CURVED I-GIRDER DESIGN

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Design data that permit rapid determination of the forces and stresses in curved bridges subjected to dead and live loads are presented. The data were developed during a recent comprehensive study on curved bridges. The data are applied in an example design.

•DURING the past 5 years there has been a trend toward designing and constructing horizontally curved highway bridges. This has created a need for methods by which such systems can be conveniently designed. Several methods have been developed to analyze curved bridge systems (1, 2, 3, 4, 5), but they are not design aids. Design aids have been obtained by using 2 analytical methods that accurately represent the response of the structural system under dead load (4) and live load (3, 5). The results are in design equations and graphs, which permits design of a curved composite girder structure. Thus the engineer can determine girder properties and diaphragm spacings before a trial-and-error analysis.

Effects of impact will be included in the presentation of the design data.

The notations used in the formulations in this paper are as follows:

- Bi = induced bimoment (lateral flange bending effect),
- b_f = flange width of steel girder,
- C = distance from bending neutral axis to any fiber,
- C_1 = bending stress correction factor = $\overline{S}/S$,
- C_2 = warping stress correction factor = $(W_n/I_w)/(\overline{W_n/I_w})$,
- E_c = modulus of elasticity of composite,
- E_s = modulus of elasticity of steel,
- f = function,
- F = maximum function,
- G_c = modulus of rigidity of composite,
- G_s = modulus of rigidity of steel,
- h = centerline distance between flanges of a steel I-girder,
- I_F = flexural impact factor,
- I_T = torsional impact factor,
- I_w = warping torsional moment of inertia,
- I_x = primary moment of inertia of girders (actual stiffness),
- K = design factor,
- K_1 = amplification factor,
- K_2 = distribution factor,
- K_3 = reduction factor,
- K_T = Saint Venant's torsional rigidity,
- L = span length,
- m = G_c/G_s,
- M = induced bending moment,
- M_b = maximum forces in 4-curved-girder system,
- M_x = internal bending moment,
- n = E_c/E_s,

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R = radius of curvature of curved girder,
S = section modulus spacing or radial diaphragm spacing,
$\overline{S}$ = wheel load factor,
w = uniform dead load applied along the span of each girder,
$W_n = b_f \cdot h/4$ = warping statical moment of steel girder section,
$\overline{W_n/I_w}$ = referenced value,
W_{n_c} = composite warping statical moment,
W_{n_s} = steel warping statical moment,
σ_b = normal stress (actual or corrected) in longitudinal direction of girder due to bending,
$\overline{\sigma}_b$ = bending stress from referenced properties,
σ_w = normal stress (actual or corrected) in longitudinal direction of girder due to warping, and
$\overline{\sigma}_w$ = warping stress from referenced properties.

GENERAL DESIGN DATA

To analyze curved girder systems, one must know the girder section properties I_x, K_T, and I_w. The designer, therefore, must estimate the size of the girders to compute these stiffnesses. This size estimation can be achieved if the primary internal girder forces M_x and Bi, which induce normal stresses, are known. Then the basic bending equation,

$$\sigma_b = \frac{M_x C}{I_x} = \frac{M_x}{(I_x/C)} \tag{1}$$

and warping equation,

$$\sigma_w = \frac{Bi W_n}{I_w} = \frac{Bi}{(I_w/W_n)} \tag{2}$$

could be applied by assuming a proportion of the design stress for σ_b and σ_w and computing the required (I_x/C) and (I_w/W_n) properties. Estimation of these stresses and forces has been obtained by performing a thorough system analysis of single-span and multispan curved bridges (4, 5).

Impact Factors

The dynamic response of single-span curved girder bridges subjected to a sprung mass vehicle was predicted by a Fourier series and lump-mass techniques (8). These techniques then were applied in determining the response of typical highway bridges. This evaluated impact factors. Partial results of these studies are shown in Figures 1 and 2 for plate girder bridges. Curves are given for various girder span lengths and central angles, $\theta = L/R$. The curves are for a vehicle speed of 60 mph (96 km/h); other curves are available for velocities of 20 and 40 mph (32 and 64 km/h) (8).

Impact factors are applied as in conventional practice:

$$M = M_{static}(1 + I_F) \tag{3}$$

$$Bi = Bi_{static}(1 + I_T) \tag{4}$$

Figure 1. Flexural impact factor.

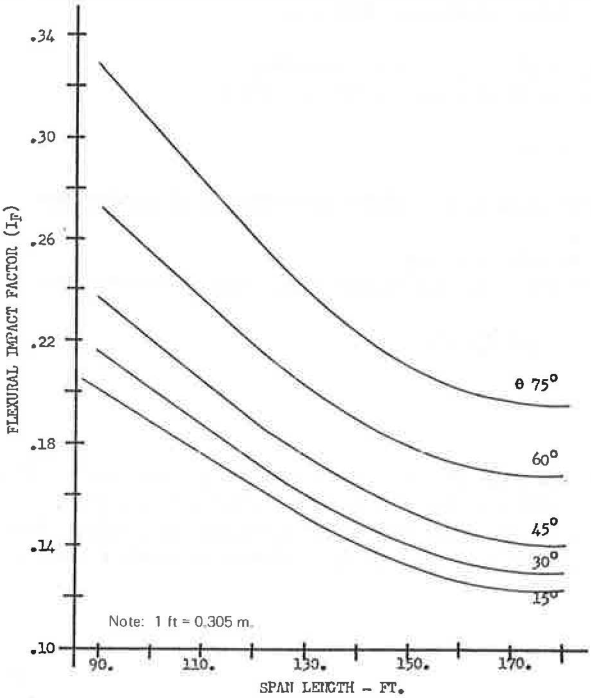
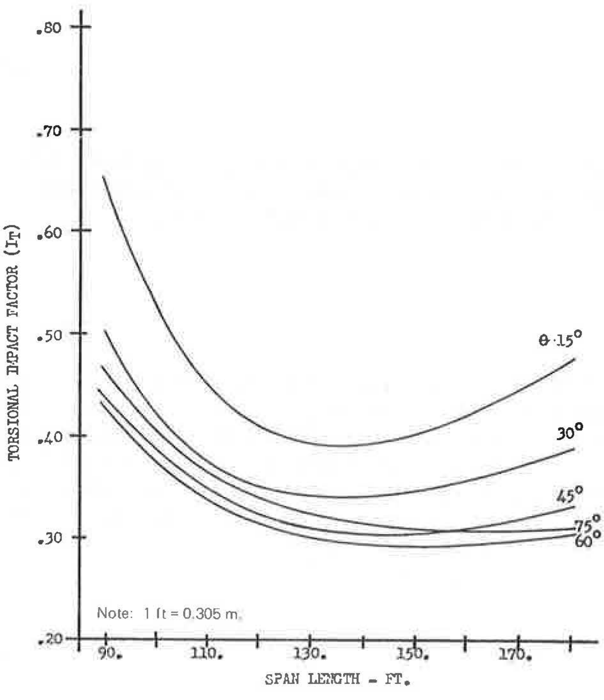


Figure 2. Torsional impact factor.



Live Load Design

Design of any bridge element requires the establishment of forces in that element. If the bridge has a straight alignment, the forces can be determined by distribution factors and simple beam theory. However, when the structure is curved, the interaction of the bending and torsional forces creates an indeterminate situation. The following equations therefore were developed by using relationships between single curved responses and the system, to permit evaluation of the live load forces developed in a curved composite-bridge I-girder system (3, 5, 6, 7). These forces then can be used to establish the induced stresses and proper girder section.

Amplification Factor, K_1

All the internal forces and deformations for a single curved girder and a single straight girder have been evaluated by using various computer programs (6, 9). The ratio of the reactions for these 2 girders gives the following:

$$K_1 = \frac{f(\text{single curved girder})}{f(\text{single straight girder})} \tag{5}$$

This factor describes the immediate effect of curvature relative to a straight member. A graphical representation and analysis of these data give the following general equations:

$$K_{moment} = \frac{0.15}{n}(L/R) + 1 \tag{6}$$

$$K_{bimoment} = [(35n)(L/R)^2 - 15(L/R)] \times 10^3 \tag{7}$$

where $n = R/100$ [$R > 100$ ft (30.5 m)].

Distribution Factor, K_2

The evaluation of true distribution of load to each girder, and realistic values of internal forces, can be considered by analyzing the curved girder as a system. The number of trucks used in the analysis would depend on the number of lanes. The ratio of these resulting maximum forces to those in a single curved girder gives

$$K_2 = \frac{f(\text{system of curved girders})}{f(\text{single curved girder})} \tag{8}$$

A plot of this ratio versus R and L will yield the following general equations:

$$K_{moment} = (n + 3)\left(\frac{0.4L}{R}\right) + 0.6 \tag{9}$$

$$K_{bimoment} = \frac{0.11}{M}(R/L) \qquad L < 70 \text{ ft } (21.3 \text{ m}) \tag{10}$$

$$K_{bimoment} = \frac{(M - 1)}{6}(R/L) \qquad L > 70 \text{ ft } (21.3 \text{ m}) \tag{11}$$

where

$n = R/100$ [$R > 100$ ft (30.5 m)], and
$M = L/50$ [$L > 50$ ft (15.25 m)].

These equations are valid for girder systems that contain 4, 6, and 8 girders and girder spacings of 7, 8, 9, and 10 ft (2.14, 2.44, 2.75, and 3.05 m).

Reduction Factor, K_3

Because many bridge structures are continuous, it is desirable to obtain some factors that can be applied to the simple span data to give preliminary forces in continuous spans. This factor can be written as

$$K_3 = \frac{f(\text{system of curved girders})N}{f(\text{system of curved girders})} \tag{12}$$

where N = number of spans.

Using a computer program, we evaluated the maximum forces in a 2- or 3-span curved bridge system of 4, 6, and 8 girders under various critical loadings. A study of all the data and the resulting K_3 values gives the values in Table 1. The data are described relative to number of spans and are independent of number of girders. It should be emphasized that the 2- and 3-span girder systems must contain equal span lengths with a maximum given span length of 100.0 ft (30.5 m). For example, for a 3-span system, the total maximum bridge length would then be $3 \times L$ or 300.0 ft (91.5 m).

EVALUATION OF GIRDER FORCES AND DEFORMATION

With the various factor equations available, one can evaluate preliminary forces in a curved girder bridge, relative to the forces in a straight girder. (All Rs and Ls refer to the centerline of the bridge system.)

1. Evaluate F for a single straight girder of length L that has been subjected to a line of AASHO wheel loads. This function would be $F_{bending}$. The function for $F_{bimoment}$ is assumed to be equal to 1.
2. Evaluate K_1 equations 6 and 7 for L and R of the bridge system.
3. Evaluate K_2 equations 9, 10, and 11 for L, number of girders in system, and R.
4. Select a reduction factor from Table 1 if system is a continuous span.
5. Determine F of curved girder system, that is

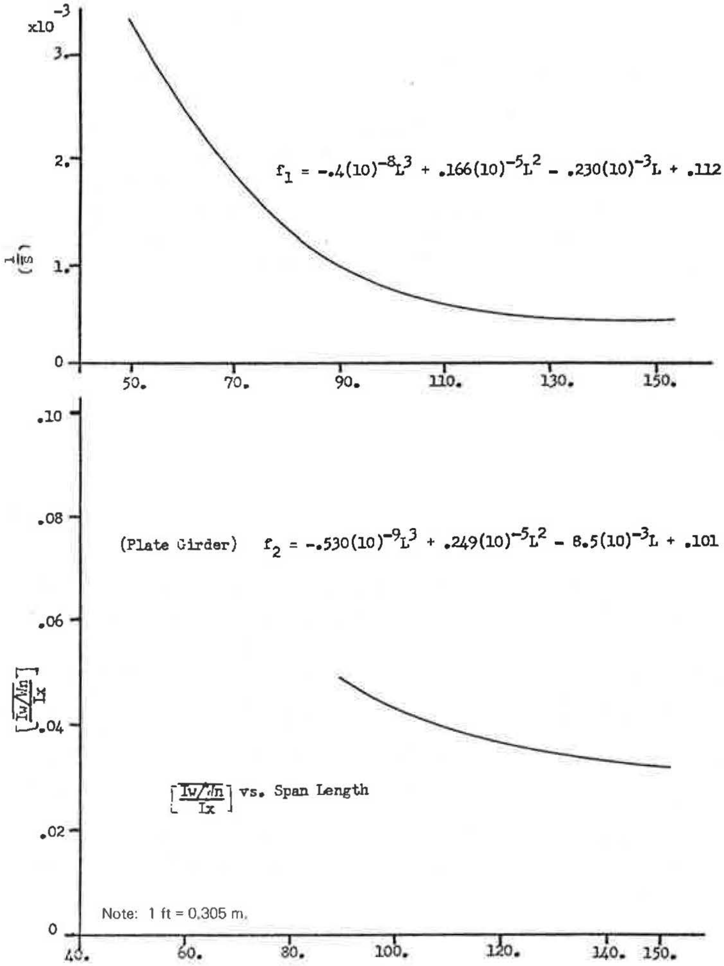
$$\text{Maximum moment}_{static} = M_{straight} \times (K_1 \times K_2 \times K_3) \tag{13}$$

and

Table 1. Reduction factors for maximum bridge function.

Number of Spans	Bending Moment	Deflection	Rotation	Saint Venant	Warping Torsion	Bimoment	Shear
2	0.75	0.70	0.70	0.75	0.45	0.35	1.00
3	0.65	0.60	0.70	0.75	0.40	0.35	0.90

Figure 3. Section properties versus span length.



$$\text{Maximum bimoment}_{\text{static}} = 1.0(K_1 \times K_2 \times K_3) \tag{14}$$

6. Multiply maximum moment by $1 + I_F$ from equation 3, and multiply maximum bimoment by $1 + I_T$ from equation 4.

To account for S, one should multiply the resulting action of a single straight girder subjected to a line of truck wheels by $\overline{S}$. $\overline{S} = 1.29$ for S = 7 or 8 ft (2.14 or 2.44 m). $\overline{S} = 1.57$ for S = 9 or 10 ft (2.75 or 3.05 m).

Dead Load Design

The dead load response of curved bridge systems has been predicted by Murphy (4), and it realistically represents the response of the bare steel frame system. This technique has been computerized and then applied in the development of design curves. The curves represent maximum σ_b and σ_w as a function of R, L, I_x, and diaphragm S. The curves were based on typical girder properties obtained from a survey of bridge design as a function of girder length, as shown in Figure 3. If the actual design properties are different from the values in Figure 3, a correction factor is required to modify the chart values. Bending stress is related by the following:

$$\sigma_b = \overline{\sigma}_b C_1 \tag{15}$$

The warping stress is related by the following:

$$\sigma_w = \overline{\sigma}_w C_2 \tag{16}$$

The design charts, Figures 4 through 9, show the induced $\overline{\sigma}_b$s and $\overline{\sigma}_w$s normalized relative to the applied dead load per length. Thus any variation in dead load may be considered in an actual design. The normalized stress versus R/L values are plotted for span lengths of 100, 125, and 150 ft (30.5, 38.1, and 45.8 m). These plots are dependent on I_x, number of girders, and diaphragm spacing. The plots of $\frac{\overline{\sigma}_b}{W}$ versus R/L values (Figures 4, 6, and 8) are independent of diaphragm spacing and I_x, as determined in the development of the curves (4). The plots of $\frac{\sigma_w}{W}$ versus R/L values (Figures 5, 7, and 9) are dependent on diaphragm spacing and I_x. These curves are limited to a 4-girder system and are for specified stiffnesses. However, for other stiffness, the following equation may be applied:

$$\left(\frac{\overline{\sigma}_w}{W}\right)_e = \left(\frac{\overline{\sigma}_w}{W}\right)_{\text{chart}} \times \frac{\overline{I}_{rx}}{I_x} \tag{17}$$

where

$\left(\frac{\sigma_w}{W}\right)_e$ = referenced modified factor, and

$\overline{I}_{rx}$ = referenced chart value given in Figures 5, 7, and 9.

Figure 4. Normalized bending stress for a span length of 100 ft (30.5 m).

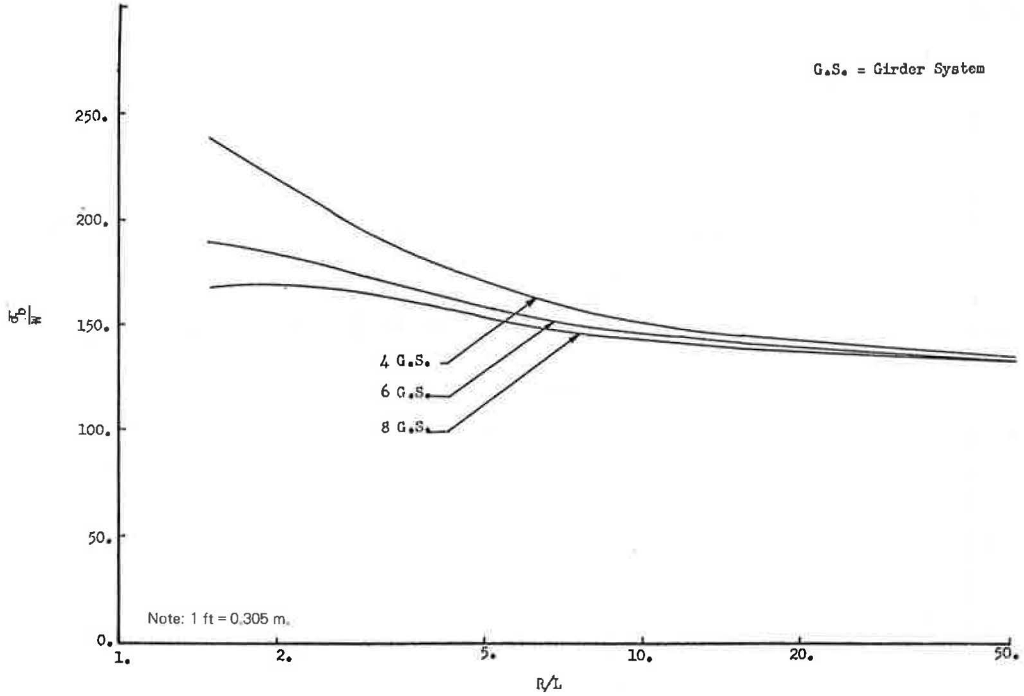


Figure 5. Normalized warping stress for a span length of 100 ft (30.5 m).

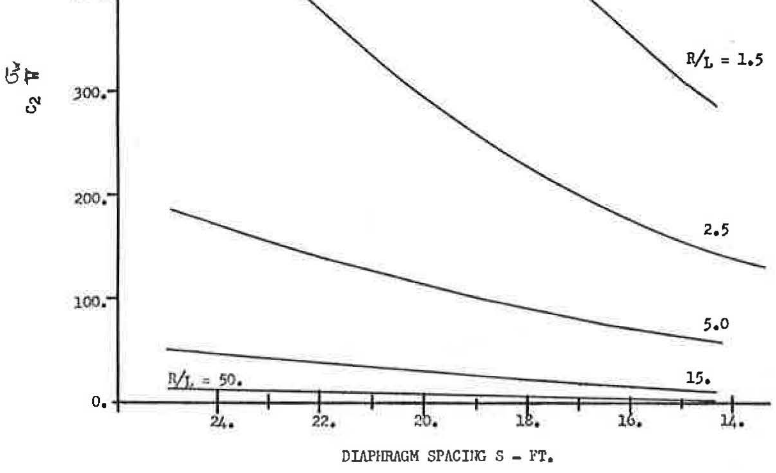


Figure 6. Normalized bending stress for a span length of 125 ft (38.1 m).

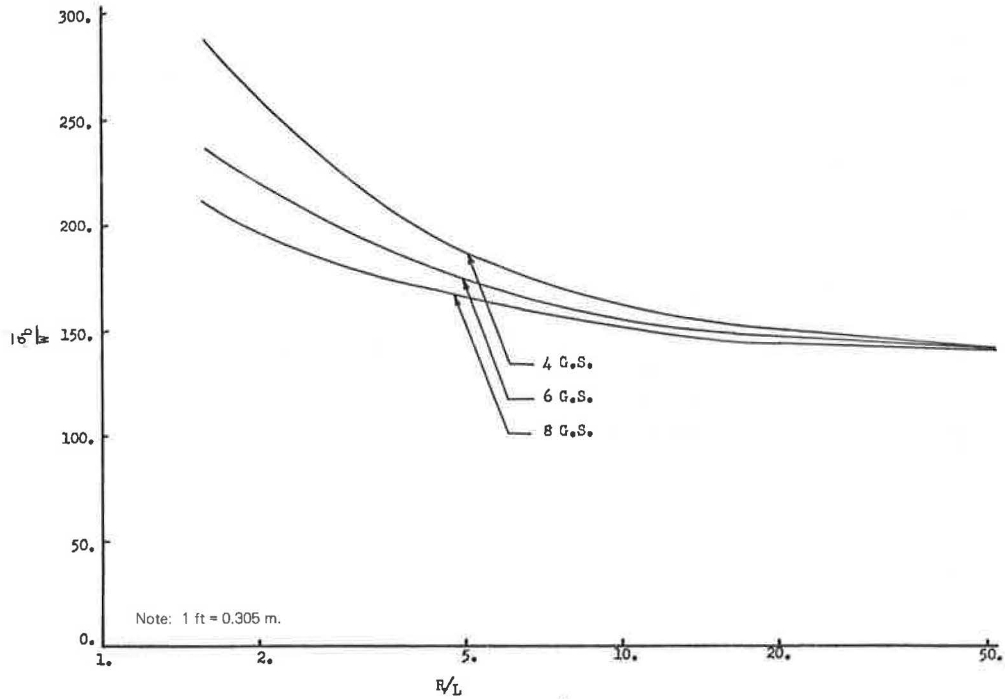


Figure 7. Normalized warping stress for a span length of 125 ft (38.1 m).

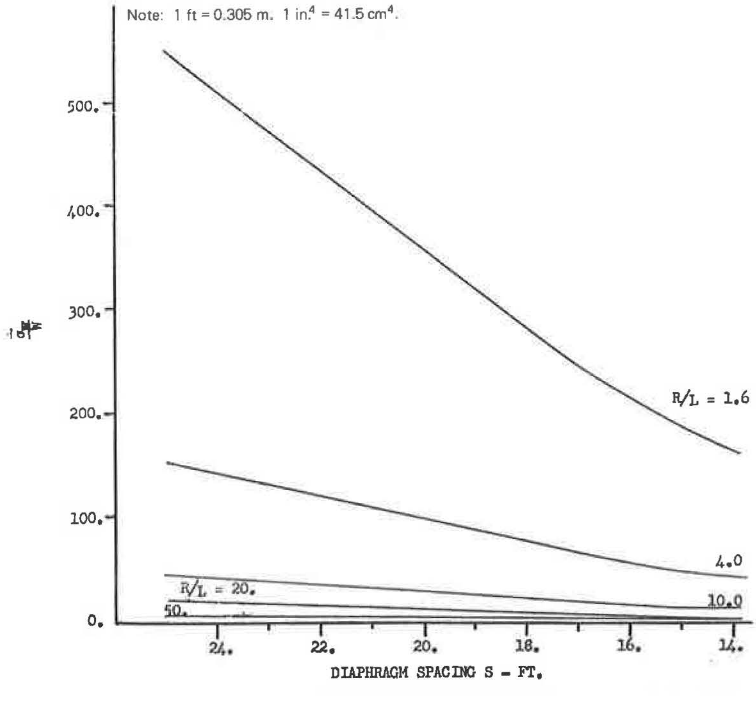


Figure 8. Normalized bending stress for a span length of 150 ft (45.8 m).

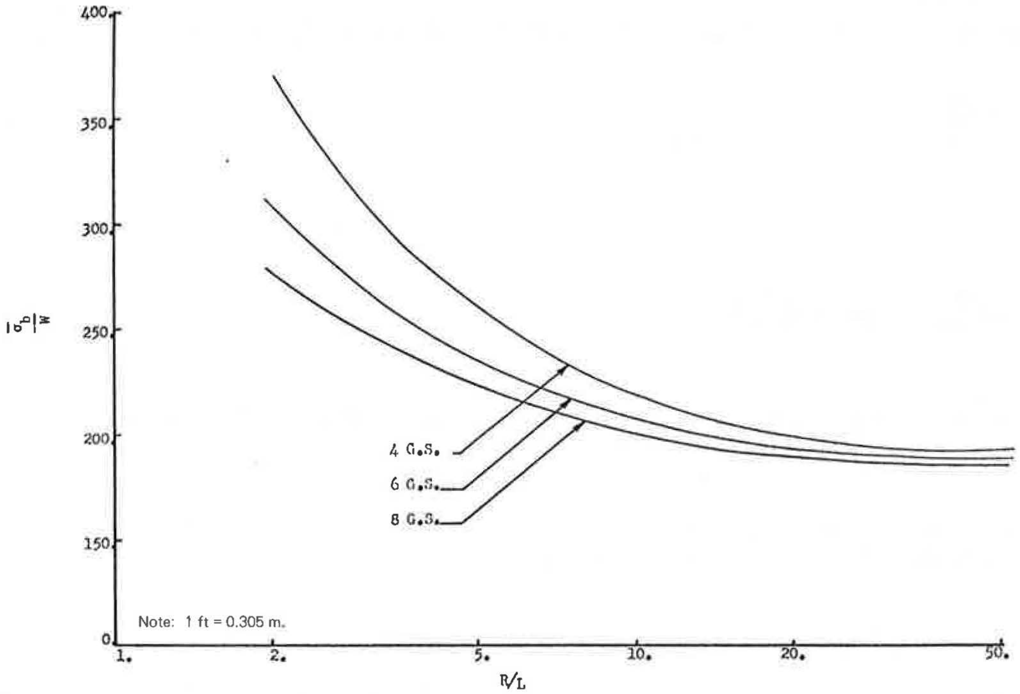
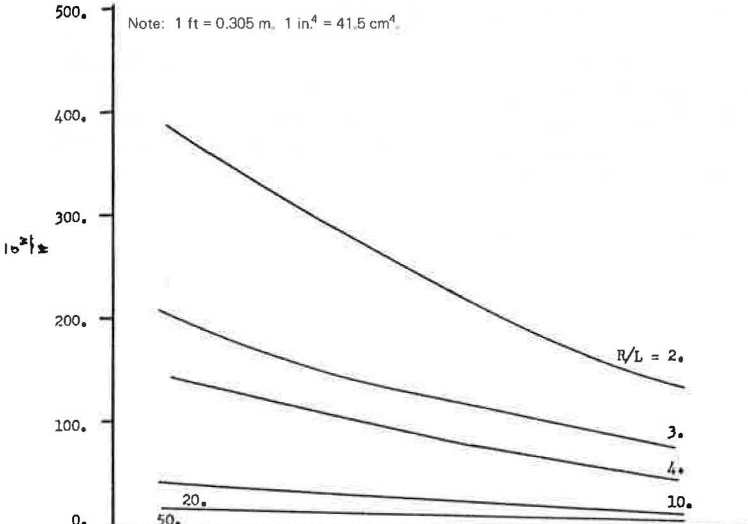


Figure 9. Normalized warping stress for a span length of 150 ft (45.8 m).



To account for the behavior of 6- and 8-girder systems, one must modify the $\frac{\bar{\sigma}_w}{W}$ chart value. These chart values are modified by the following equation:

$$\left(\frac{\bar{\sigma}_w}{W}\right)_{exact} = \left(\frac{\bar{\sigma}_w}{W}\right)_{chart} - \Delta\left(\frac{\bar{\sigma}_w}{W}\right) \tag{18}$$

where

$$\Delta\left(\frac{\bar{\sigma}_w}{W}\right) = \frac{\bar{I}_x}{I_x}\frac{S^2}{(25)^2}\Delta\left(\frac{\bar{\sigma}_w}{W}\right)', \tag{19}$$

$\Delta\left(\frac{\bar{\sigma}_w}{W}\right)'$ = change in the value of the chart because of an increase in the number of girders beyond 4 girders shown in Figure 10, and
$\bar{I}_x$ = property given in Figure 10.

The curves were developed for a girder spacing of 8.0 ft (2.44 m). However, spacings up to 10 ft (3.05 m) are acceptable.

Torsional Properties

To determine design stresses or distortions, one must know girder section properties. If the girder is subjected to torsion and bending, the torsional properties, in addition to bending properties, will be required. The exact solution of the torsional properties of composite sections has been demonstrated (10). However, by idealizing the composite section, one can develop a series of simplified equations (11).

Figure 11 shows a typical composite girder and its pertinent dimensions. If we neglect the top girder flange and modify the concrete slab thickness, as shown in Figure 12, the dimensions are defined as follows: $t_1 = nt_s$, $d_1 = b_s$, $t_2 = W$, $d_2 = d_g + t_s/2 - t_f/2$, $d_3 = b_f$, and $t_3 = t_f$. Using these dimensions, one can determine the resulting torsional properties, which are shear center, normalized warping functions, warping stiffness, and torsional constant. Shear center is

$$\alpha = \frac{d_3^3 t_3}{(d_1^3 t_1 + d_3^3 t_3)} d_2 \tag{20}$$

Normalized warping functions are

$$W_{n_c} = \frac{\alpha d_1}{2} \tag{21}$$

for slab, and

$$W_{n_s} = \frac{(d_2 - \alpha)}{2} d_3 \tag{22}$$

for beam. Warping stiffness is

Figure 10. Normalized warping stress variation.

Note: 1 ft = 0.305 m. 1 in.^{4} = 41.5 cm^4.

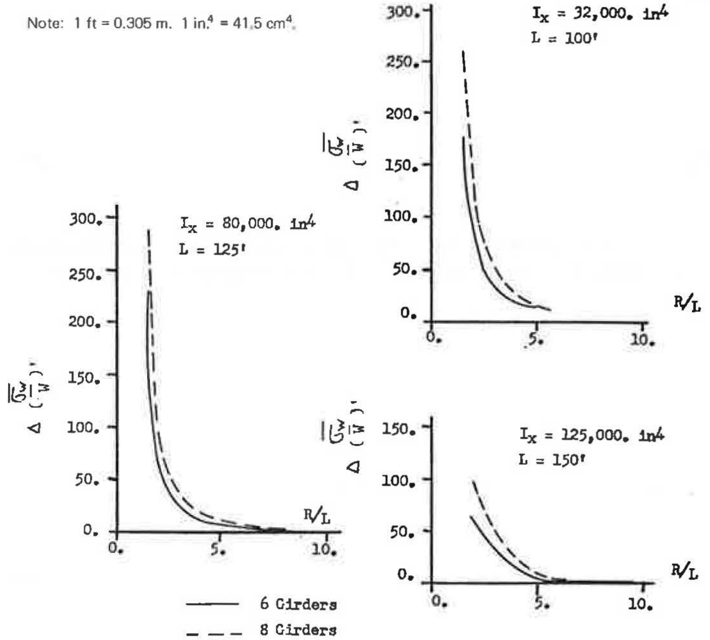


Figure 11. Typical composite section.

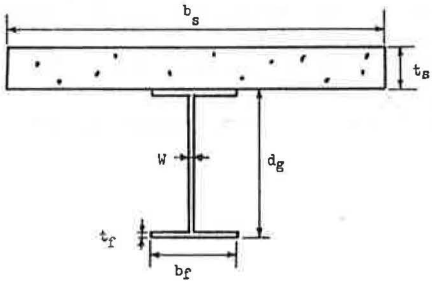
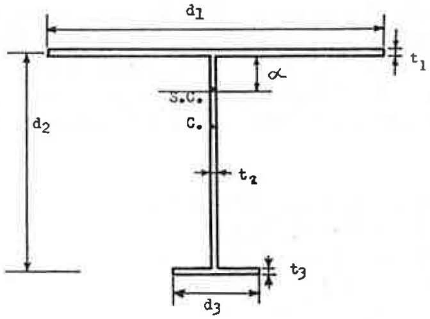


Figure 12. Idealized composite section.



$$I_w = \frac{\alpha^2}{12} t_1 d_1^3 + (d_3 - \alpha)^2 \frac{t_3 d_3^3}{12} \quad (23)$$

Torsional constant is

$$K_T = \tfrac{1}{3}(d_3 t_3^3 + d_2 t_2^3 + m d_1 t_s^3) \quad (24)$$

With the evaluation of these torsional properties, one can evaluate the resulting normal stresses in the composite section due to bimoment. Normal warping stress for slab is

$$\sigma_{w_c} = \frac{BiW_{n_c}}{I_w} \quad (25)$$

Normal warping stress for steel is

$$\sigma_{w_s} = \frac{BiW_{n_s}}{I_w} \quad (26)$$

DESIGN EXAMPLE

The design data just presented are sufficient to perform a preliminary design of a curved bridge system. Using these data, we will design a single-span, 4-girder bridge on which the girders are spaced 8 ft (2.44 m) apart at the centerline. Arc length is 98.0 ft (29.9 m); centerline radius is 588.8 ft (179.0 m). Slab thickness is 8.5 in. (21.6 cm), and the composite girder is made of A36 steel.

For live load effects, the following values, derived from the basic equations, are used:

1. R_{cl} = 588.0 ft (179.0 m),
2. S = 8.0 ft (2.44 m),
3. L_{max} = 100.0 ft (30.5 m),
4. L_{cl} = 98.0 ft (29.9 m),
5. $n = \frac{R_{cl}}{100} = \frac{588}{100} = 5.88$,
6. $M = \frac{L_{cl}}{50} = \frac{98}{50} = 1.96$,
7. $(L/R)_{cl} = \frac{98}{588} = 0.167$, and
8. $\overline{S}$ = 1.29 (7).

The evaluation of K_1 from equations 6 and 7, gives

$$K_1 \text{ moment} = \frac{0.15}{n}(L/R) + 1$$

$$= \frac{0.15}{5.88}(0.167) + 1$$

$$= 1.004$$

K_1 bimoment = $[(35n)(L/R)^2 - 15(L/R)] \times 10^3$

$= [(35 \times 5.88)(0.167)^2 - 15(0.167)] \times 10^3$

$= 3{,}250.$

Evaluation of K_2 from equations 9, 10, and 11 gives

K_2 moment $= (n + 3)\left(\frac{0.4L}{R}\right) + 0.6$

$= (5.88 + 3)(0.4 \times 0.167) + 0.6$

$= 1.194$

K_2 bimoment $= \frac{(M - 1)}{6}(R/L)$

$= \left(\frac{1.96 - 1}{6}\right)(6)$

$= 0.96$

These values can be applied to the computed single straight girder moment, which was obtained from the loading of a single set of AASHO truck wheels on a 98.0-ft (29.9-m) girder.

$M_{straight} = (1{,}488.0\ K'/2 \times 12) \times \overline{S}$

$= 744 \times 12 \times 1.29$

$= 11{,}500$ kips/in. (1300 kN/m)

Bi = 1.0

Applying equations 13 and 14, with $K_3 = 1.0$, one arrives at the following result for maximum forces in the 4-curved-girder system:

$M_b = 1.004 \times 1.194 \times 11{,}500 = 13{,}750$ kips/in. (1555 kN/m)

Bi = $3{,}250 \times 0.96 \times 1.0 = 3{,}120$ kips/in.^{2} (21 528 MPa)

Allowable design stress for A36 steel is 20 kips/in.^{2} (138 MPa). Assume that 35 percent of stress is due to live load, and 65 percent is due to dead load. Then

$$\sigma_b = 7 = \frac{M_b}{S}$$

$$S_{required} = \frac{13,750}{7} = 1,970 \text{ in.}^3 \ (32\ 200 \text{ cm}^3)$$

We will now study a composite section based on moment (10). The web is 60.0 by 0.375 in. (152 by 0.95 cm); the flanges are 16.0 by 1.5 in. (41 by 38 cm); and the slab is 8.5 by 96.0 in. (21.6 by 244 cm). The composite properties are as follows:

1. I_x = 100,976.8 in.^{4} (42 × 10^5 cm^4),
2. I_w = 2,255,951 in.^{6} (606 × 10^6 cm^6),
3. S = 1,990 in.^{3} (32 600 cm^3), and
4. W_{n_c} = 527.4 in.^{2} (3400 cm^2).

The steel girder properties are as follows:

1. I_x = 51,949.0 in.^{4} (21.6 × 10^5 cm^4),
2. I_w = 966,000 in.^{6} (261 × 10^6 cm^6),
3. S = 1,650 in.^{3} (27 100 cm^3), and
4. W_{n_s} = 246 in.^{2} (1590 cm^2).

Therefore, the induced live load stresses are

$$\sigma_b = \frac{M}{S} = \frac{13,750}{1,990} = 6.9 \text{ kips/in.}^2 \ (47.6 \text{ MPa})$$

$$\sigma_w = \frac{BiW_n}{I_w} = \frac{3,120 \times 527.4}{2,255,951} = 0.73 \text{ kips/in.}^2 \ (5.05 \text{ MPa})$$

Check dead load stresses from Figures 4 and 5. For a 4-girder system where L = 100.0 ft (30.5 m) and $(R/L)_{max} = \frac{600}{100} = 6.0$, then

$$\left(\frac{\bar{\sigma}_b}{w}\right) = 165$$

The dead load of the girder is computed as follows:

$$w = \frac{1}{12}\left\{[(60 \times 0.375) + 2(16 \times 1.5)]\frac{0.490}{144} + \left(\frac{8.5}{12} \times 8.0\right) 0.150\right\}$$

$$w = 0.0875 \text{ kips/in.} \ (1.53 \text{ kN/cm})$$

Therefore,

$$\overline{\sigma}_b = 165 \times w$$

$$\overline{\sigma}_b = 165 \times 0.0875 = 14.4 \text{ kips/in.}^2 \text{ (100 MPa)}$$

Where $\overline{S}$ is the value given in Figure 3, the coefficient $C_1 = \overline{S}/S$. From Figure 3, when L = 100.0 ft (30.5 m)

$$1/\overline{S} = 0.9 \times 10^{-3}$$

$$\overline{S} = 1{,}111.1 \text{ in.}^3 \text{ (18 200 cm}^3\text{)}$$

Therefore,

$$C_1 = \overline{S}/S = \frac{1{,}111.1}{1{,}650}$$

$$C_1 = 0.673$$

Therefore,

$$\sigma_b = \overline{\sigma}_b C_1 = 14.4 \times 0.673$$

$$\overline{\sigma}_b = 9.65 \text{ kips/in.}^2 \text{ (66.5 MPa)}$$

The induced dead load warping stress is computed from Figure 5. If we assume a diaphragm spacing of 20.0 ft (6.1 m) and if $(R/L)_{max} = 6.0$, then

$$\left(\frac{\overline{\sigma}_w}{w}\right) = 100 \times \frac{\overline{I}_{rx}}{I_x}$$

$$\left(\frac{\overline{\sigma}_w}{w}\right) = 100 \times \frac{32{,}000}{51{,}949}$$

$$\left(\frac{\overline{\sigma}_w}{w}\right) = 61.9$$

$$(\overline{\sigma}_w) = 61.9 \times 0.0875$$

$$(\overline{\sigma}_w) = 5.4 \text{ kips/in.}^2 \text{ (37.3 MPa)}$$

The coefficient $C_2 = (Wn/Iw)/(\overline{Wn/Iw})$, where Wn/Iw is the design value and the $(\overline{Wn/Iw})$ value is found from Figure 3. If we use Figure 3 and if L = 100.0 ft (30.5 m), then $(\overline{Iw/Wn})/Ix = 0.044$ or $(\overline{Iw/Wn}) = 0.044 \times 51{,}949 = 2{,}280$. The design ratio is $(Iw/Wn) = \dfrac{966{,}000}{246} = 3{,}940$. Therefore,

$$C_2 = (Wn/Iw)/(\overline{Wn/Iw}) = \frac{2{,}280}{3{,}940} = 579$$

The warping stress is then equal to

$$\sigma_w = \bar{\sigma}_w C_2$$

$$\sigma_w = 5.4 \times 0.579 = 3.12 \text{ kips/in.}^2 \text{ (21.6 MPa)}$$

Total maximum normal stress is the sum of dead load and live load. The live load stresses, however, must include impact, which is found from Figures 1 and 2. This structure has an arc of $\theta = 9.5$ deg (use 15 deg). L = 98.0 ft (29.9 m), which gives $I_F = 0.19$, $I_T = 0.52$. The live load stresses are, therefore,

$$\sigma_b = 6.9(1.19) = 8.2 \text{ kips/in.}^2 \text{ (56.5 MPa)}$$

$$\sigma_w = 0.73(1.52) = 1.11 \text{ kips/in.}^2 \text{ (7.65 MPa)}$$

The combined dead and live load stresses are equal to the following (1 kip/in.^{2} = 6.9 MPa):

Load	Bending Stress (kips/in.^{2})	Warping Stress (kips/in.^{2})
Live	8.20	1.11
Dead	9.65	3.12
Σ	17.85	4.23

The combined bending and warping stress is $\sigma = 22.08$ kips/in.^{2} (153 MPa), which indicates that a new section might be selected. However, a computer system analysis using this selected stiffness and diaphragm spacing might be sufficient for an initial trial.

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DISCUSSION

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Charts are given in the paper for use in the design of curved I-girders, and a numerical example is worked out. The example consists of a simple span on a 588-ft (179.0-m) radius curve with an arc length of 98 ft (29.9 m), 4 girders spaced 8 ft (2.44 m) apart, and an 8.5-in.-thick (21.6-cm-thick) slab.

Dead load stresses are based on the assumption that 4 girders act with diaphragms spaced at 20 ft (6.1 m) and that the slab does not participate. Live load stresses are based on composite action of the slab and girders, but no account is taken of diaphragm spacing.

For live load, the slab is depended on for resistance to the rotational effect of curvature on the girder. It is implicitly assumed that the girder will retain its cross-section shape although the rotational and lateral restraining forces are applied at the top flange only. This assumed action requires the transmission of radial forces from the top flange to the bottom flange by means of the girder web. Such action would produce a bending of the web in a radial direction of such magnitude that the cross section of the girder would be distorted; stresses would be of a magnitude that could exceed the yield point. Properly spaced diaphragms would relieve the web of practically all of this stress and distortion, and the lateral bending or warping stress in the bottom flange would vary, approximately, with the square of the diaphragm spacing. By the method in this paper, no account is taken of diaphragm spacing in the determination of live load stress.

There is an error in the dead load calculation according to the values of $1/\bar{S}$ from Figure 3. This value should be about 0.75×10^{-3} instead of 0.9×10^{-3}. Use of the corrected value changes the dead load bending stress from 9.65 kips/in.^{2} (66.6 MPa) to 11.6 kips/in.^{2} (80.0 MPa).

Live load impact was calculated as 19 percent for bending stress, and as 52 percent for warping or lateral bending stress. Because these stresses occur for the same loading at the same point and are additive, the calculations are a decided departure from past practice in design for impact.

Heins considers that failure to use diaphragms would be disastrous and that neglecting the direct effect of diaphragm spacing on live load lateral flange bending would seriously underestimate the lateral bending of the bottom flange.

AUTHOR'S CLOSURE

I appreciate Nettleton's comments and his observations concerning the effects of live load on the diaphragms. His last paragraph, however, implies that I suggested that no diaphragms were in the system during passage of the live load. This condition was neither implied nor stated. Influence of diaphragms on design is considered only under dead loading, but diaphragms are still present in the system during live load stresses. Although the thoughts given in this paragraph are interesting, they are not pertinent.

The graphs presented are for preliminary design. The value for $1/\overline{S}$ from Figure 3 given as 0.90×10^{-3} was rounded from 0.85×10^{-3}.

The impact values that were used were based on maximum effects as given in the curves. It was assumed that the worst case existed. Thus warping and bending stresses were added.

It should be realized, as I stated in the beginning of the paper, that the information presented is design data, which are helpful in selecting girder size and diaphragm spacings before computer analysis.