GENERALIZED COMBINATION METHOD FOR AREA TRAFFIC CONTROL

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A simple generalization of the British combination method is given for optimizing offsets in synchronized, traffic-signal networks of a general structure. The method then is used in a recursive procedure to determine values for the offsets along each street, the splits of green time at each intersection of the network, and the common cycle time of the controlled area. The signals' cost to travelers is evaluated as the sum of 2 components: one associated with a deterministic traffic-flow model and the other associated with randomness in traffic behavior. The deterministic component is a function of the coordination among the signals in the network and generally increases with cycle length. The stochastic component depends on the expected overflow queue at each traffic light and decreases with cycle length. It is shown that optimal settings are determined at the equilibrium point of minimum total cost resulting from the combined effect of the 2 components.

•THE PRIMARY objectives of an areawide traffic-control system are to provide smooth flow conditions for all traffic streams through the area and to reduce the delay, or travel time, incurred by users of the system. The variables of each signal program that affect the traffic flow are cycle time, splits of green time, and offsets. A coordinated traffic-signal network requires a common cycle time for all signals in the network or a cycle that is a submultiple of a master cycle. In some cases it is advantageous to partition the network into subnetworks that operate with nonsynchronous cycle times.

The conventional procedure for determining control variables is a sequential decision process. First, a common cycle time is selected for the network. Second, the splits at each intersection are determined according to the proportions of demand-capacity ratios on conflicting approaches. Third, linking of the signals is achieved by an appropriate method for selecting a fundamental set of offsets throughout the network.

Experience of researchers and practitioners in the urban traffic-control field has shown that cycle time may well be the most important control variable in a synchronized traffic-signal network (1). The approaches for selecting a cycle time can be divided into 2 classes. The first class is the node approach. Because through capacity increases with cycle length, this approach is based on analyzing the capacity requirements of each intersection in the network. Common cycle time is determined according to the requirements of the most heavily loaded intersection—the intersection with the highest sum of demand-capacity ratios on conflicting signal phases. A procedure that is used for a single intersection, such as Webster's method (2), is then used to calculate cycle length. This approach has been primarily used in conjunction with offset optimization methods such as COMBINATION and TRANSYT (3). The main deficiency in this approach is that the interaction of flows in the spatial road network structure of the area is disregarded. A formula devised for an isolated intersection, assuming that arrival

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times of cars are randomly distributed, is not necessarily valid in a network situation in which flows are fed from adjacent intersections. The result is generally a cycle time that is too long, which causes excessive delays (4, discussion). The second class is the network approach. In this case an attempt is made to select a cycle time that satisfies the capacity requirements at each intersection and is congruent with the particular network structure at hand. Simple examples in this category are the arterial progression schemes in which a cycle that produces maximal bandwidths is selected according to distance and speed data (5, 6, 7). The underlying principle is that optimal progression (offsets between signals) for a given block-length pattern is strongly dependent on cycle time. In a general network this approach is used principally by SIGOP (8). A predetermined number of cycle times are scanned in this method. For each cycle, offsets are optimized by the OPTIMIZ subroutine and performance is evaluated by a coarse simulation of traffic flow through the network (VALUAT subroutine). The optimal set of cycle and offsets is selected according to the results obtained by VALUAT. TRANSYT also indicates the possibility to iterate on cycle time in conjunction with the hill-climbing procedure for offset selection (9). However, the extensive computational requirements of this method seem to rule this out in practice. Two deficiencies of the network approach in SIGOP are apparent. First, the offset optimization procedure determines a local optimum rather than a global optimum. Second, stochastic effects on link performance are ignored. These effects do not affect the selection of offsets at a fixed cycle time, but they are of prime importance in evaluating a range of cycle times. They become pronounced as a signalized intersection approaches its capacity and, in an optimal procedure, would deter the cycle time from assuming values close to the minimum. One typical study has shown that the lower bound on cycle time was consistently selected as the optimal value. Stochastic effects conceivably would have shifted the result upward (10).

In this paper, network settings, including cycle, splits, and offsets, are determined in conjunction with a rigorous synchronization procedure (that is, one capable of determining the global optimum) that is an extension of the British combination method (CM). The combination method is an offset optimization procedure applicable to series-parallel networks; it was first introduced by Hillier (11). It was then applied by Allsop (12) to networks of a more general structure. The method was later formulated in terms of dynamic programming optimization and applied in conjunction with a computationally efficient network partitioning algorithm (13). The dynamic programming procedure for the general network is presented in this paper as a set of 2 network operation rules that are a straightforward generalization of the combination method rules for series-parallel networks. The procedure is further used as a tool in determining optimal network settings that take into account costs attributable to both the deterministic traffic-flow model and the stochastic fluctuations inherent in the traffic process.

TRAFFIC-FLOW MODEL

To illustrate the key features of the traffic-flow process, we should consider an idealized model. The discrete nature of vehicular movement would be disregarded and traffic would be thought of as continuously fluid. The following assumptions would be made:

1. All cars travel with uniform speed between adjacent intersections; and
2. Traffic flow is saturated; that is, traffic volume at each intersection equals serving capability.

Let i and j denote 2 adjacent signalized intersections in the network; cars can travel from i to j along the link connecting them. The following are definitions of the parameters shown in Figure 1:

g_j = effective green time of signal j,
r_j = effective red time of signal j,
$C = g_j + r_j$, network common cycle time,

ϕ_{ij} = offset time between signals i and j,
t_{ij} = travel time from i to j,
$F_{ij}(t)$ = instantaneous traffic flow in vehicles per unit of time, and

$$F_{ij} = \frac{1}{C}\int_0^C f_{ij}(t)dt, \text{ average traffic flow.}$$

When traffic is assumed to have a periodic arrival pattern of rectangular shape as shown in Figure 2a, it can be easily verified that the rate of delay or delay per unit of time, $d_{ij}(\phi_{ij})$, on the link i, j, is

$$d_{ij}(\phi_{ij}) = \begin{cases} F_{ij}\frac{r_j}{g_j}(t_{ij} - \phi_{ij}) & \text{if } t_{ij} - g_j \le \phi_{ij} \le t_{ij} \\ F_{ij}(\phi_{ij} - t_{ij}) & \text{if } t_{ij} \le \phi_{ij} \le t_{ij} + r_j \end{cases} \quad (1)$$

and is similarly periodic with respect to ϕ_{ij} (Fig. 2b). Examination of Figure 1 indicates that the offset ϕ_{ij} can be expressed as follows:

$$\phi_{ij} = mC + \theta_{ij} \quad (2)$$

Figure 1. Link and signal parameters.

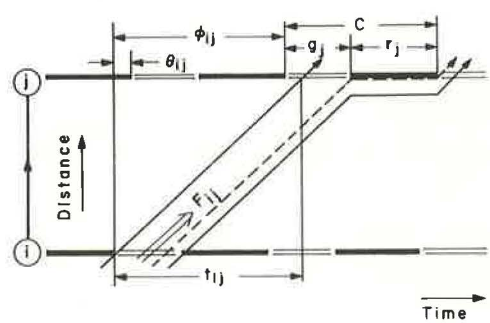
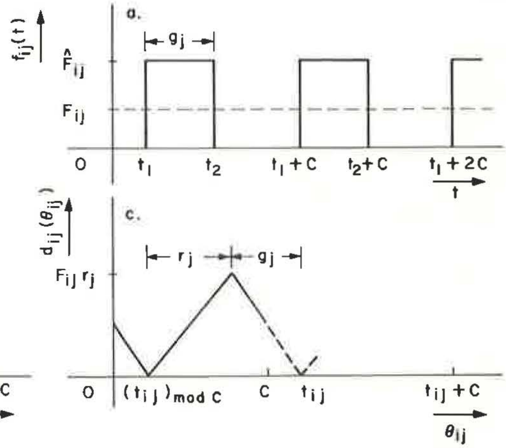
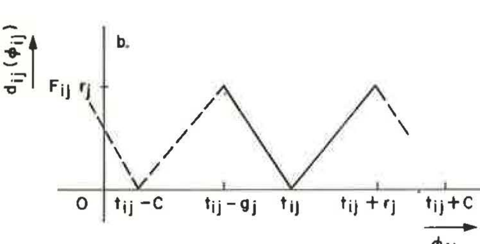


Figure 2. Idealized traffic-flow pattern and link delay functions.





where m is an integer number (in Fig. 1, n = 1). Thus we can confine offset variations to a single cycle time—$0 \le \theta_{ij} \le C$—by introducing the transformation

$$\theta_{ij} = \phi_{ij} - mC = (\phi_{ij})_{\bmod C} \qquad (3)$$

The resulting delay function $d_{ij}(\theta_{ij})$ is shown in Figure 2c. More elaborate models can be used to more closely approximate traffic conditions by taking into account secondary flows, platoon dispersion, and the like (14, 15, 16). An example of an actual traffic-flow pattern that has been measured directly by detectors on the street in the Toronto traffic-control system is shown in Figure 3a. The link delay function associated with this pattern is obtained by applying elementary queuing relationships (17) and is shown in Figure 3b. This paper primarily considers delays, but the same optimization methods can be used with a more general link performance function combining costs of delays, stops, acceleration noise, or other measures of effectiveness by using appropriate weighting factors. Huddart (18) and Chung and Gartner (19) discuss additional measures of effectiveness.

CRITERION OF OPTIMIZATION

The objective of the network optimization procedure adopted here is to determine signal settings (cycle time, splits, and offsets) that minimize total delay. In a recent report (20) it was shown that total delay in the network, D, can be regarded as a sum of 2 components as follows:

$$D = D_d + D_s \qquad (4)$$

The first component, D_d, is the delay time resulting from the deterministic traffic-flow model previously described. In a network context it is obtained by summing all the individual link delay functions such as those represented by Eq. 1 or Figure 3b.

$$D_d = \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij}(\theta_{ij}) \qquad (5)$$

where n = the number of intersections (nodes) in the network. $d_{ij} \equiv 0$ if the link i, j does not exist. For given cycle and splits this delay is a function of offsets only. The second component of delay, D_s, is due to the stochastic nature of traffic flow. It is taken to be independent of the choice of offsets in the network but is of primary importance for evaluating the best choice for cycle time because a change in cycle time involves a change in the degree of saturation at the intersection.

The procedure for optimization consists of scanning a number of cycle times that are usually in 10-sec intervals in the range of 40 to 120 sec. For each cycle time, splits at each node are calculated according to proportions of conflicting traffic streams (2), and offsets throughout the network are optimized by the generalized combination method (GCM). Another approach would be to formulate the problem in terms of an existing optimization code such as mixed-integer linear programming and to simultaneously have all the signal timings as decision variables (20).

A physical requirement of the system is that the sum of offsets around any closed loop must be equal to an integral number of cycle times. The maximum number of offsets, θ_{ij}, that can be assigned independent values in a network of n nodes is n - 1,

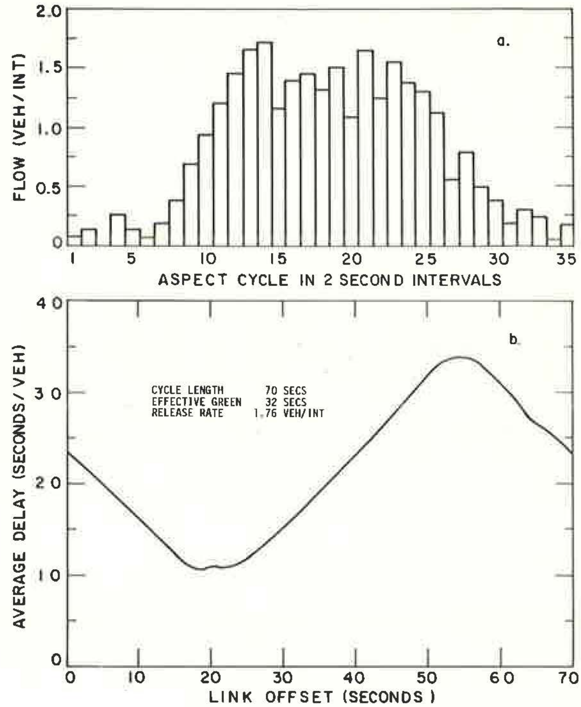


Figure 3. Actual platoon profile and link delay function.

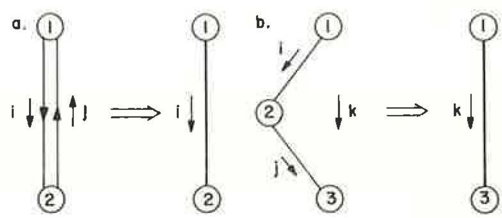


Figure 4. Series-parallel network reduction rules.

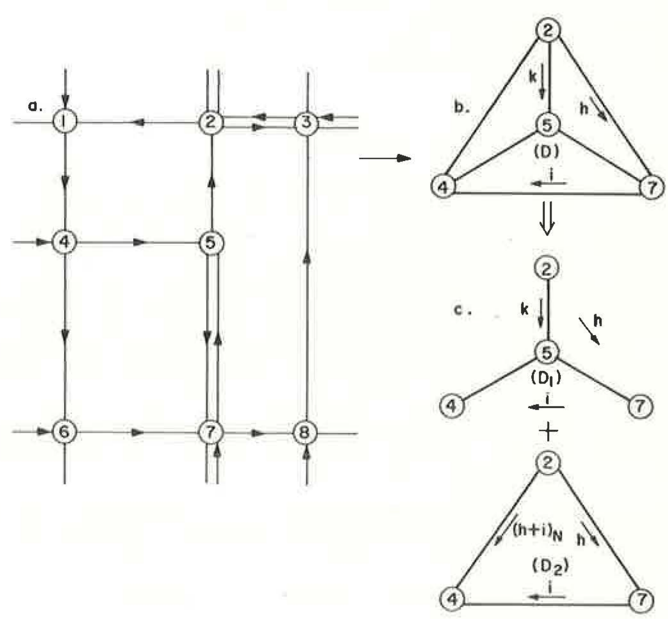


Figure 5. Signal network for example 1.

and the links across which they are defined must be in a tree pattern; in other words, they must have no loops (21). It has been shown that offset variations can be confined to the range of a single cycle's time. The computational procedure for minimizing D with respect to offsets involves the division of this range into equal N intervals. It is convenient to consider link delays to be a function of an integer number k, where k = 0, 1, . . . , N - 1, which represents the offsets at which delay is to be evaluated. To simplify notation we also should adopt the convention $(x)_{\bmod N} \equiv (x)_N$.

COMBINATION METHOD

The combination method determines offsets that minimize delays in series-parallel networks (11). The method applies a network reduction sequence to yield a total delay function for the complete network that can be represented by a single equivalent link. The optimizing offsets of the network are determined by minimizing this function. An efficient procedure for determining this sequence was developed by Robertson (22). The reduction sequence is based on 2 rules.

1. The first rule, CM 1, is the reduction of parallel links, which states that, when 2 or more links occur in parallel and join a pair of nodes, the delay functions of the individual links are added with respect to the same offset to yield a combined delay function represented by a single link between the 2 nodes. Application of this rule is shown in Figure 4a. Given $d_{12}(i)$ and $d_{21}(j)$, the combined delay, $D_{12}(i)$, for the equivalent link is

$$D_{12}(i) = d_{12}(i) + d_{21}(N - i) \tag{6}$$

for each offset i = 0, 1, . . . , N - 1.

2. The second rule, CM 2, is the reduction of series links, which states that, whenever a node is connected by 2 links to 2 other nodes, it is deleted and the 2 links are replaced by a single link. The equivalent delay function for this link is computed by minimizing the total delay for each offset between the extremities of the 2 links. At each step the procedure involves a search of all the possible offsets between one of the extremal nodes and the common node and a selection of the minimum.

Given $d_{12}(i)$ and $d_{23}(j)$, the delay function for the equivalent link 1, 3 in Figure 4b is obtained by eliminating from further consideration the offset of node 2 with respect to node 1 (offset i) through the following minimization:

$$D_{13}(k) = \min_{i} \{d_{12}(i) + d_{23}(j)\} \tag{7}$$

for all offsets k, i = 0, 1, . . . , N - 1. Because the 3 offsets i, j, and k form a closed loop, they must add up algebraically to an integral number of cycle times:

$$i + j - k = mN \tag{8}$$

or equivalently

$$j = (k - i)_{\bmod N} \tag{9}$$

Therefore, Eq. 7 can be rewritten as follows:

$$D_{13}(k) = \min_{i} \{d_{12}(i) + d_{23}[(k - i)_N]\} \tag{10}$$

for k, i = 0, 1, ..., N - 1.

GENERALIZED COMBINATION METHOD

This method relieves the series-parallel restriction imposed on the structure of networks by the ordinary combination method. By generalizing the rules stated in the preceding section it is possible to optimize networks of arbitrary layout (subject to computational considerations only).

1. The first generalized rule, GCM 1, is the combination of partial networks. Delay functions that pertain to separate parts of a network and depend on offsets between the same set of nodes are added to produce an equivalent delay function for the combined parts of the network.
2. The second generalized rule, GCM 2, is the elimination of interior nodes. An equivalent delay function for a partial network is calculated for all offsets between the boundary nodes (the nodes that disconnect a part of the network from the remainder of the network) by eliminating from the optimization process the offsets related to the interior nodes. The values of the function are determined by minimizing the total delay of the partial network for all offsets between the boundary nodes. At each step the calculation is effected by searching over all possible offsets associated with the interior nodes and selecting the minimum.

Rules CM 1 and CM 2 are special cases of rules GCM 1 and GCM 2. Recursive application of these rules defines a total delay function for the complete network for offsets between a certain final set of nodes. Optimizing offsets are determined by minimizing this function. Application of the generalized combination method is illustrated in the following 2 examples.

Example 1

The network to be optimized is illustrated in Figure 5a. Series-parallel combination produces the ∇ - Y configuration shown in Figure 5b that cannot be further reduced by these simple operations. At this stage the network is disconnected into 2 parts and a delay function is calculated for each separately (Fig. 5c). Following rule GCM 2 we obtain

$$D_1(h,i) = \min_{k} \{d_{25}(k) + d_{75}[(k - h)_N] + d_{54}[(h + i - k)_N]\}$$

This partial minimization also yields the relation $k^*(h,i)$ where k^* is the optimizing value of offset k for each combination of h and i. This relation is stored for subsequent use. Now applying rule GCM 1 we obtain

$$D_2(h,i) = d_{27}(h) + d_{24}[(h + i)_N] + d_{74}(i)$$

and D is

$$D(h,i) = D_1(h,i) + D_2(h,i)$$

Minimization of D(h, i) with respect to offsets h and i determines optimizing values h* and i*. Backtrack computation via the stored relation k*(h, i) and loop constraints yields the optimizing offsets for all links of the original network.

Example 2

The original signal network is shown in Figure 6a. After series-parallel reductions the compressed network of Figure 6b is obtained. Optimizing offsets are calculated by staged partitioning of this network and recursive application of the GCM rules at each stage. A partitioning plan that minimizes the number of operations and storage requirements for this network is given in the following table:

Stage Number	Disconnecting Nodes	Eliminated Interior Nodes
1	2, 3, 4	1
2	5, 3, 4	2
3	5, 6, 4	3
4	5, 6, 7	4, 8

The detailed minimization process is given as follows and shown in Figure 6c:

$$D_1(h, i) = \min_{j} \{d_{12}[(j - h)_N] + d_{13}(j) + d_{14}[(i + j)_N]\} + [d_{23}(h) + d_{34}(i)]$$

$$D_2(\ell, i) = \min_{j} \{d_{25}(k) + D_1[(k + \ell)_N, i]\}$$

$$D_3(n, p) = \min_{m} \{d_{36}(m) + D_2[(n - m)_N, (m + p)_N]\}$$

$$D_4(n, r) = \min_{q} \{d_{47}(q) + D_3[n, (r - q)_N]\} + [d_{56}(n) + d_{67}(r)] + \min_{s} \{d_{58}[(n + s)_N] + d_{68}(s) + d_{78}[(s - r)_N]\}$$

The delay function obtained at stage 4 represents total delay in the network for each possible combination of offsets n and r. The terminal optimization stage consists of minimizing this function with respect to n and r and calculation, by backtracking, of an independent set of optimal offsets (in this case, offsets j*, k*, m*, q*, n*, r*, s*).

NETWORK CYCLE TIME

The traffic-flow pattern on a signalized link can be regarded as the combination of a periodic component imposed by the preceding signal and a random component arising from variations in driving speeds, marginal friction, and turns. The latter component causes additional delay because of the occurrence of an overflow queue at the signal's stop line. The overflow queue represents the number of vehicles that were not cleared during the preceding green phase. Although this effect is negligible at low degrees of saturation, its predominance at high values has been proved in several studies (17, 23, 24).

Using Webster's notation for traffic-signal settings (2), we have at each node in the network the following relation

Figure 6. Signal network and optimization sequence for example 2.

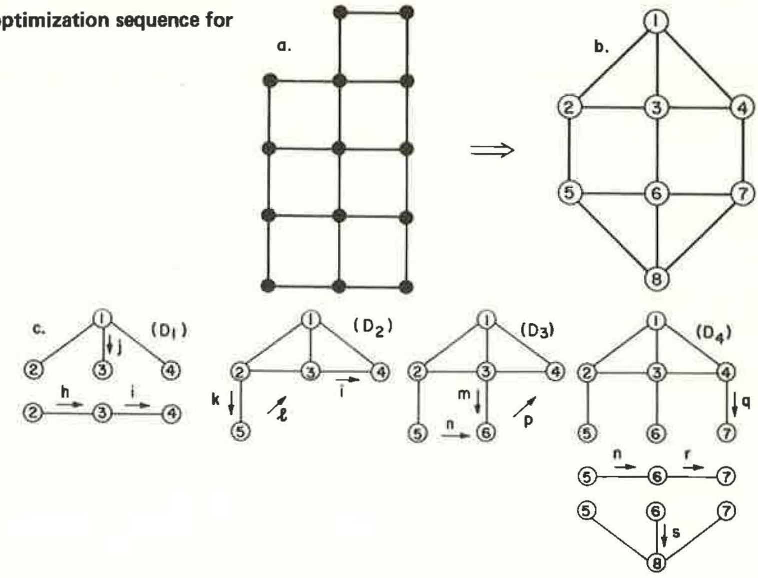


Table 1. Expected overflow queue.

Signal Capacity (vehicles per cycle)	Degree of Saturation						
	0.20	0.40	0.60	0.80	0.90	0.95	0.975
5	0.00	0.02	0.20	1.15	3.50	8.41	18.36
15	0.00	0.00	0.04	0.70	2.81	7.61	17.50
25	0.00	0.00	0.01	0.47	2.41	7.08	16.91
35		0.00	0.00	0.34	2.11	6.68	16.45
45			0.00	0.23	1.88	6.34	16.05
55					1.68	6.02	15.67

Figure 7. Overflow queue versus green-time split.

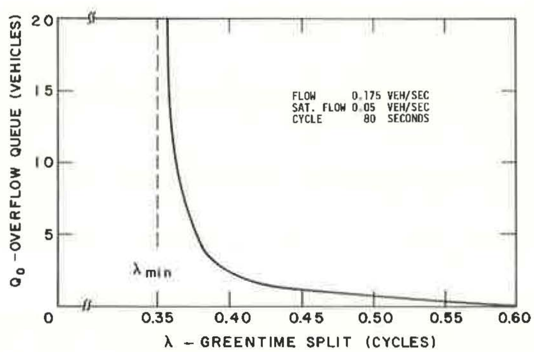
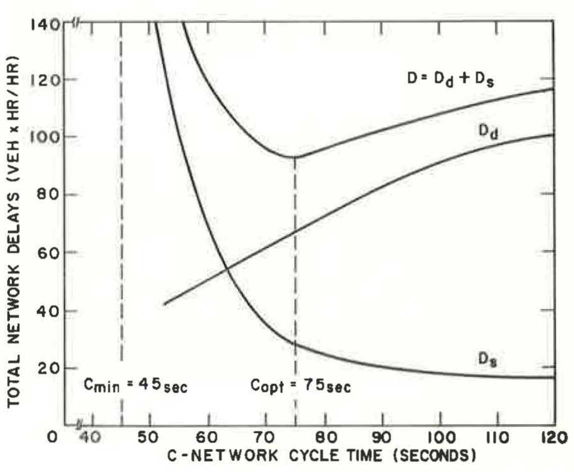


Figure 8. Variation of total network delays with cycle time.



$$\sum_j g_j = C - L \tag{11}$$

That is, the sum of effective green times on all phases equals the net green time available for movement through the intersection (cycle time less lost time). Rearranging this we obtain

$$\sum_j \lambda_j = 1 - \frac{L}{C} \tag{12}$$

where $\lambda_j = \frac{g_j}{C}$ denotes the green split j (fraction of cycle time allotted to phase j). The split, in turn, is determined as follows:

$$\lambda_j = \frac{y_j}{Y}\left(1 - \frac{L}{C}\right) \tag{13}$$

$y_j = \frac{F_{ij}}{s_j}$ is the representative ratio of flow (F_{ij}) to saturation flow (s_j) of a particular phase and $Y = \sum_j y_j$ is the sum of y-values over all phases of the intersection.

The y-values depend only on flow and saturation flow, but not on the signal settings themselves. The total lost time, L, is usually a fixed quantity at a particular intersection (3 to 5 sec for each phase). Therefore, a change in C alters the total net green time available for passage through the intersection and, consequently, its allotment to the phases—the green splits. This eventually brings about a change in the degree of saturation and, with it, the size of the overflow queue.

An estimate of the expected overflow queue, based on the capacity of the signal's approach and the degree of saturation, was calculated by Wormleighton (25) and is given in Table 1. Following field studies in Toronto, he developed a model describing the traffic behavior along a signalized link as a nonhomogeneous Poisson process with a periodic intensity function. A typical relationship between expected overflow queue and split time in this model is shown in Figure 7. Similar characteristics are used by Webster (2) in the case of the single intersection and by Robertson (9) in the TRANSYT network model.

Let us denote the expected overflow queue on link i, j with a downstream green split λ_{ij} by $Q_o(\lambda_{ij})$. The delay incurred by these queuing vehicles is simply $Q_o(\lambda_{ij})$ for any time unit that is used, such as [vehicles × hour/hour] or [vehicles × sec/sec]. The networkwide expected delay associated with the overflow queue thus will be

$$D_s = \sum_{i=1}^{n} \sum_{j=1}^{n} Q_o(\lambda_{ij}) \tag{14}$$

This provides the second component of the network objective function given in Eq. 4.

Recursive application of the GCM for different cycle times, taking into account both deterministic and stochastic effects, produces typical results as shown in Figure 8. These curves were calculated for the network shown in Figure 6a. Input links also must be included in the calculation. Although they do not affect signal coordination (calculation of offsets), they play an important role in evaluating total delay for selecting the proper cycle time.

It is evident that optimal cycle time for the network constitutes an equilibrium point between delays caused by deterministic effects and delays caused by stochastic effects. Although the former usually increase with cycle length, the latter decrease with it because of the decrease in the degree of saturation (load factor). They are asymptotic to the minimal cycle time for the network, which is the theoretical minimal cycle time for the most heavily loaded intersection that would still provide capacity if all flows were deterministic. These characteristics are completely analogous with the behavior of delay with respect to cycle time at a single intersection as studied by Webster (2). However, the results are significantly different and an analysis of a single intersection would virtually never give the optimum cycle time for the network. In the example shown in Figure 8, the optimum cycle time for the critical intersection in the network is approximately 90 sec. If this cycle time were adopted for the whole network, delay would be about 10 percent higher than optimum.

SUMMARY AND CONCLUSIONS

A systematic procedure was developed to determine signal settings (including offsets, green splits, and cycle time in a network). The basic building block of the procedure was the generalized combination method, which extended the applicability of the original combination method to networks of a general structure.

The traffic-flow model consisted of deterministic and stochastic components. The deterministic component represented periodic platoons of similar shape and size generated in a synchronized signal network. The stochastic component accounted for the variability in the characteristics of these platoons as observed in practice. A travel-cost function was associated with each component. The deterministic component cost function tended to drive cycle time down and minimized its value. On the other hand, the stochastic component cost function deterred the signal timings from approaching saturation levels at any intersection of the network and thus drove the cycle time upward. This interplay between the 2 functions was of fundamental importance in analyzing the performance of area traffic-control systems. Optimal settings in a network were determined by the least-cost equilibrium point reached as a result of this interplay.

Preliminary results obtained by applying this method to test networks indicated a potential for significant improvements in the performance of traffic-signal systems compared with other techniques in current use. As with any new model or methodology, further testing and evaluation are necessary and implementation studies are planned.

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