CYCLIC CREEP OF BITUMINOUS MATERIALS UNDER TRANSIENT, HIGH-VOLUME LOADS

Ralph Haas and Frank Meyer, Department of Civil Engineering, University of Waterloo, Ontario

Flexible pavements should be designed to limit the permanent deformation after a given number of load applications to some tolerable value. To accomplish this, the designer should be able to estimate the actual magnitude of permanent deformation for any particular design situation. The necessary predictive models for such cyclic creep should consider material characteristics; layer thicknesses; temperature; rate, intensity, and repetitions of loading; and any significant interactions of these factors. This paper presents an approach to developing cyclic creep models based on simulating field conditions in the laboratory. Equipment that can apply dynamic stresses under varying temperature conditions and that can measure accumulated permanent strains is described. The development of cyclic creep models that use data obtained from statistically designed experimental programs is described. They can be used to separately estimate permanent deformation in the compression and tension zones of a bituminous layer. The models were applied to the full-depth sections of the Brampton Test Road. Very good agreement was found between predicted and measured values of accumulated permanent deformation.

•EXCESSIVE permanent deformation of highway or airport pavements can accelerate structural deterioration and can create a safety hazard. This type of distress may well increase in the future because of the general trend toward increased traffic loads and volumes.

Pavement researchers have devoted considerable effort during the past few years to developing a working design technology for three major structural subsystems: fatigue, permanent deformation, and low-temperature shrinkage fracture. The permanent deformation subsystem is perhaps less advanced than the others. It requires the development of suitable models, so that the accumulation of permanent strains under transient, repeated wheel loads can be estimated.

The development of permanent strains, cyclic creep, depends on a variety of factors, including materials characteristics, layer thicknesses, temperature, rate of loading, and loading intensity and repetitions. A predictive model for cyclic creep should consider these factors and their interactions and should provide the pavement designer with a reliable estimate of permanent deformation as a function of time or traffic or both. He should be able to assess the effects of future traffic and the effects of various materials or layer thickness combinations for the particular design situation.

This paper describes a procedure for estimating cyclic creep in bituminous mixtures under transient, repeated load conditions. More specifically the objectives are

1. To outline the existing methodology for handling permanent deformation caused by cyclic creep in bituminous pavements,
2. To describe an approach based on laboratory simulation of field loading and environmental conditions and on the use of the experimental results in developing a predictive model,

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3. To describe the effects of certain variables such as binder consistency and air voids content on the magnitude of cyclic creep, and
4. To briefly compare predicted and measured permanent deformation in the field.

PERMANENT DEFORMATION IN ASPHALT PAVEMENTS

Need for a Predictive Methodology

Excessive permanent deformation in bituminous pavements can result in distress, loss of serviceability, and loss of safety. A design methodology should be available to predict the actual magnitude of permanent deformation under any given situation. The importance of developing such a methodology has recently been emphasized by such forums as the Third International Conference on the Structural Design of Asphalt Pavements (1) and a Corps of Engineers-University of Texas conference (2).

The need for developing predictive techniques is reinforced by the following considerations.

1. With increasing wheel loads, tire pressures, and repetitions, the extrapolation of experience is risky.
2. Reducing shrinkage fracture potential by using softer asphalts may also increase the tendency for rutting during warm periods.
3. Rational design of pavements must include economic evaluation (including trade-offs); thus, it may at times be necessary to design to some tolerable level of permanent deformation.
4. Scarcity of resources may result in the economic feasibility of new materials; a fundamental, predictive technology is needed to assess their potential behavior.

Existing Methods

There are two current working methods, indirect in nature, that are used to design for permanent deformation. One provides minimum layer thicknesses and component material strengths, stability, density, or voids content; and the other limits the vertical compressive strain on the subgrade surface to some maximum level.

The first approach occurs inherently in the structural design method being used; the second was developed by the Shell Oil Company (3). Each is directed at precluding excessive permanent deformation, rather than predicting its actual magnitude.

The following more fundamental approaches have recently been suggested:

1. Quasi-elastic approach originally proposed by Klomp and Dorman (3) and extended by Romain (4),
2. Linear-viscoelastic approach developed by Moavenzadeh and coworkers at M.I.T. (5), and
3. Rut index approach developed by Barksdale (6).

The quasi-elastic approach is promising but requires development of certain deformation laws to become operational. The linear-viscoelastic approach was considered to be the most promising by the permanent deformation and materials characterization groups at an Austin, Texas, workshop in 1970 (7). However, this approach still has some significant limitations in its present state of development (8).

The procedure developed by Barksdale appears very encouraging in that it attempts to establish relationships from actual physical measurements.

A Simulation Approach

An approach that develops cyclic creep models from statistically designed experiments that simulate field conditions was reported initially by Morris and Haas (9, 10) and in subsequent detail by Morris et al. (11). It attempts to obtain relationships from measured data, in a manner somewhat similar to that of Barksdale (6), without presupposing any laws or properties. Also, through the use of statistically based experimental designs, the models can account for interactions of variables and can provide estimates of error.

To simulate field conditions requires that what occurs in a pavement subjected to a traffic load be known. Figure 1 shows the distribution of principal vertical σ_1 and horizontal σ_3 stresses throughout the depth of the upper layer (i.e., using a full-depth section, as shown in Figure 1a). At the same time, the temperature T in the slab might vary as shown in Figure 1c.

When a series of very short duration loads is placed on the pavement, as represented by moving traffic, the elements are subjected to stress pulses. These are shown in Figures 2a and b for σ_1 and σ_3 respectively. As well, a shearing stress occurs on the element as the load passes (Figure 2c).

Each element is also subjected to strains in the directions of the stresses. If these strains are completely recoverable, i.e., if the pavement rebounds to its exact original position after the load passes, then no permanent deformation will occur. However, there will usually be a very small portion of the strain that is not recoverable. After a large number of loads have passed over the pavement, the accumulation of very small irrecoverable strains can result in significant permanent deformation along the wheelpaths.

To completely simulate these responses in the laboratory is a highly complex problem and is essentially beyond the capabilities of current technology. However, they may be quite reasonably simulated by the use of a triaxial type of apparatus that can dynamically and independently apply σ_1 and σ_3 in either tension or compression with temperature control. The development and use of such an apparatus have been described in detail (10). A general view of the equipment is shown in Figure 3. It is capable of measuring cyclic creep, both axially and laterally, at any number of load applications.

EXPERIMENTAL PROGRAM

Materials and Testing Conditions

The material chosen for testing was the asphalt concrete base used at the Brampton Test Road in Ontario (12). It consists of a ¾-in. (19-mm) maximum size coarse aggregate, as shown by the grading curve in Figure 4. The asphalts used were 85/100 and 300/400 penetration grades. Their stiffness properties at various temperatures, plus viscosity at 275 F (135 C) and stiffness properties of the asphalt concrete, are given in Table 1. The 4 by 8-in. (102 by 203-mm) asphalt concrete samples were prepared in accordance with ASTM D 1561-71. Average air voids content of the samples was 7.1 percent, which compared very favorably with average field values of 7.2 percent.

The equipment used is shown in a general sense in Figure 3. A view of the asphalt concrete sample mounted on the base of the triaxial cell is shown in Figure 5.

The testing conditions essentially involved application of dynamic vertical and horizontal stress pulses, with intervening rest periods, to the samples shown in Figure 5. The duration of stress pulses was 0.04 sec, with a 0.02-sec rest period when the vertical stress was in the compressive mode and 0.21 sec when it was in the tensile mode.

Design of Experimental Program

The design of the experimental program was based on the premise that the amount of

Figure 1. Typical pavement stress and temperature distributions in center of wheelpaths under static loading.

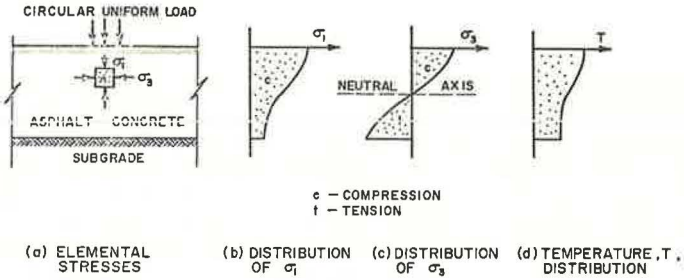


Figure 2. Stress variations in typical asphalt concrete element due to moving loads.

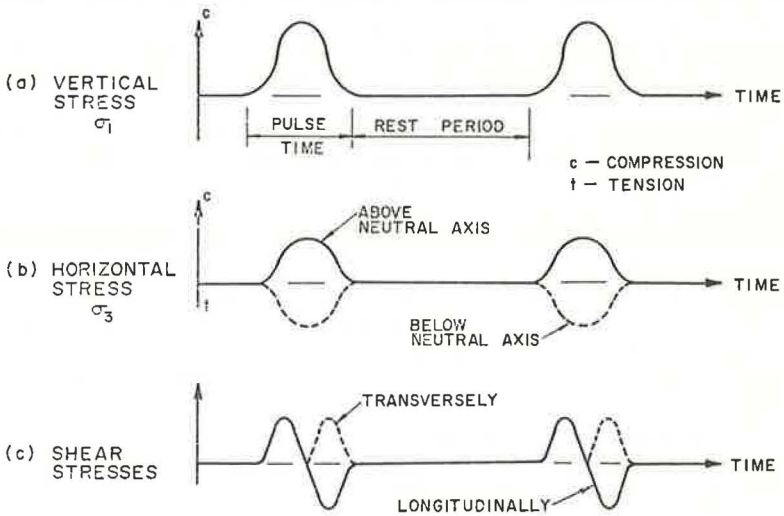


Figure 3. General view of equipment.

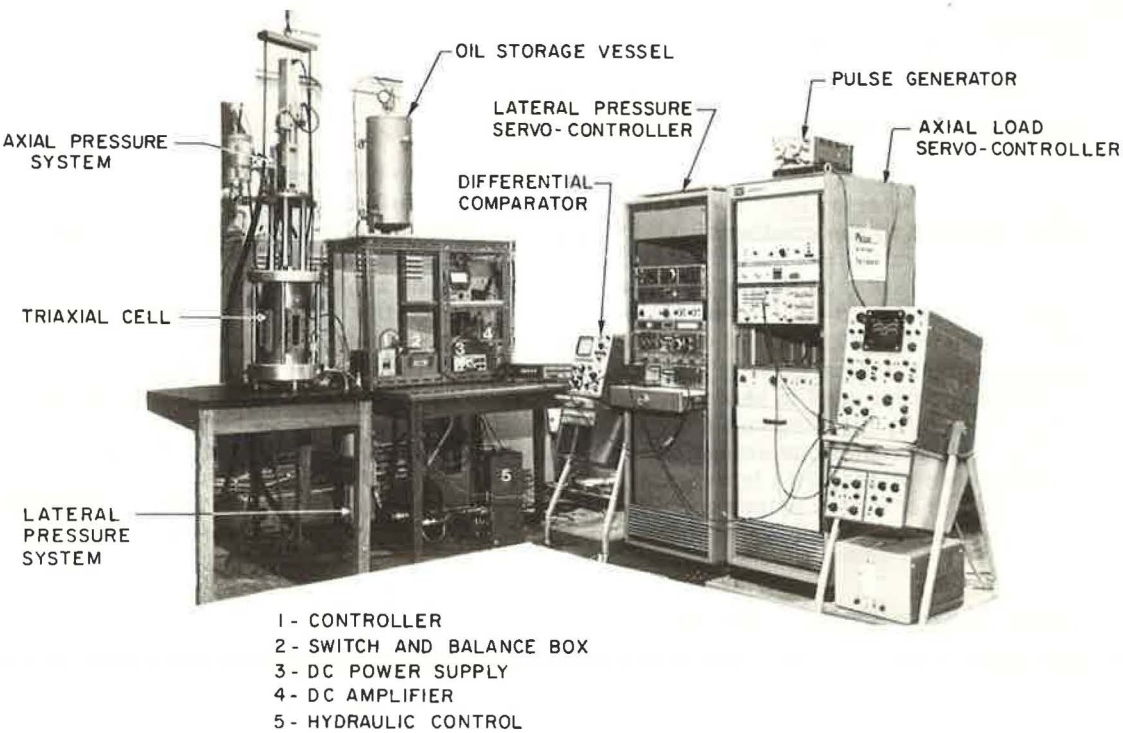


Figure 4. Average aggregate gradation curve of Brampton asphalt concrete base.

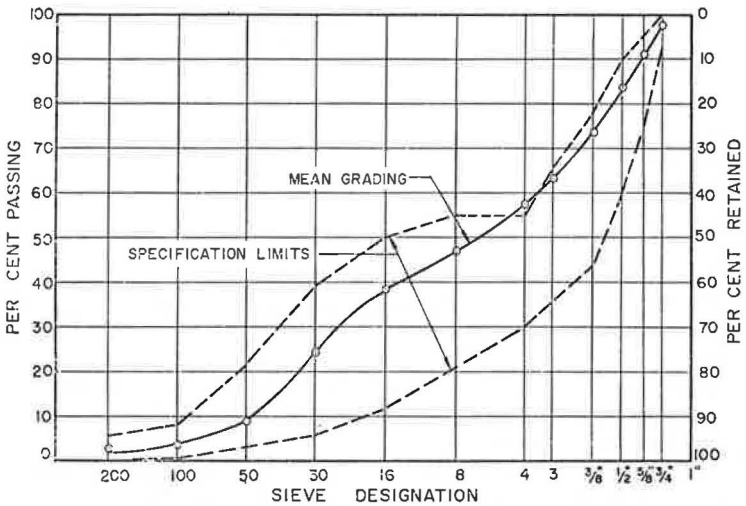
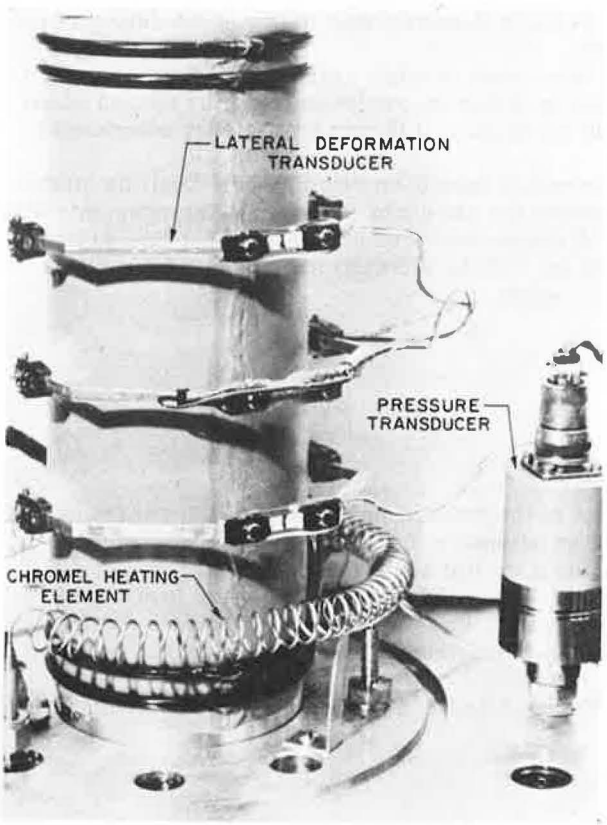


Figure 5. Lateral deformation transducers on specimen and heater.



permanent strain, ϵ_p, in an element of asphalt concrete could be expressed in the following functional form:

$$\epsilon_p = f(\sigma_1, \sigma_3, T, N) \pm E \quad (1)$$

where

σ_1 = vertical stress,
σ_3 = horizontal stress,
T = temperature,
N = number of load repetitions, and
E = estimate of error associated with any attempt to predict ϵ_p as a function of the four factors listed.

Because it was suspected that the material might respond differently in tension than in compression (i.e., below the neutral axis of Figure 1), the program made provision for independently considering these two stress modes.

The experimental program consisted of factorial arrangements of variables in two phases. Phase 1 included a 2^3 factorial (i.e., three factors each at two levels) for the asphalt concrete with 85/100 penetration grade asphalt (Table 2). The factorial was replicated, and a composite design was included (to evaluate any possible interactions and quadratic or second order effects and to provide estimates of error).

Phase 2 included a 2^4 factorial for asphalt concrete with 85/100 penetration asphalt (tension series only) and 300/400 penetration asphalt (compression series only). The functional form was the same as given in Table 2, except that high and low levels of air voids were included as the fourth factor.

Phase 1 was conducted primarily to determine whether reliable relationships could be developed and to determine the existence of any major interactions or second order effects. Phase 2 was conducted to build air voids and binder consistency effects into the relationships.

The actual arrangements of these factorials have been described in detail by Morris (8) and by Meyer (13). The range of factors for the various factorial arrangements was 30 to 67 psi (207 to 462 kPa) for σ_1 in the compressive mode, 20 to 48 psi (138 to 331 kPa) for σ_1 in the tension mode, 15 to 86 psi (103 to 593 kPa) for σ_3, 60 to 93 F (16 to 34 C) for T, and 2.5 to 10 percent for air voids.

RESULTS

Tension and Compression Test Series

Figures 6 and 7 show typical test results in the form of permanent strain versus number of load repetitions. Figure 6 shows the relationship for a compressive series (σ_1 is compression), and Figure 7 shows that for a tensile series (σ_1 is tension).

In both cases, there is a conditioning phase for the first few thousand load repetitions, followed by a stable phase (i.e., a straight line increase of permanent strain with increasing numbers of loads). The slopes of the straight line portions of the relationships were used for the analysis of the results.

The data from phase 1 are given in Tables 3 and 4, and phase 2 data are given in Tables 5 and 6.

Development of Cyclic Creep Models

Mathematical models that are linear in the unknown parameters are widely used to relate

Table 1. Asphalt properties.

Asphalt Penetration	Penetration at 77 F	Viscosity at 275 F (stokes)	Stiffness of Asphalt Cement (psi)	Stiffness of Asphalt Concrete (ksi)
85/100	89	39	4,250	420
			1,420	225
			425	110
			142	70
300/400	297	18	426	100
			7,171	30
			14.2	10
			5.7	4

Note: 1 F = 1.8 C + 32; 1 stoke = 0.0001 m^2/s; 1 psi = 6890 Pa; 1 ksi = 6.9 Pa.

Table 2. Selected experimental program.

Axial Stress	Lateral Pressure	Temperature T_0	Temperature T_1
σ_1	σ_{30}	(1)	c
	σ_{31}	b	bc
σ_{11}	σ_{30}	a	ac
	σ_{31}	ab	abc

Note: Subscript 0 = low level, 1 = high level; a = σ_1; b = σ_3; c = T. Inclusion of code letter indicates that factor is at its high level.

Figure 6. Typical permanent strain, compression series.

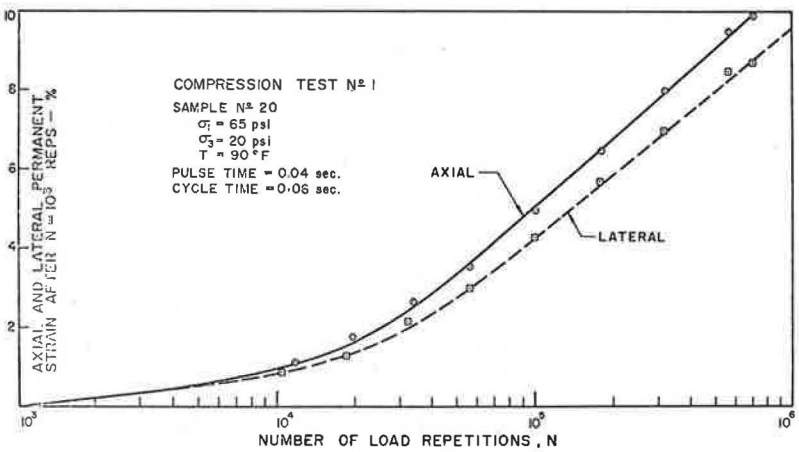


Figure 7. Axial and average lateral permanent strain, tension series.

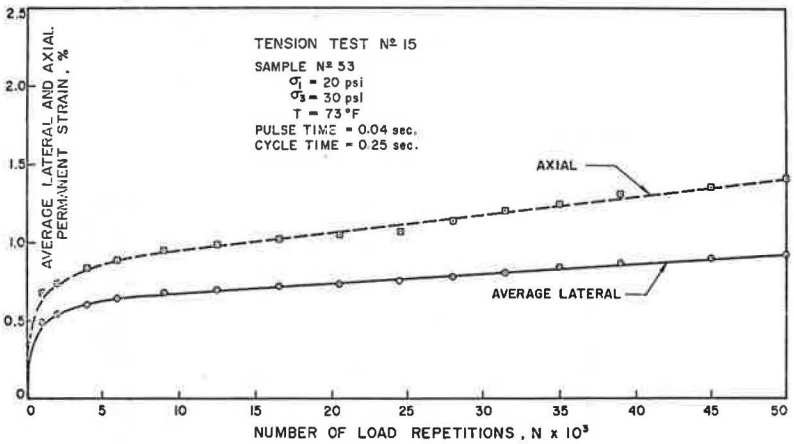


Table 3. Measured axial strain gradients/log N, compression series.

Item	Test No.	Sample No.	Condition	Gradient Percent/Log N
Main factorial	1	20	ac	5.80
	2	02	b	0.00
	3	38	c	3.10
	4	40	abc	0.10
	5	01	a	1.81
	6	23	a	0.56
	7	27	(1)	0.11
	8	26	ab	0.17
	9	42	b	0.10
	10	06	bc	-0.13
	11	09	c	1.50
	12	17	(1)	0.19
	13	08	abc	0.14
	14	14	ac	6.80
	15	44	bc	-0.27
	16	12	ab	0.05
Center and star points	17	34	σ_{10}	0.21
	18	30	Center	0.14
	19	33	Center	0.04
	20	36	T_1	0.16
	21	18	σ_{11}	0.15
	22	21	Center	0.11
	23	03	σ_{31}	0.07
	24	41	T_0	0.07
	25	07	σ_{30}	0.11

Note is as given in Table 2.

Table 4. Measured lateral strain gradients, tension series.

Item	Test No.	Sample No.	Condition	Gradient Percent/ 10^6 Cycles
Main factorial	1	59	abc	7.00
	2	69	b	0.38
	3	64	c	1.23
	4	61	c	2.10
	5	72	abc	6.58
	6	49	ac	0.63
	7	58	b	0.40
	8	50	(1)	0.20
	9	60	a	0.84
	10	47	ab	1.23
	11	74	a	1.36
	12	54	(1)	0.26
	13	48	ab	2.10
	14	55	bc	2.00
	15	53	bc	1.50
	16	45	ac	0.70
Center and star points	17	62	Center	1.60
	18	67	σ_{31}	2.14
	19	66	σ_{30}	1.00
	20	68	Center	1.74
	21	73	Center	1.40
	22	51	σ_{11}	2.60
	23	71	σ_{10}	0.47
	24	65	T_0	0.70
	25	56	T_1	2.90

Note is as given in Table 2.

Table 5. Tension results for 85/100 penetration asphalt.

Test No.	σ_V (psi)	σ_H (psi)	T (C)	Air Voids (percent)	Treatment	Percentage of ϵ_p per 10^5 Cycles
1	40	15	21	10	acd	2.1
2	20	35	16	10	bd	0.4
3	20	15	16	2.5	I	0.2
4	40	15	16	10	ad	1.35
5	40	35	21	10	abcd	9.75
6	20	35	16	2.5	b	0.6
7	40	15	16	2.5	a	1.0
8	20	15	21	10	cd	0.35
9	20	15	21	2.5	c	0.3
10	40	35	16	10	abd	2.0
11	40	35	16	2.5	ab	1.1
12	20	35	21	2.5	bc	0.4
13	20	35	21	10	bcd	2.1
14	20	15	16	10	d	0.2
15	40	15	21	2.5	ac	2.1
16	40	35	21	2.5	abc	2.2

Note: 1 psi = 6890 Pa.

variables such as those considered in this investigation. This requires the use of a number of assumptions, for purposes of simplification, as discussed by Davies (14), Yates (15), and others (8, 13).

If a good correlation between the outputs of the model and the observed data is not evident, a transformation may be necessary. Many such transformations are available, and several procedures can be found (16). In this investigation, a logarithmic transformation was satisfactory in phase 1 and phase 2.

The final models adopted from the analysis of data were as follows:

1. Compression mode, asphalt concrete with 85/100 penetration asphalt

$$\ln y = -1.7602 + 0.7\sigma_1 - 1.4281\sigma_3 + 0.4359T - 0.6790\sigma_3 T \quad (2)$$

where

y = predicted, accumulated permanent strain, in percent, per log number of load applications,
σ_1 = coded vertical stress [-1 for 45 psi (310 kPa) and +1 for 60 psi (414 kPa)],
σ_3 = coded lateral stress [-1 for 20 psi (138 kPa) and +1 for 80 psi (552 kPa)], and
T = coded temperature [-1 for 65 F (18 C) and +1 for 90 F (32 C)].

2. Tension mode, asphalt concrete with 85/100 penetration asphalt

$$\ln y = -0.1305 + 0.8404\sigma_3 + 0.3963T + 0.3261\sigma_1 + 0.2830AV + 0.2217AV\sigma_3 \quad (3)$$

where

y = predicted, accumulated permanent strain, in percent, per 10^5 load applications,
σ_1 = coded vertical stress [-1 for 20 psi (138 kPa) and +1 for 40 psi (276 kPa)],
σ_3 = coded lateral stress [-1 for 15 psi (103 kPa) and +1 for 40 psi (276 kPa)],
T = coded temperature [-1 for 60.8 F (16 C) and +1 for 69.8 F (21 C)], and
AV = coded air voids (-1 for 2.5 percent and +1 for 10 percent).

3. Compression mode, asphalt concrete with 300/400 penetration asphalt

$$\ln(y + 0.1) = -1.5424 + 0.6782\sigma_1 - 0.3472\sigma_3 + 0.2881AV - 0.2668\sigma_3 T - 0.2490\sigma_3 AV + 0.2286T + 0.2078\sigma_1 T - 0.1926\sigma_3 T\,AV + 0.1906\sigma_1 AV \quad (4)$$

where

y = predicted, accumulated permanent strain, in percent, per 10^6 load applications,
σ_1 = coded vertical stress [-1 for 30 psi (207 kPa) and +1 for 55 psi (379 kPa)],
σ_3 = coded lateral stress [-1 for 20 psi (138 kPa) and +1 for 35 psi (241 kPa)],
T = coded temperature [-1 for 64.4 F (18 C) and +1 for 78.8 F (26 C)], and
AV = coded air voids (-1 for 2.5 percent and +1 for 10 percent).

Equation 2 was derived from phase 1 in which air voids were not included as a variable; equations 3 and 4 come from the phase 2 results. The plausibility of these models was extensively evaluated in terms of lack of fit checks (8, 13). An examination of the residuals showed no patterns of inconsistency (i.e., the variances were relatively constant). Additionally, the standard error of the predicted value of y was about 0.89 for equation 2 and about 0.33 for equations 3 and 4 with an R^2 value of 0.94.

The models are reasonable in that an increase in y occurs for the following:

1. In tension, σ_1 increase, T increase, σ_3 increase, and AV increase; and
2. In compression, σ_1 increase, T increase, σ_3 decrease, and AV increase.

The models represented by equations 2 and 3 are relatively simple. Equation 4 is somewhat more complex, but this may be due to the effect of loading a material of lower stiffness (i.e., with softer asphalt). A tension model was not developed for the softer mix (with 300/400 penetration grade asphalt) because of the very low tensile stresses that would occur in a layer constructed with this material.

Field Comparisons

The basic steps required to predict permanent deformation in asphalt concrete pavements by using the models developed in this investigation are shown in Figure 8. The procedure is briefly detailed below.

1. Obtain the average temperature distribution throughout the asphalt concrete layer during those hours of the day when traffic is likely to cause permanent deformation. Depending on the degree of sophistication required and the sensitivity of the analysis, either a daily, weekly, or monthly average temperature may be selected.
2. Calculate the maximum vertical and horizontal stress distributions throughout the pavement at points immediately below the center of the wheelpaths. These can be reliably predicted by nonlinear elastic procedures such as finite element or iterative elastic multilayered solutions (18). The stiffness of the asphalt concrete is varied in accordance with the temperature distribution obtained in step 1.
3. Test the materials under simulated field conditions by using a statistically designed experimental program, and derive cyclic creep models of the form shown in Equation 1.
4. Calculate the total permanent deformation in the pavement after N load applications by dividing the layers into sublayers and summing the permanent deformations of the elements in each sublayer immediately below the center of the wheelpath. This can be expressed mathematically as

$$\Delta_p = \sum_{i=1}^{n} (\bar{\epsilon}_{pi})(\Delta_{hi}) \tag{5}$$

where

Δ_p = total permanent deformation in the pavement system,
$\bar{\epsilon}_{pi}$ = average permanent strain in the ith sublayer,
Δ_{hi} = height (or thickness) of the ith sublayer, and
n = total number of sublayers in the pavement.

These steps were used to calculate the cumulative permanent deformations of the full-depth asphalt concrete sections at the Brampton Test Road. The following simplifying assumptions were made.

Table 6. Compression results for 300/400 penetration asphalt.

Test No.	σ_v (psi)	σ_H (psi)	T (C)	Air Voids (percent)	Treatment	Percentage of ϵ_p per 10^6 Cycles
1	55	20	26	2.5	ac	0.3
2	55	20	26	10	acd	4.5
3	55	35	26	2.5	abc	0.176
4	30	20	26	10	cd	0.2
5	55	35	18	2.5	ab	0.11
6	30	20	18	10	d	0.03
7	55	20	18	10	ad	0.3
8	30	35	26	10	bcd	-0.043
9	55	35	26	10	abcd	0.255
10	30	35	18	2.5	b	-0.004
11	30	20	18	2.5	I	0.011
12	55	35	18	10	abd	0.227
13	30	35	26	2.5	bc	-0.02
14	30	20	26	2.5	c	0.01
15	30	35	18	10	bd	-0.008
16	55	20	18	2.5	a	0.1

Note: 1 psi = 6890 Pa.

Figure 8. Basic steps in simulative statistically based approach.

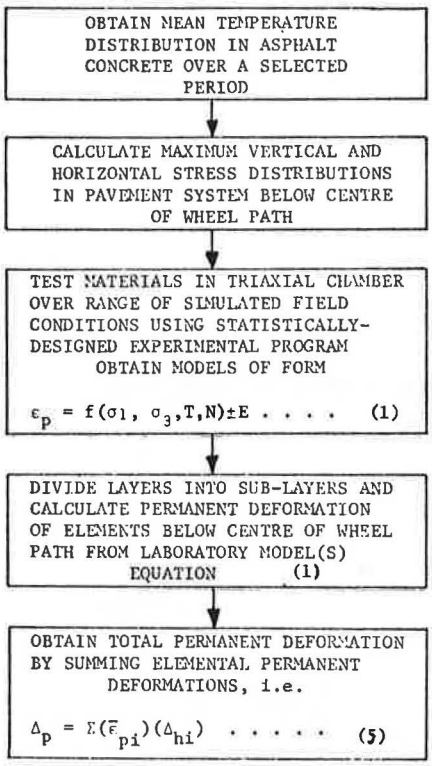


Figure 9. Average monthly pavement temperature-depth relationships.

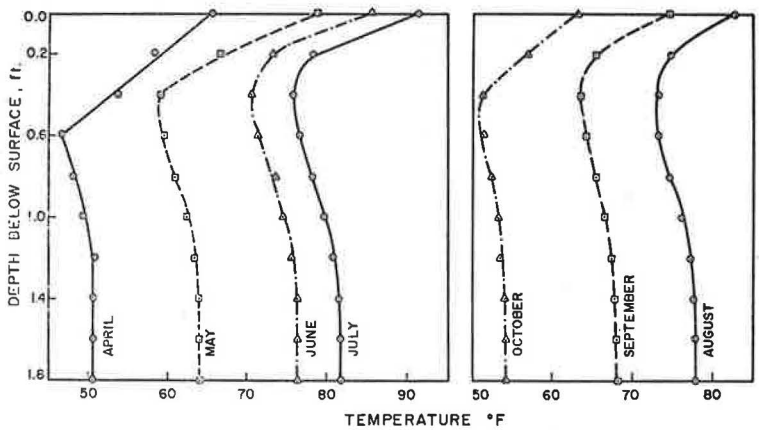


Figure 10. Vertical and horizontal stresses for 85/100 penetration full-depth asphalt concrete.

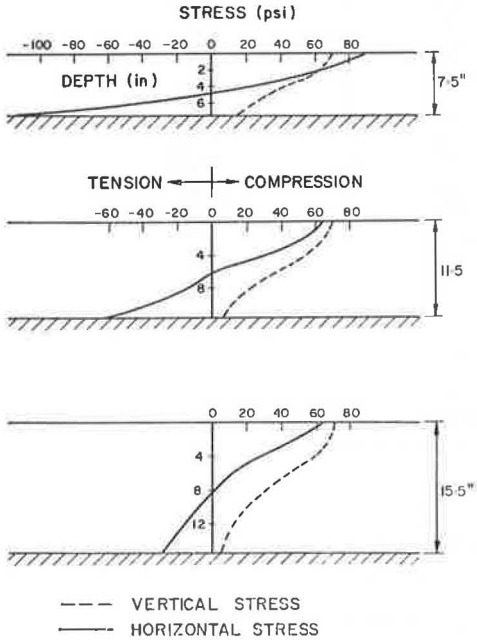


Figure 11. Vertical and horizontal stresses for 300/400 penetration full-depth asphalt concrete.

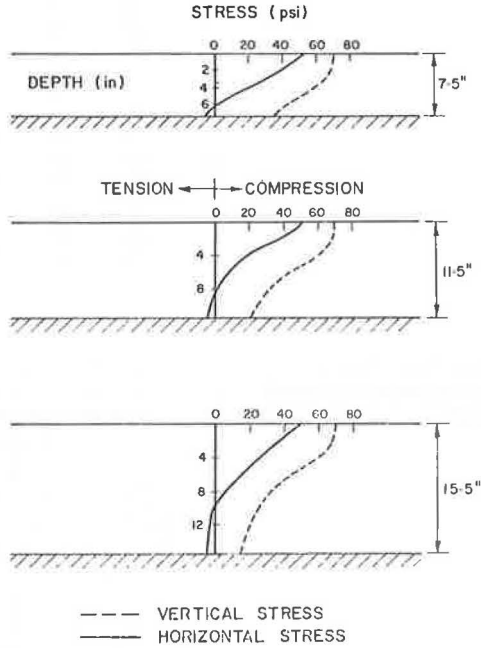
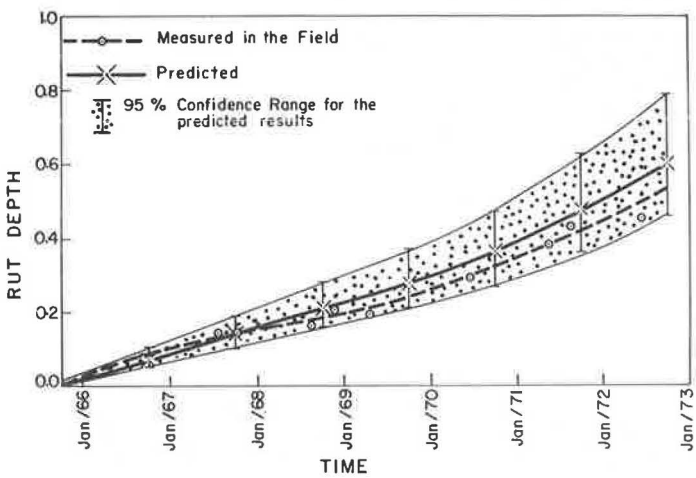


Figure 12. Rut depth-time relationship for section 3 of the Brampton Test Road.



1. Traffic data—Classification surveys including counts have been conducted on the average four times annually since the test road was constructed. Axle weight studies have also been conducted, and data on the accumulated equivalent single axle loads have been obtained. The daily variation of these loads has been shown to be relatively constant, based on this data (8).

2. Pavement temperatures—These were analyzed by the Barber method (18). The average temperature and number of load repetitions should be calculated over that interval when permanent deformation is most likely to occur. This requires consideration of the truck traffic pattern, and the temperature below which little or no permanent deformation is likely to occur. A careful review of the Brampton data led to the assumptions that permanent deformation (a) occurs daily between 7:30 a.m. and 5:30 p.m., (b) occurs only in the period from April to October, and (c) can be ignored at temperatures below 50 F (10 C). Based on these assumptions, the average monthly temperature distributions shown in Figure 9 were obtained.

3. Stress analysis—Stresses were analyzed by means of a finite element program, FEPAVE II (20). The following assumptions were made: (a) The surface was stressed to 70 psi (482 kPa) uniformly over a circular area, 12.8 in. (322 mm) in diameter; (b) the temperature in the asphalt was distributed as shown in Figure 9; (c) the resilient modulus M_R of the subgrade was a function of the deviator stress σ_d as determined by repeated-load triaxial tests; (d) the stiffness-temperature relationship for the asphalt concrete was given by McLeod's indirect method (20) at a loading time of 0.04 sec; and (e) Poisson's ratio was 0.38 and 0.43 for the asphalt concrete and subgrade respectively. Typical stress distributions for full-depth sections 7.5, 11.5, and 15.5 in. (190.5, 292.1, and 393.7 mm) thick at Brampton for the maximum summer temperatures are shown in Figures 10 and 11 for mixes with 85/100 and 300/400 penetration grade asphalt (only the 7.5- and 11.5-in. thicknesses with 85/100 penetration grade were actually constructed; other values used are for comparative purposes).

The calculated values of accumulated permanent deformation for the 11.5-in.-thick section with 85/100 penetration grade asphalt are shown in Figure 12. Comparison with actual measured values, shown in the same figure, indicates a very good agreement. If a 300/400 penetration asphalt had been used in this section, the total accumulated permanent deformation would be estimated (using equation 4) at about 0.9 in. (23 mm).

A detailed examination of these results suggests that the major amount of permanent deformation in the harder asphalt layer actually occurs because of lateral flow in the tension zone (8), i.e., below the neutral axis.

CONCLUSIONS

The major points of this paper may be summarized as follows:

1. A reliable, predictive method for estimating cyclic creep in bituminous materials is a key aspect of a permanent deformation design subsystem.
2. An approach based on simulating field loading-temperature conditions in the laboratory is a practical way of developing cyclic creep models. The necessary equipment has been developed and briefly described in this paper.
3. Cyclic creep models of acceptable reliability have been developed by using a statistically designed experimental program. The models are relatively simple and they incorporate interactions of variables as well as estimates of error.
4. The models were able to predict permanent deformation for the full-depth sections at the Brampton Test Road in close agreement with actual measured values.

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REFERENCES

1. Proc., Third International Conference on the Structural Design of Asphalt Pavements. London, 1972.
2. Conference on Methods for Prediction of Permanent Deformations in Pavement Systems. Univ. of Texas at Austin, Aug. 1973.
3. A. G. J. Klomp and G. M. Dorman. Stress Distribution and Dynamic Testing in Relation to Road Design. Proc., Australian Road Research Board, 1964.
4. J. E. Romain. Rut Depth Prediction in Asphalt Pavements. Third International Conference on Structural Design of Asphalt Pavements, London, 1972.
5. F. Moavenzadeh, J. E. Soussou, and H. K. Findakly. Synthesis for Rational Design. Final Reports for FHWA Contract 7776, Vols. 1 and 2, Jan. 1974.
6. R. D. Barksdale. Rutting of Pavement Materials. Georgia Institute of Technology, Atlanta, Aug. 1973.
7. Structural Design of Asphalt Concrete Pavement Systems. HRB Special Rept. 126, 1970.
8. J. Morris. The Prediction of Permanent Deformation in Asphalt Concrete Pavements. Transport Group, University of Waterloo, Ontario, Sept. 1973.
9. J. Morris and R. C. G. Haas. Designing for Rutting in Asphalt Concrete Pavements. Presented at Annual Conference of Canadian Technical Asphalt Association, Vancouver, 1972.
10. J. Morris and R. C. G. Haas. Characterization of Bituminous Mixtures for Permanent Deformation Predictions. Presented at ASTM Annual Meeting, June 1973.
11. J. Morris, R. C. G. Haas, P. Reilly, and E. Hignell. Permanent Deformation in Asphalt Pavements Can Be Predicted. Presented at AAPT meeting, Feb. 1974.
12. R. C. G. Haas, N. I. Kamel, and J. Morris. Brampton Test Road: An Application of Layer Analysis to Pavement Design. Ministry of Transportation and Communications, Ontario, Rept. RR182, Nov. 1972.
13. F. R. P. Meyer. Permanent Deformation Prediction of Asphalt Concrete Pavements. M.A.Sc. thesis, Univ. of Waterloo, March 1974.
14. O. L. Davies. The Design and Analysis of Industrial Experiments. Oliver and Boyd, London, 1967.
15. F. Yates. Design and Analysis of Factorial Experiments. Imperial Bureau of Soil Science, London, 1937.
16. N. R. Proper and H. Smith. Applied Regression Analysis. John Wiley and Sons, New York, 1966.
17. R. D. Barksdale and R. G. Hicks. Material Characterization and Layered Theory for Use in Fatigue Analyses. HRB Special Rept. 140, 1973.
18. E. S. Barber. Calculation of Maximum Pavement Temperatures From Weather Reports. HRB Bulletin 168, 1957.
19. J. M. Duncan, C. L. Monismith, and E. L. Wilson. Finite Element Analysis of Pavements. Highway Research Record 228, 1968.
20. R. C. G. Haas. A Method for Designing Asphalt Pavements to Minimize Low-Temperature Shrinkage Cracking. Asphalt Institute, Rept. 73-1, Jan. 1973.