STATISTICAL ANALYSIS OF CONSTRAINED SOIL MODULUS

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Well-known statistical techniques are used to formulate mathematical expressions for the stress-strain response of soil subjected to a uniaxial strain test. A special mold was used to compact 28 soils at three different densities, and uniaxial strain tests were performed on each sample. A parabolic relationship, a hyperbolic equation, and an exponential formulation are used to describe the measured stress-strain response, and the associated coefficients are assumed to be linear functions of the normalized soil parameters. Among these formulations, the exponential equation is most accurate. The relationship between the constrained modulus and the stress level is also described by several equations in which the coefficients are linear functions of the normalized soil parameters, and the modified Janbu equation is found to yield the best results. The parameters that exert the greatest influence on the constrained modulus are associated with densities, and, if the approximations assumed here are accepted, this modulus can be estimated from the results of standard laboratory compaction tests and field density measurements without an expensive series of tests.

•A QUICK and inexpensive estimate for the modulus of the soil is desirable for solving problems involving soil systems and soil-structure interaction. Notwithstanding the well-recognized need for appropriate laboratory or field tests commensurate with the required degree of accuracy for a particular problem and the degree of sophistication used in the analysis, there are many occasions when the nature of the problem or the purpose of the analysis requires only a cursory estimate of the soil modulus. For example, in most problems concerned with small-diameter, reinforced concrete pipe with cover heights of a few diameters, the response of the soil-pipe system is not sensitive to moderate variations in the soil modulus, and reasonable estimates are usually satisfactory for design purposes. Accordingly, this paper is directed toward helping the engineer obtain a reasonable estimate for the modulus of a given soil that has not actually been tested for stress-strain characteristics. Within the framework of the statistical analyses conducted on test data from the 28 soils, modulus values of similar soils can be reasonably well estimated from a knowledge of density information only.

BACKGROUND

The results of a uniaxial strain test can be used to define a constrained modulus, which is taken as the slope of the stress-strain curve. Based on the theory of elasticity, the constrained modulus M is related to the modulus of elasticity E by

$$M = E\left[\frac{1 - \nu}{(1 + \nu)(1 - 2\nu)}\right] \tag{1}$$

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where ν is Poisson's ratio. Because of the relative simplicity of conducting a uniaxial strain test in the laboratory, the constrained modulus has frequently been used as an input parameter in problems of soil-structure interaction, and its evaluation has been a subject of much research effort. Work by Schultze and his coworkers (6, 7, 8) shows that the stress-strain response and the constrained modulus determined from confined compression tests on clean sands, silty sands, silts, and ballast can be given with sufficient accuracy by

$$\epsilon = A\sigma^{B} \quad (2)$$

and

$$M = C\sigma^{D} \quad (3)$$

where A, B, C, and D can be correlated with the void ratio or porosity, water content, grain size distribution, percentage of clay (or plasticity index), and activity of the individual clay particles. Using the above expressions, Janbu (2) proposed the following relationship:

$$M = mp_a\left(\frac{\sigma}{p_a}\right)^{1-a} \quad (4)$$

where m and a are unique functions of porosity, and p_a is atmospheric pressure (introduced to maintain dimensional homogeneity). Krizek, Parmelee, Kay, and Elnaggar (3) interpreted the results presented by Osterberg (5) and suggested that the constrained modulus of a compacted soil may be a unique function of the dry density and the overburden pressure. Accordingly, a number of regression equations, including those of Schultze and Janbu, are used to examine the correlation between the constrained modulus and the stress level for a variety of compacted soils tested under uniaxial strain conditions.

EXPERIMENTS

Twenty-eight soils, ranging from gravelly sand to kaolinite, were tested so that most of the common soils encountered in nature, as well as some usually used solely in laboratory studies, could be included in the experiments. The classification of each soil was determined from standard index tests, and the compaction characteristics were determined by use of the modified Proctor test (ASTM D 1557). The maximum dry density from this test allows an evaluation of the degree of compaction, which plays an important role in the compressibility of soils. A summary of the engineering characteristics and classifications of all soils tested in these experiments is given in Table 1.

Stress-strain characteristics were determined from uniaxial strain tests, which were conducted as follows. First, each soil was compacted at its optimum water content to obtain three different dry densities (namely, the maximum dry density corresponding to the modified Proctor test, a density 10 percent above this maximum, and a density 10 percent below this maximum). These densities are qualitatively termed dense, medium, and loose states of compaction. A special mold, shown schematically in Figure 1, made of three standard consolidation rings, three related separating rings, and a confining jacket, was used to compact the samples. This thereby minimized the sample disturbance associated with trimming the specimens. The specimens were

Table 1. Soil characteristics.

Soil Classification													
Unified	AASHTO	w_L (percent)	w_P (percent)	PI (percent)	Gravel (percent)	Sand (percent)	Silt (percent)	Clay (percent)	D_{10} (mm)	D_{60} (mm)	C_u	$\frac{\gamma_d}{\gamma_w}$	w_{opt} (percent)
SP	A-2	56	28	28	0	95	3	2	0.3	0.6	2	1.89	5
CL	A-7	48	24	24	0	3	54	43	0.0006	0.005	8	1.88	16
CL	A-7	49	28	21	0	0	20	80	0.0003	0.001	3	1.40	15
CL	A-7				0	50	27	23	0.0005	0.190	380	2.08	9
CL	A-4	56	28	28	0	60	20	20	0.0003	0.07	2	1.97	10
CL	A-6				0							1.94	14
CH	A-7	69	26	43	0	4	56	40	0.0003	0.005	17	1.78	17
ML	A-4	36	27	11	0	3	91	6	0.003	0.02	7	0.79	14
SW	A-2				0	95	5	0	0.5	4.0	8	2.23	8
ML	A-7	46	23	23	0	40	35	25	0.001	0.06	60	1.73	18
SP	A-2	48	24	24	0	93	5	2	0.13	0.38	3	1.81	9
SP	A-2	48	24	24	0	88	3	9	0.003	0.34	113	1.94	9
SP	A-2	48	24	24	0	75	8	17	0.0012	0.38	317	1.99	10
SF	A-4	20	14	6	0	16	51	33	0.0009	0.009	10	2.07	10
CL	A-4	21	13	8	9	19	37	35	0.0009	0.012	13	1.96	12
CL	A-6	32	19	13	0	0	49	51	0.0008	0.003	3	1.83	16
CL	A-4	26	17	9	0	6	50	44	0.0008	0.004	5	1.97	12
CL	A-4	25	15	10	10	25	40	25	0.0012	0.04	33	1.85	15
SF	A-2				0	31	45	24	0.001	0.025	25	2.05	12
CL	A-6	37	21	16	0	4	58	38	0.0009	0.006	7	1.80	16
GW	A-1				50	46	4	0	0.08	2.4	30	1.88	14
SW	A-1				5	92	3	0	0.09	0.55	6	1.96	12
SW	A-1	37	23	14	44	40	16	0	0.05	2.2	44	1.84	14
SP	A-1				3	93	4	0	0.1	0.4	4	1.95	8
SF	A-4	19	13	6	0	27	47	26	0.001	0.025	25	1.97	11
SW	A-1				20	70	10	0	0.10	1.5	15		
CL	A-4	24	16	8									
CL	A-4	20	14	6									
CH	A-7	52	27	25									

Figure 1. Special compaction mold.

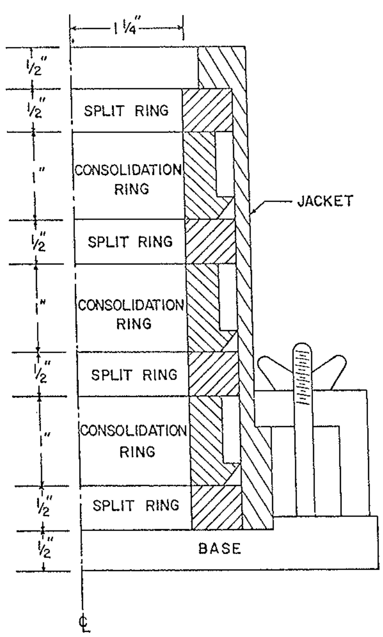


Figure 2. Stress-strain behavior for soil compacted at different densities.

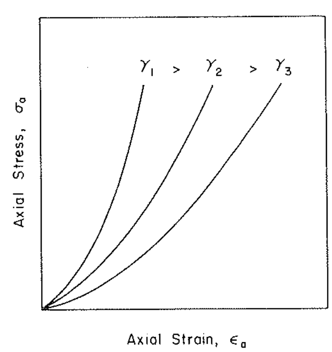
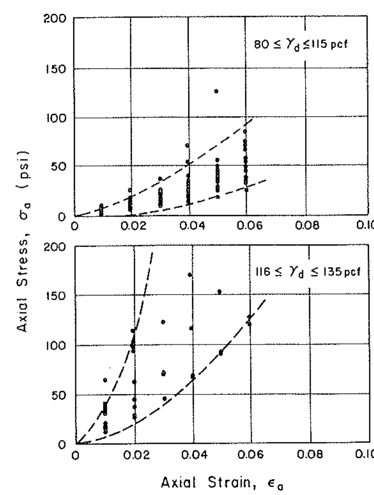


Figure 3. Stress-strain data for soil groups B and C.



loaded incrementally, and each load increment was maintained for several hours or until the increase in vertical strain essentially ceased. The typical response curves shown qualitatively in Figure 2 were obtained for maximum axial stresses at 100 psi (689 kPa) and for a few percentages of axial strains. All samples were submerged (but not necessarily saturated) for 24 hours before testing, and each sample was subjected to a sufficient load to prevent any swelling. Except for a few cases, all swelling pressures were less than 5 psi (35 kPa), and these stresses were subsequently not included in the axial stress-axial strain data. The preconsolidation stress due to compaction was not measured, and it has not been incorporated into the subsequent analyses. However, in view of the nature of most of the soils tested, the specimen thicknesses, and the fact that submerging the specimens for 24 hours tended to relieve the associated capillary stresses and relax the specimen, it is believed that these residual stresses are small in most cases. Although potentially important, the effect of water content has not been studied explicitly in this paper, except insofar as it can be back-calculated from assumptions about the dry density, specific gravity of solids, and condition of approximate saturation.

METHOD OF ANALYSIS

Among the various multiple linear regression methods of analysis, the stepwise procedure seems to offer the most practical compromise between completeness and tractability (1). This technique begins when the most correlated predictor of the response under consideration is inserted into the model. At every stage of the regression, the variables incorporated into the model in previous stages are reexamined. Therefore, a variable that may have entered the regression as the best single variable at an early stage may be ineffective at a later stage because of the relationships between it and other variables then in the regression. So that this hypothesis could be checked, the partial F-criterion for each variable in the regression at any stage of calculation was evaluated based on the ratio of sample variances and was compared with a preselected percentage point of the appropriate F-distribution. This provided a judgment of the marginal contribution made by each variable, irrespective of its actual point of entry into the model, and any variable that provided a nonsignificant contribution was removed from the model. This process was continued until convergence was achieved. The initial F-value was held constant throughout the analysis because it is simpler to choose fixed critical values that do not depend on changing degrees of freedom. The 5 percent significance level of the F-distribution corresponds to a value of 3.72 for 273 degrees of freedom, which is the initial number of degrees of freedom of the largest sample analyzed. The somewhat less discriminating value of 3.0 was selected to permit the addition of more variables to the regression equation.

ANALYSIS OF STRESS-STRAIN DATA

The compressibility of a compacted soil is affected by many variables, among which Lambe (4) lists temperature, soil composition, characteristics of the permeating material, void ratio, degree of saturation, and structure. However, many of these variables are taken into account when dry density is considered. For clay samples compacted to the same dry density, one above and one below optimum moisture content, Lambe suggests that the one compacted at the lower moisture content will exhibit a more nearly linear void ratio-stress relationship, but he does not indicate the degree of difference. Osterberg (5) suggests that this difference may not be sufficient in practice to prohibit the use of some average curve for design purposes. In addition, Lambe's observations apply primarily to clay soils, and these variations would probably be less for soils with a lower clay content since the amount of water held will be much less.

The soils tested were divided into groups before any statistical analysis was undertaken, and the subdivisions are as follows:

Group	Subdivision
A	All soils tested
B	Soils with $80 \le \gamma_d \le 115$
C	Soils with $116 \le \gamma_d \le 135$
D	Natural soils
E	Laboratory soils

Group A leads to regression equations that represent a wide variety of soils. However, groups B and C and Figure 3 show that the range of density substantially affects the nature of the stress-strain response. Groups D and E represent the most common soils encountered in nature and those special soils (such as Grundite, kaolinite, and Ottawa sand) that are used primarily for laboratory studies.

The statistical analysis of the stress-strain data was carried out by use of the following alternatives:

$$\sigma = \alpha_1 + \alpha_2\epsilon + \alpha_3\epsilon^2 \tag{5}$$

$$\sigma = \frac{a\epsilon}{1 - b\epsilon} \tag{6}$$

$$\epsilon = c\left(\frac{\sigma}{pa}\right)^d \tag{7}$$

where α_1, α_2, α_3, a, b, c and d are different linear functions of the following material properties: liquid limit w_L, plasticity index PI, plastic limit w_p, optimum water content w_{opt}, percentage of sand SAND, percentage of silt SILT, percentage of clay CLAY, specific gravity G_s, normalized maximum density γ_m/γ_w, normalized dry density γ_d/γ_w, and degree of compaction γ_d/γ_m. Equation 5 corresponds to a parabola with its axis parallel to the σ direction, equation 6 represents a two-constant hyperbola, and equation 7 is a modification of Schultz's expression. Equations 6 and 7 yield the value of the axial stress in units dictated by the empirical coefficients (in this study, they are psi); however, equation 7 is completely dimensionless since atmospheric pressure is introduced to maintain dimensional homogeneity. Not all of the material properties involved in the proposed mathematical equations are introduced into the final description; only those parameters that contribute most to the prediction of the stress-strain response are included. Tables 2 through 5 give the final regression parameters. Figure 4 shows the comparison between measured and calculated stresses from equation 7. In Tables 2 and 3, an observed value of stress will be within the standard error of the predicted value approximately two out of three times and within two standard errors approximately 95 percent of the time. In Tables 4 and 5, the statistics apply to the natural logarithm of the strain.

EVALUATION OF CONSTRAINED MODULUS

The constrained modulus is influenced by the same factors that govern the behavior of the stress-strain response; since the latter is nonlinear, the modulus will depend on the stress level at which it is evaluated. For correlation purposes in this paper, the chord modulus was determined at strain intervals of 1 percent and associated with the stress level at the middle of the interval. Although the tangent modulus calculated from the foregoing equations could have been used for these correlations, the differences between the chord modulus and the tangent modulus were sufficiently small to

Table 2. Regression parameters for equation 5.

Group	Mult. R	F-Value	Std. Error	Coefficients α_1	α_2	α_3
A	0.8245	55.62	18.31	-0.37	36.2 PI - 29.2 SAND - 12.7 SILT - 33.5 CLAY - 1,613 G_s - 1,226 γ_d/γ_w + 11,106 γ_d/γ_n	-3,705 w_L + 1,298 PI + 29,497 γ_d/γ_n
B	0.8070	63.48	16.95	-1.93	23.5 PI - 8.05 CLAY - 2,776 G_s + 9,702 γ_d/γ_n	-428.1 w_L
C	0.8712	34.65	18.78	-197.8 + 0.25 SILT + 96.2 γ_d/γ_w	72.1 w_L + 481 w_{opt} - 165 CLAY	-5,119 w_L - 7,803 PI + 4,430 SAND
D	0.8249	57.79	17.37	-86.1 + 92.2 γ_d/γ_n	-4,764 - 65.7 w_L + 95.8 PI + 65.0 w_{opt} -15.9 CLAY + 6,603 γ_d/γ_n	—
E	0.9474	120.92	11.81	40.9 - 21.2 γ_n/γ_w	34.7 SILT - 2,551 G_s + 4,060 γ_d/γ_w	966.8 PI

Table 3. Regression parameters for equation 6.

Group	Mult. R	F-Value	Std. Error	Coefficients a	b
A	0.9499	351.07	13.90	-701.4 G_s + 2,484 γ_d/γ_n	0.30 PI - 0.46 w_P - 0.20 CLAY - 12.5 γ_n/γ_w + 50.2 γ_d/γ_n
B	0.9544	439.61	11.82	1.92 SAND + 87.5 G_s	-34.7 G_s + 119.8 γ_d/γ_n
C	0.9607	220.66	14.66	46.1 w_{opt}	1.48 w_{opt} - 0.72 CLAY - 26.8 γ_n/γ_w + 0.61 γ_d/γ_n
D	0.9474	239.21	13.70	6.04 SAND + 10.62 CLAY	0.40 w_L - 0.89 w_P - 0.37 CLAY - 26.2 G_s + 103.9 γ_d/γ_n
E	0.9812	619.69	9.55	46.2 SAND	-46.8 G_s + 147.5 γ_d/γ_n

Table 4. Regression parameters for equation 7.

Group	Mult. R	F-Value	Std. Error	Coefficients log c	d
A	0.9748	501.05	0.1399	12.27 + 0.0136 w_L - 0.022 PI + 0.0021 SAND + 0.0063 CLAY - 0.0069 γ_n/γ_w + 0.0075 γ_d/γ_w - 15.51 γ_d/γ_w	1.483 + 0.0017 CLAY - 1.003 γ_d/γ_w
B	0.9779	615.67	0.1305	-1.91 - 0.003 PI - 0.001 SAND + 1.09 G_s - 2.88 γ_d/γ_n	0.66 G_s - 1.32 γ_d/γ_n
C	0.9720	256.22	0.1514	-0.47 - 0.046 w_{opt} + 0.009 CLAY - 0.54 γ_d/γ_w	0.60 - 0.015 w_{opt} + 0.004 CLAY
D	0.9791	434.43	0.1270	0.58 + 0.012 w_L - 0.021 PI - 0.002 SAND + 0.003 CLAY - 2.56 γ_d/γ_n	1.80 + 0.004 w_L - 0.007 PI - 0.002 SAND - 1.24 γ_d/γ_n
E	0.9881	396.35	0.1016	2.69 + 0.039 w_{opt} - 0.007 SILT - 0.94 G_s - 2.32 γ_d/γ_n	-0.003 CLAY + 0.60 G_s - 1.05 γ_d/γ_n

Table 5. Simplified regression parameters for equation 7.

Group	Mult. R	F-Value	Std. Error	Coefficients log c	d
A	0.9645	1,195.75	0.1636	0.22 - 2.06 γ_d/γ_n	0.57
B	0.9761	862.21	0.1348	1.05 - 2.96 γ_d/γ_n	0.60
C	0.9702	291.44	0.1552	-0.22 - 0.043 w_{opt} - 0.56 γ_d/γ_w	0.53
D	0.9603	1,155.43	0.1707	0.044 - 1.85 γ_d/γ_n	0.58
E	0.9751	695.04	0.1416	0.005 - 1.92 γ_d/γ_n	0.53

allow the two to be used interchangeably. The equations for the statistical analysis are as follows:

$$M = \alpha_1 + \alpha_2\sigma + \alpha_2\sigma^2 \tag{8}$$

$$(\log M)^2 = \alpha_1 + \alpha_2\sigma + \alpha_3\sigma^2 \tag{9}$$

$$M = \alpha_1 + \alpha_2\sigma \tag{10}$$

$$\log M = \alpha_1 + \alpha_2\sigma \tag{11}$$

$$(\log M)^2 = \alpha_1 + \alpha_2\sigma \tag{12}$$

$$\frac{M}{pa} = m\left(\frac{\sigma + p_a}{p_a}\right)^n \tag{13}$$

where the constants are linear functions of the soil parameters. The vertical stress and the constrained modulus in equations 8 through 12 must be expressed in psi. Equation 13 is a modification of the one proposed by Janbu (2), but atmospheric pressure has been added to the vertical stress to obtain an absolute pressure. The modulus is normalized with respect to atmospheric pressure, and thereby yields a dimensionless equation. Regressions of equations 8 to 13 yielded parameters for each group of soils, and selected results are given in Tables 6 through 9. Figure 5 shows the comparison of measured and calculated normalized constrained modulus from equation 13. In Tables 6, 8, and 9, the standard error refers to the natural logarithm of the modulus, and in Table 7 it refers to the squared logarithm.

CONCLUSIONS

Twenty-eight soils with a wide range of grain size characteristics were compacted at optimum water content to ±10 percent of the dry densities corresponding to a modified Proctor compactive effort. From the equations proposed to characterize the stress-strain response of these soils tested in confined compression, the power-law formulation with dimensionless coefficients and variables yielded the best results. The parabolic equations were inconsistent in maintaining the concave curvature typical for this type of response, and the hyperbolic equations only predicted the response accurately for low strains (the stress became infinitely large as the strains approached a particular empirical value). The principal soil properties that controlled this response were the degree of compaction and either the actual dry density or the maximum dry density. Since all other soil properties considered in this paper exerted a substantially lesser degree of influence, simplified expressions for material behavior were obtained by neglecting the influence of these other properties in many cases. In particular, the power-law equation, where the coefficients were functions only of the soil properties determined in the compaction test and the field density test, offered a good description of the soil behavior.

The most suitable regression equations obtained for the constrained modulus were a straight line and a parabola, both in a semilogarithmic representation, and the modified Janbu equation. The rest of the equations studied were generally unsatisfactory for a variety of reasons, including the prediction of unacceptable negative values and the

Figure 4. Measured versus calculated normalized vertical stress for equation 7.

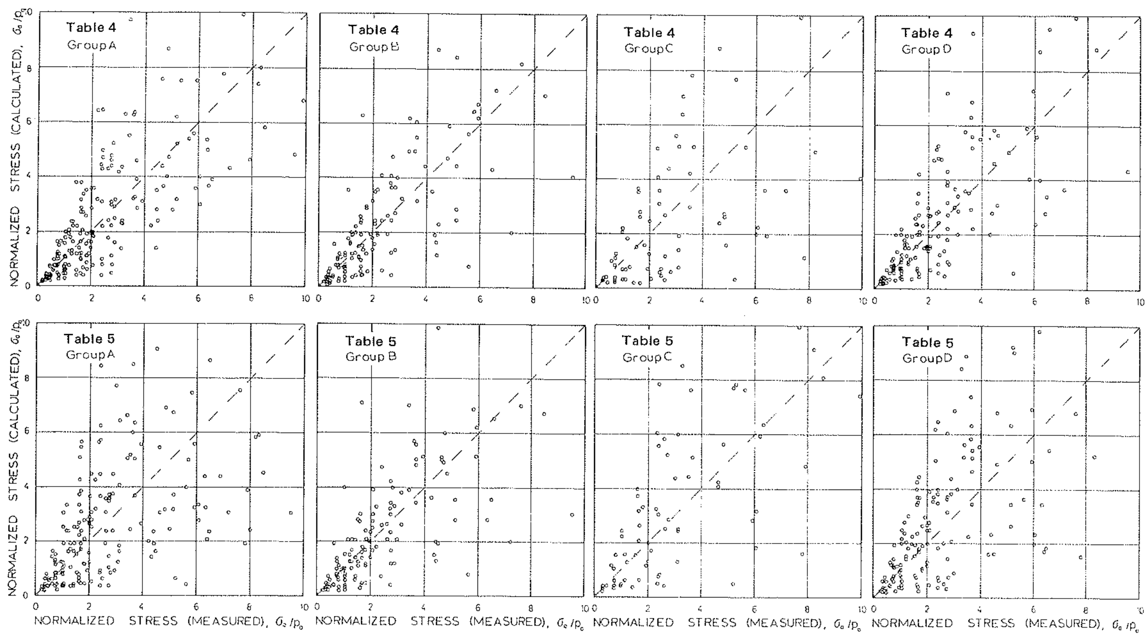


Table 6. Regression parameters for equation 11.

Group	Mult. R	F-Value	Std. Error	Coefficients α_1	α_2
A	0.8336	59.69	0.2411	$-14.92 + 0.008\ PI + 0.004\ SAND + 7.45\ \gamma_s/\gamma_w - 8.07\ \gamma_d/\gamma_w + 18.83\ \gamma_d/\gamma_s$	$0.052 + 0.00042\ w_p - 0.0016\ w_{opt} - 0.00005\ SILT - 0.028\ \gamma_d/\gamma_s$
B	0.8645	70.98	0.2163	$0.86 + 0.008\ w_L - 0.063\ w_{opt} - 0.66\ \gamma_s/\gamma_w + 3.93\ \gamma_d/\gamma_s$	$0.00012\ SAND + 0.014\ G_s - 0.033\ \gamma_d/\gamma_s$
C	0.8170	30.11	0.2519	$3.09 - 0.016\ w_L + 0.059\ PI - 0.013\ CLAY$	$0.0007\ PI - 0.0018\ w_{opt} + 0.025\ \gamma_d/\gamma_s$
D	0.8609	67.64	0.2239	$-1.18 + 0.0056\ w_L + 0.0087\ SAND + 0.0041\ CLAY + 3.56\ \gamma_d/\gamma_s$	$0.056 + 0.0004\ w_p - 0.0016\ w_{opt} - 0.032\ \gamma_d/\gamma_s$
E	0.8362	26.34	0.2296	$1.74 - 0.076\ w_{opt} - 0.010\ SAND + 2.63\ \gamma_d/\gamma_s$	$0.0012\ PI - 0.0013\ w_{opt} + 0.0003\ SAND$

Table 7. Regression parameters for equation 12.

Group	Mult. R	F-Value	Std. Error	Coefficients α_1	α_2
A	0.8353	87.40	1.4475	$-77.3 + 0.061\ PI - 0.30\ w_{opt} + 37.8\ \gamma_s/\gamma_w - 42.1\ \gamma_d/\gamma_w + 97.0\ \gamma_d/\gamma_s$	$0.0018\ w_p + 0.0010\ SAND$
B	0.8801	82.44	1.2260	$0.96 - 0.046\ w_L - 0.39\ w_{opt} - 3.78\ \gamma_s/\gamma_w + 18.42\ \gamma_d/\gamma_s$	$0.0004\ SAND - 0.0005\ SILT + 0.077\ \gamma_d/\gamma_s$
C	0.8388	35.60	1.4702	$9.72 - 0.087\ w_L + 0.35\ PI - 0.080\ CLAY$	$0.0047\ PI - 0.0116\ w_{opt} + 0.162\ \gamma_d/\gamma_s$
D	0.8855	75.86	1.2383	$-13.2 + 0.045\ PI + 0.030\ SAND + 20.3\ \gamma_d/\gamma_s$	$0.23 + 0.0033\ w_p - 0.0104\ w_{opt} + 0.0009\ SAND + 0.0007\ CLAY - 0.124\ \gamma_d/\gamma_s$
E	0.8401	27.19	1.3977	$2.03 - 0.48\ w_{opt} - 0.062\ SAND + 15.7\ \gamma_d/\gamma_s$	$0.008\ PI - 0.008\ w_{opt} + 0.002\ SAND$

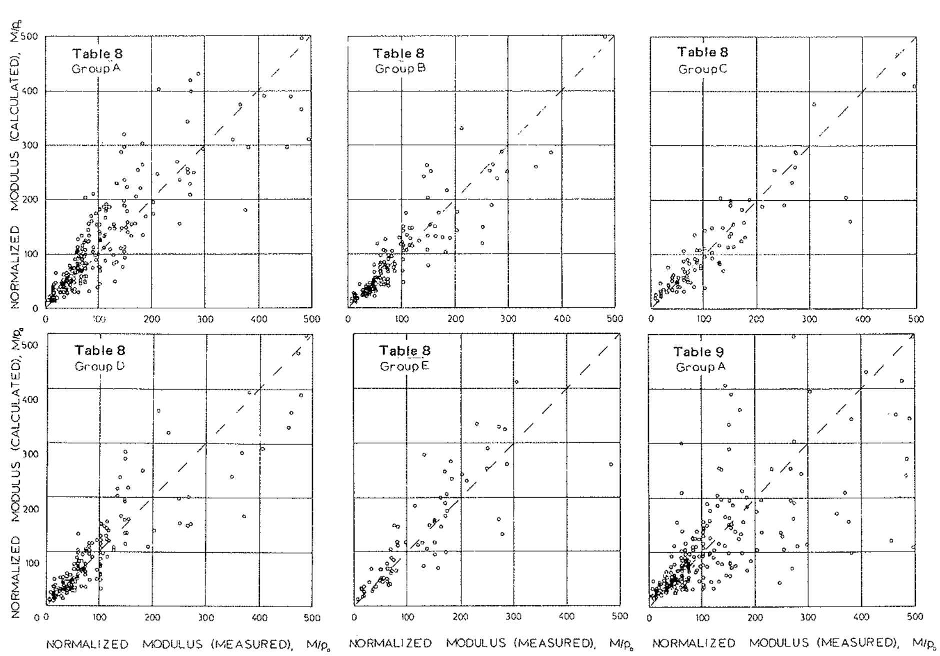
Table 8. Regression parameters for equation 13.

Group	Mult. R	F-Value	Std. Error	Coefficients: log m	Coefficients: n
A	0.8909	76.66	0.1994	$-19.0 + 0.011\ PI - 0.011\ w_p - 0.005\ CLAY + 9.0\ \gamma_s/\gamma_w - 10.0\ \gamma_d/\gamma_w + 22.8\ \gamma_d/\gamma_s$	$3.4 + 0.051\ w_p - 0.31\ w_{opt} + 0.011\ SAND + 0.010\ CLAY + 0.72\ \gamma_d/\gamma_w - 3.48\ \gamma_d/\gamma_s$
B	0.9193	153.67	0.1689	$1.34 + 0.009\ PI - 0.06\ w_{opt} - 1.14\ \gamma_s/\gamma_w + 3.00\ \gamma_d/\gamma_s$	$-0.009\ SILT + 0.92\ \gamma_d/\gamma_w$
C	0.8962	39.46	0.1971	$26.1 - 0.018\ w_L + 0.077\ PI + 0.10\ w_{opt} - 0.024\ CLAY - 9.23\ G_s$	$0.10\ PI - 0.16\ w_{opt} + 3.81\ \gamma_d/\gamma_w - 5.30\ \gamma_d/\gamma_s$
D	0.8995	114.99	0.1918	$-2.49 + 0.005\ w_L + 0.007\ SAND + 3.80\ \gamma_d/\gamma_s$	$4.02 + 0.012\ w_L - 0.13\ w_{opt} - 0.81\ \gamma_d/\gamma_w$
E	0.9089	39.17	0.1772	$-2.20 - 0.50\ w_{opt} + 0.010\ SILT + 4.25\ \gamma_d/\gamma_s$	$6.96 + 0.084\ PI - 0.16\ w_{opt} - 0.021\ SILT - 4.27\ \gamma_d/\gamma_s$

Table 9. Simplified regression parameters for equation 13.

Group	Mult. R	F-Value	Std. Error	Coefficients: log m	Coefficients: n
A	0.8049	164.98	0.2556	$-0.47 - 0.011\ w_{opt} + 2.17\ \gamma_d/\gamma_s$	$1.10\ \gamma_d/\gamma_s$
B	0.8664	172.59	0.2124	$-2.09 + 3.76\ \gamma_d/\gamma_s$	$-0.058\ w_{opt} + 2.08\ \gamma_d/\gamma_s$
C	0.7379	56.18	0.2885	$0.074 + 1.51\ \gamma_d/\gamma_s$	$0.95\ \gamma_d/\gamma_s$
D	0.8055	180.14	0.2567	$-0.57 + 2.10\ \gamma_d/\gamma_s$	$1.09\ \gamma_d/\gamma_s$
E	0.8271	77.97	0.2286	$-0.69 + 2.39\ \gamma_d/\gamma_s$	$1.07\ \gamma_d/\gamma_s$

Figure 5. Measured versus calculated normalized constrained modulus for equation 13.



restriction to small strain values. From the selected equations, the modified Janbu expression gave the highest multiple correlation coefficient, predicting moduli with reasonably good levels of accuracy. In addition, this equation is dimensionless in its coefficients and variables. A simplified equation was derived in terms of only the compaction variables and the dry density. Based on the results of this analysis, two major conclusions can be made:

1. The constrained soil modulus can be estimated with reasonably good levels of accuracy from the proposed equations; and
2. A quantitative approximation of modulus by this procedure is relatively simple since the constants in the equations are primarily functions of the compaction test parameters and the field density and, thus, expensive and time-consuming laboratory tests can be avoided.

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