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DISAGGREGATE ACCESS MODE AND STATION CHOICE MODELS FOR RAIL TRIPS

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In this study disaggregate probability choice models are developed for access mode and for access station selection. In each of the models, there are at least two alternatives available to the individual traveler. A multinomial logit model that is based on the axiom of the "independence of irrelevant alternatives" is used. Two methods of approach concerning travelers' decision-making processes are used. The first is the simultaneous approach, which assumes that the traveler may make the access mode and station choice decisions in one of two sequences: station choice preceding mode choice or mode choice preceding station choice. In the sequential approach, the choices of access mode and access station are modeled separately. Results suggest that the traveler's decision-making process for the access mode and station choices is behaviorally separate, the sequence being station choice followed by access mode choice. The study also shows that travelers do not assign the same weights to the set of transportation system attributes when making these decisions and that the pedestrian and bus modes are preferred to the automobile mode. For the station choice, the accessibility of the train station has the greatest effect on the traveler's decision.

•A PERSON planning any type of an intraurban trip makes a number of choices including those on destination, mode, and travel route. These decisions have an important bearing on transportation planning, and therefore the knowledge of how travelers go about making their decisions is essential to transportation planners.

This research discusses the access part of the rail journey. It is assumed of course that decisions on trip origin, trip destination, rail line, and so on have already been made; consequently, travelers are faced with two access choices: access mode and access station.

The main purpose of this study is to develop disaggregate choice models of the access mode and station selection for rail work trips. At the same time, this study investigates whether travelers make the two choices simultaneously or in a sequence and, if the latter, which specific sequence? Another objective of this study is to determine the types of transportation and socioeconomic attributes that affect travelers' choice decisions and how much.

FORMULATION OF ACCESS MODE AND STATION CHOICE MODELS

In this study, a multinomial logit model (15) is used because there are usually at least two alternatives available in each choice. The generalized expression of the multilogit model is

$$P_i = \frac{e^{U_i}}{\sum_j e^{U_j}} \tag{10}$$

where

P_i = probability of an individual choosing alternative $i \Sigma \{j\}$ and
U_i = utility function associated with alternative i.

The utility of an alternative travel choice is represented by attributes of the alternative (e.g., travel time, travel cost) and the socioeconomic attributes of the individual (e.g., income). A basic assumption to formulation of these models is that travelers rationalize their choices by selecting alternatives with the highest utilities.

Construction of the access mode and station choice models in this study is approached from two behavioral process assumptions: (a) the simultaneous assumption in which the choices of access mode and station are made together and in which the joint probability of the two choices, $P(m, s)$, is contained in a single model and (b) the sequential assumption in which station and access mode choices are made one at a time. In the latter case, the joint probability of the two choices is the product of a conditional probability of one choice and a marginal probability of the other choice, e.g., $P(m, s) = P(m|s) \times P(s)$ for the station-mode sequence and $P(m, s) = P(s|m) \times P(m)$ for mode-station sequence. Clearly, both sequences can be justifiably assumed, and therefore both are studied.

For the station-mode sequence, the (conditional) mode choice is modeled first, where utility is a function of the level-of-service attributes of the mode and mode-specific socioeconomic attributes of the traveler. The (marginal) station model, on the other hand, contains three types of station level-of-service attributes: station accessibility level of service; in-train level-of-service difference resulting from choosing the selected station instead of other stations in the vicinity of the trip origin; and intrinsic level of service of the station, such as parking facility. The last two types of attributes may be directly obtained. However, accessibility to a station is related to the effort that is required of the individual to reach the station by the available modes. Therefore, it is obtained by combining the probabilities of choosing the access modes with either the access mode level-of-service variables or the utility associated with each access mode. The former method results in a number of weighted modal level-of-service variables called weighted price variables. The latter method produces a single variable called the weighted inclusive price variable. In the weighted price method the same attributes may have different coefficients in the estimated models at the two choice levels, whereas in the weighted inclusive price method the values of the coefficients for the modal level-of-service variables remain unchanged. This is because the entire utility function in the access mode model is weighted in the weighted inclusive price method. From a behavioral standpoint, the weighted price method simply assumes that travelers value the same modal level-of-service attributes differently when they choose modes and stations. On the other hand, the weighted inclusive price method assumes that travelers view the relative importance of the modal level-of-service attributes equally at the two choice levels. An interesting consequence of this method of approach, as reflected in the estimated weighted inclusive price station model, is that, if this assumption is valid, then the coefficient of the weighted inclusive price variable should be 1.

The other possible sequence is the mode-station sequence in which the conditional station choice $P(s|m)$ is modeled first and then the marginal mode choice $P(m)$ is modeled.

In the simultaneous model structure, the probability that a traveler chooses access mode m and access station s is a function of the level-of-service variables of each available mode, the in-train level-of-service difference variables, and the intrinsic station variables. Again, the socioeconomic attributes of the traveler are included in the level-of-service variables.

DATA AND METHODS

Data Source and Sample Selection

The trip data used in this study were taken from an origin-destination survey conducted for the Southward Transit Area Coordination (STAC) committee in Chicago. The inner part of the STAC area (21) was chosen as the study area because most rail work trips in the area originate there. The trip origins may be identified from the survey by a ¼-square mile centroid. The access mode, access station, and the access distance to the station can also be either directly or indirectly obtained from the survey.

A total of 150 work trips were randomly selected from the Illinois Central (IC) Railroad surveys. Those samples with incomplete information were replaced with valid samples, also randomly picked. Sets of 25 samples each were selected in a similar manner from the Rock Island (RI) and the South Shore and South Bend (SS) Railroad data. Table 3 gives the number of travelers by access mode and rail line.

Construction of the access mode and station selection models is based on data on travelers' work trips on the Illinois Central. Rock Island and South Shore data are used only to test the various operational models.

The dependent variable of a multinomial logit model is the choice probability, P_i, where i is one of the alternatives in the choice set. Because only the actual choice is observed and not the probabilities, when model parameters are estimated P_i equals one for the chosen alternative and zero otherwise.

For the access mode choice models, the alternatives considered are automobile, bus, and walk. (Even though it was determined whether travelers drove or were driven, the data did not permit further detail in modeling access mode choices.) However, each person in the sample was not always considered as having all alternatives available to him. Automobile (driven or drove) was always considered a relevant alternative. Walking was considered to be unavailable to a person if walking distance to a station was more than 20 minutes. The bus mode was available if a traveler was within a ½-mile walk of a bus route.

For the station selection model, alternative stations were chosen on the basis of data and were usually near the chosen station.

Notation

The notation (22) used in the models is defined as follows:

OVT = out-of-vehicle time, the sum of walking time and waiting time during the individual's access trip to the station;
AT = automobile time, the amount of time the individual spends in an automobile during his station access trip;
BT = bus time, the amount of time the individual spends on a bus during his station access trip;
OC = operating cost of an automobile during the access trip to the station;
PC = out-of-pocket parking cost for automobile driver or bus fare for the bus user;
TC = total cost, the sum of the operating cost and the out-of-pocket cost;
LHT = line-haul time difference, the on-train travel time difference resulting from choosing the station instead of the alternative stations;
PD = parking dummy of the available parking space per automobile driver; and
S = socioeconomic attribute, the ratio of total cost to median income.

Table 3. Sample distribution.

	Railroad		
Access Mode	IC	RI	SS
Automobile			
Driver	27	4	4
Passenger	20	4	4
Walk	50	9	9
Bus	53	8	8
Total	150	25	25

Estimation Method and Evaluation Criteria

The models are evaluated in three ways: (a) The statistical significance of each variable in the model and the model as a whole is determined; (b) the reasonableness of the magnitude of the coefficients of the model variables is examined (elasticities are studied to determine the effects of the attributes on the choice probability and the value of time); and (c) the model is applied to situations different from that on which the model is estimated.

Inasmuch as the choice probabilities are not observed, a statistical test such as the estimated residue measurement (R^2) ordinarily used for linear regression analysis is not valid. The statistical tests used for the disaggregate models in this study are mainly the t-test, which determines the statistical significance of each variable in a model, and the χ^2-test, which determines the statistical significance of the entire model.

Both the sign and the magnitude of the coefficient of a variable are examined. The sign of a coefficient must be logical: The coefficients for out-of-vehicle time, automobile time, bus time, operating cost, out-of-pocket cost, weighted price, and socioeconomic variables must have negative signs, whereas the coefficients of line-haul difference, parking dummy, and weighted inclusive price variables must have positive signs.

The magnitude of the coefficients of the variables may be examined by studying the elasticities of the choice probabilities with respect to each of the variables. The mathematical expression for the direct and cross elasticities of a multilogit model are

$$E_{X_{li}} = b_l X_{li}(1 - P_i) \tag{11}$$

$$E_{X_{lj}} = -b_l X_{lj}(P_j) \tag{12}$$

where

P_i = choice probability of alternative i,
X_{li} and X_{lj} = lth explanatory variable describing alternatives i and j,
b_l = coefficient of X_l, and
$E_{X_{li}}, E_{X_{lj}}$ = direct and cross elasticities with respect to X_l.

Furthermore, the implied value of time obtained from this research is compared with the value of time obtained from other studies.

The disaggregate access mode and station models are further evaluated by applying each model to different situations. As mentioned previously, the base data for the models estimated in this study are the set of IC data; the RI and SS data are the control data and are used solely for testing the models. The service areas, operators, number of rail tracks, distances between adjacent stations, train operating frequencies, and types of signal and train facilities of the Rock Island and South Shore Railroads are different from those of the Illinois Central Railroad.

For each individual sample, the expected probability of the chosen mode or station is compared with the expected probabilities of the alternative modes or stations in their respective choice set. If the expected probability of the chosen mode or station is greater than or equal to those of the alternatives, then the model has made a correct prediction. Otherwise, the prediction is wrong. Furthermore, for the access mode model, the expected number of users of each mode is compared with the actual number of users of the same mode.

RESULTS AND EVALUATION OF ESTIMATED ACCESS TRIP CHOICE MODELS

Models were estimated for the conditional mode choice $P(m|s)$, the marginal station choice $P(s)$, and the conditional station choice $P(s|m)$. However, estimation of the marginal mode choice model $P(m)$ and the simultaneous choice model $P(s, m)$ resulted in models with incorrect signs. These models and evaluations of them are discussed below.

Conditional Mode Choice Model

Two of the estimated access mode models appeared to have the correct signs and statistically acceptable coefficients for each variable. The coefficients of these two models and other relevant information are given in Table 4. Both of these models include the out-of-vehicle, automobile, and bus times. The cost variable is different, however, in the two models. In the first model it is the operating cost (OC), and in the second model it is the total cost divided by income (S).

Statistical tests indicate that all the variables in model 1 and the model itself are significant at the 0.99 level of confidence. In model 2, the socioeconomic variable is statistically significant only at the 0.75 level of confidence. The bus time variable is statistically significant at the 0.95 level of confidence. The out-of-vehicle and automobile time variables, along with the model itself, are statistically significant at the 0.99 level of confidence. Therefore, on the whole both models are statistically acceptable.

From the coefficients in model 1, the implied values of time (in dollars per hour) are as follows:

Variable	Value	Standard Deviation
Out-of-vehicle time	0.48	0.25
Automobile time	0.75	0.50
Bus time	0.41	0.25

Comparisons of these values with the submode values of time in other studies are not available. However, the value of in-vehicle time is approximately the same in this research and in some other recent demand model studies, approximately 70 cents/hour (2, 19). However, the values of the out-of-vehicle time in this research are much lower than the value of the out-of-vehicle time of other studies, $3.00/hour and more. It should be noted, however, that the trips under investigation in this study are access trips, whereas the other studies considered either the major part of the trip or the entire trip.

From the coefficients of the second model, the implied value of automobile time is $80/hour, which is too large to be reasonable. Therefore, model 2 is considered invalid.

The direct elasticities of the access mode model, computed at the means of each variable, for each fixed probability are given in Table 5. It can be seen from Table 5 that most of the variables are elastic when computed at the means of these variables. OC, which is associated exclusively with the automobile mode, has the greatest elasticity, and AT, OVT, and BT have smaller elasticities in that order. Values of the direct elasticities in this study are not in line with the a priori knowledge of the elasticities from previous studies. However, the differences between this study and others must again be noted. Also, as the probability increases, the elasticities decrease. This suggests logically that travelers grow less concerned with changes in the transportation attributes if their chosen mode is chosen with a high probability. The model is further evaluated by applying it to both IC and RI/SS data and comparing the results. The misclassifications and the predictive rates are given in Table 6.

The expected number of travelers by mode can be computed as the sum of the expected probability values of each mode:

$$N_m(\text{expected}) = \sum_i P_m^i$$

where

P_m^i = probability of mode m being chosen by person i and
$N_m(\text{expected})$ = expected number of travelers to use mode m.

Comparisons of the expected and the actual number of travelers by mode are given in Table 7.

The comparisons show that the expected and actual values by mode are compatible. For the RI and SS data, the absolute percentage of difference for bus mode is 69 as compared to approximately 80 for the other modes. This may be attributed to the fact that bus frequencies in the areas of the RI and SS Railroads are often quite low. Though the waiting time for bus was set at no more than 8 minutes during the process of data preparation, 30-minute headways for buses in these areas are not uncommon. This may interfere with a traveler's time schedule for reaching the station and eventually the jobsite and therefore force him to choose another access mode.

Marginal Station Choice Model

Two of the estimated station models appeared to have correct coefficient signs and statistically acceptable indications. One of the models used the weighted prices and the other used the weighted inclusive price as part of their level-of-service variables.

The Weighted Price Station Model—The statistical tests of this model (Table 8) indicate that the variables are significant at the following levels of confidence:

Variable	Level of Confidence
Weighted OVT	0.99
Weighted AT	0.99
PD	0.95
LHT	0.80
Whole model	0.99

The direct elasticities are computed at the means for the weighted OVT and weighted AT, at 4 minutes for LHT, and at 1 minute for PD (Table 9).

These elasticities indicate that, when selecting the access stations, travelers are most sensitive to out-of-vehicle time and automobile time. The results also show that travelers are relatively unconcerned about the extra amount of time spent (or saved) inside the train in choosing the access station. In spite of the incompleteness of the parking availability variable, it appears that it has an effect on the choice of access station. Information on the value of time is not available, inasmuch as this model has no cost variable. Misclassifications and the predictive accuracy rates of the model are given in Table 10.

The Weighted Inclusive Price Station Model—The coefficients of the weighted inclusive price station model and other relevant information are given in Table 11. Statistical tests of this model indicate that the variables are significant approximately at the levels of confidence given below:

Variable	Level of Confidence
Weighted inclusive price	0.97
PD	0.97
LHT	0.90
Whole model	0.99

The direct elasticities are obtained in the same way as in the previous station model (Table 12). The elasticities indicate that the weighted inclusive price variable is the most important attribute to travelers selecting a station.

The coefficient of the weighted inclusive price variable in this model is 0.5850. It is tested to be significantly different from 1.0000. This indicates that the above-mentioned assumption is invalid. In other words, the traveler does assign different weights to the set of transportation system attributes when making his access mode and station choice decisions.

The misclassifications and the predictive accuracy rates of the model are given in Table 13. Comparisons of the actual number of travelers choosing a certain train station with the expected number are not made for the two access station models because the small number of travelers observed is distributed to a relatively large number of alternative stations.

Table 4. Coefficients of conditional mode models.

Variable	Model 1[a] Coefficient	Model 1[a] Standard Error	Model 2[b] Coefficient	Model 2[b] Standard Error
Out-of-vehicle time	-0.441	0.094	-0.286	0.059
Automobile time	-0.681	0.291	-1.122	0.260
Bus time	-0.382	0.120	-0.164	0.091
Operating cost	-0.556	0.193	—	—
Socioeconomic attribute	—	—	-0.084	0.111

[a] χ^2 = 99.498 with 4 degrees of freedom.
[b] χ^2 = 87.647 with 4 degrees of freedom.

Table 5. Direct elasticities of conditional mode model.

Variable	Mean (minutes or cents)	Direct Elasticity P = 0.30	P = 0.50	P = 0.70
OVT	6.7	2.08	1.49	0.89
AT	6.7	3.19	2.28	1.37
BT	10.1	2.71	1.94	1.16
OC	15.5	6.03	4.31	2.58

Table 6. Accuracy of conditional mode model.

Data	N	M	$\alpha = 1 - \frac{M}{N}$ (percent)
IC	150	12	92
RI/SS	50	7	86

Note: N = total number of observations, M = number of misclassifications, and α = predictive accuracy.

Table 7. Comparison of number of mode users.

Data	Mode	N_E	N_A	Absolute Percentage of Difference
IC	Automobile	56	47	81
	Walk	40	50	80
	Bus	54	53	98
RI/SS	Automobile	13	16	81
	Walk	15	18	83
	Bus	21	16	69

Note: N_E = expected number of travelers and N_A = actual number of travelers.

Table 8. Coefficients of weighted price station model.

Variable	Coefficient	Standard Error
Weighted OVT	-0.385	0.102
Weighted AT	-0.957	0.172
LHT	0.138	0.167
PD	0.827	0.469

Note: χ^2 = 90.391 with 4 degrees of freedom.

Table 9. Direct elasticity of weighted price station model.

Variable	Variable Value (minutes)	Direct Elasticity P = 0.30	P = 0.50	P = 0.70
Weighted OVT	6.44	1.73	1.24	0.74
Weighted AT	2.16	1.44	1.03	0.62
LHT	4.00	0.39	0.28	0.17
PD	1	0.57	0.41	0.25

Table 10. Results of application of weighted price station model.

Data	N	M	$\alpha = 1 - \frac{M}{N}$ (percent)
IC	150	32	78.7
RI/SS	50	1	98.0

Table 11. Coefficients of weighted inclusive price station model.

Variable	Coefficient	Standard Error
Weighted inclusive price	0.585	0.107
LHT	0.230	0.194
PD	1.189	0.563

Note: χ^2 = 100.4353 with 3 degrees of freedom.

Table 12. Direct elasticities of weighted inclusive price station model.

Variable	Variable Value	Direct Elasticity P = 0.30	P = 0.50	P = 0.70
Weighted inclusive price	8.07	3.31	2.36	1.42
LHT	4.00	0.64	0.46	0.28
PD	1	0.83	0.59	0.36

Table 13. Results of application of weighted inclusive price station model.

Data	N	M	$\alpha = 1 - \frac{M}{N}$ (percent)
IC	150	27	82
RI/SS	50	2	96

Table 14. Coefficients of conditional station models.

	Model 1		Model 2	
Variable	Coefficient	Standard Error	Coefficient	Standard Error
OVT	-0.664	0.180	-0.653	0.177
BT	-0.310	0.208	—	—
In-vehicle time	—	—	-0.184	0.173
OC	-0.136	0.066	-0.062	0.094
PD	0.859	0.480	0.833	0.480

Table 15. Accuracy of conditional station models.

Model	Data	N	M	$\alpha = 1 - \frac{M}{N}$ (percent)
1	IC	110	20	82
	RI/SS	23	3	87
2	IC	110	20	82
	RI/SS	23	3	87

Table 16. Marginal mode choice models.

Variable	Model 1	Model 2	Model 3
OVT	-0.083	-0.082	-0.347
BT	0.137[a]	—	—
In-vehicle time	—	0.142[a]	0.210[a]
OC	-0.189	-0.248	-0.093
D1	—	—	2.837
D2	—	—	3.638

Note: D1 and D2 are the dummy variables such that D1 = 0, D2 = 0 for automobile mode, D1 = 0, D2 = 1 for walk mode, and D1 = 1, D2 = 0 for bus mode.
[a]Incorrect coefficient sign.

Conditional Station Selection Model

Estimation of the conditional station model is the first step of the mode-station modeling sequence. The coefficients of the two models and other relevant statistical information are given in Table 14.

The values of time implied by the coefficients of model 1 are approximately \$3.00 at $\sigma = 1.73$ for OVT and \$1.40 at $\sigma = 1.12$ for BT. The values of time implied by the coefficients of model 2 are approximately \$6.00 at $\sigma = 9.94$ for OVT and \$1.80 at $\sigma = 4.20$ for in-vehicle time. The misclassifications of these two models when applied to the base and the control data are given in Table 15.

These two station models appear to be fairly good. Nevertheless, it must be noted that the station models only constitute part of the sequential modeling process. The access mode models also have to be examined before the validity of this particular mode and station decision-making sequence assumption can be determined.

Marginal Mode Choice Model

The level-of-service variables describing access to a station by mode were aggregated by the weighted price method. All the models involved incorrect coefficient signs (Table 16). The fact that no valid access mode model could be estimated raises doubt about the validity of the mode-station decision-making sequence assumption. Therefore, the marginal mode models as well as the conditional station models cannot be applied with confidence to planning problems.

Simultaneous Choice Model

No valid models could be obtained by using the simultaneous model structure. An example of the estimated simultaneous models is given below:

Variable	Coefficient
OVT	-0.0119
AT	-0.0571
BT	0.1974
PD	1.340

(Note that the coefficient for bus time shows an incorrect sign.) The results tend to suggest that the simultaneous model structure is also an invalid traveler decision-making assumption.

SUMMARY

The main purpose of this study was to develop disaggregate choice models of the access mode and access station for those travelers who make their work trip by rail.

The multinomial logit model, which is used in this study, is based on the independence of irrelevant alternatives assumption and is capable of dealing with a different number of choice alternatives for each of the behavioral units; it is considered to be the most suitable model for the situation under investigation.

The data used for the estimation and evaluation of the various probability models were obtained from the Chicago area.

It is assumed that a person makes the access mode and station choice decisions either simultaneously or in the station-mode sequence or the mode-station sequence. In the case of the sequential assumption, the joint probability of the access mode and station is separated into a conditional probability of one choice given the other choice and a marginal probability of the other choice, depending on the particular choice sequence assumed. The investigation in this study of the simultaneous model structure and the mode-station sequence structure failed to produce choice models with intuitively correct coefficient signs.

The results of the research based on the station-mode sequence revealed some interesting behavioral characteristics of individual travelers when they make their access trips. Even though some studies have reported rather high cost elasticities for the automobile mode and rather low elasticities for out-of-vehicle and in-vehicle time (2, 19), it is a common belief that travel demand is insensitive to changes in travel cost and possibly in-vehicle time, although it is quite sensitive to changes in out-of-vehicle time (2, 5, 23). It should be noted, however, that the latter are derived from travel demand models, whereas the former are so-called modal-split elasticities (i.e., trip frequency decisions were not modeled).

The results from the access mode model, $P(m|s)$, of the present study indicate that, of the travel time (modal-split) elasticities, the automobile time elasticity is the highest followed by bus time and out-of-vehicle time elasticities. Surprisingly, the automobile operating cost elasticity is the highest of all by a wide margin, and several attempts to include automobile ownership costs and bus fare in the models failed to produce plausible models. Finally, the value of automobile time was estimated at 74 cents/hour; this is in accordance with previously obtained values.

An income variable was also considered in the estimation of one of the access mode models. A very rough income figure, the median income of the traffic zone, was the only available income information. It is not known whether this is why the model in which this variable was included yielded an implausible model; income information for each individual would have been desirable.

These results indicate that paying for a car trip to a station and spending travel time inside an automobile are disliked by travelers as compared to spending travel time inside a bus. The relatively low elasticity of out-of-vehicle time (as compared to automobile operating cost) suggests, furthermore, that it should not be difficult to convince a traveler to choose access modes such as walking and even the bus for which the out-of-vehicle time may constitute a large part of the total price.

These "discoveries" appear contradictory to the information obtained from previous travel demand studies. Nevertheless, it must be realized that those studies considered the entire trip, whereas this study deals only with the access part of the trip. These two are different in nature, and consequently the behavioral responses of the travelers should not be expected to be the same.

An important consideration in this context is whether access trips can be separated from the rest of the journey. This assumption was made in this study, but it is by no means the only assumption that can be made. In similar vein, whether automobile ownership and location decisions of households should be linked with the work trip decisions must also be asked. This research does not provide answers to such questions, of course, but the somewhat counter-intuitive results tend to suggest that automobile ownership and location decisions are important in work trip decisions (and vice versa).

Still, the different behaviors may be intuitively justified. For the entire journey, the travel distance between the trip origin (home) and the trip destination (jobsite) is generally quite large. Therefore, comfort, privacy, and other advantages offered by the automobile mode become important to travelers and thus make their response insensitive to changes in automobile travel time and cost characteristics. Of course, the walk mode is usually not considered as one of the available alternatives for such a trip. On the other hand, for the access trip, the travel distance between the trip origin and the trip destination (train station) is very short. For example the average access trip travel distance of the 150 observed travelers in the set of base data is only 1.5 miles. Clearly, not much comfort or privacy can be derived by using a car for a trip of this length. On the contrary, the various inconveniences of using an automobile, such as finding a parking space, leaving the car in a parking lot close to home where it is not available for use by other members of the family, or having someone else drive the traveler to the station, become predominant disadvantages.

Access station selection models, P(s), were more in keeping with current beliefs about travel behaviors: Travelers' choice decisions are most sensitive to out-of-vehicle time followed by the automobile time, whereas bus time and travel cost variables failed to enter the model. Also, in the weighted inclusive price station selection model, the coefficient of the inclusive price variable is significantly different from 1.0, which suggests that travelers do not assign the same weights to the set of transportation system attributes when they choose access mode and station.

In regard to the simultaneous and mode-station sequential models, no conclusive explanation can be given of their failure to obtain plausible choice models. The results of this research only give empirical support to such travelers' decision processes in which the access mode and station choice is done sequentially—station choice followed by mode choice. This is, of course, a tentative suggestion.

Finally, when the access mode and station selection models were applied to different situations they produced good predictive results.

WHAT HAS BEEN LEARNED

It has been learned through this research that more detailed information than was available on the level of service of the transportation system and on the individuals in the sample is required in order to estimate disaggregate choice models effectively. Within the extent of this study, for example, the exact location of the trip origin, the individual's income, specific information on parking conditions at the stations, and most importantly relevant alternatives to the choices actually considered by each individual behavioral unit should be specifically determined when surveys are taken for disaggregate choice modeling. Of specific concern is the automobile mode, which in this study is considered a relevant alternative for every traveler.

The somewhat counter-intuitive results also suggest that there is a clear need to relate household location and automobile ownership decisions to choice of (access) mode and other (work) trip decisions if truly behavioral models are to result.

In conclusion, the disaggregate modeling technique and the information obtained were quite instructive. Only a small sample of data was required to estimate the models; thus, considerable savings of money and time can result from the use of disaggregate models. However, before disaggregate models can be confidently used in transportation planning, their transformation into aggregate travel demand models must be accomplished. To date, little work (24) has been done on forecasting aggregate travel demand by means of (transformed) disaggregate models. Of the few aggregation procedures, more empirical studies are warranted.

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DISCUSSION

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I want to clarify an issue raised in both papers that leads to serious confusion. Both papers contrast simultaneous models with what they call sequential models. Unfortunately, the urban transportation planning models of the generation-distribution-modal split-route assignment variety give particular meaning to the issue of simultaneity and sequentiality that is not the same as those presented in these papers. Whereas the papers address a sequential decision-making process, the mathematical formulations used imply no such concept. These formulations are simply a factoring of a joint probability distribution into the product of a conditional and a marginal distribution. Thus, it is a process of estimating probability functions sequentially and does not imply anything about decision-making in any sequence.

Recursive, the term used by Ben-Akiva, is far more appropriate. Although it is of course true that travelers may make sequential decisions, there is no particular reason to assume that they do. In this sense Ben-Akiva, arguing for simultaneous structures, is quite correct. On the other hand, to assume a joint probability distribution and then ask questions about conditional probabilities or marginal probabilities are perfectly natural and reasonable. In any event, I would prefer using recursive to sequential to avoid any possible ambiguity.

GENERAL REMARKS

Both papers are very interesting and appear to provide us with some very useful information. A puzzle appears when they are regarded together. In effect, Ben-Akiva says that the recursive approach to modeling does not work because there is no a priori reason to have the model go in one order, say destination to modal split, rather than the other, modal split to destination. He shows that, by changing the order, he obtains disparate empirical results. He then argues that a simultaneous estimation not only avoids the ambiguity of ordering the conditional distributions to be estimated but also is mathematically and statistically feasible for estimation.

Liou and Talvitie also look at two alternative recursive structures, one in which access mode precedes station selection and the other in the opposite order. They also find the results different, as does Ben-Akiva, but, when they attempted a simultaneous model, no useful results were obtained.

Thus, although both papers agree that the two recursive structures give different answers, one paper argues that the solution is to use simultaneous estimation and the other says that simultaneous modeling produced no useful results. Furthermore, Liou and Talvitie argue that the order is important and care should be given to the selection of the order. They even suggest that the order can be determined from the empirical results.

Neither paper really addresses the question of why the authors came out with the results they did. Neither Ben-Akiva nor Liou and Talvitie tell us why the different orderings on the recursion yield substantially different results, nor do they tell us why the results obtained by using recursive estimation differ from those that were obtained by using simultaneous equations.

Before going into the possibilities, I should state a basic inconsistency in the Liou-Talvitie paper that may be the root of at least part of the problem. They assume that the station and access mode decision process can be divorced from the selection of the basic line-haul mode. In a sense, by assuming such independence they have already assumed the appropriateness of the recursive system and may have thus "cooked" the results. This may explain in part why their simultaneous model does not work. The authors recognize this, but perhaps they do not place enough emphasis on the problem in their interpretation of the results using the recursive model.

In evaluating the reasons for discrepancies between the recursive and simultaneous approaches, I see four possibilities.

1. The theory and assumptions used in constructing the model itself may be inadequate.
2. The specification of a structure may create problems.
3. The variables used in the model may be inadequate or inappropriate.
4. The techniques used for estimating the parameters may be inadequate.

Before I begin to consider these possibilities in turn, I apologize for not having a specific answer to the confusion, but I hope my discussion can be useful as a guide to seeking the reason.

MODEL THEORY

In considering the problems of theory, we must examine probability theory and utility theory. In probability theory, the equivalence between simultaneous and recursive estimates as defined by the authors appears simple; as I indicated earlier a joint probability distribution can be expressed as the product of a conditional distribution and a marginal distribution. Furthermore, in a multiple decision framework, repeated application of this process can be used to develop a chain of probabilities that can be modeled.

In another but similar context, Manheim demonstrated the equivalence of simultaneous and sequential or recursive models (12). As for probability theory at this level, the joint probability function is independent of the order of recursion. Thus the results obtained in the papers are totally inconsistent with the probability theory, which I believe we all would accept as given.

On the other hand, the utility theory on which the models are based might be questioned. This concerns principally the assumptions of separability or additivity in the utility functions. (These assumptions are too technical to take up here, but they are extremely powerful tools for simplifying the development of models and for enhancing the power of their application.) They may, nevertheless, do violence to reality with the consequence that they lead to the kinds of results presented here. This suggests some merit in further investigation but it is not likely that this is the most fruitful first step.

MODEL STRUCTURE

Have the theoretical considerations of probability and utility been applied properly in development of a specific structure for the model? The model development leading to the logit models used by the authors and in other work appears to be quite sound. As a fairly general form for an ogive it would appear to be very robust. The specific forms of the variables entering the model, however, may not be entirely appropriate. There may be interdependencies, for example, that are not taken into full account in the types of linear functions that have been explored.

Whereas the logit model has substantial advantages because of its ease in manipulation and ease with which new alternative choices are added, it may be too simplified and we may have to forgo some of its benefits. We want to be extremely careful before discarding the model, however, because the advantages seem to be so overwhelming. In addition to the ease of adding alternatives to modes, destinations, and the like when the logit model is used, we must also consider possible future developments to add choices of an intermediate-run nature such as automobile ownership or a long-run nature such as the location of residence or place of work. The axiom of independence of irrelevant alternatives may be a mixed blessing, but before throwing it out we should be sure that we are unable to make the judicious decisions about alternatives that would avoid conflict between the axiom and reality. That assumes, of course, that we can find no similarly endowed alternative.

VARIABLE SELECTION

The variable selection and definition used in the models also may lead to the results obtained. Leaving out important variables or introducing spurious ones may have serious ramifications for model misspecification. It is well known that model misspecification can lead to peculiar results; all model estimation may suffer some from this problem. One problem is the attribution of effects to the wrong variables. In the case at hand the problem may be more subtle and serious. In particular, it seems that Liou and Talvitie's failure to distinguish between automobile driver and automobile passenger trips seriously compounds the problem. Also, their use of aggregate zonal income may explain some of the inconsistencies. This is suggested, for example, by the very poor statistical results obtained for their socioeconomic variable, the ratio of total cost to median income.

The idea of a combined price or cost variable with income is an interesting one; it was also used by Ben-Akiva in his dissertation (2). The inability of such a variable to pick up separate price and income effects, however, is a serious weakness (although there may very well be a relationship between the level of income and the size of the price effect).

An additional problem may arise in the Liou and Talvitie study because no variable is provided for a difference in rail fares between the alternative stations. If there is no difference, they should tell us so, but one might suppose that such a difference (at least at the margin) would have an effect on station choice.

It seems that a fruitful avenue of exploration to explain the differences in results as we change the direction of recursion and between recursive and simultaneous estimation approaches might lie in better variable selection, which may isolate effects that are currently being related improperly.

ESTIMATION TECHNIQUES

The last area that may lead to discrepancies between recursive approaches and between the recursive approach and the simultaneous approach is in the estimation techniques themselves. The idea of an inclusive price developed by Charles River Associates (4) was to simplify the estimation process. The results obtained through the use of such a variable were extremely encouraging. In terms of parameter estimates obtained, the inclusive price was statistically quite significant, and the weight seemed to be reasonable. Ben-Akiva's dissertation (2) reaffirmed those characteristics. The use of such a function, if it is consistent with the theory, is extremely beneficial. It reduces the number of variables that must be considered at any single stage of the estimation and may make an otherwise intractable estimation problem relatively simple. Nevertheless, it may be that the process using an inclusive price creates estimation problems.

Ben-Akiva informs us that the estimation times for the recursive and simultaneous models were not very different when input-output differences were considered, the simultaneous being somewhat longer, but not significantly so. On the other hand, when the computing time itself is considered, the simultaneous model took nearly four times as long as the recursive model. With larger samples, more variables, and more stages of the recursion to be processed, such as trip frequency and time of day, the differences in computation time between methods may be substantial. Furthermore, estimating equations with large numbers of variables is an extremely difficult procedure. I am not fully convinced that we should discard recursive estimation in favor of simultaneous estimation. Rather, we should explore further the use of the inclusive price, including the possible introduction of constraints in the estimation process that will ensure consistency between the different recursion orders and between recursive and simultaneous estimation techniques.

DECIDING AMONG THE MODELS

An important question remaining is, How can we decide which of the models are good? The tests suggested by Liou and Talvitie seem to be good ones. They suggest we look at statistical significance, reasonableness, and application of the model to a new set of data to compare the estimated and observed values. I would put the reasonableness test first and insist that the results agree with theory and that parameter values have correct signs and be of reasonable magnitude with respect to a priori considerations. Statistical significance without reasonableness produces very questionable results.

Application of the three tests suggested may help us to reject one model over another; however, it does not tell us what is wrong with the rejected model nor indeed what is possibly wrong with a model that is accepted. Unfortunately, we cannot pick models in any general way by using these techniques. We can only compare model results with results in the real world. Even here, we must be extremely cautious because theory is all we have. If the results of statistical estimation are inconsistent with our theory, we may wish to explore the theory itself or, alternatively, go back to the entire estimation process to locate the difficulty. Indeed, the results reported in both papers appear to have problems with statistical significance.

USING THE MODELS

In closing, there are two problems that need a great deal more attention before we will be able to use disaggregate models generally in the planning process. Both papers indicate that further research is essential to develop these models for general application. This is not to say that we have not learned a great deal through the development and estimation of the models to date. We are making extraordinary advances in model development for urban transportation planning purposes, but it is important that we explore the causes for the problems cited in these papers.

Both authors make brief mention of the aggregation problem in the use of disaggregate models. The problems are not trivial. They cannot be dealt with by simple handwaving and suggesting that aggregation can be accomplished by estimation followed by addition of the results for many individuals. Some basic requirement is called for to examine the possibility of developing aggregate models from the disaggregate or alternatively learning how to use parameter estimates from the disaggregate models for broader, more aggregative decision-making. Indeed, this has been the genesis of these models where Lisco, Thomas, Stopher, and others attempted to measure values of time and where others have been concerned with estimates of price and service level elasticities.

The fundamental problem confronting us is, "Can we develop a model structure that does not do violence to our theories or to reality yet is both mathematically and statistically tractable?"

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The impression that is gained from reading these two papers is that the authors draw diametrically opposing conclusions. Ben-Akiva claims that a simultaneous model of destination and mode choices for a shopping trip can be constructed and is to be preferred over a recursive procedure comprising two models for destination and mode choices. In contrast, Liou and Talvitie are unable to construct a satisfactory simultaneous model of station and access mode choices for the access segment of a work trip and conclude that these choices must be modeled recursively.

In my opinion, neither of these conclusions is sufficiently supported by the papers, and I must conclude that there is no basis for accepting either conclusion at this time.

These papers may be discussed from a number of viewpoints. I have elected to consider the statistical evidence supplied and will leave it to other discussants to consider matters of philosophy, structure, and the like.

First, both papers are lamentably deficient in the reporting of statistical measures of assessment and comparisons for the models. Hence, many of my comments are in the form of requests for more information. Ben-Akiva bases his conclusions on the fact that the coefficients of identical variables are numerically different in the simultaneous model from those in the recursive model. However, he does not establish whether these differences are statistically significant. Liou and Talvitie dismiss the simultaneous choice model on the grounds of an incorrect sign for one variable in each simultaneous model built, but do not establish whether the incorrectly signed coefficient is statistically significant from zero. Further testing of differences between the models is largely left alone because of the incorrectly signed coefficient. The question of statistical differences between the other coefficients remains open.

Both papers make model comparisons on the basis of derived travel time values. These values are obtained by computing the ratio of the coefficients of travel time and travel cost. Both papers report a standard deviation for these computed travel time values, but neither paper reports on the method used for computing this standard deviation. Correctly, this is determined (25) as

$$V\left(\frac{a_1}{a_2}\right) = \frac{a_2^2 V(a_1) + a_1^2 V(a_2) - 2a_1a_2 \operatorname{cov}(a_1, a_2)}{a_2^4}$$

where

a_1 and a_2 = coefficients whose ratio is being determined,
V( ) = variance of, and
cov( ) = covariance of.

If both $E(a_1)$ and $E(a_2)$ are nonzero, then this variance has a distribution. However, this distribution is likely to be seriously skewed (26) and will not permit the standard deviation to be used as a means of establishing confidence limits in the normal manner. Hence, the reporting of values of travel time and their standard deviations provides little or no information for comparison between models. Based on the actual values derived by the authors, and the untenable assumption that the ratio is normally distributed, none of the values of time reported in either paper is significantly different from zero and hence each other. Thus, I must dispute the statement by Ben-Akiva that "Estimated values of time from the simultaneous model are greater than those estimated from a mode choice model (given destination) and smaller than those estimated from a destination choice model (given mode)."

Neither of the two papers reports the values of one or more constants for the multiple-choice models. However, the estimation of one or more constants permits coefficient estimates to be made with minimal bias and also allows overall goodness-of-fit measures to be determined. In binary choice models, the constant determines the position of the logit curve in relation to the values of the fitted linear function. It serves an identical purpose (in more dimensions) for a multiple-choice logit model. In conceptual terms, the constant may be considered as providing some of the information lost by an improperly specified model (27). The lack of a constant therefore presupposes a fully specified model (i.e., a constant equal to zero) and also changes significantly the meaning of the chi-square test of model fit.

A further problem with both papers concerns the data base for the choices. In both cases, it appears that the choice sets have been assigned to the individuals in the data set and the alternative options have been provided with engineering measures of the relevant variables. Many previous studies have shown this to be behaviorally incorrect. First, it is necessary to define the perceived choice set for each individual and, second, it is the perceived attributes of the alternatives that determine behavior. Because neither of these sets of perceptions was determined, the data bases of both studies must be considered suspect. The computation of models based on relative measured attributes for an arbitrarily defined set of potential alternatives provides no behavioral information and consigns the exercises to academic esoterica.

Finally, it should be noted that, in Ben-Akiva's paper, the coefficients of the variables relating to characterization of alternative destinations are generally not significantly different from zero. This is indicated by the fact that the reported standard errors of the coefficients are generally more than half the value of the coefficients for the variables EMP_d and $DCBD_d$. However, the coefficient of each variable is a function of the variances and covariances of all variables used in the model, including those that yield nonsignificant coefficients. The presence of these variables in the simultaneous model may be the sole cause for the difference in the coefficient values from the mode choice model, where the destination variables do not appear. Similarly, the destination choice model has nonsignificant coefficients for the destination variables, which makes comparisons between the recursive and simultaneous models trivial.

In summary, neither of the conclusions drawn by these papers can be accepted unless the authors can provide much more evidence. Given the questions raised here concerning the data base and the significance of the destination descriptors in Ben-Akiva's models, it is doubtful whether the conclusions can be accepted under any circumstances. The primary contributions of the two papers are, first, to highlight the problem of model structure in disaggregate, behavioral, travel demand models and, second, to demonstrate that it is methodologically possible to formulate simultaneous models within this approach. The failure of both authors to achieve statistically and conceptually acceptable simultaneous models is more likely to be a function of the data available than to be a major methodological problem. No matter how convincing the statistical evidence may be, the final test of recursive models versus simultaneous models is their comparative predictive accuracy and ease of operation. Neither paper addresses these questions. Hence, I conclude that the matter of the preferred structure of travel demand models remains a matter for future research, preferably the near future.

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These papers represent important research in disaggregate demand modeling and its application to transportation planning. The papers emphasize both the theoretical structure of these tools and model calibration using conventional travel data.

The previous discussions have concentrated on the differences in the findings of these studies, particularly the authors' results on the sequentiality or simultaneity of model structures. However, I am impressed more with the similarities between the authors' approaches and their general conclusions than with the differences in their empirical findings.

1. Both authors investigate in detail whether sequential or simultaneous structures are more valid for modeling travel behavior and rely heavily on multinomial disaggregate techniques for studying these problems.
2. Both authors conclude, generally speaking, that choice of a model has considerable influence over the coefficients that will be obtained and very likely the conclusions that may be drawn based on such models. Therefore, both authors call for great care in the selection of models and in their use.
3. With respect to data bases, both authors suggest the use of very small data sets, concentrating on the detail within records rather than the collection of large data bases. It is interesting to note that both research efforts were conducted with less then 150 observations, extremely small by current standards.
4. Both authors mention the aggregation problem as one that needs to be addressed as these models are applied in transportation planning.
5. Finally, both authors suggest a variety of detailed applications as a key element of further research.

The differences in the specific findings of these studies only serve to emphasize the authors' own caveats concerning the models and their use. We are dealing here with two very different kinds of problems, as the authors have pointed out. In Ben-Akiva's case a commonly studied travel component (off-peak shopping trips) is investigated within the context of several prior travel decisions (purpose and time of day). In Liou and Talvitie's case, a rather specialized problem in transportation planning (rail commuter trips) is further broken down for study into only the access portion. Given such differences, it is not surprising that the empirical findings of these studies are different. In fact, I would have been surprised had they been identical, given the great differences in the contexts being studied.

Of particular interest are the implications in this research for use of these tools in transportation planning. It seems that a number of events must occur before the tools described here will be included in the general repertoire of transportation planning procedures. The first of these is that the profession must know a great deal more about the kinds of transportation problems to which such tools can be logically applied. This is particularly true with complicated choice combinations, including purpose, time of day, route, mode, and destination choices. There are not many cases in which a planner would want to model that entire choice sequence as one set of simultaneous choices. Ben-Akiva's results, it seems, obtain partially because he chooses a problem that by its nature lends itself easily to the assumptions of the Luce model. On the other hand, Liou and Talvitie's findings probably stem from the fact that they study a problem drawn from the context of a broader but integral choice decision. It is doubtful whether either of these authors would have obtained the same results had they extended their choice contexts to, say, trip purpose choices in Ben-Akiva's case or primary mode choices in Liou and Talvitie's case. The models constructed by these authors have only been demonstrated and tested in problems involving logically paired choices and have not been extended to more complicated sequences.

This is a situation the profession will have to live with for some time. It appears unreasonable to expect that the simultaneous models suggested by Ben-Akiva will immediately be applicable to a broad range of intricate transportation choices. There are certainly choice contexts to which these models can be applied, but perhaps first these tools should be embedded within the overall multichoice transportation planning procedure currently in common use, perhaps replacing one or more of those steps.

This phase is probably required for practical reasons as well. Except for a few individuals primarily in the academic and consulting environments, transportation planners are not, generally speaking, well acquainted with the underlying theory and application of disaggregate techniques. By and large, the profession consists of individuals trained in conventional UTPS procedures. There is a general dissatisfaction with the conventional UTPS approach, but the disaggregate techniques are not perceived to hold the answer to these problems, at least not yet. What is required now are demonstrations of the application and potential savings of these procedures, with particular attention to computer processing and institutional constraints to model implementation. It is simply not an easy job to bring on line a disaggregate model and to make a case in an agency for its use, as opposed to a currently used conventional procedure. Parallel use of both tools is more feasible in the short run. In the interim, it seems probable that these procedures will remain relatively unused until the profession is more convinced of their utility.

Of particular concern here is the relevance of models to the profession in general. Most transportation planners are much more concerned with the usefulness of models to their work than they are with the theoretical niceties of their structure. If models do not have the right policy variables or cannot be used to address questions at issue today, then whether they are constructed by using disaggregate techniques or conventional aggregate procedures will be equally irrelevant, for in neither case will the models be used. Therefore I suggest that the most important step we can take to ensure that the results of research such as this will be used is to ensure that our models are capable of addressing relevant policy questions. This means not just the study of demand model structures, although that is important, but also the specification of appropriate variables and identification of particular problem contexts in which those variables can be used to predict behavior and to estimate impact. How many of us today can say we have on-line demand models capable of addressing questions related to the energy crisis, car pooling, pricing schemes, fuel policies, and parking modal interface? I doubt that many of us can.

AUTHORS' CLOSURE

Moshe Ben-Akiva

The discussions by Kraft, Stopher, and Hartgen deal with three important aspects of the proposed travel demand models: model structure and estimation procedure, empirical evidence, and applicability to transportation planning. Before discussing these topics, I will briefly restate my line of argument and in particular the precise purpose of the empirical study.

RESEARCH STRATEGY

The research begins with the assumption (which was not questioned by any of the discussants) that the choices of frequency, destination, mode, and time of day for a specific trip purpose (e.g., shopping) are elements in a single decision. In other words, it is assumed that a potential traveler compares alternative trips and therefore jointly selects a frequency, destination, mode, and time of day for a given travel purpose. Any sequence assumed for choices that are elements of a single decision is arbitrary. For some decisions, a conditional decision-making process implying a specific sequence of choices may be a realistic assumption. However, a joint decision-making assumption is more realistic for nonwork trips such as shopping trips.

Alternative assumptions about the decision-making process or about the causal relationships among choices result in different model specifications. A simultaneous structure is used to represent jointly determined choices, whereas a recursive structure is used to represent a specific choice sequence. In addition to differences in their mathematical formulations, the alternative models are estimated differently. In a simultaneous structure a model for the joint probability is estimated directly, whereas models for the conditional and marginal probabilities in a recursive structure are estimated separately.

Based on a priori reasoning, the simultaneous model is superior to a recursive model because it is a more realistic representation of the behavioral process. We do not expect these different models to produce identical estimation results. The value of empirical evidence is in determining the consequence or practical significance of the differences between the models. The differences are in the estimation results and potentially also in the ease and cost of estimation and application.

The empirical study that was conducted and reported in the paper was therefore not designed to test which model is better. Rather, it was designed to determine the feasibility of the simultaneous model and the practical significance of the differences in the estimation results between the alternative models. The empirical study indicated that a simultaneous model is feasible and that the differences in parameter estimates are of significant practical importance. Based on these results, it was recommended that travel demand models be developed by using the simultaneous model structure.

Since completion of this study, several other empirical studies have strengthened these conclusions (28, 30, 34). The results of Liou and Talvitie also support the conclusions that the estimation of a simultaneous model is feasible and that the alternative models produce differences in estimation results that are of practical importance. (The fact that the estimation results of their simultaneous model were not satisfactory can be attributed to a poor specification.)

MODEL STRUCTURE AND ESTIMATION PROCEDURE

Kraft states that models formulated with a recursive (or conditional) structure do not imply a sequential (or conditional) decision-making process but "are simply a factoring of a joint probability distribution into the product of a conditional and a marginal distribution." Furthermore, the fact that alternative model structures produced different estimation results leads Kraft to conclude that the results obtained are inconsistent with probability theory.

There are no inconsistencies between the results and probability theory. The differences between the simultaneous and the recursive models are the direct result of different mathematical formulations and different estimation procedures.

Any given model, whether simultaneous or recursive, can be expressed mathematically as a joint probability or as a sequence of marginal and conditional probabilities. However, the mathematical expression for a joint probability derived from a recursive model will, in general, be different from the formulation of the joint probability in a simultaneous model. Likewise, marginal probability derived from a joint model will in general be different from its specification in a recursive model. The reason for these differences is the need to introduce additional assumptions in a recursive model in order to formulate composite variables. This is the basic difference between simultaneous and recursive structures. An additional behavioral assumption of a sequence of choice is embedded within a composition rule. Thus, the two structures are mathematically different, and we should not expect identical results.

This is illustrated by using the logit model and the example of mode and destination choice as in my paper. We can estimate a single logit model that explains directly the joint probability of shopping destination and mode choice as follows:

$$P(d,m:DM) = \frac{e^{U_{dm}}}{\sum\limits_{d'm' \epsilon DM} e^{U_{d'm'}}} \tag{13}$$

This model treats the choices of mode and destination jointly (i.e., simultaneously), does not require any sequence assumptions, and allows for a realistic representation of choice between complete alternatives (e.g., shopping trip to the CBD by bus versus a car trip to a suburban shopping center).

Using the logit model for each choice separately in one specific sequence, we estimate the following two models:

$$P(d:D) = \frac{e^{U_d}}{\sum\limits_{d' \epsilon D} e^{U_{d'}}} \tag{14}$$

$$P(m:M_d) = \frac{e^{U_{m|d}}}{\sum\limits_{m' \in M_d} e^{U_{m'|d}}} \tag{15}$$

The basic difficulty with a recursive structure is the representation of variables that vary across more than one choice in the utility function of the marginal probability. Travel time, for example, varies between mode and destination combinations. Therefore, this variable enters directly into the joint utility U_{dm} in Eq. 13 and the conditional utility $U_{m|d}$ in Eq. 15, but it cannot be directly represented in the marginal utility U_d in Eq. 14 because the mode is indeterminate. Denoting this variable as X_{dm}, for a joint model we can write the following [for simplicity other variables in the utility functions are not explicitly included (2, ch. VI)]:

$$U_{dm} = U(X_{dm}) \tag{16}$$

where the travel time variable, for example, enters directly the utility function. For the sequential model, we can write the conditional utility as

$$U_{m|d} = U^m(X_{dm}) \tag{17}$$

and the marginal utility as

$$U_d = U^d(X_d) \tag{18}$$

where X_d is assumed to represent the values of X_{dm} by all alternative modes. Thus,

$$X_d = g(\{X_{dm}, \forall m \in M_d\}) \tag{19}$$

where g( ) is some composition rule and X_d is a composite variable of X_{dm} across modes. An example of such a definition is

$$X_d = \sum_{m \in M_d} X_{dm} \times P(m:M_d) \tag{20}$$

This is the rule that was used in the shopping model estimated by CRA (4). It implies a sequence assumption, and it requires that a lower level conditional probability (to compute these composite variables) be estimated before a higher level marginal probability can be estimated or predicted.

The assumption implied for the variable of travel time, for example, is that destination choice is based on expected travel time across modes. This means that the actual choice of mode is indeterminate when the destination choice is made; it is assumed that destination is chosen first, and then conditional on the destination alternative chosen a mode is chosen.

Thus, modeling a set of choices, which are realistically assumed to be made jointly, in a recursive structure with composite variables will result in errors due to model misspecifications.

The difference in estimation procedures between the simultaneous and the recursive structures also contributes to differences in the results. The differences can be attributed to both efficiency issues and specification errors. Consider a simultaneous model such as the one for the joint probability of mode and destination in Eq. 13. We can mathematically derive the expression for any conditional or marginal probability. Therefore, although the model is specified as simultaneous we can estimate the model coefficients in two ways. We can either estimate the joint probability directly or estimate any sequence of marginal and conditional probabilities, say P(d) and P(m|d). What are the differences between these two estimation procedures? [The answer to this question is discussed in detail by Ben-Akiva (2, ch. IV).]

First, the estimation results obtained by the direct estimation of the joint probability are more efficient than those obtained for the marginal and conditional probabilities. This is because all the data are used to estimate the coefficients appearing in $P(m|d)$ in the first case, whereas only the data on alternative modes to the chosen destination are used in the second case to estimate the same coefficients. In addition, the coefficients estimated for the conditional probability are used to create the composite variables used in the estimation of the marginal probability. Thus, there is also a propagation of errors in a sequential estimation, where the randomness in lower stage estimates shows up as measurement errors in the higher stage models. Thus, in the absence of specification errors, the difference in the estimation results can be explained by random variation. The direct estimation of the joint probability provides the most reliable estimates.

Second, the differences between the two estimation procedures are also attributed to specification errors. Specification errors will affect both estimation procedures. Because the effect of the error on the joint probability estimation will be different from that on a conditional and a marginal probability estimation, it contributes to the differences that will be observed between the two procedures. A more careful selection of variables may reduce the specification errors and therefore reduce the differences between the estimation results of the joint and conditional probabilities. However, even if the model is fully specified, the differences between simultaneous and recursive models that result from use of composite variables in a recursive structure will still be present.

To overcome these difficulties, Kraft suggests that constraints be included in the estimation process that will ensure consistency between different sequences. This suggestion is implemented in the direct estimation of the joint probability of a simultaneous model.

The logical answer to Kraft's question, Can we develop a model structure that does not do violence to our theories or to reality yet is both mathematically and statistically tractable? is the simultaneous model recommended in my paper. If we consider a wider range of travel-related choices, such as employment location, automobile ownership, and residential location in addition to short-run choices, the appropriate model structure may be termed block recursive. In this structure, the blocks of long-run and short-run choices are recursive with respect to each other. However, within each block the choices are made jointly and modeled in a simultaneous structure.

EMPIRICAL EVIDENCE

The essence of Stopher's discussion is that the conclusions are not supported by the supplied statistical evidence. However, as noted earlier, my conclusions are not based on hypothesis testing or solely on comparisons of goodness-of-fit measures. From a theoretical point of view the simultaneous model is superior to the recursive model. The empirical evidence is used to demonstrate the feasibility of the simultaneous model and to determine the practical significance of the differences that we expect a priori.

Stopher states that I did not establish that the differences of the estimates of the recursive and the simultaneous models are statistically significant. (It is not clear to me what specific statistical test can be used for this purpose because the models have different nonlinear mathematical formulations. The goodness-of-fit measures of the different models are almost identical.) Suppose for the moment that the differences are not statistically significant at a reasonable significance level. Should this be a reason to revise my conclusions? One knows a priori that there are differences due to different mathematical formulations and estimation procedures; therefore, it can be concluded that the lack of significant statistical differences is due to the small sample size. Furthermore, the coefficient estimates of the simultaneous model are more reliable because of the direct estimation of the joint probability. Inasmuch as a simultaneous model does not cost much more to estimate and apply than a recursive model, which has been indicated, the simultaneous model structure should be recommended.

Stopher further questions the empirical evidence due to the use of so-called engineering rather than perceived choice sets and attribute values. He goes so far as to say that using engineering data in model estimation "provides no behavioral information and consigns the exercises to academic esoterica." Alternatively, the use of perceived values can be considered the academic esoterica, inasmuch as the models are intended to provide decision-makers with forecasts of the impacts of alternative policies or plans. A prerequisite for the use of a model estimated with perceived (or reported) values for forecasting is a set of relationships between perceived and engineering values. Furthermore, perceptions are only intermediate variables formed by individuals on the basis of physical objects and characteristics that are measured by the engineering values. Thus, a model that uses engineering estimates explains directly the individual's reaction to physical objects and characteristics, circumventing the need to deal with the intermediate variables of perceptions. It is clear, however, that the model functional form and parameters embody both the formation of perceptions and the behavioral response to these perceptions. This rationale is the basis for a large body of econometric literature, and I am not aware of any studies that have shown it to be "behaviorally incorrect" as Stopher claims.

I agree with Stopher's statement that a lack of constants in a choice model specification presupposes a fully specified model. Constants can be excluded from the model only if it can be shown that they have no explanatory power; i.e., the constants are equal to zero. (The specification used by Liou and Talvitie did not include constants. This may explain their unsatisfactory estimation results for the simultaneous model.) This was not done in this study. The constants included are a mode-specific constant, DA, and a CBD destination constant, DCBD. It does not make sense to use a constant for every possible destination because there are so many.

In summary, the conclusion that the simultaneous structure is preferred does not depend on additional statistical evidence. Future effort should be directed at the steps necessary to make fully specified simultaneous travel demand models available for application rather than at further comparisons of alternative model structures.

APPLICABILITY TO TRANSPORTATION PLANNING

The applicability of the proposed models to transportation planning, the essential issue of research in travel demand modeling, is the focus of Hartgen's discussion. I agree with Hartgen's conclusion that "what is required now are demonstrations of the application and potential savings of these procedures, with particular attention to computer processing and institutional constraints to model implementation." Furthermore, the overall direction for the development of travel demand models should be toward operational models that are more reliable in a forecasting context.

Several studies that have been completed since my paper was written have further shown the practicality of estimating simultaneous choice models in situations with more than two dimensions of choice and with a very large number of alternatives and observations. First, the simultaneous model of destination and mode for the shopping trip has been extended to include frequency as well in the simultaneous structure (29). A similar simultaneous model was also applied successfully with a data set from the Netherlands (30). Simultaneous disaggregate choice models have also been successfully applied to the automobile ownership and work mode choices (31) and are currently being applied to the entire set of long-run locational and automobile ownership choices (33).

From the point of view of aggregate forecasting the use of simultaneous models places no additional burden on data requirements and computational efforts. Two ongoing research efforts at M.I.T. focus on the application of simultaneous travel demand models to aggregate forecasting. The first study is investigating alternative procedures of using these models for aggregate predictions (32). The second study (sponsored by DOT University Grant Research Program) is implementing these models and procedures for a transportation planning case study.

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The issues raised by the discussants are well taken, and perhaps we can clarify some of the questions that have been brought up.

It is true that mathematically the joint probability should have the same value as the product of a conditional probability and a marginal probability and be independent of any specific mathematical formulation; e.g., $P(s, m) = P(s|m) \times P(m) = P(m|s) \times P(s)$. However, the conditional probability, by definition, means the probability of one event taking place given that the other event has already taken place. Therefore, from the behavioral viewpoint, decisions of the access mode and station choices may be approached from three directions: the simultaneous decision-making process and the two sequential decision-making processes. It appears in this study that empirically it is possible to estimate choice models based on the station-mode sequential assumption. Further research is necessary to determine whether the joint probabilities would indeed be the same had it been possible to estimate choice models based on the other sequential structure or based on the simultaneous modeling structure.

With regard to the modeling aspect of this study, the importance of including one or more constants in the utility function was investigated. In fact, in the various access mode and station models, mode-specific dummy variables were included to indicate the access mode with which the in-vehicle time was associated. Nevertheless, all the models estimated in this fashion involved (significant) coefficients with incorrect signs. One such model is in Table 16 (model 3).

Another point concerning the modeling aspect is the access mode alternatives. Although it was observed in the survey whether a traveler drove or was driven to a station, the detail of the data did not permit separating these two modes. Consequently, the automobile mode was viewed as a single mode and the value of each level-of-service variable associated with the automobile mode was obtained by averaging the values by automobile-driver mode and automobile-passenger mode (22).

In formulation of the station selection models, differences in train fare among alternative stations were not included because the alternative stations were generally adjacent to each other and were located within the same fare zones.

With regard to the model evaluation aspect of this study, two points need to be mentioned. As reported in the paper, no valid models could be obtained by using the simultaneous model structure. The estimated models were considered invalid on the basis of incorrect coefficient signs. The coefficients with wrong signs were statistically significant; therefore, further testing between the models was not carried out. For the simultaneous model, for example, the standard error of the bus time variable is 0.0427, whereas the variable coefficient is 0.1974. The second point concerns the standard error of the implied value of time. The variances of the implied values of time were determined in the same way as suggested in one of the discussions (19). It was not known, however, whether their distributions were normal or skewed.

Another important aspect discussed concerns the applicability of the multinomial logit modeling technique to transportation planning and forecasting. This is a laudable goal for transportation research. However, it seems that the objective is not only to replace the existing UTPS package with a new package but to replace it with a better and more valid modeling system or concept. Therefore, it ought to be realized that transportation planning should be done on the basis of methodology and assumptions that are consistent with travel choice behavior and consumer theory, especially when such planning involves both short-range predictions and long-range projections extending 20 years into the future. This would require a full and extended knowledge and understanding of the extent and limitations of this and other modeling techniques.

In closing, we would like to emphasize that formulation of the access mode and station choice models in this study was based on the assumption that the modeling of the access trip can be separated from the rest of the journey. Further research is necessary to determine whether this consideration is proper. On the other hand, the behavioral decision-making process involved in making a trip is so complex that structuring a single simultaneous model to include all the trip choices such as purpose and household location on the one hand and trip frequency, time of day, trip destination, travel mode, and route on the other is clearly not advisable, if not impossible. Some assumptions have to be made to separate the various trip choices and thus simplify the modeling process. This study only explores one of the many such assumptions that could be made.

Finally, it is stressed in all the discussions that further research in this area is warranted. As pointed out, knowledge of the relevant alternatives as perceived by individual travelers and specific socioeconomic information (e.g., individual income, household expendable income) are important in formulating behavioral models. Unfortunately, these types of information were not available for this study. Furthermore, the validity and the effects of a number of assumptions that are essential for disaggregate multinomial modeling such as the separability assumption and of course the axiom of independence of irrelevant alternatives should be investigated in the near future.