CLARIFYING THE AMBIGUITIES OF INTERNAL RATE OF RETURN METHOD VERSUS NET PRESENT VALUE METHOD FOR ANALYZING MUTUALLY EXCLUSIVE ALTERNATIVES

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Many engineering economists have attempted to demonstrate that proper use of either the net present value method or the internal rate of return method to analyze mutually exclusive alternatives will result in identical and correct economic decisions. Unfortunately, however, the internal rate of return method, even when properly applied, often will result in either ambiguous or incorrect economic decisions. The purpose of this paper is to illustrate more completely and definitively the ambiguities that can occur and to show that the 2 methods cannot be reconciled without additional calculations, which, by definition, go beyond the internal rate of return method as strictly and properly applied.

•THE LITERATURE of economics and engineering economics is rich with articles and books dealing with the various methods of economic analysis for assessing and comparing alternative, mutually exclusive investment projects. Although the engineer's interest and knowledge in this subject has been sharpened, more often than not the various articles and books appearing within the engineer's domain are misleading, incorrect, or incomplete.

Consequently, I shall review 2 of the most popular benefit-cost analysis methods and, hopefully, demonstrate which method is appropriate or inappropriate for certain conditions and why.

BENEFIT-COST ANALYSIS DATA REQUIREMENTS

Let us assume that some given number of mutually exclusive engineering projects are being analyzed. Each of them will, in turn, lead to a series of present and future cost outlays (capital or operating) and to a stream of present and future benefits. The planning or analysis period and the minimum attractive rate of return (MARR), which also is known as the cutoff rate or opportunity cost of capital, will need to be known.

Analysis or Planning Period

It is important to analyze various investment proposals over the same analysis period to properly account for reinvestment of any earnings or benefits accrued before the end of the analysis (or replacement) period especially when one project may have a shorter terminal date than another (whether replaced or not) (2, pp. 74 ff.; 1, p. 233). There are, of course, many ways to ensure that projects are compared for the same periods of analysis. Some are explicit and some are not. For example, if project A has some capital items whose service life is so short that they must be replaced or terminated before the end of the planning or analysis period, then the application of a capital recovery factor to the initial capital outlay for rolling stock or other capital items will result in the implicit assumptions that (a) the capital items are perpetually replaced at the end of their service life and (b) the replacement costs of the capital items in future years will be exactly the same as they were when the project was started. A more appropriate analysis method would simply list the year-by-year cost outlays and benefits (or revenues, where appropriate) that are expected to occur in planning or analysis regardless of whether they change. This latter method at least permits both factor price and technological changes to be accounted for properly.

If a project is terminated rather than replaced before the end of planning, then benefit-cost comparisons will be valid as long as either the discounted benefit-cost ratio or net present value (NPV) methods are used and calculated with an appropriate discount or interest rate. The benefit-cost ratio method will not be discussed in detail in this paper, but, if it is properly applied, the decisions among alternatives will not differ from those of the net present value method when either discounted or equivalent annual benefits and costs are used even though more calculations will be required with the benefit-cost ratio method. On the other hand, use of the internal rate of return method will not always permit valid comparisons to be made among alternatives in the same case (1, pp. 234-241), or its use will result in ambiguities.

The following are essential points with which the analyst is concerned:

1. Examining the benefit and cost conditions expected to occur over the same analysis or planning period for all alternatives regardless of replacement or early termination and
2. Based on expected future benefits and costs, determining whether any initial capital outlays should be made at the present and, if so, which level of outlay is best.

For item 1, if a project among the set of alternatives is terminated early, the analyst must be concerned with other available opportunities for using the capital funds that would have been used for replacement and what returns (benefits or revenues) can be accrued from them. Similarly, when benefits or revenues are accrued in early years either before the end of the analysis period or before the terminal date of any project, the analyst cannot ignore the problem of properly accounting for the reinvestment or use of the early year benefits or revenues. Some of these matters will be clarified in later examples (1, pp. 234-241).

Opportunity Cost of Capital or Appropriate Discount or Interest Rate

In this paper, no attempt will be made to fully describe the difficulties and problems associated with choosing an appropriate discount or interest rate for use in some of the benefit-cost analysis methods (5, pp. 116-151). For each of the methods, though, an interest rate must be specified directly or indirectly. Often, and especially for the internal rate of return method, the interest rate to be specified is referred to as the minimum attractive rate of return, which reflects the interest that can be earned from foregone alternative opportunities. This term is equivalent to that used by economists, which is the opportunity cost of capital or an interest rate that reflects the earnings that will be foregone from other investment opportunities if the capital is to be committed to a project in question. To a large extent, the specification of an appropriate interest rate or MARR or opportunity cost of capital is arbitrary and thus open to question. Consequently, the analysis should be carried out for a range of interest rates. This range may reflect private market rates at one extreme and judgments about the social rate of discount at the other extreme. The range may vary widely from 3 to as much as 25 percent. However, 1 point is clear: The rate to be used in any analysis is usually not equal to the borrowing rate for bonds that must be floated to raise capital for a project.

METHODS OF BENEFIT-COST ANALYSIS

The net present value or net present worth and the discounted internal rate of return methods can be most easily described analytically.

Let

i = interest or discount rate (minimum attractive rate of return or opportunity cost of capital) in decimal form,
n = length of analysis period or planning horizon in years,
$C_{x,t}$ = expected cost outlays (capital or operating) for project x during year t, and
$B_{x,t}$ = expected benefits or revenues from project x during year t.

For convenience, it will be assumed that $B_{x,t}$ or $C_{x,t}$ will be accrued or committed in lump sum at the end of t. Typically, for other than the do nothing or abandonment alternative, that is, when $C_{x,0} = 0$, some initial cost outlays will occur in the beginning of the first year (when $t = 0$); benefits or revenues will not usually begin to accrue until at least a year later (when $t \geq 1$). In any case, though, the formulation is perfectly general and will apply to all situations. The cost and benefit streams during the n year planning period for any project x will look the same as those shown in Figure 1. In Figure 1, it is assumed that costs or benefits are incurred or accrued in a lump sum at the end of year t and that the costs or benefits during any year t can be 0.

A year-by-year cash flow tabulation of the benefits and costs for all alternatives in which, say, there are m alternatives and thus x varies from x = 1, 2, . . ., m could be displayed in much the same manner as that indicated for project x in Figure 1. However, the m alternatives should be ordered or ranked in ascending order so that alternative 1 is the alternative having the lowest initial cost in year t = 0 (x = 1), and alternative 2 is the alternative having the next lowest initial cost in year t = 0 (x = 2). The alternative having highest initial cost in year t = 0 is alternative m (x = m). These ranking or ordering rules can be applied to all the benefit-cost methods, but they are not necessary for the net present value method.

Net Present Value Method

With the net present value method, the benefits and costs are discounted to their present value or present worth, that is, to their value in year t = 0, and then netted to determine the resultant net present value. Determined analytically for project x, $NPV_{x,n}$, the net present value for the n-year analysis period, is

$$NPV_{x,n} = \sum_{t=0}^{n} \frac{1}{(1+i)^t} \cdot B_{x,t} - \sum_{t=0}^{n} \frac{1}{(1+i)^t} \cdot C_{x,t} \tag{1}$$

or

$$NPV_{x,n} = \sum_{t=0}^{n} \frac{1}{(1+i)^t} \cdot (B_{x,t} - C_{x,t}) \tag{2}$$

where

$\frac{1}{(1+i)^t}$ = present worth factor for year t, which is a factor for reducing future benefits or costs to present day values, and
i = minimum attractive rate of return or opportunity cost of capital in decimal form.

For each alternative, from x = 1 to x = m, the net present value must be determined.

In turn, the alternative having the highest nonnegative net present value is selected as best from an economic standpoint.

The net present value method is straightforward and guarantees that public or private agencies will maximize their net benefits or profits for any type of measurement, planning period, or interest rate. When the opportunity cost of capital (discount rate for other investments) is unknown or questionable, the calculations can be repeated for different rates, and the final results can be compared for similarities or differences in ranking. Also, if one should move from a lower initial cost alternative to a higher initial cost alternative, the net present value increases, and one may be certain that the discounted incremental or extra benefits outweigh the discounted extra costs.

There is no more easily applied, unambiguous, and less tedious benefit-cost analysis method than the net present value method. Moreover, the method is just as applicable to situations in which there is a budget constraint and the problem is to select the most worthwhile set of projects among a larger group of alternatives. In such a case, one simply combines those projects whose total initial costs are less than or equal to the budget constraint but whose combined total net present value is largest.

Discounted Internal Rate of Return Method

The discounted internal rate of return method has been popularized increasingly by engineering economists although, oftentimes, it has been improperly explained or used. Most important, though, this method can result in the making of improper or incorrect economic choices. More recently, Bergmann (6, p. 81) outlined a method that attempted to reconcile the results of the internal rate of return and net present value methods and avoid the ambiguities that can result from use of the internal rate of return method. He developed a rank ordering technique for alternatives that appeared to obviate the ambiguity that can result with certain investment cases; he did note, however, that his method was not general and "... applies only to situations where the rates of return on both the basic and incremental investments for each alternative are unique." The 3 examples to be contained in this paper will demonstrate that his special rank ordering technique indeed only applies when rates of return are unique and thus does not avoid the ambiguities and reconcile the different decisions that result from the rate of return and net present value methods in situations in which nonunique solutions occur.

As a consequence, a fully general technique will be outlined in this paper that will be explained in more detail for the more general 3-year and n-year cases. Hopefully, these examples and the accompanying explanation can clarify the matter and thus permit analysts to discard those methods that give incorrect or ambiguous answers when they evaluate mutually exclusive projects.

The discounted internal rate of return method has 2 essential steps (3, pp. 65-66; 1, pp. 230-232). In the first step, a MARR or opportunity cost of capital must be stated. This discount rate serves as the cutoff rate for accepting or rejecting projects being analyzed. Given this, the next step is to compute the internal rate of return for the lowest initial cost alternative ($x = 1$). The internal rate of return, r_x, for any project x can be determined analytically or iteratively by determining the rate of return value or discount rate, in decimal terms, that satisfies the following formulation. Find r_x, so that

$$\sum_{t=0}^{n} \frac{1}{(1+r_x)^t} \cdot B_{x,t} = \sum_{t=0}^{n} \frac{1}{(1+r_x)^t} \cdot C_{x,t} \tag{3}$$

where $1/(1+r_x)^t$ = discount factor for internal rate of return method. If r_x is at least as large as the MARR, then alternative x is judged to be economically acceptable by this method. (A later example will show that this is not necessarily correct.)

The r_x for individual projects is determined, starting with alternative x = 1, until the lowest initial cost project having an acceptable internal rate of return ($r_x \geq$ MARR) is ascertained. This alternative, say, alternative x, then becomes the lowest cost-acceptable alternative.

The second step in the internal rate of return method is to determine the internal rate of return on increments of investment or initial cost over the lowest acceptable initial cost alternative. Again, if alternative x is the lowest acceptable initial cost alternative, then the internal rate of return on the increase in initial cost between x and the next higher initial cost alternative (x + 1) must be determined. Find $r_{x/x+1}$, the internal rate of return on the increase in investment or initial cost between alternative x and the next higher initial cost alternative x + 1, so that

$$\sum_{t=0}^{n} \frac{1}{(1+r_{x/x+1})^t} (B_{x+1,t} - B_{x,t}) = \sum_{t=0}^{n} \frac{1}{(1+r_{x/x+1})^t} (C_{x+1,t} - C_{x,t}) \qquad (4)$$

where $1/(1+r_{x/x+1})^t$ = discount factor for internal rate of return on increment in initial cost. When the lowest initial cost alternative, say, x, having an acceptable rate of return ($r_x \geq$ MARR) is determined, then paired calculations for increasingly higher initial cost alternatives are made by using equation 4; if $r_{x/x+1}$ is at least as large as the MARR, then alternative x + 1 is accepted as a better alternative. If not, then alternative x + 1 is rejected, and a paired comparison is made between x and x + 2 and so forth until the highest initial cost alternative that satisfies both sets of rate of return calculations is determined. Under the internal rate of return method, the highest initial cost alternative satisfying these conditions will be selected as the best economically.

However, if the internal rate of return formula (equation 3) for any alternative x and the internal rate of return formula for the increment in initial cost found when one compares alternative x with x + 1 (equation 4) are rearranged, 2 formulations will result. Find r_x so that

$$\sum_{t=0}^{n} \frac{1}{(1+r_x)^t} \cdot (B_{x,t} - C_{x,t}) = 0 \qquad (5)$$

This is identical to saying: The internal rate of return for any alternative x is exactly equivalent to the discount rate at which the net present value is 0. (Compare equations 3 and 5.) Find $r_{x/x+1}$ so that

$$\sum_{t=0}^{n} \frac{1}{(1+r_{x/x+1})^t} \cdot (B_{x+1,t} - C_{x+1,t}) = \sum_{t=0}^{n} \frac{1}{(1+r_{x/x+1})} \cdot (B_{x,t} - C_{x,t}) \qquad (6)$$

This is identical to saying: The internal rate of return for increments of investment or initial cost between 2 alternatives is exactly equivalent to the discount rate at which the net present value of the 2 alternatives being compared is equal. (Compare equations 4 and 6.)

APPLICATION AND CRITIQUE OF NET PRESENT VALUE AND INTERNAL RATE OF RETURN METHODS

To apply and critique net present value and internal rate of return methods, let us consider 3 examples, 1 of which has been widely discussed (4, pp. 38-54; 1, pp. 241-243; 6) but is somewhat oversimplified and 2 others that are less well known but underscore the ambiguities of the internal rate of return method. For the last of these, the data were obtained from Bierman and Smidt (4, p. 55, problem 3-2).

Example 1

Assume the 2-year stream of benefits and costs given in Table 1. The 2 alternatives having equal initial costs but different benefits and costs in the following 2 years have been ranked according to the method suggested by Bergmann (6, p. 81) so that the first alternative, x = 1, is that which has the highest benefits during the first year. For the data in Table 1, r_x for alternative x = 1 is 25 percent; r_x for x = 2 is 20 percent; $r_{1 \text{ versus } 2}$ is ≈10.9 percent. These rates are given in percentages for convenience. Bergmann argues that, for alternatives having equal initial costs and a unique solution, unambiguous results occur whether one uses the net present value or whether one uses the internal rate of return method as long as the alternatives are ranked in the fashion that he suggests. That is, if the initial costs are equal, the first alternative is that which has the highest earnings or lowest costs, whichever applies, during the first year when t = 1. For this very special situation, one which is hardly applicable generally, Bergmann's ranking does produce identical results and reconcile the methods. But it would be misleading to suggest that the methods can be reconciled generally. For the data given in Table 1, the internal rate of return method would result in the selection of alternative x = 2 as long as the MARR was about 10.9 percent or less. For a MARR value above 10.9 percent but equal to or below 25 percent, alternative x = 1 is best. For higher MARRs, neither alternative would be selected. Calculations of the net present value at different interest rates would give identical decision results for these data and this case as shown in Figure 2.

Example 2

Assume the 2-year stream of benefits and costs given in Table 2. They will be ranked according to the rule outlined by Bergmann (6, p. 81). For the data in Table 2, r_x for alternative x = 1 is 20 and 1,580 percent; r_x for x = 2 is 25 and 1,022 percent; $r_{1 \text{ versus } 2}$ is ≈10.9 percent. Although the data used in Table 2 are not typical, they nonetheless will demonstrate the ambiguity that can result from using the internal rate of return method in the usual fashion even when alternatives are ranked in the manner outlined by Bergmann. For instance, given the data shown in Table 2 and the internal rates of return, a confusing and ambiguous set of conclusions will result if the analyst insists on applying the rate of return method in straightforward fashion without additional calculations. To be specific, he or she presumably would come to 1 of 2 sets of conclusions.

1. The high rates of return for alternatives x = 1 and x = 2 (1,580 and 1,022 percent respectively) should be ignored or rejected as unmeaningful. This means that (a) alternative x = 2 is best for a MARR equal to or less than 10.9 percent; (b) alternative x = 1 is best for a MARR between 10.9 and 20 percent; (c) alternative x = 2 is best for a MARR over 20 percent; and (d) no alternative is acceptable for a MARR greater than 25 percent.
2. The lower rates of return figures for alternatives x = 1 and x = 2 (20 and 25 percent respectively) should be ignored. This means that (a) alternative x = 2 is best for a MARR equal to or less than 10.9 percent; (b) alternative 1 is best for a MARR between 10.9 and 1,580 percent; and (c) no alternative is acceptable for a MARR greater than 1,580 percent.

Figure 1. Cost-benefit streams.

YEAR t	COSTS DURING YEAR t	BENEFITS DURING YEAR t
t = 0	$C_{x,0}$	$B_{x,0}$
1	$C_{x,1}$	$B_{x,1}$
.	.	.
.	.	.
t	$C_{x,t}$	$B_{x,t}$
.	.	.
.	.	.
t = n	$C_{x,n}$	$B_{x,n}$

Table 1. Cost and benefit data for example 1.

	Alternative x = 1 (dollars)		Alternative x = 2 (dollars)	
Year	$B_{1,t}$	$C_{1,t}$	$B_{2,t}$	$C_{2,t}$
t = 0	0	100	0	100
t = 1	100	0	20	0
t = 2	31.25	0	120	0

Note: Some numbers have been rounded for convenience.

Figure 2. Plot of net present value versus interest rate for example 1 data.

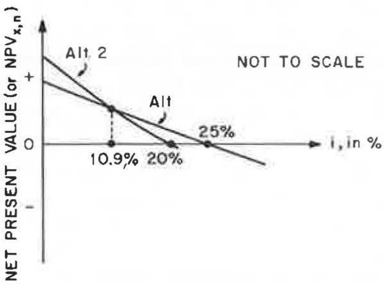


Table 2. Cost and benefit data for example 2.

	Alternative x = 1 (dollars)		Alternative x = 2 (dollars)	
Year	$B_{1,t}$	$C_{1,t}$	$B_{2,t}$	$C_{2,t}$
t = 0	0	100	0	100
t = 1	1,800	0	1,247	0
t = 2	0	2,016	0	1,403

Note: Some numbers have been rounded for convenience

Figure 3. Plot of net present value versus interest rate for example 2 data.

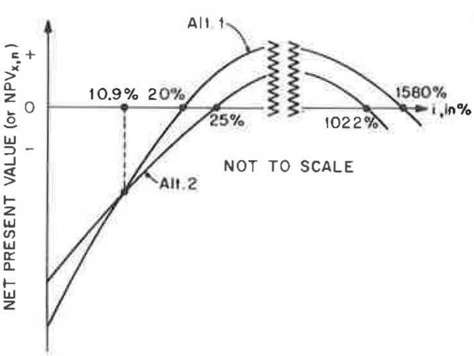
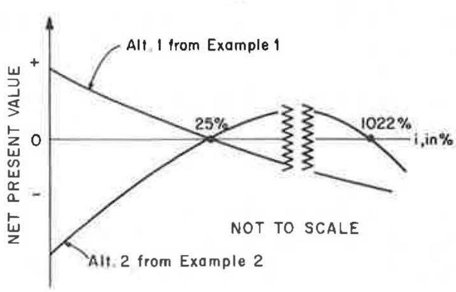


Figure 4. Plot of net present value versus interest rate for 2 examples when both have identical initial costs and basic internal rates of return.



It is evident that the 2 sets of conclusions are different. Thus which alternative is best under certain conditions is ambiguous. More important, though, is that both sets of conclusions are incorrect. To demonstrate this fact (not only for this example, but all others as well), one simply needs to tabulate or plot, approximately to scale, the net present values for each alternative versus the interest rate. That is, determine and plot the net present values for different MARRs. Figure 2 shows the plot for the data in example 1. Figure 3 shows the plot for the data in example 2. From the curves in Figure 3, it is simple to conclude the following for example 2: (a) No alternative is acceptable for a MARR below 20 percent; (b) alternative x = 1 is best for a MARR between 20 and 1,580 percent; and (c) no alternative is acceptable for a MARR higher than 1,580 percent. Clearly, this set of conclusions is far different from that which resulted from use of the internal rate of return method.

One cannot always properly interpret simple (discounted) internal rate of return calculations (including those for increments of investment over the lowest initial cost-acceptable alternative) without having additional information, which, by definition, is not part of the internal rate of return method. Thus, in some, if not most, cases, the methods and answers cannot be reconciled. One might argue that to end up with situations in which the net present value for alternatives can be negative at a zero discount or interest rate is hardly possible and certainly not typical. However, given that (a) both highway and transit systems are long lived (b) heavy capital outlays often must be made for 5 to 10 years before benefits begin to accrue, and (c) heavy capital and operating outlays are required in future years (for rolling stock, resurfacing, and repairs and maintenance), this eventuality seems possible, if not probable. In any case, the possibility of this occurrence alone should convince the engineering economist to abandon the deceivingly simple but sometimes inaccurate or ambiguous internal rate of return method.

Another way to highlight the ambiguities and the reasons for them is to compare alternative x = 1 from example 1 with alternative x = 2 from example 2. For both, the lowest basic internal rate of return was 25 percent and the initial cost was $100; for alternative x = 2 from example 2, the higher basic rate of return was 1,022 percent. In 1 case, the cutoff rate of return should be interpreted one way, in the other case it should be interpreted in another way. For instance, in Figure 4, the net present value versus interest rate curves have been plotted for these two alternatives. From this diagram it is obvious that alternative x = 1 from example 1 will be acceptable only if the MARR is 25 percent or less; alternative x = 2 from example 2 is acceptable only if the MARR is between 25 and 1,022 percent. This is apart from considering any changes associated with examining the return from increments of benefit and cost between alternatives.

These points can be made even more strongly by considering a third example, an example that has positive net present values at a 0 discount rate and no negative future benefits, but which covers a 3-year period and is seemingly more straightforward and generally applicable.

Example 3

Assume a 3-year stream of benefits and costs for the 3 alternatives as given in Table 3. The internal rates of return for the 3 alternatives were computed by using equation 3, and the paired internal rates of return for increasingly higher cost alternatives were computed using equation 4. For the data in Table 3, r_x for alternative x = 1 is 24 percent; r_x for alternative x = 2 is 20 percent; and r_x for alternative x = 3 is 21 percent. $r_{1 \text{ versus } 2}$ is 19.7 percent; $r_{2 \text{ versus } 3}$ is 15.7 and 271 percent; and $r_{1 \text{ versus } 3}$ is 20.7 percent. The questions are: Under what conditions is which alternative best according to internal rate of return and net present value methods, and what is the reason for the differences when the answers differ?

If we apply the internal rate of return method, we first cannot fail to note that there are multiple rates of return (2 real and positive discount rates) that satisfy equation 4 when we compare the extra costs and extra benefits of alternatives 2 and 3. Thus we are faced with an obvious ambiguity about which rate is the correct cutoff rate or which one to use in what instance. Moreover, the ambiguity cannot be clarified without carrying out at least some net present value calculations to supplement the rate of return results previously given. At any rate, before doing additional calculations, one can draw certain conclusions about which alternative is best by strictly applying the internal rate of return method.

1. If the MARR or opportunity cost of capital is equal to or less than 24 percent, then alternative x = 1 is acceptable.

2. If the MARR is equal to or less than 15.7 percent, then alternative x = 1 is acceptable (because r_1 is greater than 15.7 percent); in turn, because the return on the increment of investment between alternatives x = 1 and x = 2 is 19.7 percent and is greater than 15.7 percent, alternative x = 2 should be selected as being more acceptable than x = 1. Similarly, because the return on the extra investment of alternative x = 3 over alternative x = 2 is either 15.7 or 271 percent, it would appear that alternative x = 3 may be preferable to x = 2 and is acceptable (because r_3 is greater than 15.7 percent). Nevertheless, the answer is ambiguous.

3. If the MARR is greater than 15.7 percent but equal to or less than 19.7 percent, then alternative x = 1 will be acceptable (because r_1 is greater than 19.7 percent); also, because the return on the extra investment between alternatives x = 1 and x = 2 is equal to the highest MARR value (because $r_{1\text{ versus }2}$ is 19.7 percent), then clearly alternative x = 1 should be rejected in favor of alternative x = 2. On the other hand, because the return on the extra investment between alternatives x = 2 and x = 3 is either 15.7 or 271 percent and because which rate applies under what conditions is ambiguous, it is difficult to say whether alternative x = 2 or alternative x = 3 is better for a MARR range greater than 15.7 percent but equal to or less than 19.7 percent.

4. If the MARR is greater than 19.7 percent but equal to or less than 20.7 percent, then alternative x = 1 is acceptable (because r_1 is greater than 19.7 percent); but the additional investment to move to alternative x = 2 is economically unacceptable because $r_{1\text{ versus }2}$ is not more than 19.7 percent and thus alternative x = 2 must be rejected in favor of x = 1. In turn, on examining the return on the additional investment in going from x = 1 to x = 3, we find that the extra return, or $r_{1\text{ versus }3}$, is 20.7 percent and thus is acceptable; accordingly, for this MARR range, alternative x = 3 is judged to be the best acceptable alternative.

5. If the MARR is greater than 20.7 percent but equal to or less than 24 percent, then alternative x = 1 is clearly acceptable. But, because the return on the increment of investment from x = 1 to x = 2 is less than 20.7 percent ($r_{1\text{ versus }3}$ = 19.7 percent), alternative x = 2 must be rejected in favor of alternative x = 1. Similarly, because the return on the extra investment between alternatives x = 1 and x = 3 ($r_{1\text{ versus }3}$ = 20.7 percent) is less than the previously stated MARR (which is more than 20.7 percent), alternative x = 3 must be rejected and alternative x = 1 must be accepted as the best acceptable alternative.

6. If the MARR is more than 24 percent, then all alternatives must be rejected because r_x for x = 1, 2, 3 are all equal to or less than the MARR.

The results for the internal rate of return analysis, strictly applied, are given in Table 4. If the analyst had failed to note the multiple rates of return when comparing alternatives x = 2 and x = 3, and had simply overlooked the seemingly unrealistic 271 percent rate of return (which is a valid root), the results would have been even more misleading, and, in fact, incorrect. Specifically, if the 271 percent rate for alternative x = 2 versus alternative x = 3 had been ignored and only the 15.7 percent figure considered, then the Table 4 results would have indicated that alternative x = 3 was best for a MARR less than 15.7 percent and that alternative x = 2 was best for a MARR greater than 15.7 percent but equal to or less than 19.7 percent; for other ranges of interest the answers would not differ. However, as one can see from Figure 5 and other items to be discussed, these results would definitely be incorrect and would cause bad economic decisions.

For this particular example, where the net present values for all alternatives were

Table 3. Cost and benefit data for example 3.

Year	Alternative x = 1 (dollars)		Alternative x = 2 (dollars)		Alternative x = 3 (dollars)	
	$B_{1,t}$	$C_{1,t}$	$B_{2,t}$	$C_{2,t}$	$B_{3,t}$	$C_{3,t}$
t = 0		1,000		10,000		11,000
t = 1	505	0	2,000	0	5,304	0
t = 2	505	0	2,000	0	5,304	0
t = 3	505	0	12,000	0	5,304	0

Note: Some numbers have been rounded for convenience.

Table 4. Best alternatives under internal rate of return analysis.

Range for MARR (percent)	Best Acceptable Alternative
≤15.7	x = 2 or x = 3[a]
>15.7 but ≤19.7	x = 2 or x = 3[a]
>19.7 but ≤20.7	x = 3
>20.7 but ≤24	x = 1
>24	None

[a]Answer is ambiguous.

Figure 5. Plot of net present value versus interest rate for example 3 data.

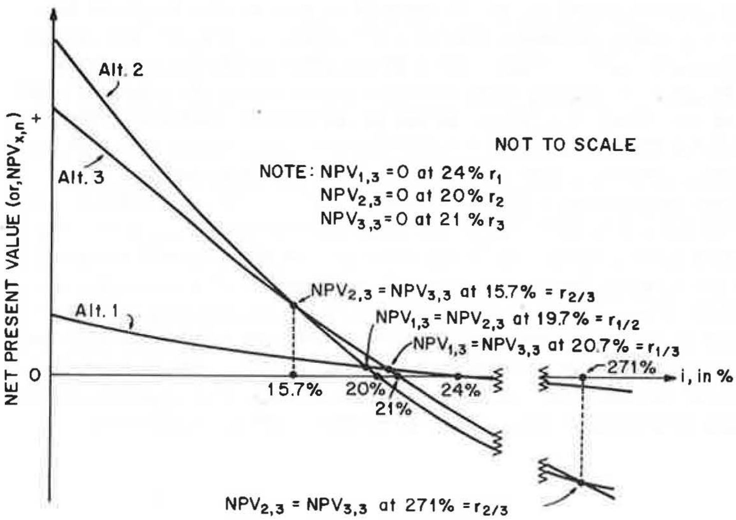
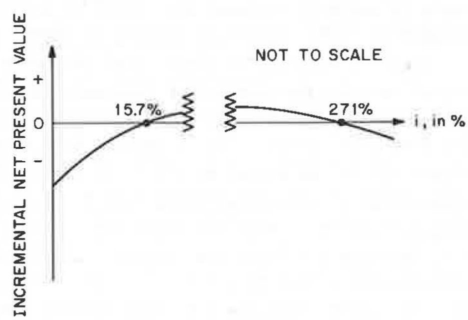


Figure 6. Plot of incremental net present value versus interest rate for incremental costs and benefits between alternatives x = 2 and x = 3 for example 3.



positive at a 0 discount rate ($i = 0$), the ambiguities arise from the complications associated with interpreting the multiple rates of return when comparing alternatives $x = 2$ and $x = 3$. Again, a simple plot of net present values versus interest rates for the incremental benefits and costs between alternatives $x = 2$ and $x = 3$ would have quickly resolved the problem. Figure 6 shows that plot and clearly demonstrates that the return from the increment in cost in moving from alternative $x = 2$ to $x = 3$ is acceptable (increases the net present value) only if the MARR or opportunity cost of capital is between 15.7 and 271 percent. For a MARR below 15.7 percent, alternative $x = 2$ is the best among the 3 alternatives, but, for a MARR between 15.7 and 271 percent, alternative $x = 3$ is the best among alternatives $x = 2$ and $x = 3$ although it is still unacceptable for a MARR greater than 21 percent. Also, for a MARR between 20.7 and 24 percent, alternative $x = 1$ is the best. But, without having this plot or other net present value computations in addition to the normal set of basic and incremental rates of return, the decisions among these 3 alternatives can only be ambiguous or wrong.

The ambiguities or inaccuracies among the alternatives noted in the examples not only can but often will result in a comparison of the benefit and cost streams for different alternatives. These examples, although they seem contrived, should not necessarily be regarded as atypical or trivial. They serve to emphasize in the strongest possible way that either ambiguous or wrong answers can occur when the internal rate of return method is stictly applied. However, when the net present value method is applied, in all cases (including those with different or equal initial costs and those with different terminal or replacement dates), answers will always be clear-cut and unambiguous.

The reasons for ambiguities occurring with the internal rate of return method have been discussed amply and thoroughly in both the economics and engineering economics literature (1, 2, 3, 4, 5). They hardly need much more than a brief discussion here. Problems arise because the internal rate of return method assumes that earnings or benefits accrued before the end of a project replacement date or planning period are reinvested at the internal rate of return rather than at the minimum attractive rate of return or opportunity cost of capital. This assumption hardly seems sensible because, by definition, MARR defines the return that other alternative investment opportunities will provide for funds that are released at any time during the period of analysis.

CONCLUDING REMARKS

Hopefully these examples and comments will prove that strict application of the internal rate of return method can lead to incorrect or ambiguous answers even if the special ordering technique suggested by Bergmann (6, p. 81) is applied.

Also it should be clear that the simplest and most unambiguous way to carry out benefit-cost analyses for mutually exclusive alternatives is merely to calculate the net present values for each of the alternatives over the entire range of relevant interest or discount rates. None of the iterations, multiple solutions, and complicated calculations of the internal rate of return method is required. Should the range of interest rates being considered be large, then numerous calculations may be required. Nonetheless, they are easily carried out, and the results can be plotted on a set of curves showing for each alternative the net present value versus the interest rate, or they can be displayed in tabular form. Such a set of curves, similar to those shown in Figure 5, or a table will indicate the alternative that has the highest positive net present value under certain interest rate conditions. The method is complete, avoids complications and ambiguities, and provides the maximum of benefit-cost information to the policy maker.

Finally, I emphasize the reality of the 3 examples and problems I have discussed. Admittedly, the numbers used in the 3 illustrations were hardly typical (they were contrived for computational ease) and the 2- and 3-year analysis periods were much too short to apply to the usual public project analysis. However, the ambiguities that did stem from these examples and the multiple solutions that did occur with the internal rate of return analysis method are realistic and not necessarily atypical. In fact, because of the long service lives of public projects and because of the almost certain probability that in some future years outlays will outweigh benefits, it appears that multiple solutions and ambiguities would be even more likely in actual project analysis than they were in the examples if one were to employ the rate of return method. This simply underscores the importance of rejecting the internal rate of return method in favor of the more certain and straightforward net present value method.

ACKNOWLEDGMENT

My appreciation is gratefully extended to Dietrich R. Bergmann for pointing out a serious error that appeared in an earlier version of this paper; substantial improvement of the paper resulted from his discovery. I do not burden him with any subsequent errors of fact or judgment to the extent there are any. I am also grateful for Gerald Kraft's earlier contributions, which improved the paper in important ways.

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DISCUSSION

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It is useful to clarify methodological ambiguities that exist in the literature. Therefore, Wohl is to be complimented on his effort to further identify and illustrate the difference between the rate of return method and net present worth method in comparing mutually exclusive investment alternatives.

Wohl's paper appears to be a reply to my Highway Research Record paper of 1973 (6). In that paper I indicated that many authors held in disrepute the rate of return method in comparing mutually exclusive alternatives for two reasons.

1. Where the solution for the rate of return is unique, rate of return methodology is alleged to occasionally yield a conclusion that is directly opposite to that produced by application of the net present worth method. This has often been alleged to be the case for alternatives with equal investments.
2. Where the solution for the rate of return is not unique, conclusions stemming from application of the rate of return method are ambiguous whereas those stemming from net present worth methodology are always unambiguous.

My 1973 paper (6) indicated that reason 1 is invalid. Illustrations that have been published to point out alleged inconsistencies between the 2 methods were examined and found to be devoid of any analysis of incremental cash flows, which is a fundamental step in comparing mutually exclusive alternatives by the rate of return methodology. Because most of those illustrations involved alternatives having identical initial investments, a detailed procedure was presented in that paper that showed exactly how to handle such cases. The reader will find that the procedure is a straightforward extension of the rate of return method for comparing mutually exclusive alternatives as it is already defined in standard textbooks on engineering and managerial economics.

With regard to reason 2, my paper (6), near the end, recognized the possibility of multiple solutions for rate of return and pointed out that, in fact, both methods are ambiguous in such cases. Furthermore, it was clearly pointed out that the algorithm presented in that paper reconciled problems dealing only with situations that other authors have categorized as being covered by reason 1.

Wohl's paper includes 3 examples, the first of which corresponds to the first example presented in my paper (6). He suggests that the logic I presented there does not apply to a period of analysis beyond 2 years. Such a suggestion is clearly without foundation.

In the review of his second example Wohl refers to the method summarized in my paper (6, p. 81) and asserts that it yields ambiguous conclusions. This assertion is without basis because the method summarized applies only to situations in which the rate of return is unique for the basic investment and the incremental investment. Each alternative presented in Wohl's second example involves not 1 but 2 solutions for the rate of return. With regard to the second example, it should be noted also that the approach Wohl uses to complete the analysis of the 2 alternatives by what he refers to as the "rate of return method" is one that I have not seen before.

The third example in Wohl's paper rather nicely illustrates a situation in which the rates of return are unique for basic investments but not unique for incremental investments. The procedure in my paper (6, p. 81) does not apply here for the same reason that it does not apply in Wohl's second example.

Close to the conclusion of his paper, Wohl suggests that the reader consult the literature (1, 2, 3, 4, 5) for further background and support of his viewpoints. A review of these references may be of interest to the reader. I suggest that the reader consult my paper (6) before drawing any conclusions from Wohl and Martin (1). Lorie and Savage (3, Table 2) apply both the rate of return and net present worth methods to the evaluation of 2 investment alternatives that are mutually exclusive from a capital budgeting perspective rather than a physical perspective. The reader will find that computation of the rate of return for the increment between the cash flows of the 2 alternatives will resolve the apparent inconsistency discussed there.

Solomon (2) deals with 2 illustrations, the first of which falls into the general category of situations covered by reason 1 and the second of which involves a situation covered by reason 2. With regard to his first example, straightforward application of the method introduced in my paper (6, p. 81) again results in a decision consistent with those given by the net present worth method and by both of the other 2 approaches that Solomon suggests as alternates to the rate of return method.

Bierman and Smidt (4) present results that I also have referenced (6).

Hirshleifer, DeHaven, and Milliman (5, Chapter 7, p. 167) compare net present worth and rate of return methods in situations covered by reason 2. Their illustration and claim (5, pp. 170 and 171) regarding the failure of the rate of return method involve reason 1. Note, though, that their claim is resolved by using the approach defined in my paper (6, p. 81).

From the foregoing review, it is readily apparent that the reason 1 illustrations cited by Wohl and his references are all resolved by applying the approach defined in my paper (6, p. 81). However, we have a different situation when encountering reason 2 situations; this is shown by Wohl's second and third examples. It is indeed appropriate to say that the rate of return method is not well defined and is ambiguous for such cases. It should be added, though, that the net present worth method, although always unambiguous in such cases, can be deceptive and misleading in its simplicity and straightforwardness. For instance, in Wohl's second example, blind adherence to the net present worth criterion would result in rejection of both alternatives if the MARR is 15 percent, and further consideration of each alternative if the MARR is increased to around 26 percent. It is a strange investment alternative whose total outlays exceed its total receipts and which becomes profitable only as the MARR increases. This point is not new; it has been made by Teichroew, Robichek, and Montalbano (7). Their treatment of the problems associated with both methods when the solution for the rate of return is not unique is extremely comprehensive and extends the contributions made by Solomon (2) and Lorie and Savage (3). I highly recommend the Teichroew, Robichek, and Montalbano (7) paper to readers seeking further perspective on situations involving multiple solutions for the rate of return.

ACKNOWLEDGMENT

The viewpoints expressed in this discussion are my own; they do not necessarily represent those of my employer nor any organization with which I am affiliated.

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AUTHOR'S CLOSURE

In many respects I do not think Bergmann's discussion is worthy of further comment. However, because the evaluation techniques in question are widely used and misused in practice and because Bergmann's paper (6) and discussion are both terribly misleading (the latter is even incorrect in some respects), I will respond to 4 aspects of his discussion.

1. In the third paragraph of his discussion, Bergmann says that reason 1 for rejecting the rate of return method is invalid. This, of course, is misleading. Specifically, correct results and decisions resulting from using the rate of return method (with equal initial costs and a unique solution) will necessarily result only if Bergmann's special ranking method (6, p. 81) is employed also. Because this ranking method is not an inherent feature of the rate of return method as widely discussed and employed, I can only regard his flat statement as misleading, if not inaccurate. For instance, if the data for alternatives 1 and 2 in example 1 and Table 1 of my paper are reversed and the commonly applied rate of return method (that without special ranking techniques, which Bergmann admits is an "extension of the rate of return method") is used, then the incorrect alternative will be selected even if the analysis of incremental cash flows is incorporated. I ask: How many engineering economists have ever heard of, much less understand, Bergmann's ranking technique, and how many practicing analysts understand that unique solutions do not always occur?

2. In the sixth paragraph of Bergmann's discussion, he states that I asserted that the method summarized in his paper (6, p. 81) yields ambiguous conclusions. He says that this is without basis because the method summarized by him applies only to situations in which the rate of return is unique for the basic investment and the incremental investment. First, the assertion is entirely correct. Second, I said (somewhat differently than implied by Bergmann) that the data in example 2 and Table 2 of my paper "will demonstrate the ambiguity that can result from using the internal rate of return method in the usual fashion even when alternatives are ranked in the manner outlined by Bergmann." I did not imply that Bergmann would find a different result; to do so would be inaccurate and misleading. In short, Bergmann's comment is without redeeming value.

3. In the eighth paragraph of Bergmann's discussion, he correctly notes that Wohl and Martin (1, section 8.7) failed to compute the rate of return for the increment between the cash flows of the 2 alternatives. This I freely acknowledge; I did so in the first paragraph of my paper. (It is worth noting, though, that this failure was not repeated when using the same data within example 1; this is a point that Bergmann overlooks.) However, once again, Bergmann is incorrect in saying that "the reader will find that computation of the rate of return for the increment between the cash flows of the 2 alternatives will resolve the apparent inconsistency." Although carrying out this additional computation is necessary, it is not sufficient. If you reverse the data for alternatives 1 and 2 in example 1 and Table 1 of my paper and only compute the rates of return for the alternatives and the incremental cash flows between the 2 alternatives, you will obtain an incorrect result. You must also rank the alternatives as Bergmann suggested. Thus, once again, Bergmann has misled the reader.

4. Bergmann, in the last paragraph of his discussion, comments on the applicability of the rate of return and net present worth methods to situations having multiple rates of return (nonunique solutions). In part, he said: "It is indeed appropriate to say that the rate of return method is not well defined and is ambiguous for such cases." For once, we can agree. But then he adds: "It should be added, though, that the net present worth method, although always unambiguous in such cases, can be deceptive and misleading in its simplicity and straightforwardness." Surely Bergmann is not serious. If the rate of return method is ambiguous and the net present worth is always unambiguous (and straightforward), then what better reason is there to reject the rate of return method outright? Also, why is an unambiguous and straightforward method deceptive and misleading? I take it that Bergmann feels that the net present worth method is deceptive and misleading because certain investment situations can result in negative net present worths if the discount or interest is 0. Although he says this is a strange investment alternative, he makes no effort to justify the comment. Consequently, because this situation can, and in all likelihood will, occur (because of heavy future expenditures relative to benefits, for example), Bergmann's comment must be dismissed, at least as a general proposition, because it has no basis in fact.

Bergmann's paper was entitled Evaluating Mutually Exclusive Investment Alternatives: Rate-of-Return Methodology Reconciled With Net Present Worth (6). After carefully reading my paper, as well as Bergmann's paper and discussion, it should be perfectly clear that the 2 methods can always be reconciled if and only if the rates of return for both the alternatives and incremental cash flows between alternatives are unique and if the alternatives are ordered in the fashion that Bergmann suggests (6, p. 81). Thus to imply as the title of Bergmann's paper does that the 2 evaluation alternatives can be reconciled for the general case is both misleading and deceptive. As a consequence, I wrote my paper and this closure to clarify this and other misconceptions.