ANALYTIC EQUILIBRIUM MODEL FOR DIAL-A-RIDE DESIGN

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Dial-a-ride is a demand-responsive transportation system in the experimental stages of development. Previous analyses of the system have been dominated by relatively expensive, supply-oriented simulation models and crude, insensitive demand predictions. This paper presents an analytic equilibrium model that has minimal data and computational requirements and is suitable for use in designing future dial-a-ride systems. The model is used to test the sensitivity of level of service and net operating cost to changes in demand model parameters and fares. The results demonstrate the important effects of decisions such as fleet size, service area, and fare levels on the economic and noneconomic prospects of a potential dial-a-ride system. In dial-a-ride as in many other transportation systems, the interrelations between design parameters and demand response are so complex that only an equilibrium model can predict the impacts of a specific design.

•BY THE END OF 1973, about 20 demand-responsive urban bus systems were operating in North America (1, 2). These systems are designed to provide high-quality service at a premium fare. Dial-a-ride systems have been implemented in widely dissimilar locations, ranging from small independent cities (Batavia, New York) to commuter suburbs (Haddonfield, New Jersey; Bay Ridges, Ontario) to sectors of large cities (Regina, Saskatchewan; Ann Arbor, Michigan; and Rochester, New York) to new communities (Columbia, Maryland). As awareness of the potential of this new system increases, many other localities will likely consider the implementation of demand-responsive services. In this type of planning environment it is important that modeling tools be available to help answer questions such as, What service area is best? What are the implications of a given fare on ridership, profit, and service? How many vehicles should be operated to provide a desired quality of service?

To date, the most frequently used analysis tool for aiding in the design of dial-a-ride systems is the detailed computer simulation model (3). Although simulation can be very effective, it suffers from 2 major deficiencies in this application. First, it is generally an expensive tool, requiring extensive software development and involving large amounts of computational resources in the application. More important, however, is the fact that dial-a-ride simulation models have been supply oriented. In these models demand must be exogenously determined; traditionally it has not been considered an explicit function of the quality and cost of service provided by dial-a-ride or competing modes. These models may be accurately described as defining a supply surface rather than determining an actual operating point.

This paper describes an analytic model that builds from the existing models to overcome their weaknesses so that it is suitable for assisting in the design of future dial-a-ride systems. This model uses an equilibrium framework in which dial-a-ride ridership is assumed to be a function of the average fare, wait time, and in-vehicle time of the dial-a-ride system and a function of automobile travel time. The model has minimal data and computational requirements and can therefore be used to test a broad range of policy options at extremely low cost. Since the model is discussed elsewhere in detail (4, 5), this paper summarizes the model system and presents some test results.

MODEL SYSTEM

The model system consists of 3 basic components: a supply model, a demand model, and a net cost model. The supply model determines the quality of service that can be provided in an area by a specified vehicle fleet at a given level of ridership. The demand model predicts the level of ridership that will result from a given quality of service and fare level. The net cost model determines the financial implications of the service. The supply and demand models are solved simultaneously and yield the equilibrium level of ridership and quality of service. By using the models repeatedly, one can determine the implications of selecting different numbers of vehicles, fares, or service areas. In this analysis, fare, number of vehicles, and service area are key policy variables. To use the models, the planner must specify the following inputs:

1. Average vehicle speed,
2. Average trip length,
3. Total number of minutes per day during which dial-a-ride operates,
4. Factor input prices such as labor wage rates and vehicle capital and operating costs,
5. Size of the service area,
6. Total number of vehicle trips made in the service area during the time the dial-a-ride system operates,
7. Time needed for a passenger to exit a vehicle, and
8. Time needed for a passenger to board a vehicle.

From these parameters, the model determines daily dial-a-ride ridership, revenues, costs, average travel time, and average wait time.

Supply Model

The supply model is formulated to predict average wait time and average travel time for a given system. The aim is to develop good structural relations that can then be calibrated with simulation model results. The model should be accurate over the reasonable operating range of dial-a-ride, but because of the objective of minimal computational requirements the full complexity of dial-a-ride operating decisions cannot be included. The travel time model is derived by treating each vehicle as a queue. The act of picking up a passenger corresponds to the arrival of a user at the end of the queue, and the act of dropping off a passenger is analogous to the user's being served and his leaving the queue.

The rate of arrivals per vehicle per minute is defined by λ, which is determined in the demand model. The rate at which passengers are serviced, μ, depends on the vehicle speed, the distance between drop-offs, and the time required to actually pick up and drop off a passenger.

The wait-time submodel was based on a simple assumption about the dispatching algorithm: The vehicle is routed to move toward a waiting passenger's origin as directly as it moves toward an in-vehicle passenger's destination. From the travel time submodel, the mean velocity toward any point, V_{EFF}, can be estimated as the ratio of the average trip length to the average travel time. Given the average distance between the vehicle that is assigned to the new demand and the demand origin L_W, the expected wait time is simply L_W/V_{EFF}.

This 2-component supply model was calibrated by the adaptation of a detailed simulation model and the testing of 27 hypothetical systems. These test results were then used to develop an expression for the mean vehicle interstop distance and L_W and to select the most appropriate queuing model form.

The interstop distance was modeled as a linear function of the average trip length and the demand arrival rate λ, which together measure the efficiency with which tours can be put together. L_W was modeled as a function of the vehicle density and the demand density in the service area. Both equations yielded reasonable fits for linear forms and had coefficients with the expected signs.

Both the single-server queuing models tested tended to underpredict travel and wait times for highly congested systems. However, the range of demand rates and vehicle densities over which the model was valid was quite well defined. All the results reported in this paper are within the range of model validity.

In general, the M/M/1 model, which assumes a Poisson process for the server, resulted in predictions that better matched the simulated data and so it was used in the supply model.

Demand Model

At present, there are few comprehensive data on the demand for dial-a-ride service. For this reason, a relatively simple incremental demand model form was selected (6).

Total daily travel within the service area is assumed to be fixed, and the dial-a-ride modal split is determined as a function of fare, wait time, and the ratio of dial-a-ride in-vehicle time to automobile travel time. The model assumes a known base-point modal split denoted as MS^0, which corresponds to a known base fare, wait time, and travel time ratio, denoted as f^0, tw^0, and TTR^0 respectively. The modal split at other fares, wait times, and travel ratios is expressed as follows:

$$MS = MS^0 \left[1 + e_w\left(\frac{tw - tw^0}{tw^0}\right) + e_{TTR}\left(\frac{TTR - TTR^0}{TTR^0}\right) + e_f\left(\frac{f - f^0}{f^0}\right)\right]$$

where e_w is the elasticity of demand for dial-a-ride with respect to wait time, e_{TTR} is the elasticity of demand for dial-a-ride with respect to the travel time ratio, and e_f is the elasticity of demand for dial-a-ride.

Simply stated, this model predicts changes in modal split from the base point as the weighted sum of 3 effects: the fraction deviation of wait time from the base point, the fraction deviation of the travel time ratio from the base point, and the fraction deviation of fare from the base point. The coefficients for these 3 variables are their respective elasticities.

The base point selected was a 2 percent modal split for a wait time of 15 minutes, a travel time ratio of 2.0, and a fare of $0.60. This is based on the records of the Batavia, New York, system for the early months of operation in the fall of 1971.

The elasticities used have a great deal of uncertainty associated with them. The figures chosen are based on the attitudinal survey of Golob and Gustafson (7). They derived a set of demand curves from these surveys; however, these models gave predicted modal splits that seem far too high when compared with the market shares observed in cities with dial-a-ride service. Rather than use these demand curves directly and seriously overestimate demand, we used only the elasticities implied by their work. These elasticities are rough averages over the range of levels of service and fare considered. The elasticities used are as follows:

$$e_{TTR} = -0.3$$

$$e_w = -0.3$$

$$e_f = -1.1$$

The service elasticities are lower than those often used, and the fare elasticity is quite high. This may reflect the tendency for the elderly, poor, and young to use the system. Such socioeconomic groups are likely to be more fare sensitive and less service sensitive.

The service elasticities for travel time and wait time were roughly equal in Golob and Gustafson's demand curves. This is somewhat unusual in that wait time is generally regarded as being more onerous than is vehicle time (8). However, dial-a-ride wait time is generally spent in the passenger's home rather than at a bus stop or transit station. Furthermore, the arrival of the dial-a-ride vehicle is likely to be quite reliable since the telephone operator at the control center can often give the passenger an expected vehicle arrival time. Because the service elasticities are well below other estimates, such as the -0.593 value found in a model calibrated by Domencich and Kraft, extensive sensitivity analysis was done to determine whether elasticities would greatly affect the predictions made (9).

The fare elasticity, although higher than those generally assumed for public transportation, seems reasonable since the dial-a-ride fare is generally substantially higher than fares for conventional public transit. Golob and Gustafson's survey work indicates that fare elasticity tends to increase with fare. Analysis of the results of a fare increase in the Peoria Premium Special subscription bus service also indicated a fare elasticity near unity (10).

For the dial-a-ride system to be in equilibrium, both the supply and demand relations must be satisfied concurrently. The simultaneous solution of these equations results in a third order polynomial expression in λ, the demand arrival rate. The coefficients of this polynomial are functions of the trip length, vehicle speed, and the coefficients of the equations for the mean interstop distance and L_w.

Net Cost Model

The cost for any given dial-a-ride system was divided into 4 major categories (11):

1. Customer communications, including handling and processing incoming calls;
2. Vehicles, including capital and operating costs and driver wages;
3. Dispatching, including computer rental, space, maintenance, and programming; and
4. Overhead.

Each of these categories was further disaggregated into space, labor by job type, phone rental, and other subcategories. Wage rates and other factor input prices were derived from a number of sources and represent reasonable values for the northeast United States where there is unionized labor.

In the cost analysis, true demand-responsive service operated only during off-peak hours; more efficient subscription bus service operated during peak hours. Thus, a portion of the cost was allocated to these peak-hour activities. The entire model system was developed for a typical weekday of operation. Thus, some fraction of fixed costs was allocated to weekend and holiday dial-a-ride service.

PARAMETRIC TEST CASE

The entire model system was used to test the effects of various dial-a-ride systems and the sensitivity of the model to a range of parameters. The sizes of the 3 hypothetical areas considered were 2 by 2, 2.8 by 2.8, and 3 by 4 miles. The average trip length, fare, fleet size, demand elasticities, and base modal split were all varied. Only increases in the magnitude of the travel time ratio and wait time elasticities were considered because of the unusually low value of these elasticities implied by attitudinal survey research. The following variations were examined for all systems.

1. Trips per day: 16,000, 2 by 2 miles; 32,000, 2.8 by 2.8 miles; and 48,000, 3 by 4 miles.
2. Trip lengths: $\frac{1}{6}(h_1 + h_2)$, $\frac{1}{3}(h_1 + h_2)$, and $\frac{1}{2}(h_1 + h_2)$, where h_1, h_2 are the dimensions of the service area.
3. Fare elasticities: -0.8, -1.1, and -1.3.
4. Base modal splits: 1, 2, and 3 percent.
5. Fares: \$0.25, \$0.50, \$0.75, \$1.00, and \$1.25.
6. Wait time and travel time ratio elasticities: -0.3, -0.3; -0.5, -0.5; and -0.7, -0.7.

The following parameters were held constant.

1. Total service time per day: 480 minutes.
2. Vehicle speed: 0.25 miles per minute.
3. Base fare: \$0.60.
4. Base travel ratio: 2.0.
5. Base wait time: 15 minutes.

For systems characterized by both high fares and high fare elasticities, no positive equilibrium solution could be found. This probably resulted from the inadequacy of the constant elasticity assumption used in developing the demand model. Occasionally, when the high-fare, high-elasticity system did yield a positive volume, the results were completely unreasonable in that the predicted dial-a-ride travel time was less than the automobile travel time. However, these systems were a small fraction of those tested and were characterized by input values far beyond the range of values for which the supply model was calibrated.

No system tested showed a profit. This appears reasonable in light of existing operational experience and when one considers that only the off-peak hours were considered. Efficient peak-hour subscription bus service could offset some or all of the off-peak loss.

Figures 1 through 6 show some of the results of the test runs for various representative systems. Two basic statistics were considered. First, the ratio of total dial-a-ride travel time and automobile travel time is termed the level of service. This measure reflects the overall quality of service provided by the system, and its value increases as the actual quality of service declines. Second, the daily deficit of the system is an economic performance measure. In general, there is a trade-off between improved service and reduced deficit.

Figures 1 and 2 show that the fare is a significant design variable. Higher fares imply lower demand, which results in improved service, which encourages more demand, which to some extent offsets the impact of increased fare. However, because the fare elasticity is high while the wait time and travel ratio elasticity is low, this offsetting effect is quite small. The deficit curves for various fare levels are U-shaped, and the minimum deficit lies between $0.75 and $1.00 per trip, depending on the fare elasticity. In general, this fare is somewhat higher than is currently being charged by most existing dial-a-ride systems.

Figures 3 through 6 show the effects of various demand parameters on the level of service and net daily deficit. The base modal split is a major determinant of service quality and economic performance. For example, the deficit for an 8-vehicle system is almost $100 per day less at the 2 percent base modal split than at the 1 percent (Fig. 3). The magnitude of this differential tends to increase with vehicle fleet size. In general, a 1 percent increase in modal split produced a 10 to 20 percent decrease in daily deficit. The significant effect that the base modal split also has on quality of service is shown in Figure 4.

Figures 5 and 6 show the effect of the travel time ratio and wait time elasticities for various fleet sizes. In general, because most of the system tested operated at wait times and travel time ratios considerably below the base points of 15 minutes and 2.0 respectively, higher service elasticities implied higher demand and resulted in a lower operating deficit.

Relatively small increases in the service elasticities had substantial effect on the size of the deficit. For example, Figure 5 shows that a shift in the elasticities from -0.3 to -0.5 resulted in a 12 to 15 percent decrease in deficit, depending on the size of the vehicle fleet. In general, the size of the deficit decrease was a constant proportion of the total deficit, independent of the vehicle fleet.

The effect of increases in service elasticity on the quality of service is shown in Figure 6 for the same system as was used in Figure 5. The increase in demand implied by higher elasticities resulted in poorer quality service. To maintain the same level of service when the service elasticities shifted from -0.3 to -0.5 would have required the addition of 2 to 3 vehicles. Shifts from -0.3 to -0.7 imply the addition of 3 to 5 vehicles to maintain an equivalent level of service.

CONCLUSIONS

In previous analyses of dial-a-ride systems, either simulation has been used to analyze supply characteristics or attitudinal or empirical analysis has been used to predict demand. This analysis shows that both supply and demand parameters are important and must be considered in an integrated framework in designing dial-a-ride systems.

Figure 1. Fare versus deficit for various elasticities.

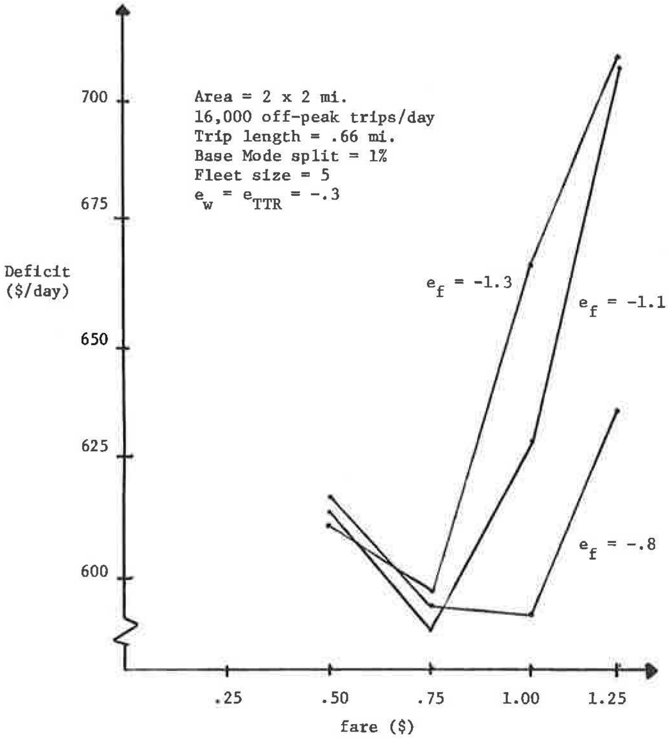


Figure 2. Fare versus level of service for various elasticities.

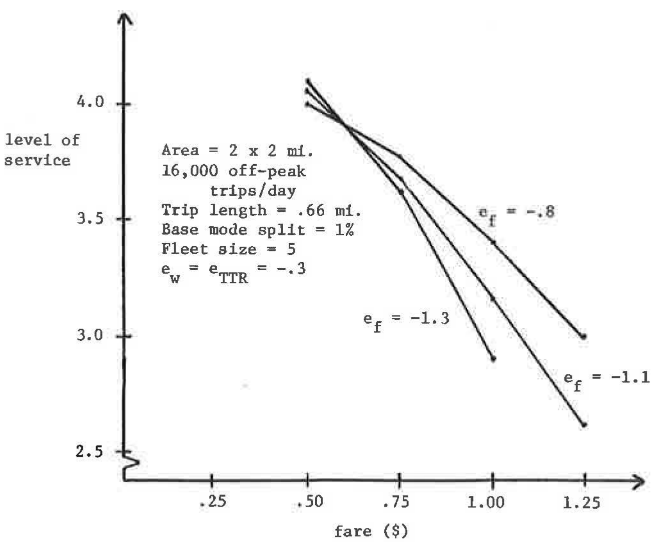


Figure 3. Fleet size versus deficit for various base modal splits.

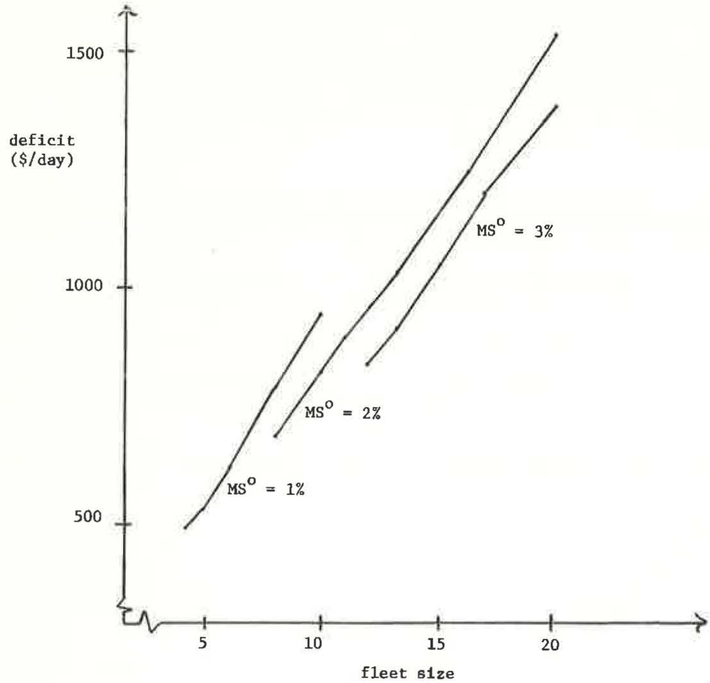


Figure 4. Fleet size versus level of service for various base modal splits.

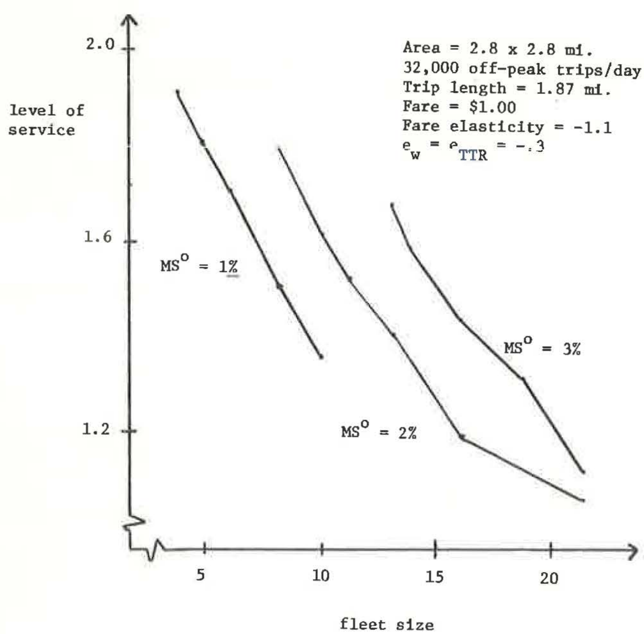


Figure 5. Fleet size versus deficit for various service elasticities.

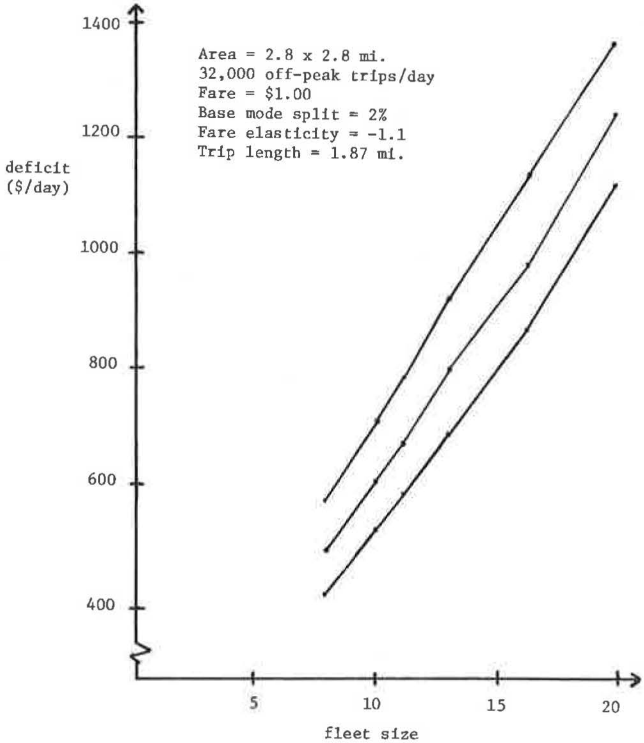
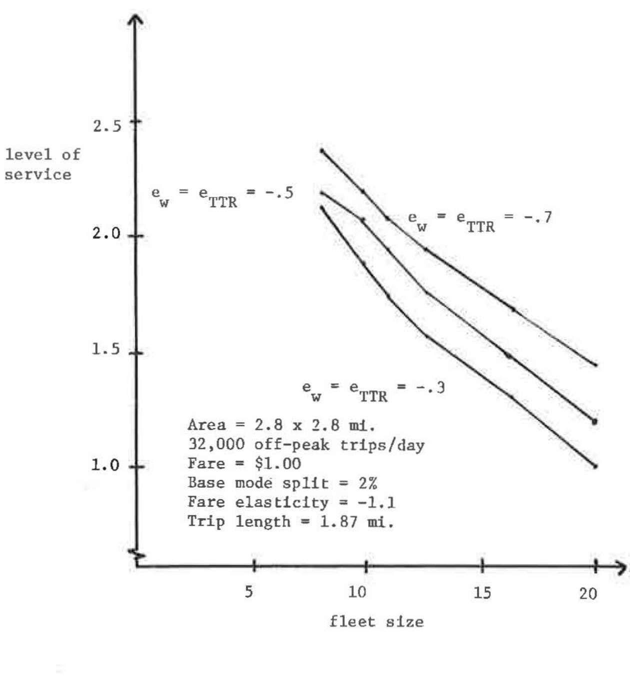


Figure 6. Fleet size versus level of service for various service elasticities.



This paper presents a model system based on an equilibrium framework that requires that both supply and demand be satisfied. Furthermore, the analytic form of all of the model components greatly reduces the computation required to evaluate a broad spectrum of design options.

The model system developed is of necessity somewhat crude, but it is sensitive to the types of system design options that are probably most relevant and is useful in analyzing changes in both short-run operating policy such as fare and long-run investment decisions such as fleet size and service area.

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