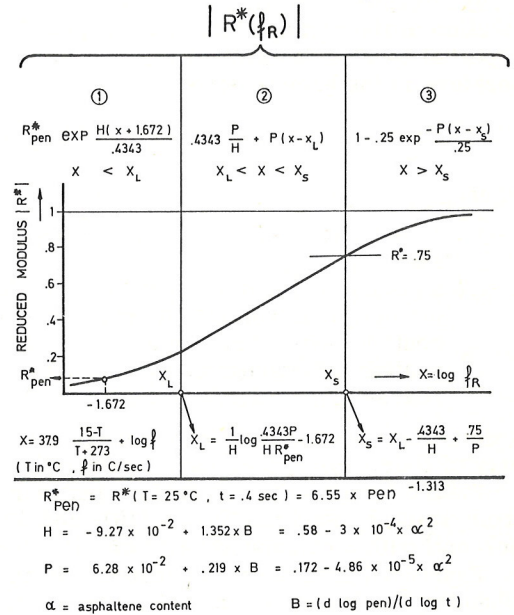
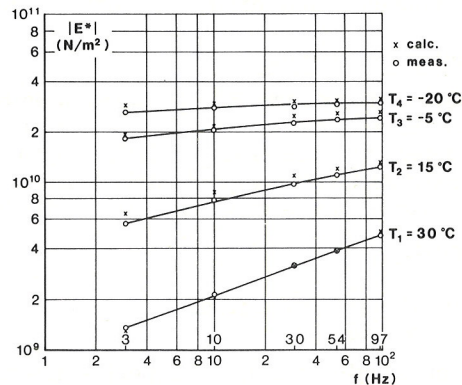


Figure 3. Comparison of observed and calculated values of |E\*|.



$\sigma$  = assigned stress,  
 $|E^*|$  = initial modulus,  
 $\epsilon_r(N)$  = initial strain,  
 $N$  = number of cycles to produce failure,  
 $K$  = a factor depending on the mix considered but practically independent of the temperature and frequency, and  
 $a$  = an empirical factor.

Values of  $\log K$  and  $a$  for the 42 mixes are also given in Table 5. For the mixes considered, the  $a$  slopes of the straight lines [ $\log \epsilon_r(N)$  and  $\log N$ ] are nearly the same ( $\bar{a} = 0.21$ ; standard deviation = 0.02). On the basis of this, the fatigue law presented by Verstraeten (5) may thus be replaced by the approximation

$$\epsilon_r(N) = K' \times N^{-0.21} \quad (6)$$

$K'$  was chosen so that Eqs. 5 and 6 would yield the same  $\epsilon_r$  value for  $N = 10^6$ .

#### Predicting Fatigue Laws

The results given in Table 5 suggest that the factor  $K'$  in Eq. 6 depends on the composition of the mix and bitumen. Explicitly, the fatigue laws of bituminous mixes in repeated bending (controlled tests) may be written independently of frequency and temperature in the general form

$$\epsilon_r(N) = \bar{\Lambda} \times G \times [V_L/(V_L + v)] \times (N/10^6)^{-0.21} \quad (7)$$

where

$\bar{\Lambda}$  = a coefficient given by Figure 4 depending on the asphaltene content of the bitumen and

$G$  = an empirical factor equal to unity when the following conditions are fulfilled:

- (a) aggregate volume content is between 78 percent and 85 percent;
- (b) bitumen content, without having an excessive value is such that the aggregates are fully coated;
- (c) aggregates are made up of fine aggregates and at least 50 percent coarse aggregates; and
- (d)  $0.5 \times 10^{-4} \leq \epsilon_r(N = 10^6) \leq 1.3 \times 10^{-4}$ .

This is the case for most mixes used in road construction; therefore, the case  $G \neq 1$  is only of academic interest and is not further discussed.

Figure 5 shows the comparison between the experimental values of  $\epsilon_r(N)$  derived from Table 5, Pell and Taylor (10), and Taylor (11) for 40-50 bitumen mixes ( $\alpha \geq 20$  percent) with a calculated value of  $\epsilon_r(N)$  derived from Eq. 7 for  $N = 10^5$ ,  $10^6$ , and  $10^7$ .

Equation 7 thus permits prediction of the fatigue law in bending of a bituminous mix when the  $V_A$ ,  $V_L$ ,  $v$ , and  $\alpha$  in the bitumen are known. The influence of factors such as grading, nature and shape of aggregates, and mode of compaction on the critical strain  $\epsilon_r(N)$  is taken into consideration through the factor  $V_L/(V_L + v)$ .

#### Criteria for Selecting the Bitumen

The factor  $\bar{\Lambda}$  of the fatigue law—Eq. 7—can also be related to other characteristics of the bitumen besides  $\alpha$ , for example,  $T_{AB}$ ,  $\eta_o$  at 25 C, and  $\eta_o$  at 25 C/ $\eta_\infty$  at 25 C. Consideration of the values of these characteristics given in Table 2 and of the coefficient  $\bar{\Lambda}$  given in Table 5 shows that a high and fairly constant value of  $\bar{\Lambda}$  generally is obtained. Therefore, the following may be taken as criteria for the selection of the bitumens:

1.  $\alpha \geq 18$  percent,
2.  $T_b \geq 48$  C,
3.  $\eta_o$  at 25 C  $\geq 1.3 \times 10^5$  Ns/m<sup>2</sup>, and
4.  $\eta_o$  at 25 C/ $\eta_\infty \geq 1.75$ .

Furthermore, observations made on experimental wearing courses (3) give rise to the following criteria, which complement the preceding ones:

Figure 4. Variation of  $\Lambda$  by  $\alpha$ .

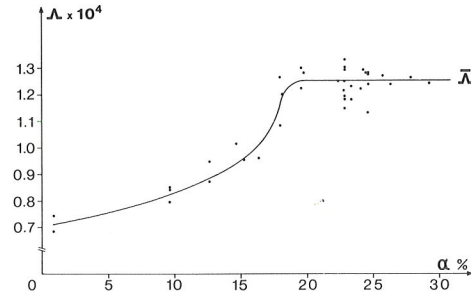


Figure 5. Comparison of measured and calculated values of  $\epsilon_r(N)$ .

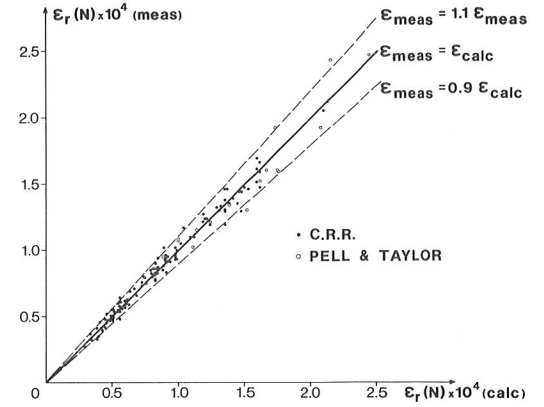


Table 5. Fatigue laws.

Mix	log K	a	$\epsilon_r(N = 10^6)$ $\times 10^4$	$\Lambda$ $\times 10^4$	$\bar{\Lambda}$ $\times 10^4$	$\frac{\Lambda}{\bar{\Lambda}}$	G
P1r	2.800	0.20	1.00	1.263	1.250	1.010	1.0
P2r	2.680	0.22	1.00	1.269	1.250	1.015	1.0
P3r	2.957	0.19	0.80	1.015	0.935	1.086	1.0
P4r	2.693	0.22	0.97	1.235	1.250	0.988	1.0
P5r	2.822	0.20	0.95	1.214	1.250	0.971	1.0
P6r	2.925	0.20	0.75	0.953	0.960	0.993	1.0
P7	2.892	0.19	0.93	1.292	1.250	1.034	1.0
P8	2.640	0.27	0.55	0.876	0.880	0.996	1.4
P9	2.781	0.23	0.69	0.945	0.880	1.074	1.0
P10	2.729	0.25	0.59	1.082	1.160	0.933	1.4
P11	2.671	0.22	1.02	1.298	1.250	1.038	1.0
P12	2.888	0.24	0.47	0.686	0.710	0.966	1.0
P13	2.917	0.22	0.58	0.850	0.825	1.030	1.0
P14	2.882	0.19	0.95	1.293	1.250	1.034	1.0
P15	2.832	0.20	0.93	1.177	1.250	0.942	1.0
P16	2.887	0.19	0.94	1.294	1.250	1.035	1.0
P17	2.772	0.21	0.93	1.193	1.250	0.954	1.0
P18	2.626	0.21	1.30	1.235	1.250	0.988	1.5
P19	2.641	0.24	0.83	1.146	1.250	0.917	1.0
P20	2.871	0.20	0.85	1.191	1.250	0.953	1.0
P21	2.974	0.19	0.77	1.129	1.250	0.903	1.0
P22	2.697	0.24	0.73	1.250	1.250	1.000	1.0
P23	3.141	0.15	0.91	1.240	1.250	0.992	1.0
P24	2.919	0.20	0.76	1.299	1.250	1.039	1.0
P25	3.009	0.19	0.71	1.330	1.250	1.064	1.0
P26	2.926	0.19	0.86	1.334	1.250	1.067	1.0
P27	2.957	0.19	0.80	1.336	1.250	1.069	1.0
G1	2.786	0.21	0.90	1.264	1.160	1.090	1.0
S1	2.828	0.24	0.54	0.742	0.710	1.045	1.0
S2	2.992	0.21	0.56	0.795	0.825	0.964	1.0
S3	2.806	0.21	0.86	1.237	1.250	0.990	1.0
S4	2.941	0.19	0.83	1.251	1.250	1.000	1.0
S5	2.712	0.23	0.81	1.221	1.250	0.977	1.0
S6	2.860	0.22	0.66	0.960	1.010	0.951	1.0
S7	2.866	0.20	0.86	1.280	1.250	1.024	1.0
S8	2.788	0.22	0.78	1.179	1.250	0.943	1.0
S9	2.736	0.22	0.88	1.231	1.250	0.985	1.0
S10	2.739	0.19	1.32	1.281	1.250	1.025	1.5
L1	2.920	0.19	0.87	1.275	1.250	1.020	1.0
L2	2.821	0.21	0.83	1.223	1.250	0.978	1.0
L3	2.888	0.23	0.54	0.842	0.825	1.021	1.0
PV1	2.686	0.23	0.86	1.283	1.250	1.026	1.0



1.  $\alpha \leq 27$  percent and
2. Pen  $> 40$ .

These 2 criteria aim at limiting the unfavorable effects of aging and effects caused by an increase of the rigidity of the mixes and excessive brittleness.

### CONCLUSIONS

The research results presented in this paper give rise to several conclusions.

1. The  $|E^*|$  moduli =  $E_\infty \times R^*(f_R)$  of a bituminous mix can be predicted, for different temperatures and frequencies, on the basis of knowledge of the volumetric composition of the mix, which influences the value of the modulus of elasticity, and characteristics of the bitumen (B or  $\alpha$  and Pen), which influence the value of the reduced modulus. On the basis of the formulas presented for this prediction and to facilitate their use, a computer program giving the components  $|E^*|$  and  $\phi$  of the complex modulus for any temperature or frequency has been written.
2. The fatigue law in sinusoidal bending (controlled stress tests) of a bituminous mix can be predicted on the basis of knowledge of the volumetric composition and asphaltene content of the bitumen.
3. From the point of view of fatigue strength, the following criteria for the selection of the bitumen are brought out:  $\alpha$  from 18 to 27 percent,  $T_{AB} > 48$  C,  $\eta_0$  at 25 C  $\geq 1.3 \times 10^5$  N/m<sup>2</sup>,  $\eta_0$  at 25 C /  $\eta_\infty$  at 25 C  $\geq 1.75$ , Pen  $> 40$ .
4. Experiments are now in progress to complement the results obtained in repeated bending (cracking) with results in repeated compression (rutting). For this purpose a special apparatus was developed at the Centre de Recherches Routières (12, 13).

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