DESIGN TECHNIQUE FOR PRIORITY-ACCESS RAMP METERING

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One proposed method to provide preferential treatment for buses and car pools on freeways would allow buses and car pools immediate access and would meter the rest of the entering traffic. This paper presents a ramp-metering design technique that is appropriate for a situation in which all ramps under consideration have a priority-vehicle access lane. The technique is optimal in that it minimizes total passenger travel time in a limited corridor. An example design is presented to illustrate the procedure. In addition, conclusions are drawn relating to ramp queues as incentives to make travelers shift to car pools or buses.

•IN THE PAST few years, transportation planners have increasingly emphasized the consideration of alternative transportation modes on freeways. Among these alternatives are various forms of preferential treatment for buses and car pools (2, 3, 5, 6, 9, 10).

This paper is concerned particularly with the mode in which priority vehicles (buses and car pools) are provided preferential access to the freeway and nonpriority vehicles are subject to ramp metering. Recent studies have indicated the feasibility of this mode. In Los Angeles, an unused lane on a metered on-ramp was painted to indicate only car-pool access (Fig. 1)(3). In Minneapolis, following a proposal first developed by the Texas Transportation Institute, special ramps were constructed to allow preferential access to buses (Fig. 2)(6, 10).

This paper develops a ramp-metering design technique appropriate for situations in which preferential access is provided at every ramp by modifying a ramp-metering design technique developed by Payne and Thompson (9). One important aspect of the evaluation provided by this technique is the prediction of ramp waiting times for nonpriority vehicles. Because time saved not waiting in an on-ramp queue may serve as an impetus to form a car pool or to use a bus, identifying ramp waiting times is of interest.

METHODOLOGY

The design of a priority-access ramp-metering plan is approached by optimizing the performance of the traffic pattern generated by a ramp-metering plan. This involves a traffic model in which volumes are taken to be constant over time slices and route selections are made by traffic assignment. The performance measure employed is the total passenger travel time within each time slice.

Freeway Corridor Model

The freeway corridor model that we used was developed by Payne and Thompson (9). It is composed of a series of freeway links and parallel street links connected at interchanges. The network of surface streets between interchanges is aggregated into 1 equivalent street link. This aggregation of flows on the street link is the average of the traffic conditions as seen from the freeway on-ramps. Although some detail is lost, our concern is the relationship between neighboring street volumes and freeway on-ramp volumes. A portion of this network is shown in Figure 3.

The freeway corridor with N interchanges consists of 2 N nodes, 1 for the freeway

Figure 1. Shared-ramp priority access for car pools.

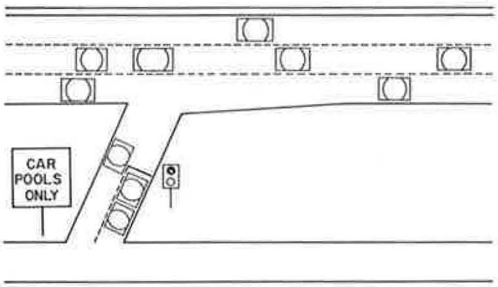
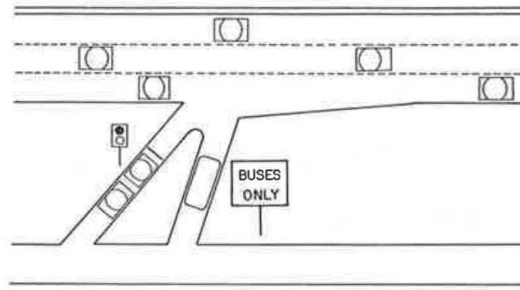


Figure 2. Bus on metered freeway ramp.

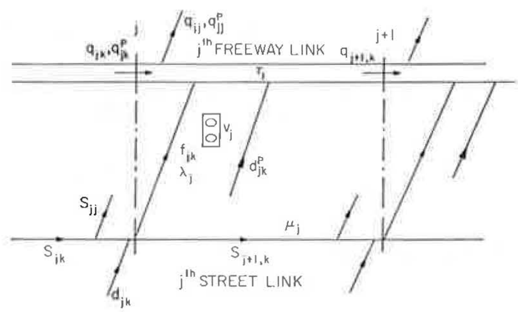


and 1 for the equivalent street. The nodes are connected by N - 1 freeway links and N - 1 street links to a section corresponding to an on-ramp, an off-ramp, or a change in freeway geometry.

Demand volumes are distinguished by origin-destination pairs. Freeway on-ramp and street-link volumes are distinguished by components that represent total flow to a destination; information relating to the origin is not retained.

Priority vehicles are allowed immediate access to the freeway. When they are on the freeway, priority and nonpriority vehicles are assumed to travel at the same speed.

Figure 3. Typical corridor interchange.



We now will put the freeway corridor model into a mathematical format. The following variables are defined for J = 1, 2, ... N and K = 1, 2, ... N, the interchange numbers:

q_{JK} = volume of nonpriority vehicles passing J on the freeway destined for the off-ramp at K $(K \geq J)$,
q^P_{JK} = volume of priority vehicles passing J on the freeway destined for the off-ramp at K $(K \geq J)$,
f_{JK} = volume of nonpriority vehicles entering the freeway at J destined for the off-ramp at K $(K \geq J)$,
d_{JK} = volume of nonpriority vehicles entering the freeway corridor at J destined for K on a surface street $(K \geq J)$, and
d^P_{JK} = volume of priority vehicles entering the freeway corridor at J destined for K on a surface street $(K < J)$.

We assume that a stationary traffic pattern exists and that flows do not vary in time. Thus there is no storage in a link. By conservation of flow, the flow into a node must be equal to the flow out of a node. For priority vehicles

$$q^P_{J+1,K} = q^P_{JK} + d^P_{JK}, \quad 1 \leq J < K \leq N$$

For nonpriority vehicles

$$q_{J+1,K} = q_{JK} + f_{JK}, \quad 1 \leq J < K \leq N$$

A capacity constraint is placed on the flow in each freeway link, and we define the following variables:

$$q_J = \sum_{K=J}^{N} q_{JK} = \text{total volume of nonpriority vehicles passing J on the freeway, and}$$

$$q_J^P = \sum^{N} q_{JK}^P = \text{total volume of priority vehicles passing J on the freeway.}$$

For each link in the freeway, the capacity constraint is

$$q_J + q_J^P \leq C_J, \quad J = 2, \ldots, N$$

In a similar manner for the street link, we define the following variables:

S_{JK} = volume of vehicles on the street approaching J that are destined for K $(K \geq J)$,

$\tilde{d}_{JK} = d_{JK} + S_{JK}$ = total volume or accumulated demand of nonpriority vehicles on the street at J destined for K $(K \geq J)$,

$S_J = \sum_{K=J}^{N} S_{JK}$ = total volume of nonpriority vehicles on street at J.

For the street network

$$S_{J+1,K} = S_{JK} + d_{JK}, \quad 1 \leq J < K \leq N$$
$$= \tilde{d}_{JK} - f_{JK}$$

Traffic Assignment Algorithm

To determine the set of ramp-metering rates that minimize the performance measure in the freeway corridor, one must determine the resulting traffic pattern for a set of fixed ramp-metering rates. Priority vehicles are assumed to enter at the first available on-ramp so that there will be no difficulty in identifying their routes. Traffic assignment is used to determine the remaining drivers' route choices. The method we used was developed by Payne and Thompson (9) and is similar to that developed by Yagar (13) in that each provides for a queue at each on-ramp. In addition, the method we used produces a traffic pattern that is user optimized. A user-optimized traffic pattern is consistent with Wardrop's first principle (12) that states that, "Journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused route." Under fixed ramp metering, a queue of vehicles will result if the ramp-metering rate is less than or equal to the vehicle arrival rate. If the ramp-metering rate is less than the vehicle arrival rate, the queue will grow extremely large and cause vehicles in the queue to have long delays. When the ramp-metering rate is equal to the vehicle arrival rate, the queue will remain at a fixed length. When the ramp-metering rate is greater than the vehicle arrival rate, there will be no queue. After drivers experiment with alternate routes and when an equilibrium is established, the vehicle arrival rate at the freeway on-ramp will be less than or equal to the ramp-metering rate. Accumulated demand, which is the sum of new demand and existing street traffic in excess of the ramp-metering rate, will be diverted by a street route to the next downstream interchange where drivers may be allowed access to the freeway or diverted again to the street.

In allocating accumulated demand to the freeway on-ramp, one diverts first the vehicles making the shortest trips. Choosing component ramp volumes on this basis is done because drivers having the farthest to travel are more likely to wait in the queue. Allocating accumulated demand to the on-ramp is continued until the ramp-metering rate equals demand. If demand does not exceed the ramp-metering rate, then all accumulated demand is allowed to enter the freeway. If this allocation is performed at interchange J, there exists an interchange number, $\ell(J)$, downstream of interchange J, such that traffic destined for an interchange upstream of $\ell(J)$ will be diverted to the street link, and all traffic destined for an interchange downstream of $\ell(J)$ will be allocated to the freeway on-ramp (Fig. 4).

The traffic assignment algorithm is divided into 3 parts. The first part selects the component ramp volumes, f_{JK}. The choice of component ramp volumes is based on the fact that drivers making the shortest journey will be diverted first. The second part computes link travel times and checks to determine that the freeway capacities (reduced by the volumes of priority vehicles) have not been exceeded. The third part computes ramp queue lengths and travel times.

On-ramp queue lengths are determined by the solution of an equilibrium equation. To facilitate explanation of this equation, we introduce the following variables:

θ_J = time required to cross the jth on-ramp,
λ_J = queue length at the jth on-ramp,
V_J = fixed ramp-metering rate at the jth on-ramp,
τ_{JK} = freeway travel time from interchange J to K $(K > J)$,
t_{JK} = total travel time from interchange J to K on the best route comprised of street and freeway links $(K > J)$, and
μ_J = travel time over street-link J.

The equilibrium equation for interchange J is

$$\theta_J + \lambda_J/V_J + \tau_{J,\ell(J)} = t_{J+1,\ell(J)} + \mu_J$$

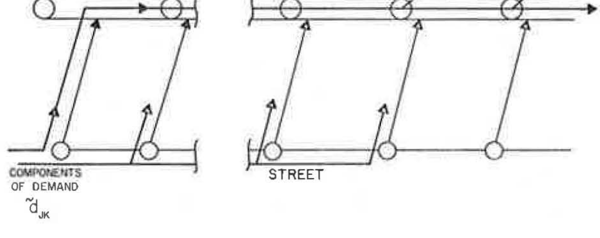
where

$\theta_J + \lambda_J/V_J + \tau_{J,\ell(J)}$ = travel time to $\ell(J)$ when on-ramp J is used, and
$t_{J+1,\ell(J)} + \mu_J$ = travel time to $\ell(J)$ when street-link J is used.

This equation embodies Wardrop's first principle (12). Because drivers traveling to $\ell(J)$ may travel over either of 2 routes, the respective travel times must be the same. This equation is solved for λ_J.

The complete algorithm (9) involves a recursive computation of λ_J, τ_{JK}, and t_{JK} that starts at the downstream end of the corridor and moves to the upstream end.

Figure 4. Bifurcation of ramp demand.



Optimal Allocation

The allocation problem is the determination of the set of ramp-metering rates that minimize total passenger travel-time rate in the freeway corridor. The total passenger travel-time rate is defined in terms of the following variables and those defined earlier.

$\tau_J = \tau_J(q_{J+1} + q^P_{J+1})$ = travel time across freeway link J, a function of link volume,
ρ = average occupancy of nonpriority vehicles, and
ρ^P = average occupancy of priority vehicles.

Total passenger travel-time rate is formulated as

$$\sum_{J=1}^{N-1} (\rho^P q^P_{J+1}\tau_J) + \rho(q_{J+1}\tau_J + S_{J+1}\mu_J + \lambda_J)$$

where

$\rho^P q^P_{J+1}\tau_J$ = passenger travel-time rate in priority vehicles, and
$\rho(q_{J+1}\tau_J + S_{J+1}\mu_J + \lambda_J)$ = passenger travel-time rate in nonpriority vehicles.

Passenger travel-time rate in nonpriority vehicles consists of vehicle travel time on streets, the freeway, and the on-ramp queue. Because priority vehicles are allowed access to the freeway with no wait, no travel time is associated with queues or streets.

If we define the passenger travel-time rate at interchange J as $g_J(q^P_{J+1}, q_{J+1}, S_{J+1}, \lambda_J)$, then the optimal allocation problem would be formulated as

$$\underset{V_1,\ldots,V_{N-1}}{\text{Min}} \sum_{J=1}^{N-1} g_J(q^P_{J+1}, q_{J+1}, S_{J+1}, \lambda_J)$$

subject to the following constraints:

$$q_{J+1} + q^P_{J+1} \le C_{J+1},\ J = 1,\ldots,N-1$$

$$0 \le V_{JMIN} \le V_J \le V_{JMAX},\ J = 1,\ldots,N-1$$

The 4 variables in the performance measure are q^P_J, the total volume of priority vehicles in freeway link J; q_J, the total volume of nonpriority vehicles in freeway link J; S_J; and λ_J. q_{J+1}, S_{J+1}, and λ_J are determined by initial freeway and street flow and the set of ramp-metering rates through the traffic assignment algorithm. q^P_{J+1} does not vary with the ramp-metering rate because priority vehicles are allowed direct access to the freeway. q_{J+1}, S_{J+1}, and λ_J can be viewed as state variables dependent on the set of control variables, ramp-metering rates, and upstream corridor conditions. There is a constraint on total freeway volume in each link, and there is an upper and lower bound on each ramp-metering rate. The constraint on freeway capacity is a function of freeway design. There is no constraint on total street-link volume because we assume that capacity is infinite.

For a problem involving 10 or more interchanges, direct optimization (8) would impose a heavy computational burden. One approach to the solution is to formulate the allocation problem as a dynamic programming problem (1). However, this approach is infeasible because of the large number of states. The number of state variables to be considered at a given interchange is equal to the number of downstream interchanges plus 1. A computationally effective compromise developed by Payne and Thompson (9) is to take a suboptimal approach. This greatly reduces the number of states to be considered.

The suboptimal approach considers discrete levels of ramp-metering rates, each of which bifurcates accumulated demand at an interchange between the street and the freeway link at a level corresponding to 1 of the downstream interchanges. For instance, at interchange J, there are N - J possible metering rates as follows:

$$\tilde{d}_{JN},\ \tilde{d}_{JN} + \tilde{d}_{J,N-1}, \ldots, \sum_{k=J+1}^{N} \tilde{d}_{JK}$$

Only those rates that meet minimum and maximum metering-rate constraints are considered.

Details of the algorithm that provides optimal bifurcation metering rates are given elsewhere (9).

DESIGN EXAMPLES

Freeway Location and Geometry

The segment of freeway used in this example is part of northbound I-405 (San Diego Freeway) in Los Angeles County, California, from the Vermont Avenue on-ramp to a point that is just upstream of the eastbound Imperial Highway off-ramp. Figure 5 shows this section of the freeway.

Fixed ramp metering has been used on this freeway segment since 1972 (5). For our purposes, the freeway segment is divided into 10 links. The upstream boundary of each link corresponds to the first off-ramp in the interchange. Link 6, however, has no off-ramp. Links 8, 9, and 10 have 2 on-ramps, and link 10 has 2 off-ramps. The on-ramps and off-ramps in these links correspond to eastbound and westbound street-traffic on-ramps and off-ramps near interchanges 9 and 10 and northbound and southbound street-traffic on-ramps and off-ramps near interchange 8.

Table 1 gives some additional information about the freeway geometry. Each freeway link has 4 lanes except for link 9, which has 5. Freeway capacity is assumed to be 2,025 vehicles per hour (vph) per lane. Link 9's capacity, which is reduced, is 1,800 vph per lane. This lower value was chosen because we assumed that the higher freeway capacity in the other links could not be maintained where merging was anticipated. The upper bound on on-ramp rates is 900 vph per lane.

Traffic Engineering Data

The volume and density data used to estimate the speed-volume relationship and origin-destination pairs were derived from 1-min averages of loop-sensor data. The loop-sensor data for this segment of freeway for April 23, 1974, from 6:30 to 9:00 a.m. were obtained from the California Department of Transportation.

A parabolic relationship between speed and volume was used. The region of the curve representing uncongested flow was used for subsequent computations of speed from volume. A freeway loop-sensor station nearest the upstream boundary of a link was used to derive the speed-volume relationship for that link by a least squares fit. A sample of the least squares fit of the speed-volume relationship is shown in Figure 6.

The volume data for the upstream boundary of the freeway segment and the volume

Figure 5. Freeway segment for design example.

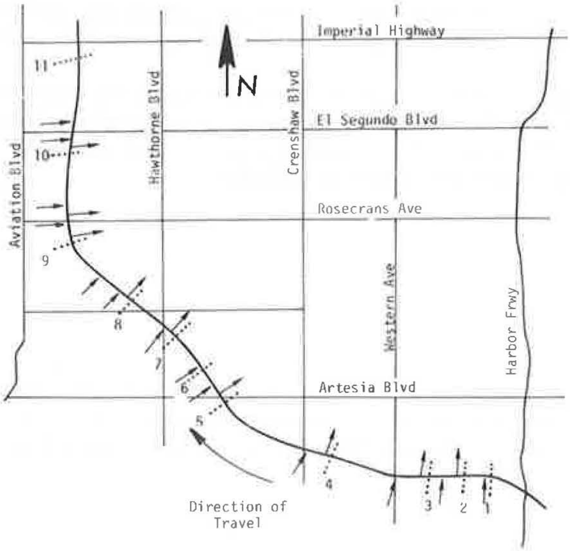
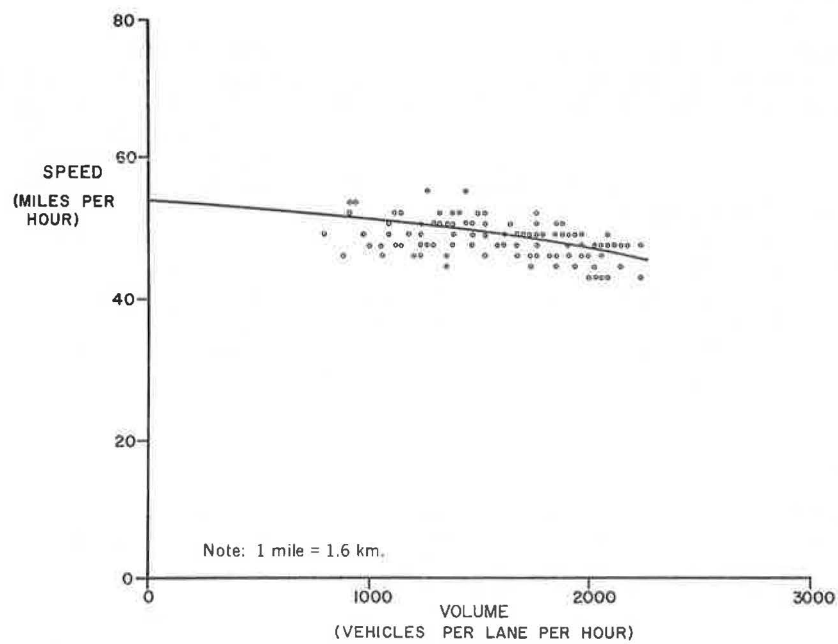


Table 1. Corridor geometry.

Interchange Link	Link Length (miles)
1. Vermont Avenue	0.408
2. Normandie Avenue	0.542
3. Western Avenue	1.051
4. Crenshaw Boulevard	1.294
5. Artesia Boulevard	0.431
6. Redondo Beach Boulevard	0.428
7. Hawthorne Boulevard	0.647
8. Inglewood Boulevard	0.983
9. Rosecrans Avenue	0.896
10. El Segundo Boulevard	0.790

Note: 1 mile = 1.6 km.

Figure 6. Speed-volume relationship.



data for the on-ramp and off-ramp volumes were averaged over 15-min intervals starting at 6:30 a.m. Origin-destination data, which were compatible with observed data, were estimated.

Comparison to Present Design

The traffic assignment algorithm was used to determine the traffic pattern and performance measure in the corridor that corresponded to current ramp-metering design and estimated origin-destination data. A new ramp-metering design was determined by the optimal allocation algorithm for the same origin-destination data. In both applications of the methodologies explained previously, identical speed-volume relationships and freeway geometry data for each link were used as inputs to the 2 algorithms. With the 2 algorithms, a comparison was made of the difference between the current design and optimal design from 6:30 to 8:00 a.m. Comparisons of current ramp-metering design and optimal ramp-metering design are given in Tables 2 and 3. The comparison shows that there is a difference in the 2 designs. The optimal allocation algorithm tends to fill an unused portion of the freeway. Note, however, that the current design was developed from a different set of origin-destination data. Table 4 gives a comparison of the performance measure for the 2 different designs. The difference in the total passenger travel time of the 2 designs is not significant, which indicates that the optimal ramp-metering design produces only marginal changes in freeway corridor performance. An Appendix[1] includes additional comparisons of link speeds, travel times, and ramp queue lengths.

Table 2. On-ramp metering rates for links 1 through 5.

	Link 1		Link 2		Link 3		Link 4		Link 5	
Time	Current Design	Optimal Design	Current Design	Optimal Design	Current Design	Optimal Design	Current Design	Optimal Design	Current Design	Optimal Design
6:30 a.m.	180	172	180	172	180	195	240	424	180	551
6:45 a.m.	180	475	180	188	360	212	240	472	240	548
7:00 a.m.	140	470	220	176	300	244	360	632	240	180
7:15 a.m.	200	551	140	172	260	280	580	508	480	208
7:30 a.m.	120	596	100	172	260	180	520	508	460	256
7:45 a.m.	140	184	120	338	220	176	540	384	140	500

Note: Values are in vehicles per hour.

Table 3. On-ramp metering rates for links 6 through 10.

	Link 6		Link 7		Link 8		Link 9		Link 10	
Time	Current Design	Optimal Design	Current Design	Optimal Design	Current Design	Optimal Design	Current Design	Optimal Design	Current Design	Optimal Design
6:30 a.m.	180	216	180	552	720	551	1,580	1,273	1,000	320
6:45 a.m.	180	236	180	581	720	596	720	1,388	960	344
7:00 a.m.	180	196	180	368	540	632	1,440	1,276	1,000	352
7:15 a.m.	560	208	350[a]	344	540	1,177	1,280	1,368	980	368
7:30 a.m.	560	188	350[a]	342	840	1,454	1,360	1,600	1,240	400
7:45 a.m.	500	352	660	360	1,040	568	840	1,760	800	360

Note: Values are in vehicles per hour.

[a]Design on ramp meter changed to reflect observed values.

[1] The original manuscript of this paper included an appendix. This appendix is available in Xerox form at cost of reproduction and handling from the Transportation Research Board. When ordering, refer to XS-55, Transportation Research Record 533.

Table 4. Total passenger travel-time rates.

Time	Current Design (passenger hours)	Optimal Design (passenger hours)
6:30 to 6:45 a.m.	2,055	2,047
6:45 to 7:00 a.m.	2,100	2,059
7:00 to 7:15 a.m.	2,201	2,183
7:15 to 7:30 a.m.	2,061	2,035
7:30 to 7:45 a.m.	2,135	2,100
7:45 to 8:00 a.m.	2,245	2,233
6:30 to 8:00 a.m.	3,199	3,164

Ramp-Metering Plans Under Modal Shift

An analysis of the effect of modal shift on total passenger travel time within the corridor was performed by using the optimal allocation algorithm. Modal shift is the difference in the percentage of passengers using priority vehicles from the initial distribution of passengers in vehicles. The analysis is performed to study the differential between the passenger travel times for priority and nonpriority vehicles, which is of particular interest because this difference may act as an incentive to drivers to shift to the priority-vehicle mode. It is assumed that the results are measured after a modal shift has occurred.

The distribution of vehicle occupancy is as follows:

Passengers per Vehicle	Percentage of Vehicles
1	80
2	15
3	5

The average occupancy of this distribution is 1.25 passengers per vehicle. It can be seen from this distribution that 12 percent of the passengers in the freeway corridor would be given preferential treatment, if no modal shift occurred and preferential access were allowed to vehicles with 3 or more passengers. The method for calculating the reduction of vehicles by a change in the mode of travel of passengers for a nonpriority vehicle to a priority vehicle is as follows. Five new priority vehicles containing 3 passengers are created from 3 nonpriority vehicles containing 2 passengers and 9 nonpriority vehicles containing 1 passenger. The change in mode of travel is done uniformly throughout the corridor. The reduction in vehicular demand occurs both for upstream components of total freeway volume and for components of total on-ramp demand. During the analysis of different levels of modal shift, the passenger demand from an origin to a destination remains constant. However, as the modal shift increases, demand, in terms of vehicles, decreases.

The optimal allocation algorithm was used to determine the ramp-metering plan and the resultant traffic pattern for different levels of modal shift. Figure 7 shows a comparison of passenger travel times for increasing modal shifts. The greatest decrease in passenger travel time occurs when 24 to 36 percent of the passengers are in priority vehicles.

If we reallocate passengers by the method previously described, a 12 percent increase in passengers using priority vehicles would produce a 7 percent decrease in total corridor demand. Estimated demand in the traffic corridor would exceed freeway capacity by 7 to 14 percent because on-ramp queues are reduced significantly after a modal shift of 12 percent. Table 5 gives queue waiting time for different levels of modal shift. It is apparent from Table 5 that there is little incentive after a modal shift of 12 percent. However, this level of modal shift is sufficient to reduce demand in the corridor to freeway capacity.

A sensitivity analysis was performed to determine the effect of overestimation or underestimation of modal shift on total passenger travel time in the corridor. A set of optimal ramp-metering designs for a specific modal shift were chosen, and the traffic pattern and performance measure for different modal shifts were determined. Table 6 gives the results of the sensitivity analysis.

It can be seen from the data given in Table 6 that, if modal shift is overestimated, if optimal ramp-metering design is based on a modal shift that is greater than actual

Figure 7. Passenger travel time, 6:30 to 8:00 a.m.

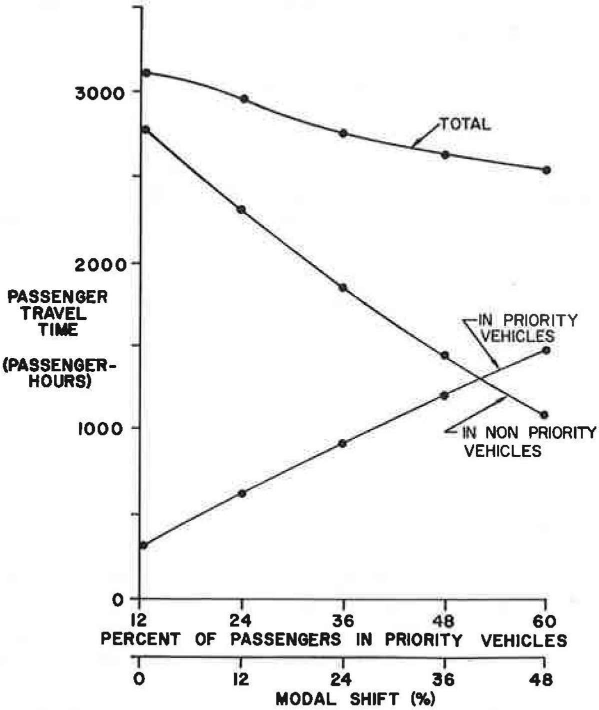


Table 5. On-ramp waiting times.

Level	Wait (min)									
	Link 1	Link 2	Link 3	Link 4	Link 5	Link 6	Link 7	Link 8	Link 9	Link 10
No priority access	3.3	2.8	2.6	2.2	1.3	1.3	1.1	0.6	0	0
Priority access										
No modal shift	3.2	2.8	2.6	2.1	1.3	1.3	1.1	0.6	0.1	0
12 percent modal shift	0.4	2.8	2.6	2.2	1.3	1.0	0.7	0.3	0.1	0
24 percent modal shift	0	2.4	2.2	1.6	0.8	0.4	0.2	0	0.1	0
36 percent modal shift	0	0	0	0	0	0	0	0	0	0

Table 6. Sensitivity of performance to estimates of modal shift.

Modal Shift	Ramp-Plan Modal Shift			
	None	12 Percent	24 Percent	36 Percent
None	2,012	—[a]	—[a]	—[a]
12 percent	1,977	1,857	—[a]	1,994
24 percent	1,844	1,851	1,777	1,929
36 percent	1,870	1,800	1,798	1,713

Note: Values are in passenger hours.

[a]Traffic demand exceeds freeway capacity.

modal shift, freeway links may be congested. The greater the difference is between actual modal shift and modal shift assumed in developing the design, the greater the degradation is in performance when modal shift is underestimated.

When modal shift is overestimated, the freeway segment may become congested. The optimal ramp-metering design used is based on a demand that is less than actual demand. In this case, ramp-metering rates are chosen that allow more vehicles on the freeway because upstream freeway volume is reduced. When modal shift is underestimated, the freeway segment is underused. The optimal ramp-metering design allows fewer vehicles on the freeway because upstream freeway volume is assumed to be larger than actual volume. As a consequence, vehicles are unnecessarily denied access to the freeway from on-ramps.

The sensitivity analysis and comparison of corridor performance when preferential access is presented are based on estimated origin-destination data. The estimated origin-destination data do not encompass latent demand within the corridor. If preferential access is allowed, reduced corridor demand might attract more vehicles that currently are not entering the freeway corridor.

CONCLUSIONS

In this paper, techniques for evaluating and designing priority-access ramp-metering plans have been presented. The design technique presented generates a ramp-metering plan that minimizes total passenger travel time in a freeway corridor and predicts the traffic pattern, including ramp queues, that would result. These techniques are limited to use in situations in which every on-ramp has priority access. To extend these techniques to situations in which only certain ramps have priority access would require new methodological development.

The presence of ramp queues might serve to induce drivers without passengers to form or join car pools or to use buses. The methodology presented here includes a prediction of on-ramp waiting times so that one might assess this factor as a motivation for a modal shift. In the example presented here, there was little incentive to form additional car pools after a modal shift of about 12 percent.

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