ELASTIC LAYER ANALYSIS RELATED TO PERFORMANCE IN FLEXIBLE PAVEMENT DESIGN

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From experience in Ontario with flexible pavements and from results of the AASHO Road Test, it was found that the calculated subgrade deflection under a standard wheel load is the best indicator of performance of the pavement as a whole when it is compared with other stress, strain, and deformation values calculated by elastic layer theory. In the calculations, layer equivalencies obtained from experience and variations in subgrades were expressed in terms of elastic moduli. Subgrade deflections can be calculated more simply by using Odemark's concept of equivalent layer thickness. Expressions for load equivalency factors were derived from AASHO Road Test data by using this simplified deflection calculation. Finally, a functional relationship between subgrade deflection, number of standard load applications, and present serviceability index was established. The findings constitute major parts of a design subsystem to be used within a management system for flexible pavements.

• THROUGH a process of continual pavement evaluation, pavement design engineers in Ontario were able to compile a table of successful thickness designs (1). The table recognizes differences in thickness caused by the traffic, road class, and type of subgrade. Elastic layer analysis was used to examine the table to find a more rational method of flexible pavement design. It was hoped that a possible clue to the success of the conventional designs listed in the table might be found.

The method of investigation was to assign values of elastic moduli to each pavement layer and subgrade class and to calculate stresses, strains, and deflections in each layer for a standard wheel load. The elastic moduli assigned to each pavement layer and to each class of subsoil were selected after a study of available literature. The calculated stresses, strains, and deflections were examined for a constant value of these parameters within each traffic or highway class. A constant value within the same road class over the six major subgrade types identified within Ontario could indicate a common distress mechanism and would provide a practical criterion for design.

In this process of calculation, in which the Chevron computer program was used, many different sets of moduli were assigned to the pavement layers and subgrades. Through this procedure, it was discovered that only the vertical deflection on top of the subgrade emerged as the response value, which could be made to remain constant within each traffic or road class. Several sets of assumed moduli were successful in this respect. Subgrade deflections were also calculated by a simplified method that uses the principle of equivalent layer thickness as proposed by Odemark (3).

The course of investigation was then directed to the best documented experiment available.

AASHO ROAD TEST

Two sets of moduli, which had been applied successfully to the Ontario designs,

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were assigned to the layers of the main factorial designs of the AASHO Road Test (5, 6), and the subgrade deflections were calculated for both the applied single-axle load in each loop and the standard 18-kip (80 kN) axle load. A statistical analysis of these calculated deflections not only resulted in a formula for load equivalency factors but culminated in finding a relationship between the loss of performance or serviceability and the number of equivalent standard load applications for given values of subgrade deflection.

By using sets of elastic moduli for calculating subgrade deflections, we demonstrated that this deflection is linked to standards of performance or serviceability. The design subsystem of this research is shown in Figure 1. An equation was derived for determining the necessary total equivalent granular thickness so that the design method could be completed.

EXPLORING SUCCESSFUL ONTARIO DESIGNS

Ontario's successful designs, which have survived an average of about 11.5 years, are given in Tables 1 and 2. The lines in the table pertain to traffic or road classes indicated by approximate average daily traffic values. The columns of the table pertain to the types of subgrade soils as they are classified in Ontario.

For each of the calculations, basically two sets of moduli were assumed and subsequently varied and modified into different sets with which calculations were continued. These two sets were the subgrade moduli E_s, which constitute a decreasing sequence from hard subgrades (granular) to soft subgrades (soft clay) and the layer moduli E_1, E_2, and E_3 for asphaltic hot mix, granular base, and sand subbase. Cases 1, 2, 3, and 4 of these calculations were finally assembled, which may be thought of as being based on true or realistic relations between the assumed moduli. The moduli E_1, E_2, and E_3 of these four cases are related to the layer equivalencies, valid in Ontario, as follows:

$$\text{hot mix : base : subbase} = 1 : 2 : 3 = \frac{1}{\sqrt[3]{E_1}} : \frac{1}{\sqrt[3]{E_2}} : \frac{1}{\sqrt[3]{E_3}} \quad (1)$$

The relationship between the set of layer moduli and the set of subgrade moduli is different in all four cases, and this indicates insensitivity about this relationship.

For a wheel load of 9,000 lb (40 kN) and a pressure area radius of 6.4 in. (16.3 cm), all stresses, strains, and deflections at the layer interfaces were calculated. The most important of these are shown in Figure 2 and their values for cases 3 and 4 are given in Tables 3 and 4. In all four cases assembled, only the deflections on top of the subgrade were approximately equal for each of the five traffic or road classes. This indicates that this deflection could be a powerful design criterion.

SUBGRADE DEFLECTION AS DESIGN CRITERION

The calculations on the successful Ontario designs revealed that the most promising design parameter for flexible pavements was the vertical deflection on top of the subgrade. This hypothesis is in line with previous research findings (2) in which the vertical compressive strain on the subgrade was declared the dominating design parameter. These findings were based on the AASHO Road Test, which was carried out on the same subsoil. For constant subgrade modulus the two criteria are indeed equivalent, but the strain criterion obviously breaks down if a wide range of subgrades is considered. The same is true for the corresponding stress.

Tensile stress or strain in the asphaltic layer must be considered, although it is probably a secondary design criterion. For instance, the thickness of the asphaltic layers, as a portion of the total equivalent thickness, could possibly be determined by the magnitude of tensile strain under repeated loads (fatigue) and under varying temperature conditions, whereas the total thickness is still determined by the subgrade deflection.

If only subgrade deflections are needed, then it is more economical to calculate them by the method suggested by Odemark (3, 4). The deviations of the following design equations based on subgrade deflections can be studied in more detail in the

Figure 1. Pavement design subsystem.

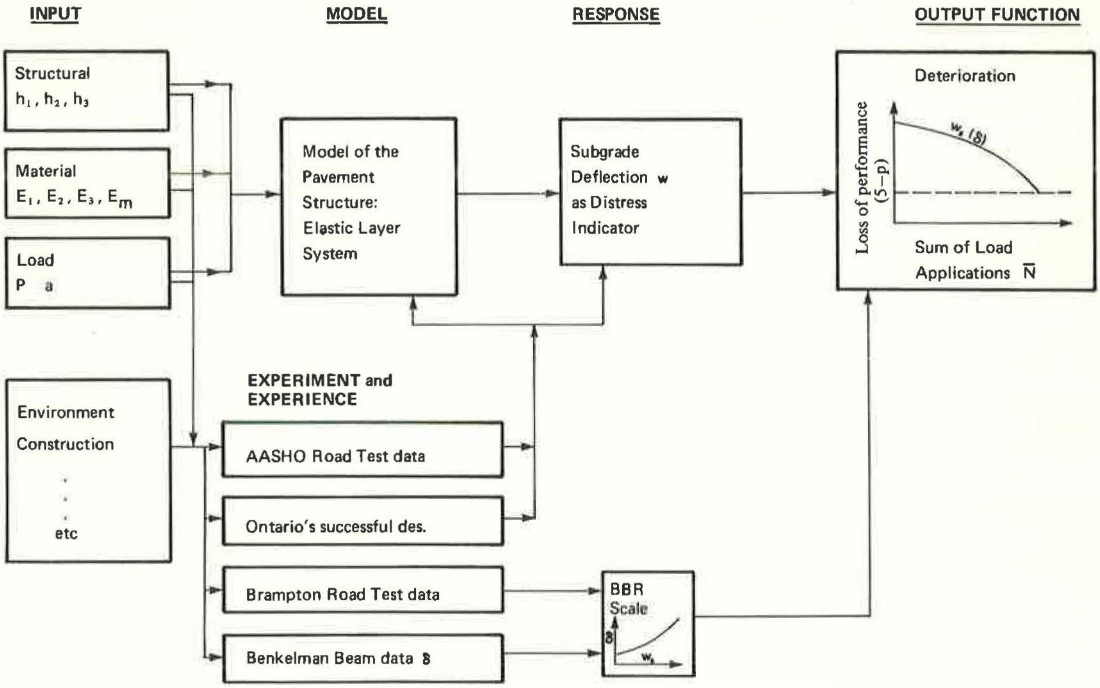


Table 1. Moduli of successful Ontario designs.

		Subgrade Material, psi					
		Grain Type of Materials Suitable as Granular Borrow	Sandy Silt and Clay, Loam Till			Clay	
Case	Modulus		Silt <40, Very Fine Sand and Silt <45	Silt 40 to 50, Very Fine Sand and Silt 45 to 60	Silt >50, Very Fine Sand and Silt >60	Hard Lacustrine	Soft Varved and Leda
1	E_1	400,000	400,000	400,000	400,000	400,000	400,000
	E_2	50,000	50,000	50,000	50,000	50,000	50,000
	E_3	–	15,000	15,000	15,000	15,000	15,000
	E_s	15,000	8,500	7,000	5,700	7,500	3,800
2	E_1	320,000	320,000	320,000	320,000	320,000	320 000
	E_2	40,000	40,000	40,000	40,000	40,000	40,000
	E_3	–	12,000	12,000	12,000	12,000	12,000
	E_s	15,000	8,400	6,900	5,700	7,600	3,900
3	E_1	400,000	400,000	400,000	400,000	400,000	400,000
	E_2	50,000	50,000	50,000	50,000	50,000	50,000
	E_3	–	15,000	15,000	15,000	15,000	15,000
	E_s	11,000	6,000	5,000	4,000	5,300	2,700
4	E_1	600,000	600,000	600,000	600,000	600,000	600,000
	E_2	75,000	75,000	75,000	75,000	75,000	75,000
	E_3	–	22,000	22,000	22,000	22,000	22,000
	E_s	11,000	6,000	5,000	4,000	5,300	2,700

Note: 1 psi = 6.8948 kPa.

Table 2. Average subgrade deflections of successful Ontario designs.

Class and Road	Thickness	Subgrade Material Thickness, in.: Grain Type of Materials Suitable as Granular Borrow	Sandy Silt and Clay Loam Till: Silt <40, Very Fine Sand and Silt <45	Silt 40 to 50, Very Fine Sand and Silt 45 to 60	Silt >50, Very Fine Sand and Silt >60	Clay: Hard Lacustrine	Soft Varved and Leda	Average Deflection Values, in.[b]			
King's highways											
Multilane	h_1	5.5	5.5	5.5	5.5	5.5	5.5	0.0128	0.0136	0.0163	0.0144
	h_2	7.5, 6.5[a]	6	6	6	6	6				
	h_3	—	15	21, 20[a]	27	18	42	0.0119	0.0129	0.0152	0.0133
Two lanes,	h_1	4.5	4.5	4.5	4.5	4.5	4.5	0.0136	0.0145	0.0172	0.0151
AADT	h_2	7.5	6	6	6	6	6				
>2,000	h_3	—	15	21, 20[a]	27	18	42	0.0128	0.0138	0.0162	0.0142
Two lanes,	h_1	3.5	3.5	3.5	3.5	3.5	3.5	0.0167	0.0178	0.0212	0.0186
AADT	h_2	6	6	6	6	6	6				
<2,000	h_3	—	12, 11[a]	15	21	12	30	0.0159	0.0170	0.0206	0.0177
Secondary roads,	h_1	1.5	1.5	1.5	1.5	1.5	1.5	0.0202	0.0217	0.0259	0.0227
AADT >1,000	h_2	6, 6.5[a]	6	6	6	6	6				
	h_3	—	9	12	21, 18[a]	12	30, 27[a]	0.0200	0.0200	0.0260	0.0230
Township roads,	h_1	1.5	1.5	1.5	1.5	1.5	1.5	0.0268	0.0286	0.0347	0.0297
AADT >200	h_2	4	6	6	6	6	6				
	h_3	—	4	6	9	6	18	0.0260	0.0280	0.0338	0.0298

Note: 1 in. = 2.54 cm.

[a]Modified thicknesses only used for cases 3 and 4. [b]Upper values for each entry set are for Chevron; the bottom for Odemark.

Table 3. Calculated criteria for case 3.

Class of Road	Type of Criterion or Distress Indicator	Subgrade Material: Grain Type of Materials Suitable as Granular Borrow[a]	Sandy Silt and Clay Loam Till: Silt <40, Very Fine Sand and Silt <45[b]	Silt 40 to 50, Very Fine Sand and Silt >45[c]	Silt >50, Very Fine Sand and Silt >60[d]	Clay: Hard[e]	Soft[f]	Average Deflection Values (in.)
King's highways								
Multilane	Subgrade deflection, Chevron, in.	0.0157	0.0159	0.0163	0.0168	0.0162	0.0171	0.0163
	Subgrade deflection, Odemark, in.	0.0152	0.0151	0.0158	0.0153	0.0153	0.0155	0.0152
	Total deflection, Chevron, in.	0.0186	0.0236	0.0249	0.0265	0.0246	0.0284	0.0244
	Total deflection, Odemark, in.	0.0168	0.0191	0.0194	0.0198	0.0194	0.0199	0.0191
Two lanes, >2,000 AADT	Subgrade deflection, Chevron, in.	0.0167	0.0170	0.0177	0.0176	0.0172	0.0179	0.0172
	Subgrade deflection, Odemark, in.	0.0161	0.0162	0.0172	0.0162	0.0164	0.0162	0.0162
	Total deflection, Chevron, in.	0.0203	0.0262	0.0274	0.0289	0.0271	0.0308	0.0268
	Total deflection, Odemark, in.	0.0182	0.0211	0.0213	0.0214	0.0213	0.0214	0.0208
Two lanes, <2,000 AADT	Subgrade deflection, Chevron, in.	0.0207	0.0208	0.0206	0.0207	0.0216	0.0228	0.0212
	Subgrade deflection, Odemark, in.	0.0200	0.0199	0.0199	0.0198	0.0210	0.0211	0.0203
	Total deflection, Chevron, in.	0.0245	0.0306	0.0319	0.0334	0.0321	0.0370	0.0316
	Total deflection, Odemark, in.	0.0223	0.0251	0.0255	0.0256	0.0261	0.0267	0.0252
Secondary road, paved >1,000 AADT	Subgrade deflection, Chevron, in.	0.0262	0.0263	0.0264	0.0250	0.0254	0.0263	0.0259
	Subgrade deflection, Odemark, in.	0.0263	0.0266	0.0267	0.0253	0.0257	0.0257	0.0260
	Total deflection, Chevron, in.	0.0316	0.0402	0.0418	0.0426	0.0408	0.0460	0.0405
	Total deflection, Odemark, in.	0.0305	0.0354	0.0360	0.0353	0.0352	0.0357	0.0346
Township road, paved >200 AADT	Subgrade deflection, Chevron, in.	0.0334	0.0357	0.0340	0.0344	0.0327	0.0326	0.0347
	Subgrade deflection, Odemark, in.	0.0334	0.0337	0.0346	0.0351	0.0332	0.0332	0.0338
	Total deflection, Chevron, in.	0.0376	0.0487	0.0463	0.0488	0.0449	0.0505	0.0461
	Total deflection, Odemark, in.	0.0362	0.0400	0.0415	0.0427	0.0403	0.0416	0.0404
King's highways								
Multilane	Vertical subgrade stress, psi	-6.88	-2.09	-1.51	-1.03	-1.70	-0.532	—
	Vertical subgrade strain, in.	-0.000505	-0.000314	-0.000269	-0.000223	-0.000287	-0.000167	—
	Radial asphalt stress, psi	142.0	142.0	140.0	139.0	141.0	137.0	—
	Radial asphalt strain, in.	0.000204	0.000204	0.000202	0.000200	0.000203	0.000198	—
Two lanes, >2,000 AADT	Vertical subgrade strain, psi	-7.82	-2.44	-1.72	-1.14	-1.96	-5.69	—
	Vertical subgrade strain, in.	-0.000583	-0.000370	-0.000311	-0.000251	-0.000334	-0.000179	—
	Radial asphalt stress, psi	153.0	159.0	157.0	156.0	158.0	154.0	—
	Radial asphalt strain, in.	0.000227	0.000233	0.000231	0.000230	0.000232	0.000227	—
Two lanes, <2,000 AADT	Vertical subgrade stress, psi	-11.7	-3.72	-2.55	-1.64	-3.14	-8.98	—
	Vertical subgrade strain, in.	-0.000849	-0.000535	-0.000467	-0.000369	-0.000543	-0.000285	—
	Radial asphalt stress, psi	179.0	173.0	171.0	169.0	172.0	167.0	—
	Radial asphalt strain, in.	0.000269	0.000262	0.000260	0.000258	0.000262	0.000256	—
Secondary road, paved >1,000 AADT	Vertical subgrade stress, psi	-18.9	-6.03	-4.30	-2.51	-4.45	-1.22	—
	Vertical subgrade strain, in.	-0.000138	-0.000925	-0.000793	-0.000574	-0.000776	-0.000395	—
	Radial asphalt stress, psi	82.0	74.9	73.1	73.6	74.0	74.4	—
	Radial asphalt strain, in.	0.000186	0.000176	0.000174	0.000175	0.000175	0.000176	—
Township road, paved >200 AADT	Vertical subgrade stress, psi	-28.3	-10.6	-6.99	-4.73	-7.24	-1.95	—
	Vertical subgrade strain, in.	-0.001870	-0.001570	-0.001270	-0.001080	-0.001240	-0.000645	—
	Radial asphalt stress, psi	135.0	117.0	72.3	68.7	73.2	69.4	—
	Radial asphalt strain, in.	0.000264	0.000223	0.000172	0.000168	0.000174	0.000170	—

Note: Modulus of (a) hot mix asphalt E_1 = 400,000; (b) the base E_2 = 50,000; and (c) the subbase E_3 = 15,000. 1 in. = 2.54 cm. 1 psi = 6.8948 kPa.

[a]E_m = 11,000. [b]E_m = 6,000. [c]E_m = 5,000. [d]E_m = 4,000. [e]E_m = 5,300. [f]E_m = 2,700.

Figure 2. Diagram of multilayer structure.

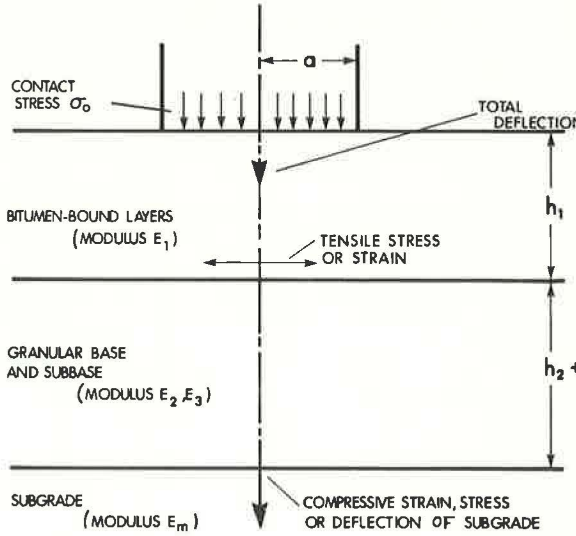


Table 4. Calculated criteria for case 4.

		Subgrade Material						
		Grain Type of Materials Suitable as Granular Borrow[a]	Sandy Silt and Clay Loam Till			Clay		Averag Deflect Values (in.)
Class of Road	Type of Criterion or Distress Indicator		Silt <40, Very Fine Sand and Silt <45[b]	Silt 40 to 50, Very Fine Sand and Silt >45[c]	Silt >50, Very Fine Sand and Silt >60[d]	Hard[e]	Soft[f]	
King's highways								
Multilane	Subgrade deflection, Chevron, in.	0.0138	0.0141	0.0145	0.0150	0.0144	0.0146	0.0144
	Subgrade deflection, Odemark, in.	0.0134	0.0132	0.0133	0.0134	0.0134	0.0136	0.0134
	Total deflection, Chevron, in.	0.0159	0.0195	0.0205	0.0217	0.0202	0.0225	0.0200
	Total deflection, Odemark, in.	0.0143	0.0154	0.0157	0.0159	0.0157	0.0161	0.0155
Two lanes, >2,000 AADT	Subgrade deflection, Chevron, in.	0.0146	0.0150	0.0153	0.0157	0.0153	0.0152	0.0151
	Subgrade deflection, Odemark, in.	0.0142	0.0143	0.0143	0.0142	0.0144	0.0142	0.0143
	Total deflection, Chevron, in.	0.0172	0.0214	0.0223	0.0235	0.0221	0.0243	0.0218
	Total deflection, Odemark, in.	0.0153	0.0170	0.0171	0.0172	0.0171	0.0172	0.0168
Two lanes, <2,000 AADT	Subgrade deflection, Chevron, in.	0.0182	0.0181	0.0181	0.0184	0.0189	0.0201	0.0186
	Subgrade deflection, Odemark, in.	0.0177	0.0175	0.0175	0.0174	0.0184	0.0185	0.0178
	Total deflection, Chevron, in.	0.0210	0.0250	0.0260	0.0271	0.0262	0.0299	0.0259
	Total deflection, Odemark, in.	0.0190	0.0204	0.0206	0.0206	0.0213	0.0217	0.0206
Secondary road, paved >1,000 AADT	Subgrade deflection, Chevron, in.	0.0231	0.0230	0.0230	0.0220	0.0221	0.0233	0.0227
	Subgrade deflection, Odemark, in.	0.0235	0.0235	0.0235	0.0223	0.0226	0.0226	0.0230
	Total deflection, Chevron, in.	0.0270	0.0327	0.0337	0.0341	0.0328	0.0368	0.0328
	Total deflection, Odemark, in.	0.0261	0.0286	0.0289	0.0280	0.0281	0.0283	0.0280
Township road, paved >200 AADT	Subgrade deflection, Chevron, in.	0.0249	0.0314	0.0297	0.0299	0.0286	0.0288	0.0297
	Subgrade deflection, Odemark, in.	0.0302	0.0298	0.0305	0.0309	0.0293	0.0290	0.0299
	Total deflection, Chevron, in.	0.0332	0.0408	0.0384	0.0400	0.0372	0.0411	0.0384
	Total deflection, Odemark, in.	0.0320	0.0335	0.0345	0.0353	0.0334	0.0339	0.0338
King's highways								
Multilane	Vertical subgrade stress, psi	-5.43	-1.64	-1.20	-0.824	-1.34	-0.426	–
	Vertical subgrade strain, in.	-0.000388	-0.000242	-0.000207	-0.00172	-0.000220	-0.000130	–
	Radial asphalt stress, psi	151.0	144.0	142.0	140.0	143.0	137.0	–
	Radial asphalt strain, in.	0.000142	0.000137	0.000136	0.000134	0.000136	0.000132	–
Two lanes, >2,000 AADT	Vertical subgrade stress, psi	-6.17	-1.91	-1.36	-0.910	-1.54	-0.455	–
	Vertical subgrade strain, in.	-0.000447	-0.000286	-0.000238	-0.000193	-0.000257	-0.000139	–
	Radial asphalt stress, psi	161.0	161.0	158.0	156.0	159.0	153.0	–
	Radial asphalt strain, in.	0.000157	0.000157	0.000155	0.000153	0.000156	0.000151	–
Two lanes, <2,000	Vertical subgrade stress, psi	-9.29	-2.92	-1.99	-1.29	-2.54	-0.715	–
	Vertical subgrade strain, in.	-0.00656	-0.000412	-0.000358	-0.000280	-0.000419	-0.000217	–
	Radial asphalt stress, psi	190.0	175.0	172.0	169.0	174.0	167.0	–
	Radial asphalt strain, in.	0.000188	0.000176	0.000174	0.000172	0.000176	0.000170	–
Secondary road, paved >1,000 AADT	Vertical subgrade stress, psi	-15.10	-4.75	-3.36	-1.95	-3.49	-0.966	–
	Vertical subgrade strain, in.	-0.001070	-0.000722	-0.000611	-0.000435	-0.000600	-0.00298	–
	Radial asphalt stress, psi	80.6	69.1	67.9	69.6	68.7	71.1	–
	Radial asphalt strain, in.	0.000122	0.000112	0.000112	0.000113	0.000112	0.000114	–
Township road, paved >200 AADT	Vertical subgrade stress, psi	-23.30	-8.44	-5.48	-3.67	-5.68	-1.51	–
	Vertical subgrade strain, in.	-0.000490	-0.001240	-0.000984	-0.000822	-0.000965	-0.000480	–
	Radial asphalt stress, psi	150.0	114.0	66.2	62.9	67.2	65.5	–
	Radial asphalt strain, in.	0.000173	0.000146	0.000110	0.000107	0.000110	0.000110	–

Note: Modulus of (a) the hot-mix asphalt E_1 = 600,000; (b) the base E_2 = 75,000; and (c) the subbase $E_3 \approx$ 22,000. 1 in. = 2.54 cm. 1 psi = 6.8948 kPa.

[a]E_m = 11,000. [b]E_m = 6,000. [c]E_m = 5,000. [d]E_m = 4,000. [e]E_m = 5,300. [f]E_m = 2,700.

Appendix. The variable measurements may be either U.S. customary or metric units.

$$w = \frac{P}{2E_m z} \times \frac{1}{\sqrt{1 + \frac{a}{z}}} \tag{2}$$

where

$$z = 0.9 \times \sum_{i=1}^{m-1} h_i \sqrt[3]{\frac{E_i}{E_m}} \tag{3}$$

and where

- w = subgrade deflection in inches;
- m = number of layers including subgrade;
- h_i = thickness of layer i in inches;
- E_i = modulus of layer i in psi;
- E_m = subgrade modulus in psi;
- a = radius of loaded area in inches; and
- P = wheel load in lb.

The deflections w calculated in Eqs. 1 and 2 differ slightly from the subgrade deflections calculated with the Chevron program (Tables 2, 3, and 4). The correlation coefficient r between the two calculated deflections, however, was found to be close enough to unity (r = 0.993 to 0.997) so that the much simpler method of calculation by Eqs. 2 and 3 is justified. The correlation between the two deflections of case 3 is shown in Figure 3.

SUBGRADE DEFLECTIONS OF AASHO ROAD TEST SECTIONS

Subgrade deflections w have been calculated for all designs given in Tables 1 and 2 and, for the moduli of cases 1 through 4, these deflections were approximately equal for each highway traffic class. In these calculations the applied load was constant, but the subgrade material E_m was one of the main variables.

In contrast to this, the main factorial design sections of the AASHO Road Test were built on a uniform subgrade material (soft clay), but were exposed to a variety of axle loads (5). By using Eqs. 2 and 3, subgrade deflections w were calculated for these AASHO Road Test designs. The wheel loads P of the single-axle weights in each loop were assumed to be uniformly distributed over a circle of radius a according to recorded tire pressures (6).

Based on a scale (7, fig. 28) and a soil support value of S = 3, the modulus of the subgrade was assumed to be E_m = 3,000 psi (20.7 MPa). The moduli of the pavement layers were assumed to be the same as in the calculations on the Ontario designs (Tables 3 and 4).

The number of weighted, i.e., seasonably adjusted, load applications N for a terminal present serviceability index (PSI) p = 2.5 and the corresponding values of $N_{1.5}$ for p = 1.5 are given elsewhere (5, table 8; 5, table 6 respectively). Correlation regression analyses were performed on all four sets of data ($N_{2.5}$ and $N_{1.5}$, cases 3 and 4) for loops 3, 4, 5, and 6 separately, and the results are given in Tables 4 and 5.

If separate plots for each loop in each case are made and if each regression equation in the tables is drawn and modified, the regression analyses could be harmonized into the following expression based on a constant rounded average value of six for the slopes (exponent of w).

$$N = \frac{1}{w^6 \times 10^{K-0.09P}} \tag{4}$$

Figure 3. Correlation of subgrade deflections.

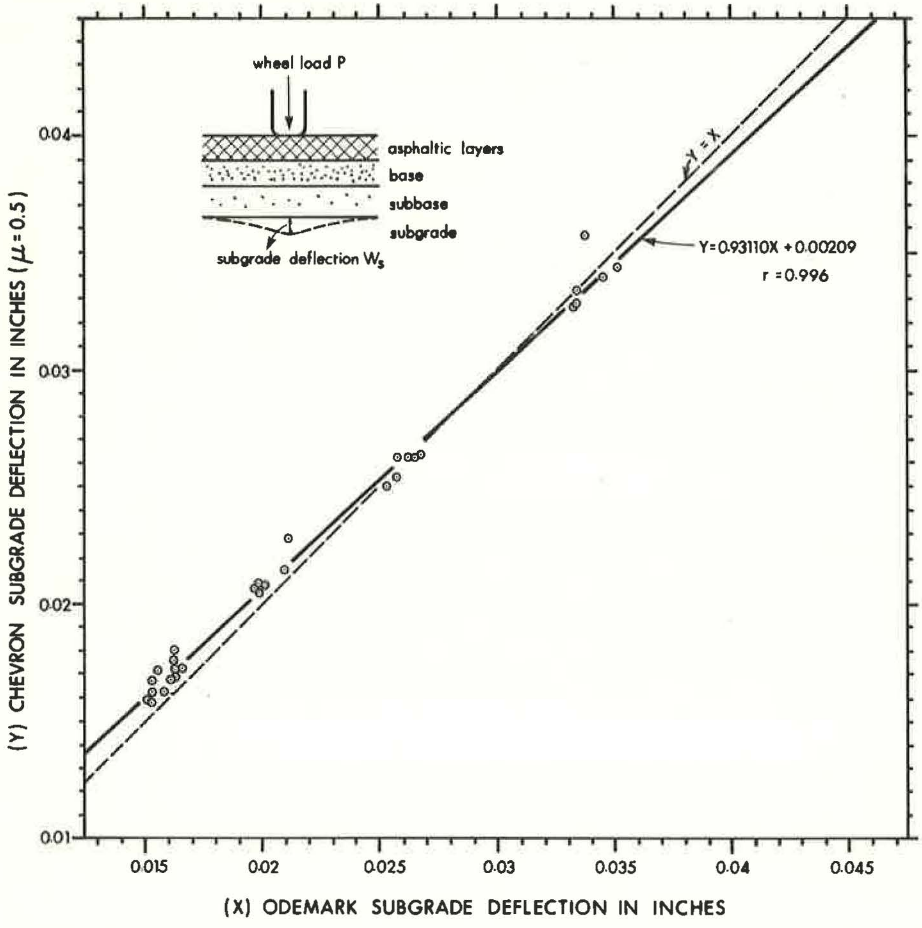


Table 5. Correlation regression equations for AASHO Road Test results (p = 2.5).

Loop Number	Axle Load (kips)	Equations for Case 3	Equations for Case 4	Sample Size
3	12	log N = -4.567 log w - 1.529	log N = -4.520 log w - 1.715	27
4	18	log N = -5.843 log w - 2.892	log N = -5.795 log w - 3.151	28
5	22.4	log N = -5.745 log w - 2.729	log N = -5.672 log w - 2.959	27
6	30	log N = -6.156 log w - 2.857	log N = -6.118 log w - 3.152	27
Suggested predicting equation[a] for $\overline{N}$ = e N		log $\overline{N}$ = -6 log w_s - 3.22	log $\overline{N}$ = -6 log w_s - 3.56	109

Note: Measurement of w and w_s is in inches. 1 in. = 2.54 cm. 1 kip = 4.448 222 N.

[a]Standard error of prediction of log $\overline{N}$ = 0.26; standard error in slope = 0.19. Standard error in Y-intercept = 0.58; correlation coefficient = -0.95.

where the following are values for the constant K

Values	Case 3	Case 4
For p = 2.5	4.03	4.37
For p = 1.5	3.94	4.28
Difference, $K_{2.5} - K_{1.5}$	0.09	0.09

and the wheel load P is to be measured in 1,000-lb (4.45 kN) units.

LOAD EQUIVALENCY FACTOR

Equation 4 was established for a wide range of wheel loads P. The number of load applications N and the subgrade deflections w pertain to wheel load P. Equation 4 is also valid for the standard wheel load P_s, which is 9,000 lb (40 kN) ($P_s = 9$) or for any other value within the range of the loads being investigated. If a load of $P_s = 9$ is applied on any design section, the calculated subgrade deflection will be w_s, and, with these two values, Eq. 4 will predict the number of equivalent standard axle load applications N_s. From these considerations, the load equivalency factor $e = N_s/N$ can be derived and was found to be

$$e = \left(\frac{w}{w_s}\right)^6 \times 10^{-0.09\,(P - P_s)} \tag{5}$$

The following equation is presented for large values of z and for a constant radius of tire pressure area $a = a_s$ = constant (which is the same for P and P_s):

$$e = \left(\frac{P}{P_s}\right)^6 \times 10^{-0.09\,(P - P_s)} \tag{6}$$

(If P and P_s are metric, then the -0.09 coefficient changes accordingly.) Equation 6 has been plotted in Figure 4 for $P_s = 9$ [9,000 lb (40 kN)] together with equivalency factors derived by Shook and Chastain (8, 9). If Eq. 6 is true, it follows that the destructive effects of heavy axle loads $P > P_s$ have usually been overestimated.

PREDICTION OF EQUIVALENT STANDARD AXLE LOAD APPLICATIONS

The weighted axle load applications $N_{2.5}$ and $N_{1.5}$ (5, tables 6 and 8) were converted into numbers of equivalent standard 18-kip loads ($\bar{N}_{2.5} = e \times N_{2.5}$ and $\bar{N}_{1.5} = e \times N_{1.5}$) by using equivalency factors e calculated by Eq. 5.

$\bar{N}_{2.5}$ and $\bar{N}_{1.5}$ were then correlated with all the calculated deflections of loops 3, 4, 5, and 6 for cases 3 and 4. The results of these correlation regression analyses, each based on over 100 pairs of values $w_s - \bar{N}$, are as follows:

1. For case 3, p = 2.5: $\log \bar{N}_{2.5} = -5.93 \log w_s - 3.12$;
2. For case 3, p = 1.5: $\log \bar{N}_{1.5} = -5.94 \log w_s - 3.06$;
3. For case 4, p = 2.5: $\log \bar{N}_{2.5} = -5.90 \log w_s - 3.41$; and
4. For case 4, p = 1.5: $\log \bar{N}_{1.5} = -5.92 \log w_s - 3.35$.

In all four cases correlation coefficients $r \approx -0.95$, errors of prediction ≈ 0.26, 95 percent confidence limits of the slopes are approximately 5.5 to 6.3, and average standard error of the slope ≈ 0.19. The errors of prediction (≈ 0.26) compare favorably with the root-mean-square residual of the AASHO Road Test data, which is 0.31.

These correlation regression equations were then harmonized as before based on a constant slope of 6. The same equations were obtained as from Eq. 4 for P = 9 kips (40 kN). They are given in log form on the bottom of Tables 5 and 6. Plots of the points and the regression lines for cases 3 and 4 and for p = 2.5 are shown in Figures 5 and 6.

Thus, the subgrade deflection principle or model has been successfully applied to the AASHO Road Test data even with gross assumptions for the elastic moduli and layer

Figure 4. Load equivalency factor versus wheel load.

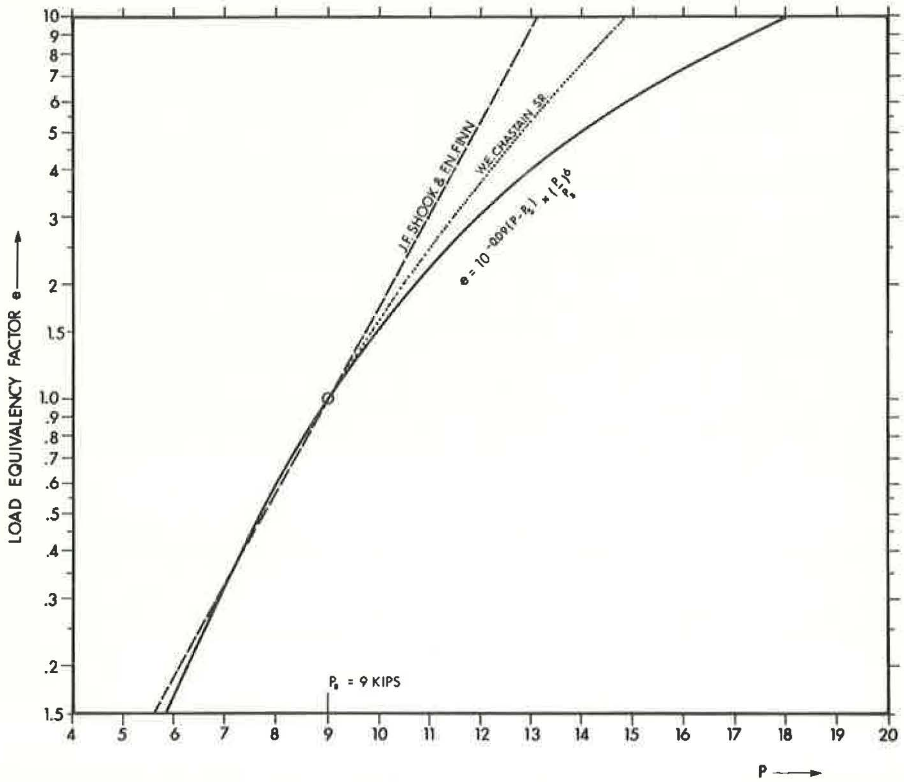


Table 6. Correlation regression equations for AASHO Road Test results (p = 1.5).

Loop Number	Axle Load (kips)	Equations for Case 3	Equations for Case 4	Sample Size
3	12	log N = -4.358 log w - 1.174	log N = -4.214 log w - 1.212	25
4	18	log N = -5.838 log w - 2.805	log N = -5.785 log w - 3.056	25
5	22.4	log N = -5.766 log w - 2.647	log N = -5.652 log w - 2.823	25
6	30	log N = -5.891 log w - 2.414	log N = -5.849 log w - 2.689	22
Suggested predicting equation[a] for $\overline{N}$ = e N		log $\overline{N}_{18}$ = -6 log w_s - 3.13	log $\overline{N}$ = -6 log w_s - 3.47	97

Note: Measurement of w and w_s is in inches. 1 in. = 2.54 cm. 1 kip = 4.448 222 N.

[a]Standard error of prediction of log $\overline{N}$ = 0.26; standard error in slope = 0.20. Standard error in Y-intercept = 0.59; correlation coefficient = -0.95.

Figure 5. Verification of predicting Eq. 4 for case 3.

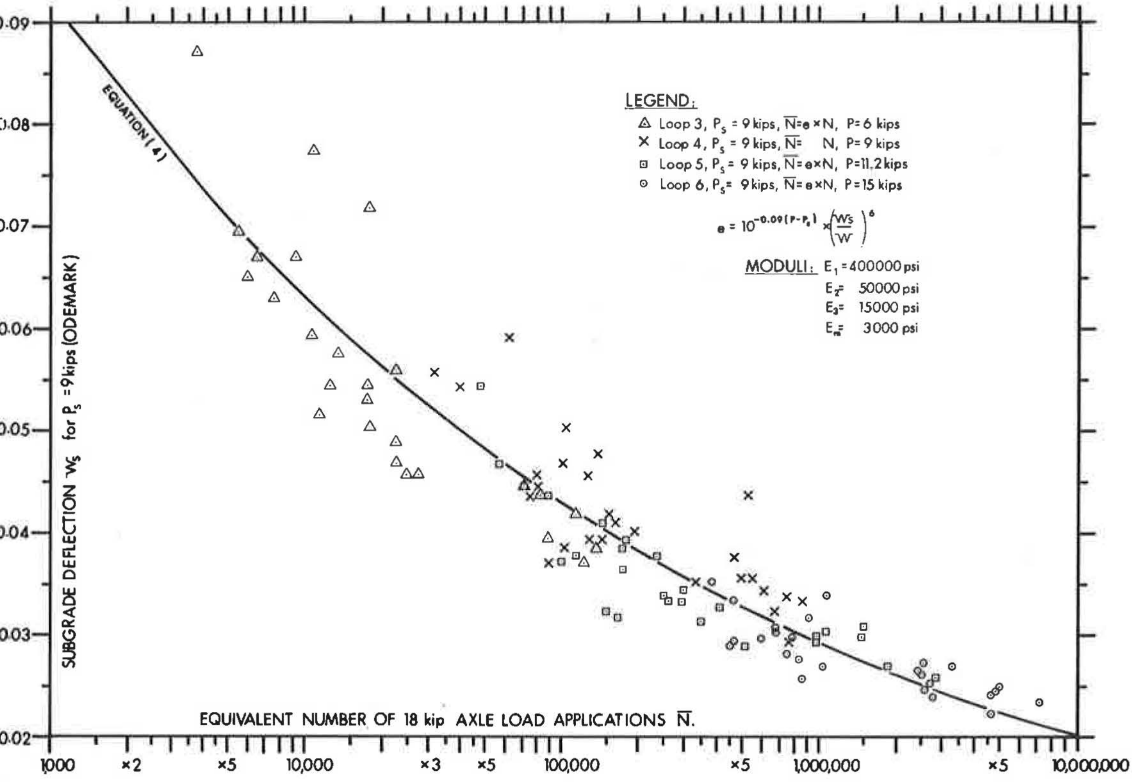
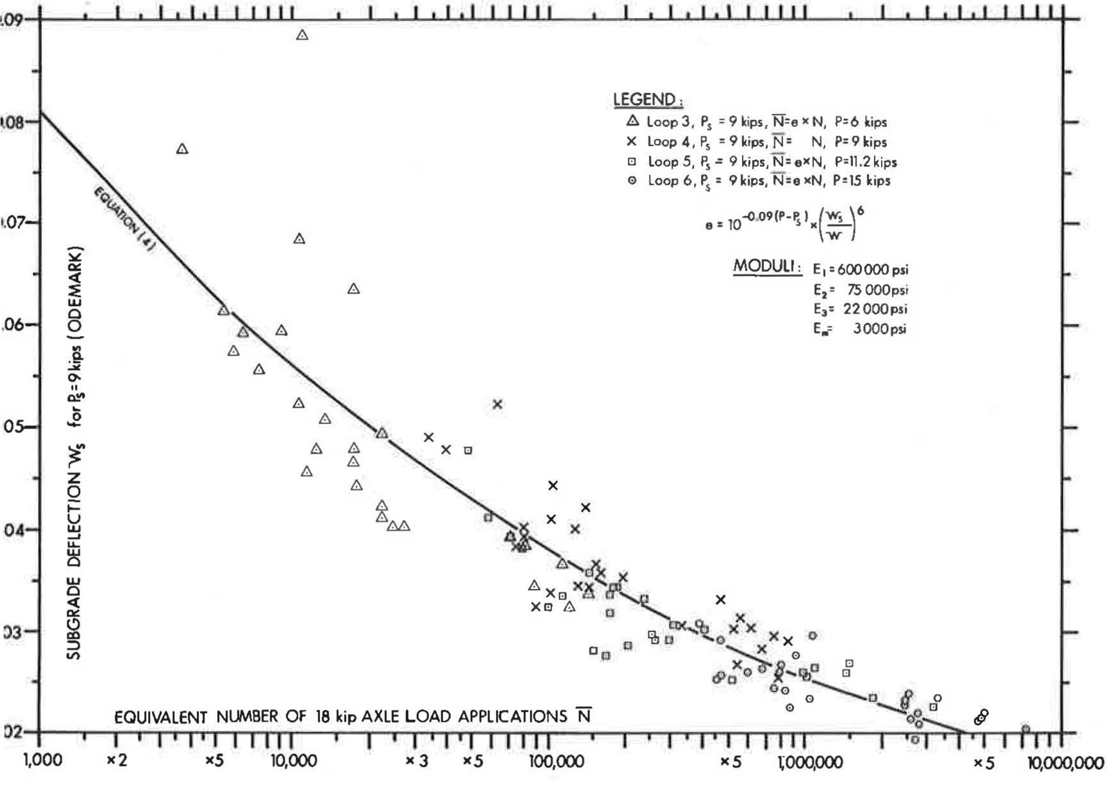


Figure 6. Verification of predicting Eq. 4 for case 4.



equivalencies. Equations 4, 5, and 6 and the regression equations were derived concurrently for both cases 3 and 4 with concordant results. This shows that the subgrade deflection model is not sensitive about the relation between subgrade and pavement layer moduli. From here on, investigations are restricted to case 3 as an example only.

LOSS OF SERVICEABILITY

The number of equivalent 18-kip axle load applications $\bar{N}$ for the two terminal levels of serviceability p = 2.5 and 1.5 (PSI) can be calculated by Eq. 4 by setting P = 9 kips (40 kN). This substitution leads to two expressions that have been combined into one performance equation relating $\bar{N}$ to the subgrade deflection w_s and to the loss in performance. With Eq. 4, and by using the K-values of case 3, by setting P_s = 9 kips (18-kip axle) [40 kN (80 kN)], and by assuming an initial value of p_0 = 4.2 (5), one can derive the following equation by connecting the three points p_0 = 4.2, p_1 = 2.5, and p_2 = 1.5 by a cubic parabola:

$$p = 4.200 - (1.22275\,\psi + 4.4024\,\psi^3) \tag{7}$$

where

$$\psi = 1000 \times w_s^6 \times \bar{N} \quad \text{for } w_s \text{ in inches} \tag{8}$$

or

$$\psi = 3.7238\, w_s^6 \times \bar{N} \quad \text{for } w_s \text{ in cm} \tag{9}$$

and where

- w_s = deflection on top of the subgrade as a design parameter for the standard wheel load P_s = 9 kips (40 kN),
- p = PSI, and
- $\bar{N}_p$ = number of equivalent 18-kip (80 kN) axle weight applications.

The last term of Eq. 7 can be interpreted as the loss in PSI because of traffic loading.

$$p_L = 1.2228\,\psi + 4.402\,\psi^3 \tag{10}$$

In this form, the predicting equation could eventually be used more universally, for instance for other initial values p_0 and in other environments by including another loss term to account for additional losses from environmental forces, a concept which at present is being applied to the results of the Brampton Road Test (10, 11, 12). Figure 7 shows the losses p_L as a function of $\bar{N}$ and w_s.

REQUIRED EQUIVALENT GRANULAR THICKNESS

Equation 2 can be solved explicitly for z, and the resulting equation, with Eq. 3, can be multiplied by

$$\sqrt[3]{E_s/E_{2g}} \tag{11}$$

where E_{2g} is the modulus for granular A base material. In this way, a design equation may be derived:

$$H_e = \frac{1}{0.9} \times \sqrt{\left(\frac{P_s}{2E_s w_s}\right)^2 - a^2} \times \sqrt[3]{\frac{E_s}{E_{2g}}} \tag{12}$$

where H_e is the required granular thickness for the particular design in terms of granular A material. This thickness requirement H_e is the sum of all layer thicknesses multiplied by layer equivalency coefficients.

$$H_e = c_1h_1 + c_2h_2 + c_3h_3 + \ldots \tag{13}$$

Figure 7. Loss of performance or serviceability because of traffic loading (mean values).

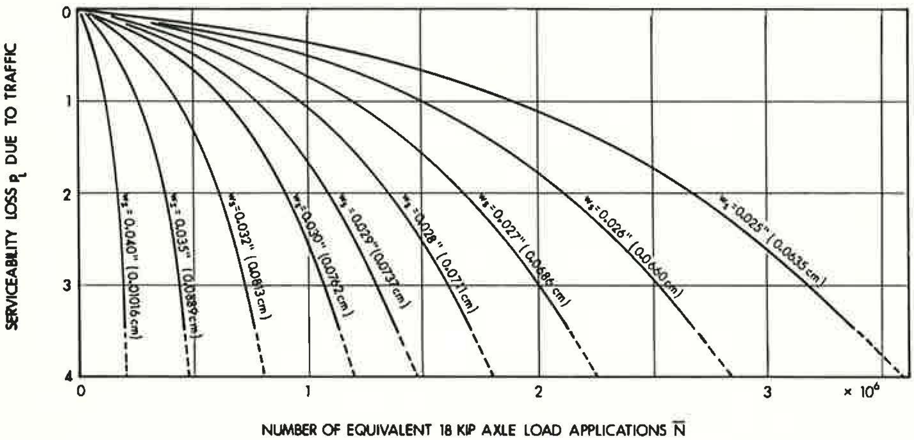
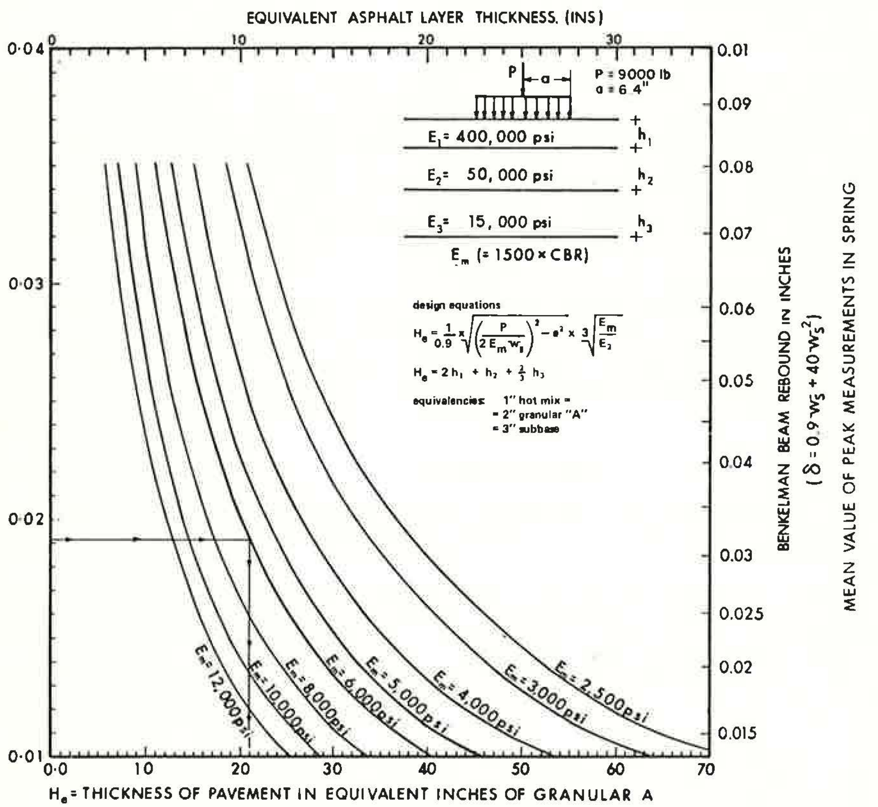


Figure 8. Design chart for flexible pavements in Ontario.



TYPICAL SUBGRADE MODULI IN ONTARIO

GRAN. TYPE MATERIALS SUITABLE AS GRAN. BORROW	SANDY SILT AND CLAY LOAM TILL			LACUSTRINE CLAYS	VARVED AND LEDA CLAYS
	SILT <40 V.F. Sa and Si. <45	SILT 40 - 50 V.F. Sa. and Si. 45-60	SILT >50 V.F. Sa. and Si. >60		
psi	psi	psi	psi	psi	psi
11,000	5,000 TO 7,000	4,000 TO 6,000	3,000 TO 5,000	3,500 TO 6,000	2,000 TO 4,500

These coefficients express the effect of each layer in resisting load P_s to generate a vertical deflection w_s on the subgrade, which is the design parameter. Therefore, they are (as in Eq. 3) related to the pavement layer moduli as follows:

$$c_1 = \sqrt[3]{\frac{E_1}{E_{2_s}}} \quad c_2 = \sqrt[3]{\frac{E_2}{E_{2_s}}} \quad c_3 = \sqrt[3]{\frac{E_3}{E_{3_s}}} \tag{14}$$

In this paper, coefficients were based on experience gained in Ontario, especially from the Brampton Road Test results (10): $c_1 = 2$, $c_2 = 1$, and $c_3 = 2/3$. They determine the relation E_1:E_2:E_3 of the pavement layer moduli (Eq. 1) within the subgrade deflection concept (Eqs. 2 and 3). In other words, the pavement layer moduli were based on layer equivalencies determined from experience. This is justified if the performance is linked to the subgrade deflections w calculated by Eqs. 2 and 3. [The similarity of design Eqs. 12 and 13 with the Kansas formula (13, 14) is recognized.]

A design chart for determining the required total thickness in terms of H_e was drawn with Eq. 12 and is shown in Figure 8. The following example may show how to use the chart. The assigned Odemark subgrade deflection is $w_s = 0.019$ (to be taken from a suitable performance diagram similar to Fig. 7). The subgrade is a clay loam till with 30 percent silt and with very fine sand and silt of about 40 percent; therefore, select $E_s = 6{,}000$ psi (41.4 MPa) from the table in Figure 8. The required granular A thickness from the same figure is $H_e = 21$ in. (53 cm).

CONCLUSIONS

A practicable system of flexible pavement design, which is a subsystem of the whole pavement management system, can be based on simple concepts of linear elastic theory. An elastic layer system can serve as a structural design pavement model. The subgrade deflection for this model was found to be the most relevant distress indicator for the loss of performance of the pavement as a whole. The link between the response of this model, in terms of vertical deflections on the subgrade, and the output function, in terms of loss of performance, was established by considering past experience with successful Ontario designs and the AASHO Road Test.

The material characterizations and load applications of the input variables of this model, although not definitely established, were demonstrated and exemplified. Thus, experiences in Ontario were mainly used to establish realistic relations between layer and subgrade moduli, and AASHO Road Test data were used to exemplify the necessary range of loads.

REFERENCES

1. Phang, W. A., and Slocum, R. Pavement Decision Making and Management System. Ministry of Transportation and Communications, Ontario, Rept. RR174, Oct. 1971.
2. Dormon, G. M., and Edwards, J. M. Shell 1963 Design Charts for Flexible Pavements, An Outline of Their Development. Shell International Petroleum Co. Ltd., London, O.P.D. Rept. 232/64M, April 1964.
3. Odemark, N. Investigations as to the Elastic Properties and Soils and Design of Pavements According to the Theory of Elasticity. Statens Vaeginstitut, Stockholm, 1949.
4. Shook, J. F., and Finn, F. N. Thickness Design Relationships for Asphalt Pavements. Proc., International Conference on the Structural Design of Flexible Pavements, Univ. of Michigan, Ann Arbor, Aug. 1962, p. 52.
5. The AASHO Road Test, Report 5: Pavement Research. HRB Special Rept. 61 E, 1962.
6. The AASHO Road Test, Report 6: Special Studies. HRB Special Rept. 61 F, 1962.
7. Evaluation of AASHO Interim Guides for Design of Pavement Structures. NCHRP Rept. 128, 1972.
8. Schnitter, G., and Jentasch, R. J. Designing Flexible Road Pavements. Proc., International Conference on the Structural Design of Flexible Pavements, Univ. of Michigan, Ann Arbor, Aug. 1962, p. 537.

9. Secor, K. E., and Monismith, C. L. Viscoelastic Properties of Asphalt Cement. HRB Proc., Vol. 41, 1962, pp. 299-320.
10. Kamel, N. I., Morris, J., Haas, R. C. G., and Phang, W. A. Layer Analysis of the Brampton Test Road and Application to Pavement Design. Highway Research Record 466, 1973, pp. 113-126.
11. Phang, W. A. Four Years' Experience at the Brampton Test Road. Ministry of Transportation and Communications, Ontario, Research Rept. RR153, Oct. 1969.
12. Phang, W. A. The Effect of Seasonal Strength Variation on the Performance of Selected Base Materials. Ministry of Transportation and Communications, Ontario, Research Rept. IR39, April 1971.
13. De Barros, S. T. A Critical Review of Present Knowledge of the Problem of Rational Thickness of Design of Flexible Pavements. Highway Research Record 71, 1965, pp. 105-128.
14. Yoder, E. J. Principles of Pavement Design. John Wiley and Sons, New York, 1959.
15. Szechy, K. Der Grundbau, Vol. 1. Springer-Verlag, Vienna, 1963, p. 249.

APPENDIX

DESIGN FORMULA BASED ON SUBGRADE DEFLECTIONS

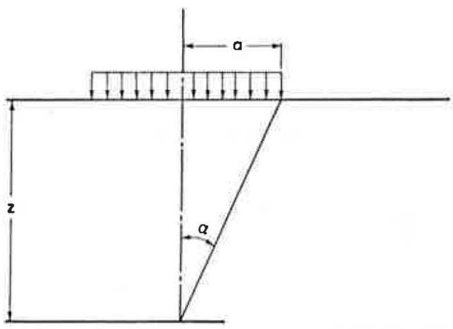
A design formula based on subgrade deflections can be derived by using various existing concepts such as the solution of an elastic stress analysis for the isotropic half space and the equivalent layer thickness suggested by Odemark (3, 8).

Newmark (15) gives a formula for the vertical deflection in the center of a wheel load that is equally distributed over a circular contact area at depth z of a uniform elastic half space.

$$w_s = (1 + \mu) \times \frac{\sigma_o a}{E} \times \left[\sin \alpha + (1 - 2\mu) \frac{1 - \cos \alpha}{\sin \alpha}\right] \quad (15)$$

where

w_s = vertical deflection at the top of the subgrade;
μ = Poisson's ratio;
σ_o = tire pressure, uniformly distributed over a circular area;
a = radius of the loaded circular area;
α = angle as indicated in the figures; and
$\alpha = \arctan \frac{a}{z}$.



Equation 15 is rewritten so that an important simplification can be achieved:

$$w_s = K \times \frac{\sigma_o a}{E} \times \sin \alpha \tag{16}$$

where

$$K = (1 + \mu) \left[(1 - 2\mu) \times \frac{1 - \cos \alpha}{\sin^2 \alpha}\right] \tag{17}$$

For $\mu = 0.25$ to 0.50 and for $\alpha = 0$ to 40 degrees the coefficient varies only slightly from $K = 1.5$ to $K = 1.6$, and a constant value can be selected. In particular, the coefficient K increases slightly by decreasing Poisson's ratio ($\mu < 0.5$) and by increasing α. A fixed value of $K = 1.5708 = \frac{\pi}{2} > 1.5$ is suggested.

For a Poisson's ratio of $\mu = 0.5$, Eq. 15 is simply

$$w_s = 1.5 \times \frac{\sigma_o a}{E} \times \sin \alpha \tag{18}$$

This is a well-known equation (2, 3, 4). By referring to Eq. 16, the following substitution can be made

$$\sin \alpha = \frac{\tan \alpha}{\sqrt{1 + \tan^2 \alpha}}, \quad \tan \alpha = \frac{a}{z}, \quad \text{and } P = \pi a^2 \sigma_o$$

$$w_s = \frac{KP}{\pi E z} \times \frac{1}{\sqrt{1 + \left(\frac{a}{z}\right)^2}} \tag{19}$$

Solving for z,

$$z = \sqrt{\left(\frac{KP}{\pi E w_s}\right)^2 - a^2} \tag{20}$$

where P = design wheel load $= \pi a^2 \sigma_o$, and $\frac{K}{\pi} = \frac{1}{2}$.

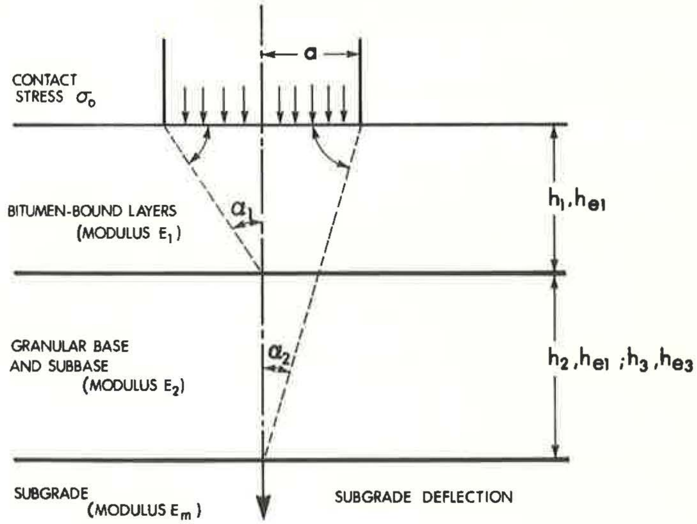


Figure 9. Diagram of elastic layered system.

According to Odemark (3), an elastic layered system as shown in Figure 9 can be transformed into a uniform elastic half space by introducing an equivalent layer thickness h_{ei}.

$$h_{ei} = n h_i \times \sqrt[3]{\frac{E_i}{E_m}} \tag{21}$$

where

E_i = modulus of layer i,
E_m = modulus of subgrade = reference modulus,
h_i = thickness of layer i,
h_{ei} = equivalent thickness of layer i, and
n = reduction factor, for flexible pavements = 0.9.

For flexible pavements, Odemark (3) has suggested a value of n = 0.9. This was verified by numerous comparative calculations.

The depth z can be expressed by Eq. 21 as

$$z = \sum_{i=1}^{m-1} h_{ei} = n \sum_{i=1}^{m-1} h_i \sqrt[3]{\frac{E_i}{E_m}} \tag{22}$$