WHERE ARE THE KINKS IN THE ALIGNMENT?

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This paper uses the tangent method to determine the maximum length of an approach to a curve so that the driver does not see a kink (a sudden change in direction) in the alignment. Equations are presented for plan curves, crest curves, sag curves, and combinations of these curves. The picture presented by the road to the driver can be analyzed with the equations, and, thus, the road design engineer is able to relate important road properties to driver experiences. It is suggested that using this method to check road alignment design for kinks reduces the need for the time-consuming process of drawing perspectives.

•ANY curve, when seen from far away in perspective, looks like a sudden change in direction, a kink. This is even true for planned curves with large radii if they are observed from a long enough distance. At some distance, however, there is a transition between seeing the curve as a sudden direction change and seeing the curve as open, the critical distance. The kink does not give the driver sufficient information to accurately regulate the car's speed in anticipation of the curve to come. The development of the tangent method of drawing road perspectives has provided the road design engineer with many equations that allow analysis of the picture presented by the road to the driver. It allows a scientific means to relate important road properties to driver experiences. This method has been applied to find the maximum length of an approach to a curve so that the driver does not see a kink in the alignment. This knowledge can then be used to check road alignment design for kinks, and there will be a reduced need for the time-consuming process of drawing perspectives.

TANGENT METHOD

The tangent method (1, 2) assumes close similarity between circular curves on roads and parabolas. The perspective images of these parabolas have the form of hyperbolas. This allows the drawing of perspectives by a simple graphical method making use of the asymptotes of the hyperbola. One of the useful properties of the hyperbola is that the tangent point divides the tangent between the asymptotes into two equal parts (Figure 1, VP to T = T to H). Some other properties of a perspective image of a circular curve are shown in Figure 1 (distance D is shown at the reversal point). The equation follows from central projection theory and the parabolic assumption used in the tangent method.

The tangent method was used to draw all the perspectives in this paper and was found to be simple in its application, especially for road curves of large radii.

The notation used in the tangent method is as follows:

a = distance in meters from driver to road edge;
a_r = distance in meters from driver to road edge on driver's right;
A = clothoid spiral parameter, $LR = A^2$;

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d = perspective viewing distance in meters (usually 1.0 m);
D = perspective distance in meters;
h = height in meters of the driver's eye above the pavement;
H = corner point, intersection between the tangent to the hyperbola at the terminal of the geometric element and the asymptote - 0;
M = midpoint, intersection of two asymptotes of a hyperbola;
R_H = radius in meters of horizontal circle or plan circle;
R_V = radius in meters of vertical circle;
R_{Vc} = radius in meters of vertical crest curve;
R_{Vs} = radius in meters of vertical sag curve;
T = terminal of geometric element;
V = speed of vehicle in kilometers per hour;
VP = vanishing point of direction at terminal of geometric element;
X = distance in meters on the road plan;
Y = level difference in meters on the road profile;
Z = total distance in meters from driver to point on road;
Z_1 = distance in meters from driver to terminal point of first geometric element; and
Z_2 = length in meters of second geometric road element.

DISTANCE OF REVERSAL POINT FROM DRIVER

It is assumed that drivers judge their expected speed-driving behavior for an oncoming curve by the visual shape of the reversal point formed by the inside curve edge of pavement (Figure 1). It is therefore of some value to the driver to be provided with an informative view of this reversal point at a distance. This allows the driver to draw conclusions and take action accordingly. This applies to circular, spiral, or parabolic curves in the vertical or horizontal direction.

A systematic analysis of the different situations follows.

PLAN CURVES

So that all possibilities can be covered fully, the curves are considered in groups:

1. Plan circular curves to the right,
2. Plan circular curves to the left,
3. Spiral curves, and
4. Plan circular curves on upslopes and downslopes.

Plan Circular Curves to Right

Plan circular curves to the right will be treated as the basic situation for the derivation of equations. The plan for such a situation is shown in Figure 1.

It is relatively simple to calculate the distance from a driver to the point where he or she sees the reversal in the inside curbline. This distance can be shown to be equal to Z, where

$$Z^2 = 2R_H a_r + Z_1^2 \qquad (1)$$

The derivation of equation 1 can be found in the Appendix.

With equation 1, it is possible to calculate the distance from a viewpoint to the place on the road edge where the driver sees the reversal curve. Where the driver is in fact already in the curved section,

$$Z_1 = 0$$

and the equation becomes

$$Z^2 = 2R_H a_r \tag{2}$$

This indicates a constant distance, as could be expected.

Plan Circular Curve to Left

The same equations as for plan circular curves to the right apply except that the distance a_r is replaced by a_L.

Spiral Plan Curves

For a clothoid spiral, the distance X as in Figure 1 would be approximately

$$\frac{Z_2^3}{6A^2} \tag{3}$$

and

$$D = \frac{a_r + \frac{-(Z - Z_1)^3}{6A^2}}{Z} \tag{4}$$

This equation cannot be reduced as simply as the equation for circular curves was. However, if the spiral is seen as a form between a straight line and a circular curve, it may be noted that for a straight line the reversal point lies at infinity.

For a circular curve, the reversal point lies at a constant distance from the beginning of the curve (equation 2). The spiral therefore must have its reversal point at a distance greater than the constant for the circular curve but less than infinity.

Driving toward a spiral presents the driver with a picture where the distance to the reversal point decreases to the circle constant. This provides the driver with a good optical guidance into the curve.

Plan Curves on Upslopes and Downslopes

From equations 1 and 2, note that the eye height of the observer does not have any effect on distance Z. It can also be shown that the actual perspective view of a sloping road is almost the same as that of a horizontal road because the distance of the vanishing points above or below the horizon is very small (Figure 2). Since the slope of the road is rarely steeper than 10 percent, the Y distances are practically less than 0.1 d. Arguments based on the idea that the observer looks against the road in the case of an upslope and with the road in the case of the downslope would favor total curve readability slightly in the case of the upslope. However, the improvement is so marginal that it can be neglected.

Figure 1. Plan and perspective view of road edge on right side of driver.

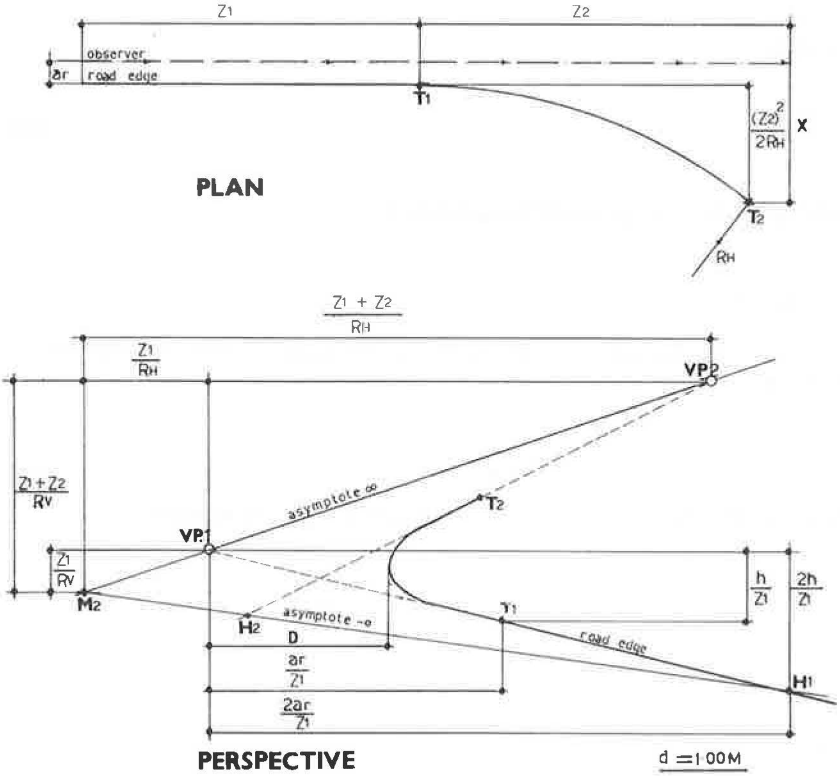
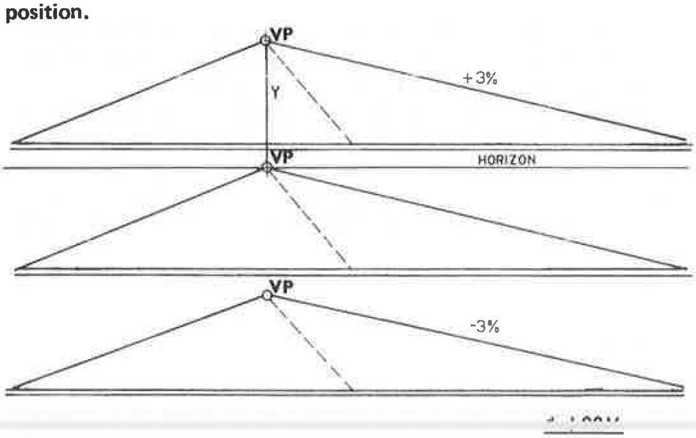


Figure 2. Perspectives of sloping roads as seen from same observation position.



VERTICAL CURVES

Crest Curves on Straight Roads

The detailed analysis in the Appendix shows that, for crest curves (Figure 3), the distance from observer to the visible top of the crest equals Z, where

$$Z^2 = 2R_v h + Z_1^2 \tag{5}$$

This is of course similar to equation 1. The reversal point is in fact the visible top of the crest pavement.

Sag Curves on Straight Roads

In the sag curve there was no particular point that could be compared with the reversal point in the horizontal curve or with the top of the visible pavement at the crest. In fact, the total length of the pavement is visible. Still kinks can make the sag look somewhat disjointed, and this may be used to indicate maximum allowable lengths of straight roads to the sag curve.

KINKS IN PLAN CURVES

In an attempt to find the critical distance from the driver to the curve, the approach distance, 30 perspectives were drawn of curves with radii from 600 to 4000 m and with approach lengths from 100 to 600 m. A portion of one of the series is shown in Figure 4.

When the perspective was observed from the correct viewing distance of 1 m, it became clear that the kink was related to the sharpness of the reversal point. In each series, the perspective with the shortest approach distance to the clearly visible kink was chosen as the critical case. Similarities between the perspectives of critical case were then studied.

As is indicated in Figure 4, the critical case has the angle β (between the two asymptotes) at about 2.5 deg; therefore, when β is larger than 2.5 deg, the curve will be observed as open. The following relationship can thus be established.

$$\tan \beta = \frac{(2h/Z)}{(Z/R_H)} = \frac{2R_H h}{Z^2} \tag{6}$$

or

$$Z^2 = 2R_H h/\tan \beta \tag{7}$$

for $\beta = 2.5$ deg,

$$\tan \beta = 0.044 \tag{8}$$

so that

$$Z^2 = 46R_H h \tag{9}$$

Figure 3. Profile of crest curves on road.

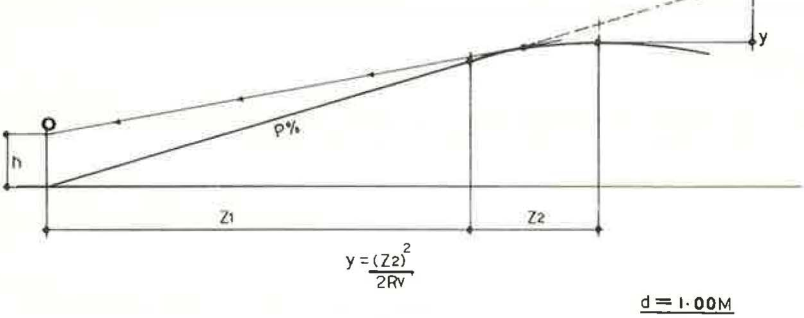
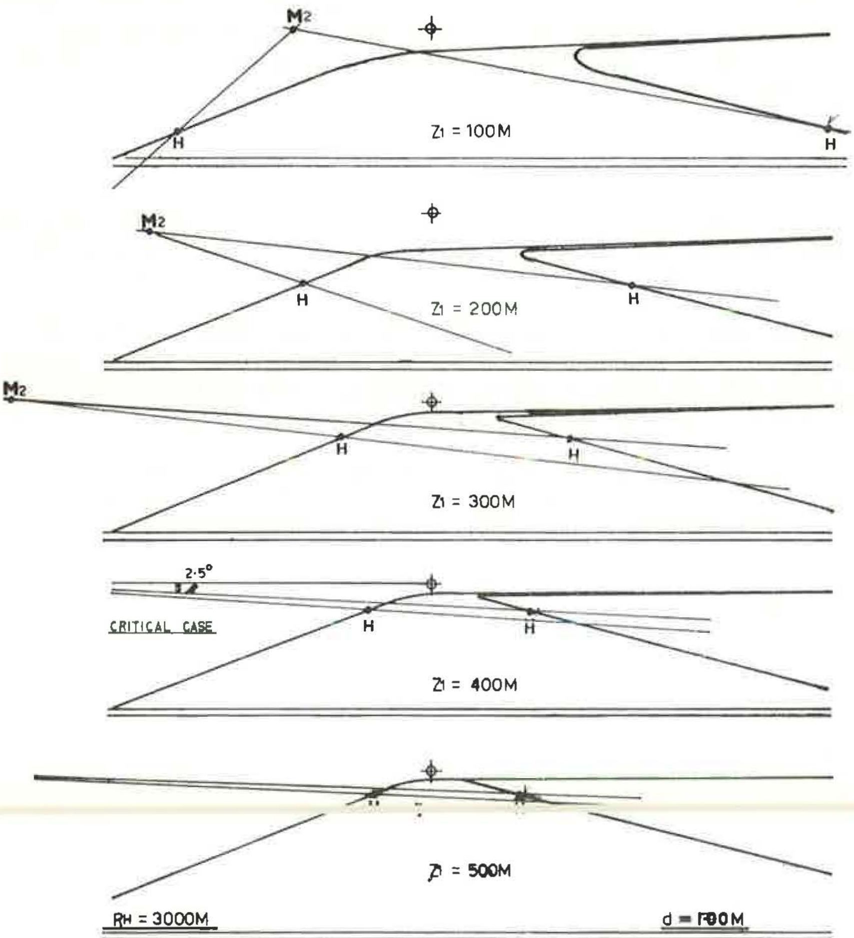


Figure 4. Series of horizontal curves on equal radii from different approach distances.



This result is similar to the conclusion drawn from an investigation in the Netherlands; the minor difference lies in the manner of derivation (1). Earlier it was shown that

$$Z^2 = 2R_H a_r + Z_1^2 \tag{1}$$

By substitution, the following general relationship for all horizontal curves can be derived.

$$Z_1^2 = R_H\,(46h - 2a) \tag{10}$$

where a is the distance from the driver to road edge, to the left for a curve to the left and to the right for a curve to the right.

Equation 10 shows that the maximum approach length to a curve depends on the curve radius and the road width. An approach longer than this maximum will present the curve to the driver as a kink. When approaching nearer than Z_1, the driver will see the curve as open and can adjust his or her driving if necessary. Figure 5 shows curves of different radii and approach lengths where the kinks are similar.

From equation 10 it follows that, the wider the road is, the shorter the critical approach length will be.

CREST CURVES

It was observed earlier that crest curves show no kink. However, it has long been recognized that a driver must see a portion of the crest before he or she gets the feel of the curve (3, 4).

Thirty perspectives were produced of crests with different radii and approach lengths. A portion of one series of these is shown in Figure 6. Similarities were sought between those perspectives that just contained enough information for the driver to read the crest. Again the angle between the two asymptotes was the common factor. At α = 70 deg, all crests began to be readable.

From Figure 6,

$$\tan\alpha = \frac{2a_r}{Z_1} \qquad \frac{2hR_v + Z_1^2}{Z_1 R_v} \tag{11}$$

For α = 70 deg,

$$Z_1^2 = R_{vc}\,(0.73a_r - 2h) \tag{12}$$

In this case, the acceptable approach will be longer when the road is wider. I think the driver gets more clues from the road edge on the driver's side.

SAG CURVES

Thirty perspectives were drawn, and the angle between the asymptotes was the common critical factor. The critical angle was measured to be α = 35 deg (Figure 7). This leads to

Figure 5. Different plan curves from their critical approach distances.

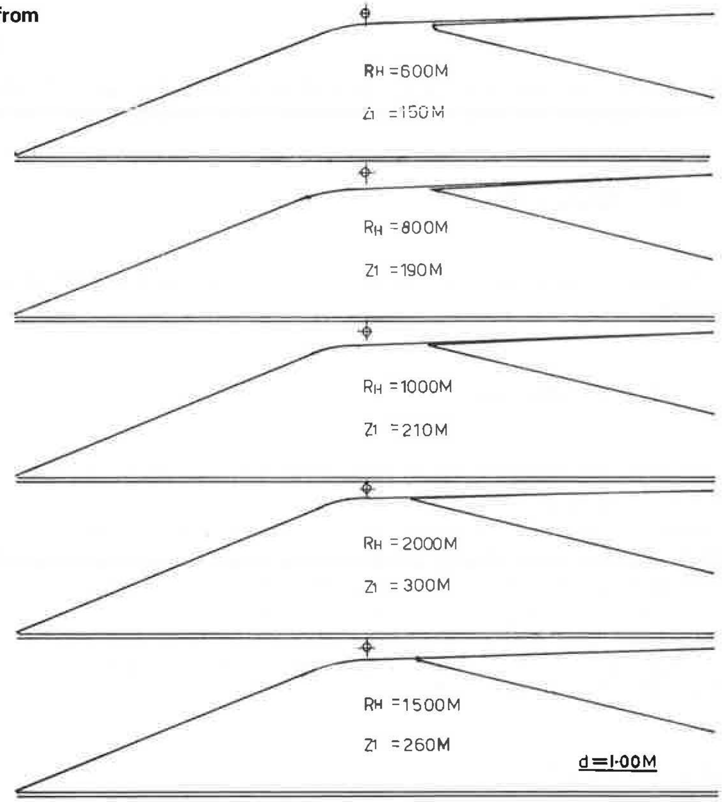
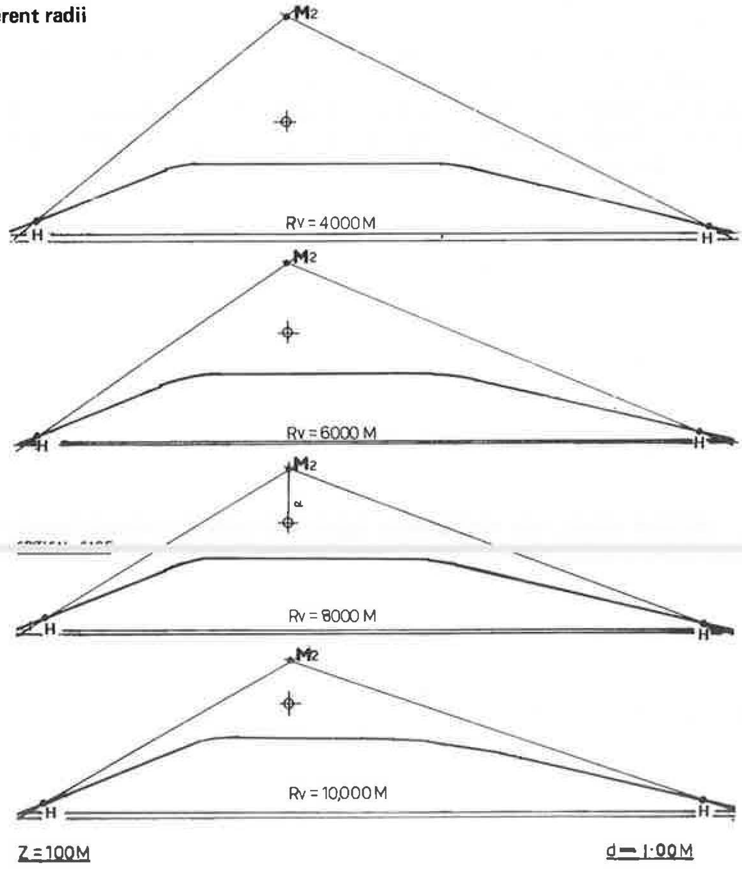


Figure 6. Series of crests of different radii but equal approach distances.



$$Z_1^2 = R_{vs}\ (2.6a + 2h) \tag{13}$$

As before, the wider the road is, the longer the acceptable approach will be. In addition, the approach to sag curves can be much longer than the approach to crests when all other conditions are equal.

COMBINED HORIZONTAL AND VERTICAL CURVES

Whenever the plan curve is combined with a crest or a sag curve, whether they are of equal length and in phase or not, there will always be a section of the road that can be seen as a combined curve. This combined curve can be assumed to be a helix for the purpose of this discussion (1).

Therefore, in a long plan curve with a vertical curve somewhere along its length, the section would be analyzed as one part circular plan curve, one part combined curve, and one part circular plan curve.

The simple plan or vertical portions of curves have been discussed previously. The following attempts to analyze the visual consequences of approaches to combined curves.

Crest-Plan Curve

Other authors have already indicated that for crest-plan curves the section of horizontal curve visible to the driver must have at least 2.5 deg deflection (3, 4, 5). At the same time, the maximum approach to the crest must be such that the driver reads its vertical curvature. There is no need to consider the kink in the plan curve because the crest is at the end of the length of visible pavement.

This leads to four equations that must be fulfilled.

$$Z_1^2 = R_{vo}\ (0.73a_r - 2h) \tag{12}$$

$$Z_2 = 0.044\ R_H \quad \text{(for a deflection of 2.5 deg)} \tag{14}$$

$$Z = Z_1 + Z_2 \tag{15}$$

$$Z^2 = 2R_v h + Z_1^2 \tag{5}$$

Combination of these equations leads to a relationship that indicates the maximum ratio between R_H and R_v.

$$R_H = \sqrt{R_v}\ (\sqrt{380\ a} - \sqrt{380\ a - 1004\ h}) \tag{16}$$

On a two-lane rural road, where a_r = 5 m and h = 1.2 m, equation 16 becomes

$$R_H = 17\sqrt{R_v} \tag{17}$$

and

$$Z_1 = 1.11\sqrt{R_v} \tag{18}$$

This relationship between R_H and R_v must be understood in the context of equations 17 and 18.

Provided the plan curve radius is smaller than $17\sqrt{R_v}$, the driver will be able to read the crest at an approach distance of $1.11\sqrt{R_v}$ while the deflection in the visible section of the horizontal curve is larger than 2.5 deg.

Although equation 12 seems excessively complex, design policies would carry, of course, values of R_H with relation to R_v for specific road widths because h is uniform.

Sag-Plan Curve

The full length of the sag-plan curve can be seen from the approach road. A kink, as a result of the plan or of the vertical curve, may be visible and, therefore, needs to be considered.

Only if the approach length is less than the smallest of those permitted for simple plan or vertical curves will no kink be seen by the driver. Therefore,

$$Z_1 \begin{cases} Z_1 = R_H\,(46h - 2a) \\ Z_1 = R_v\,(2.6a + 2h) \end{cases} \tag{19}$$

A critical combination curve is reached when

$$R_H\,(46h - 2a) = R_v\,(2.6a + 2h)$$

or

$$\frac{R_H}{R_v} = \frac{2.6a + 2h}{46h - 2a} \tag{20}$$

Both curves will show a kink when seen from a critical distance. This rather cumbersome equation is simplified for a rural road, where a = 5 m and h = 1.2 m, to

$$\frac{R_H}{R_v} = 0.35 \tag{21}$$

For the ratio to be larger than 0.35, the equation for the vertical curve should be used. When the ratio is smaller than 0.35, the equation for the horizontal curve should be used to calculate maximum length of the approach road.

A somewhat different approach would be to calculate the dominant of the two curves. Combined curves are shown in Figure 8: dominant horizontal in 8a, critical case in 8b, and dominant vertical in 8c. The borderline case between these is found when the combined curve asymptote, $-\infty$, coincides with the extension of the image of the outside edge of the road pavement in the curve (2).

From the figure it can be seen that, if $\tan\alpha < \tan\beta$, the curve is a dominant plan curve, and, if $\tan\alpha > \tan\beta$, the curve is a dominant vertical curve.

In the critical case, $\tan\alpha = \tan\beta$ or

$$\frac{R_H}{R_v} = \frac{h}{a} \tag{22}$$

where a is to outside curbline.

Figure 7. Sag-curve perspective.

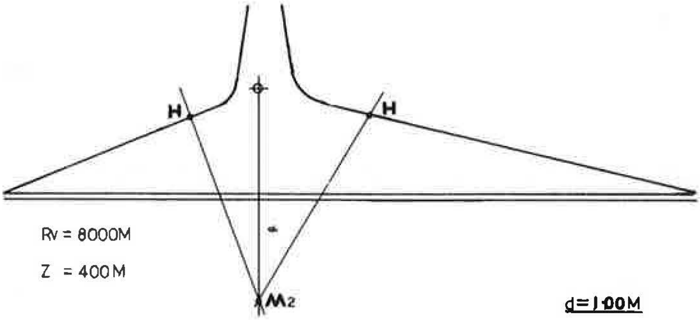
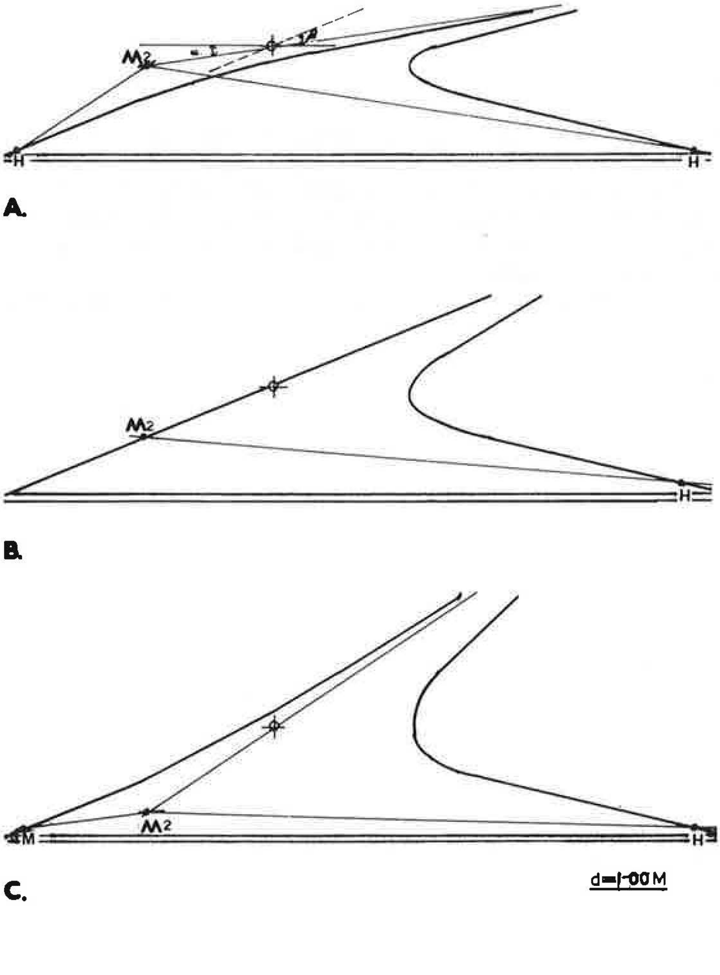


Figure 8. Combined curves: (a) dominant horizontal, (b) critical case, and (c) dominant vertical.



Therefore, when

$$R_H > \frac{h}{a} R_V \tag{23}$$

the combined curve is a dominant-vertical curve, and the approach length must be calculated based on R_V. When

$$R_H < \frac{h}{a} R_V \tag{24}$$

the combined curve is a dominant-plan curve, and the approach length must be calculated based on R_H.

For a rural road with a = 5 m and h = 1.2 m, the critical ratio works out to be 0.24 (2).

REACTION TIME AND KINKS

It is often assumed that the driver reads the road for a distance ahead that is proportional to the speed of the car. Lorenz and Springer (1, 4) suggest that this distance is approximately equal to 10 s of driving time. This agrees with values adopted by me earlier (6). It may not always be the road designer's wish to eliminate all visible kinks in the road alignment. However, it is reasonable to insist that all curves observed from a 10-s distance should be open and readable.

This leads to a relationship between the driver's speed and the road geometry. The 10-s distance is

$$Z_1 = 2.78V\,(m) \tag{25}$$

The critical approach length to the curve in plan is

$$Z_1^2 = R_H\,(46h - 2a) \tag{7}$$

Substituting and rounding off gives

$$R_H > \frac{V^2}{6h - 0.25a} \tag{26}$$

Similarly, for crest and sag curves respectively, we can derive

$$R_{vc} > \frac{10V^2}{a - 2.5h} \tag{27}$$

$$R_{vs} > \frac{3V^2}{a + 0.75h} \tag{28}$$

The curve radii calculated by equations 27 and 28 are the minimum to be used for a given speed so that an open, nonkink curvature at approach distances equal to up to 10 s of driving can be provided, always provided the curves can actually be seen from these distances.

RESULTS

The results are expressed in a form that would indicate mathematical precision. It should not be overlooked, however, that they are based on some degree of subjective judgment of simplified road perspectives. The outcome of calculations of critical distances should, as a result, be regarded as a guide figure and not as a precise distance. Since this is really all that a designer requires to help in decisions, this limitation on the result is not serious.

Note also that, for a designer to use the results, decisions would have to be made about whether it is necessary to provide alignments without kinks. A critical approach time (such as 10 s) may also be set during which the driver must see the curve as open. If these requirements are to be satisfied, small approach straights will require following curves of much larger radii than the traditional considerations of driver dynamics would prescribe. For a design speed, where $V = 100$ km/h, $a_r = 4.7$ m, and $h = 1.2$ m,

$$R_{H_{open}} > \frac{(100)(100)}{7.20 - 1.25} = 1680 \text{ m} \quad (D_c < 1.04 \text{ deg}) \tag{29}$$

Dynamic considerations would give

$$R_H > \frac{V^2}{127(E+F)} > \frac{10\,000}{127(0.20)} = 393.7 \text{ m} \quad (D_c < 4.44 \text{ deg}) \tag{30}$$

CONCLUSIONS

The following relationships have been derived from the theory developed in the tangent method of perspective drawing of road pavements.

For plan, crest, and sag curves respectively, the maximum approach distances to curves so that they are readable are

$$Z_1^2 = R_H\,(46h - 2a) \tag{10}$$

$$Z_1^2 = R_{Vc}\,(0.73a_r - 2h) \tag{12}$$

$$Z_1^2 = R_{Vs}\,(2.6a + 2h) \tag{13}$$

For the combined sag-curve plan, the same equation can be used considering that when

$$\frac{R_H}{R_V} > \frac{2.6a + 2h}{46h - 2a} \tag{31}$$

the curve is treated as a vertical curve and when

$$\frac{R_H}{R_v} < \frac{2.6a + 2h}{46h - 2a} \tag{32}$$

the curve is treated as a horizontal curve.
For crest-plan curves,

$$\frac{R_H}{R_v^{1/2}} < (380a)^{1/2} - (380a - 1000h)^{1/2} \tag{33}$$

The curve will be open and give sufficient visual information to the driver for him or her to read the properties of the crest at the critical distance for the crest.

So that drivers are given a timely indication of the properties of the curve, head alignment radii should be chosen so that, for plan, crest, and sag curves respectively,

$$R_H > \frac{V^2}{6h - 0.25a} \tag{26}$$

$$R_{vc} > \frac{10\ V^2}{a - 2.5h} \tag{27}$$

and

$$R_{vs} > \frac{3V2}{a + 0.75h} \tag{28}$$

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APPENDIX

DERIVATIONS

The derivation of equation 1 in which the distance from the driver to a reversal point is determined follows. From Figure 1, x on the plan view is shown as D on the perspective.

From the plan view,

$$x \approx a_r + \frac{Z_2^2}{2R_H} \tag{34}$$

and

$$Z_2 = Z - Z_1 \tag{35}$$

where Z_1 is a constant.

From the perspective view,

$$D = \frac{x}{Z} = \frac{a_r + \frac{Z_2^2}{2R_H}}{Z} = \frac{2R_H a_r + (Z - Z_1)^2}{2R_H\, Z} \tag{36}$$

At the reserval point, D will have a minimum length, or

$$\frac{dD}{dZ} = 0 \tag{37}$$

This differentiation leads to

$$Z^2 = 2R_H a_r + Z_1^2 \tag{1}$$

The analysis of equation 5 in which the distance from the driver to the visible top of a crest is determined follows. From Figure 3, the form of the crest is

$$\text{Crest form} = PZ_2 - \frac{Z_2^2}{2R_v} \tag{38}$$

The slope of the tangent to the crest is

$$\text{Tangent slope} = P - \frac{Z_2}{R_v} \tag{39}$$

The slope of a light ray from the crest to the driver is

$$\text{Light-ray slope} = \frac{ZP - \left(\frac{Z_2^2}{2R_v}\right) - h}{Z} \tag{40}$$

and must be equal to the slope of the tangent. Therefore,

$$\frac{ZP - \left(\frac{Z_2^2}{2R_v}\right) - h}{Z} = P - \frac{Z_2}{R_v} \tag{41}$$

where

$$Z_2 = Z - Z_1 \tag{42}$$

This leads to

$$Z^2 = 2R_v h + Z_1^2 \tag{5}$$

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