DEVELOPMENTS IN THE PREDICTION OF POTENTIAL STRENGTH OF CONCRETE FROM RESULTS OF EARLY TESTS

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Relationships among degree hours of maturity and compressive strengths of untreated, 1 to 3-day-old concrete cylinders are used to develop equations for predicting minimum potential 28-day strength. The one-sided confidence limit on values is approximately predicted at f_c - 600 psi (f_c - 4140 kPa). From the results of early tests, one equation produces predicted values for mixtures of the known maturity and the known cement factor, one requires only that specimens be tested at a constant maturity, and one requires only that the maturity be known. Equations are derived by statistical methods and a regression analysis. The data were obtained from a designed experiment made up of 200 cylinders, 4 cement factors, and tests at 1, 2, 3, and 28 days. Equations are based on the nonlinear time-temperature, or maturity, concept of the rate of gain of strength first advanced by Plowman. Predicted results are limited to 28 days and have an average coefficient of variation of 12 percent. Also discussed are the findings that predictions based on the compressive strengths of untreated cylinders, cured at about 73 F (22.8 C), are as accurate as those based on cylinders conditioned by a 3½-hour immersion in a 200 F (93.3 C) water bath. It was found that the compressive strengths not exceeding 1,800 psi (12 400 kPa) of early test specimens tested with Celotex pressure pads are nearly equal to and have about the same standard deviation as those tested with sulfur mortar caps.

•THIS paper presents findings of a continuation of previously published work (1) to investigate the practicality of using early tests of concrete cylinders for the quality control of concrete. The scope of the study described includes concrete of four cement constants proportioned to simulate field conditions where part of the cement was inadvertently omitted. One to 3-day-old specimen cylinders were tested. Results of tests, at early ages, based on sulfur mortar caps were compared with results based on Celotex pressure pads. The findings indicate that 1 to 3-day-old specimens can be tested without pretreatment or special conditioning to provide reliable estimates of minimum compressive strengths at 28 days.

In addition, the findings indicate that these specimens can be prepared for tests by means of disposable pressure pads in lieu of the usual sulfur mortar caps and at a savings of time and equipment requirements. The data obtained made possible the simplification of the previously developed prediction equation (1, 2) and the development of two alternate prediction equations.

OBJECTIVES

The objectives of the work described were to determine the possibility of further increasing the practicality of the applications of previous research (1, 2) and to develop prediction equations for concrete mixtures not previously investigated.

The specific objectives established for our study are as follows:

1. To determine the feasibility of basing predictions of potential 28-day strength on concrete cylinder specimens made and cured at ambient temperatures of approximately 73 F (23 C). In previous work (1, 2), the majority of the specimens were subjected to conditioning by immersion in hot water or by heating in an oven or in an autoclave for short periods before the test. Testing concrete compressive specimens without pretreatment accomplishes appreciable savings in time and equipment cost. This would be particularly advantageous under field conditions where equipment and personnel for pretreatment may not be available.
2. To determine the feasibility of substituting pressure pads of semicompressible material for the usually specified sulfur mortar caps (AASHO T 231) with which test specimens were prepared in previous work (1, 2). The purpose is to eliminate otherwise required equipment or labor and to reduce the time involved in making early tests.
3. To develop equations for predicting from the results of early tests the potential minimum 28-day strength of concrete made with significantly reduced cement factors. The purpose is to develop prediction equations appropriate for use when early tests indicate abnormally low compressive strengths.

RESEARCH APPROACH

Prediction of Concrete Strength

Since the 1930s, much work has been done to determine whether or not concrete is of satisfactory quality before the results of tests are made known a considerable time after the placement of the concrete in construction. A report by the Ontario Department of Highways (3) lists 87 references relating to methods for use in estimating the later strength of concrete from early test results. Interest in these methods has been intensified by the adoption of the concepts of quality assurance by many segments of the highway construction industry. The availability of early test results, which can be used as a basis for immediate action, is inherent to the optimization of quality assurance systems.

As previously mentioned, much work has been done in this area. Investigations by others (4) have had various objectives and have resulted in procedures and prediction equations appropriate to these objectives. In general, the underlying concept has been that the gain of strength of concrete from the time of final set is a function of a nonlinear time-temperature relationship. Accordingly, previous work has involved acceleration of the gain of strength by some method of increasing the temperature of the concrete specimens after a short waiting period following their fabrication. This was accomplished by immersion in hot-water baths, low-pressure steam curing, oven heating, or autogenous curing (AASHO C 684-71T). In some cases, where the objective was to obtain an early estimate of the 28-day strength of laboratory-mixed design mixtures, the plastic concrete or concrete that had reached the condition of final set was heated; in others the concrete was heated after curing at ambient temperatures. After pretreatment, the specimens were usually tested in compression, and a mathematical formula was used to predict the strength at some later period, usually 28 days. However, at least in one case an attempt was made to predict the ultimate strength of the concrete (5).

A review of the literature indicated that most previous approaches to the problem of relating the strength of concrete at an early age to that at 28 days suffered from several deficiencies. Procedures often required that the early tests be made at fixed intervals after the specimens were formed. Most of these procedures were not very adaptable to conditions associated with highway construction where there are differences in early curing temperatures and variations in the time required to transfer specimens from the job site to the testing facility. The methods of preparing specimens for the test were sometimes inconvenient or required extensive equipment. In addition, the mathematical equations used for prediction of potential strength had not proved to be entirely reliable, and the degree of confidence that could be placed in the prediction was uncertain. However, in view of the normal variation of results of tests of concrete made and cured under standard conditions and the knowledge that, for quality control purposes, all that is required is a prediction of minimum potential strength with an acceptable degree of confidence, it was believed that a satisfactory method could be developed of evaluating concrete quality at early ages that would be comparable in significance with 28-day tests.

Accordingly, a relatively new quality assurance procedure for determining the early strength of concrete was developed and evaluated by laboratory experiment and a limited amount of field application (1, 2). The evaluation of these investigations indicates that the procedure used and the formula developed for predicting potential strength are reliable over the range of conditions investigated.

In the work previously reported (1, 2), 0 to 72-hour-old specimens were pretreated by heating in a water bath, oven, or autoclave before testing. It was found that the results of tests of specimens conditioned by any of these methods could be used for the purpose of predicting 28-day strengths with acceptable accuracy. However, in one series of tests where the cylinders were tested at 24, 48, and 72 hours without any pretreatment or heating, predictions based on the results were as accurate as, if not more accurate than, those obtained from specimens pretreated by any of the heating methods.

The equation that was fitted to the experimental data was a modification of the Plowman equation and has the form

$$S_M = S_m + b\,(\log M - \log m) \qquad (1)$$

where

S_M = predicted normal compressive strength at maturity M;
S_m = measured compressive strength at maturity m;
M = degree hours of maturity under standard conditions, i.e., when cured at about 73 F (23 C);
m = degree hours (F hours) of maturity of specimen at time of early test after conditioning (hours of age × ambient temperature) + C;
C = degree hours (F hours) of maturity determined by autogenous heating, method of preparation or conditioning, residual temperature at time of test, and possible unknown factors; and
b = slope of prediction line.

Equation 1 plots as a straight line on semilog graph paper and differs from the Plowman equation in that the line of prediction is projected from the actual value of the early test S_m instead of from the intercept, which was given a constant value by Plowman. This increases the accuracy of prediction since both early and 28-day strengths are affected by sample-to-sample variation. Because of this variation, plotted lines representing tests on different samples are not likely to have the same intercept.

Capping of Concrete Test Specimens

The previous work proved the practicality of predicting the potential strength of the normal concrete from the results of early tests with a reasonable degree of assurance when sulfur mortar caps were used. It would appear that the practicality of the test procedure for quality assurance purposes would be increased if it were further simplified by eliminating the capping operation. This would result in a savings of time, equipment, and materials and would be of particular advantage when tests were made in a field laboratory.

From Hudson's personal communication with Walker in the 1930s, it was learned that some experimentation had been conducted using Celotex pressure pads with some degree of success. A literature search failed to give any specific information on this matter; however, there is some discussion to the effect that sheet materials had been used, that there was a reduction in strength, and that the reduction was less when low-strength cylinders were tested. Limited experimentation indicates that concrete with 1,900 to 3,700-psi (13 000 to 26 000-kPa) observed strengths will have observed strengths reduced by approximately 8 percent when tested with corrugated cardboard pads in lieu of standard sulfur mortar caps and that the coefficient of variation is less than 10 percent. A reduction in observed strength of this magnitude would not be unacceptable in this particular application since the objective is not necessarily to obtain a precise measure of strength but a value from which future minimum strengths can be predicted.

Prediction Equation 1

Although equation 1 developed in the course of the previously reported work (1, 2) has been shown to produce reasonably accurate estimates of potential 28-day compressive strengths, under normal conditions, there is a theoretical disadvantage. The increased accuracy of prediction of this equation stems from the use of the early test results as the start of the line of prediction. The size of the right side of the equation, representing the estimate of the increased strength due to additional maturity, depends in part on the cement constant of the concrete. Therefore, concrete that was seriously deficient in cement content (evidenced by a very low early strength) could be shown by the equation to have a satisfactory 28-day predicted minimum strength since the rate of gain of strength with increased maturity would be based on normal cement content. For this reason, one of the objectives of this series of tests was to obtain data suitable for deriving a prediction equation that would be applicable to concrete mixtures deficient in cement.

Designed Experiments

The experiment designed to accomplish the objectives of the research reported here was designated as series V. The primary purpose of series V was to determine the effect of omitting cement from a concrete mixture designed for a six-bag cement constant and to provide data for deriving an equation appropriate for predicting potential minimum 28-day strength from the results of early tests under these conditions.

In addition to these primary objectives, the designed experiment included a provision for use of data from series V for determining the effects of substituting pressure pads for sulfur mortar caps when early tests are made. The data were also intended for use in testing the fit of equation 1 to tests made at early ages without pretreatment of the specimens to accelerate the gain of early strength.

Specimen Preparation

The preparation, curing, and testing of concrete test specimens were conducted by the Materials Control, Soil and Testing Division of the West Virginia Department of Highways.

The order of making batches was pseudorandom so that there was no consistent pattern of a batch with a certain cement content being made after a batch with another cement content was made. Mix proportions are given in Table 1.

After mixing, the concrete from all batches was tested for slump and air content. Concrete used for these purposes was discarded. Cylinders were made by the group method in cardboard molds outside the moisture room and were then carried to the

Table 1. Mix proportions for series V.

Cement Constant	Cement, Marquette Type 1	Fine Aggregate, Ohio River Sand	Coarse Aggregate, No. 67 Acme Limestone	City Water	Air Content, Master Builders Vinsol Resin (percent)
Three bags	282	1,379	1,745	197 ± 7	7.5 to 10.4
Four bags	375	1,379	1,745	226 ± 4	5.5 to 8.1
Five bags	470	1,379	1,745	237 ± 3	5.7 to 7.5
Six bags	564	1,283	1,690	265 ± 3	5.2 to 5.8

Note: Values are in pounds per cubic yard. 1 lb/yd^3 = 0.59 kg/m^3.

moisture room for curing at standard temperature of approximately 73 F (23 C). All cylinders were tested without pretreatment and at storage temperature. One-half the cylinders were prepared for test with sulfur mortar (Atlas Vitrobond) caps. The other half were prepared by placing Celotex pressure pads ½ in. (12.7 mm) thick by 7 in. (178 mm) square on both ends of cylinders at time of test.

Companion cylinders were made from 30 batches, and 120 comparisons were made of the strengths obtained on 1 to 3-day-old specimens. The average strengths of specimens made from the same batch and the standard deviations are given in Table 2. The average strengths obtained by using sulfur mortar caps and pressure pads are given in Table 3. The overall reduction in strength due to the use of the pressure pads was less than 5 percent; most of the reduction in strength occurred in specimens testing in excess of 1,800 psi (12 000 kPa) as shown in Figure 1. Variation in strength, as measured by the standard deviation of tests within batches, was approximately the same for the Celotex pressure pads as for the sulfur mortar caps over the entire range of strengths. Because of the small differences, the two sets of data were pooled for some purposes.

Prediction Equation 2

For predicting minimum potential compressive strength at 28 days, equation 1 simplifies to

$$S_{28} = S_a + b\ (4.699 - \log m) \qquad (2)$$

where 4.699 is the log of the maturity of the concrete specimens stored for 28 days at approximately 73 F (23 C). The other parameters are the same as for equation 1 except that in the previous work where the specimens were conditioned by heating in a 200 F (93 C) water bath, it was necessary to determine a constant C, which accounted for the effects of heating and other possible undefined variables in equations 1 and 2. In series V, where no specimens were heated, the use of the constant C was not required for practical applications. The value of b was found by simple regression and by substituting actual compressive strength values and actual maturities based on temperature measurements in the equation $S_M = b\ (\log m) - a$.

The graphical relationship of maturity (F hour) to compressive strength [psi (kPa)] of the concrete at early ages and at 28 days for series V is shown in Figure 2. As shown in the figure, equation 2 is capable of making reliable predictions of minimum 28-day strengths providing that the temperature history (maturity) and the cement content of the mixture are known.

Comparison of Measured and Predicted Compressive Strengths

A comparison was made of measured 28-day compressive strengths [psi (kPa)] deter-

Table 2. Compressive strength of test specimens.

Cement Constant	Measurement	24 Hours SM[a]	24 Hours PP[b]	48 Hours SM[a]	48 Hours PP[b]	72 Hours SM[a]	72 Hours PP[b]	28 Days SM[a]
Three bags	Mean	273	282	503	458	657	670	1,572
	Standard deviation	45	48	61	52	27	19	205
	Coefficient of variation	16.4	18.2	12.1	11.1	4.2	2.8	13.0
Four bags	Mean	616	620	974	986	1,279	1,278	2,794
	Standard deviation	141	34	142	134	109	202	421
	Coefficient of variation	22.9	5.5	14.6	13.6	8.5	16.8	15.1
Five bags	Mean	934	919	1,402	1,441	1,717	1,776	3,831
	Standard deviation	142	73	110	77	182	102	247
	Coefficient of variation	15.2	7.9	7.8	5.3	10.6	5.8	6.4
Six bags	Mean	1,631	1,566	2,446	2,339	2,894	2,673	5,182
	Standard deviation	149	161	180	174	203	287	231
	Coefficient of variation	9.1	10.3	7.4	7.4	7.0	10.7	4.5

Note: Values are in pounds per square inch. 1 psi = 6.9 kPa.

[a]Sulfur mortar caps. [b]Celotex pressure pads.

Table 3. Average compressive strengths of concrete cylinders with sulfur mortar caps and pressure pads.

Sulfur Mortar Caps	Pressure Pads	Distribution of Differences Amount	Distribution of Differences Percent[a]	Sulfur Mortar Caps	Pressure Pads	Distribution of Differences Amount	Distribution of Differences Percent[a]
273	265	-8	-2.9	1,717	1,776	+59	+3.4
503	468	-35	-7.0	1,758	1,653	-105	-6.0
616	620	+4	+0.7	2,057	1,955	-102	-5.0
657	670	+13	+2.0	2,448	2,339	-109	-4.5
934	919	-15	-1.6	2,632	2,570	-62	-2.4
974	986	+12	+1.2	2,894	2,673	-221	-7.6
1,301	1,193	-108	-8.3	3,001	2,821	-180	-6.0
1,402	1,441	+39	+2.8	3,133	2,990	-143	-4.6
1,631	1,566	-65	-4.0	3,686	3,210	-476	-12.9
				$\bar{X}$ = 1,756	1,673	-83	-4.7
				σ = 1,021	928	93	-9.1

Note: Results are based on averages of six or seven test results. Values are in pounds per square inch. 1 psi = 6.9 kPa.

[a]Of sulfur mortar caps.

Figure 1. Compressive strength results for sulfur mortar caps versus pressure pads.

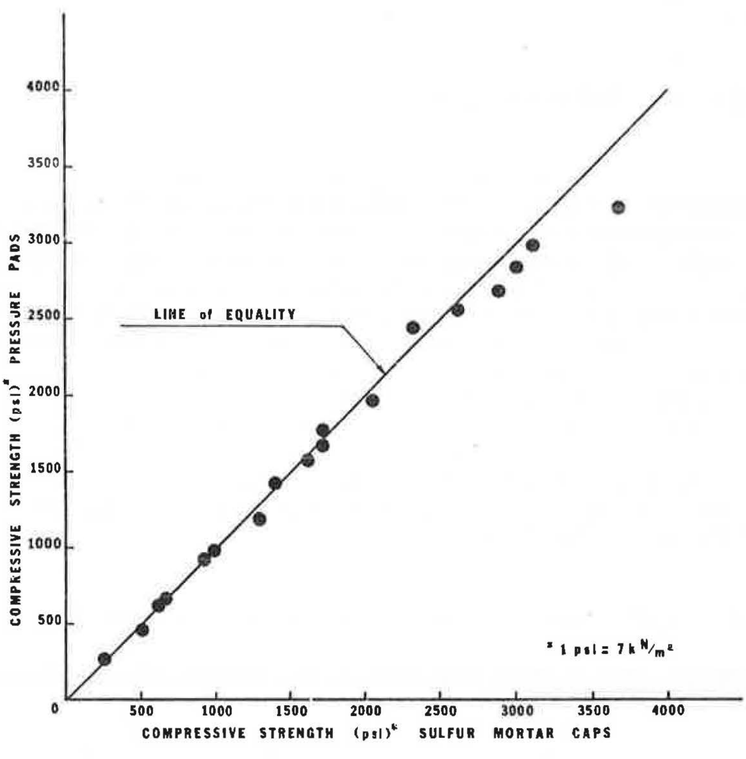


Figure 2. Early and 28-day compressive strengths based on equation 2.

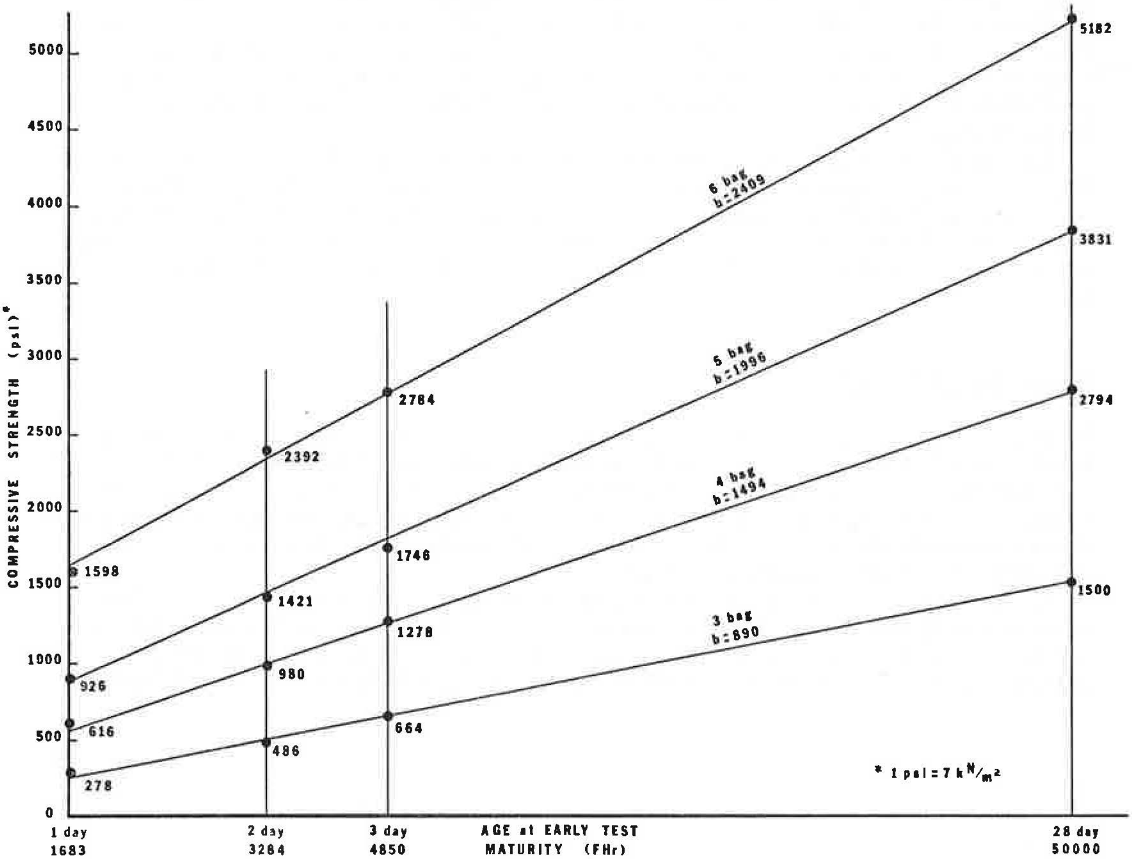
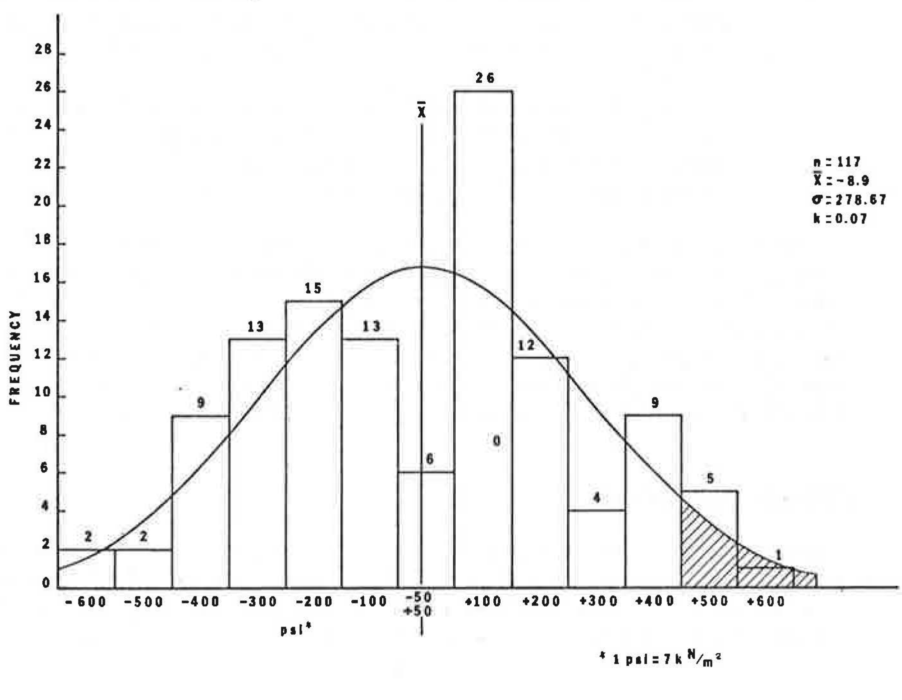


Figure 3. Frequency distribution of differences for values predicted by equation 2 and measured values.



mined in series V and the strengths estimated by equations 1 and 2. The distribution of the differences Δ between the measured values and the predicted values is shown in Figure 3. This shows that, if the predicted 28-day strength (S_{28}), computed by the correct form of equation 2, exceeds the critical acceptance value L by 1.645 σ_e or 460 psi (3200 kPa), there is a 95 percent probability that the true average strength is greater than L.

The standard deviation σ_e of differences between measured 28-day strengths and the strengths predicted by equations 1 and 2 from the results of tests of specimens not conditioned by heating is about 278 psi (1900 kPa). This is about 45 percent less than the σ_e value of 493 psi (3400 kPa) obtained from the results of tests of specimens conditioned by heating to 200 F (93 C) for 3½ hours as reported for the previous work (1).

Prediction Equation 3

As stated in the objectives, the primary purpose of series V was to obtain data that could be used to derive an equation that would predict the potential strength of concrete mixtures deficient in cement from the results of early tests. Equations 1 and 2 were satisfactory for predicting the potential 28-day strength of concrete made with a known cement constant but were unreliable when low strengths were due to a significant deficiency of cement of unknown amount.

In studies made to determine the possibility of developing a prediction equation, alternate to equations 1 and 2, it was found that results of early tests of concrete at a particular maturity were highly correlated with 28-day compressive strengths (9). The general form of the equation of relationship is for a parabola and can be written as

$$S_{c_{28}} = a\,(S_{c_m})^b \tag{3}$$

where

$S_{c_{28}}$ = compressive strength in psi (kPa) of specimens cured for 28 days under standard conditions and prepared for test by means of sulfur mortar caps,

a = value of the intercept of the line of relationship with the Y ordinate on a log-log plot,

S_{c_m} = compressive strength in psi (kPa) of specimens tested at a particular age or degree of maturity without pretreatment and prepared for test with pressure pads in place of sulfur mortar caps, and

b = slope of the line of relationship on a log-log plot.

As shown in Figure 4, equation 3 can be used to predict minimum 28-day strengths independently of cement content, providing that the early-age specimens are tested after 24, 48, or 72 hours of curing at about 73 F (23 C). The distribution of the differences between the measured values and the values predicted by equation 3 is shown in Figure 5. This figure shows that, if the predicted 28-day strength exceeds the critical acceptance limit L by 1.645 σ_e or 570 psi (3900 kPa), there is a 95 percent probability that the true average strength is greater than L.

Prediction Equation 4

Since equations 1 and 2 appeared to be more suitable for predicting potential 28-day compressive strength of concrete with a known cement content over a range of maturities, as shown in Figure 2, and equation 3 was suitable for prediction of potential 28-day strength of concrete when tested at a particular maturity, as shown in Figure 4,

Figure 4. Early and 28-day compressive strengths of concrete based on equation 3.

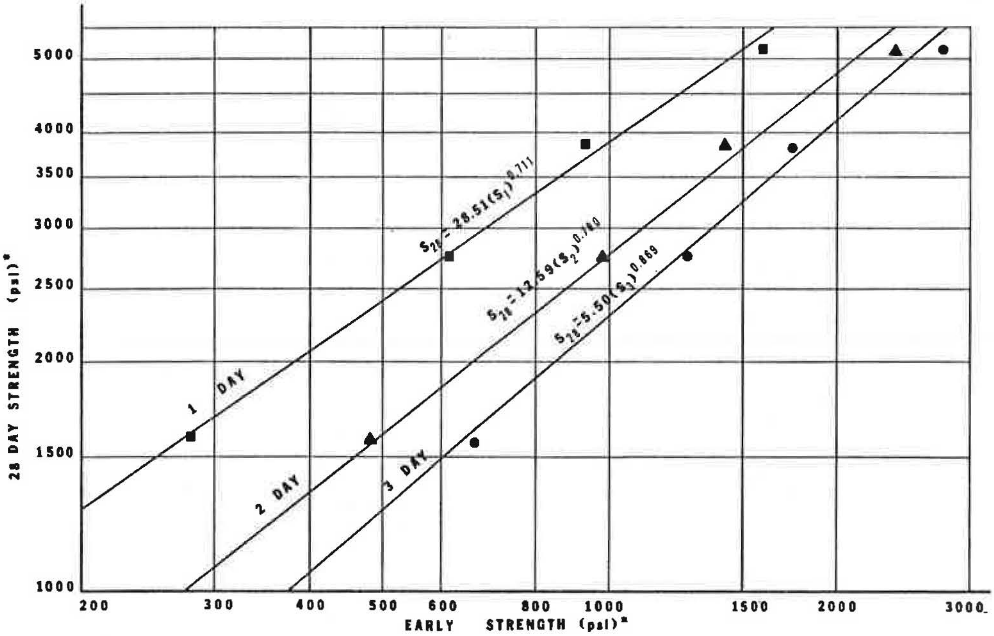


Figure 5. Frequency distribution of differences for values predicted by equation 3 and measured values.

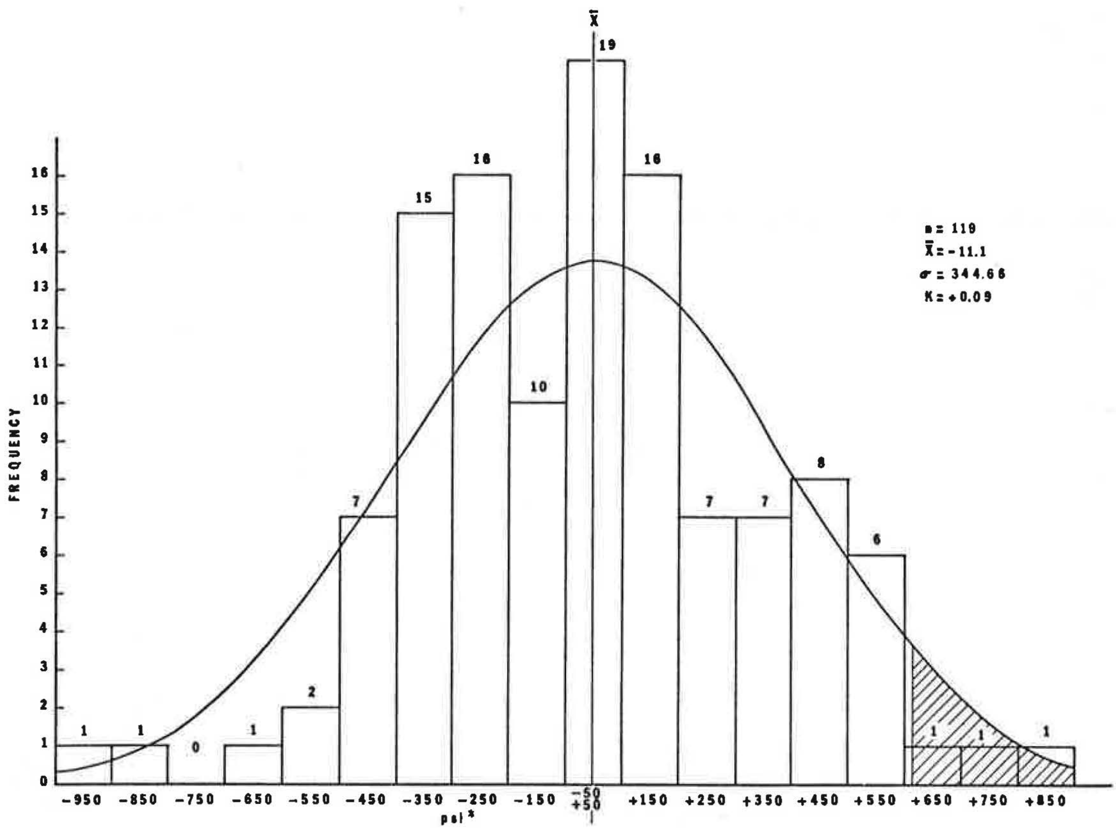


Figure 6. Nomograph for estimating potential 28-day compressive strength from results of early tests.

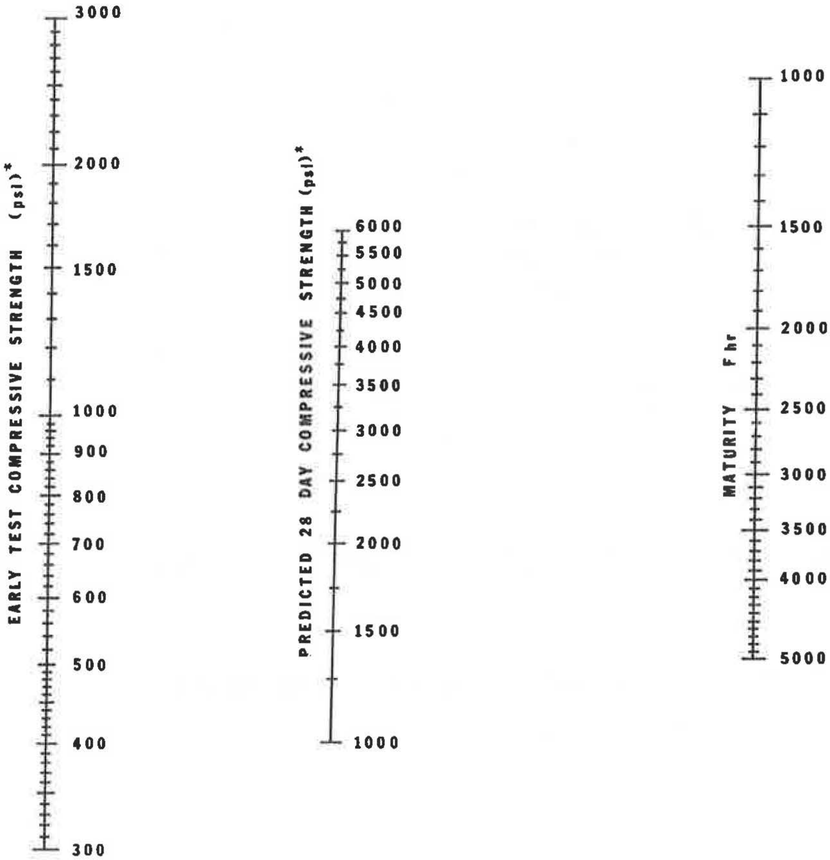
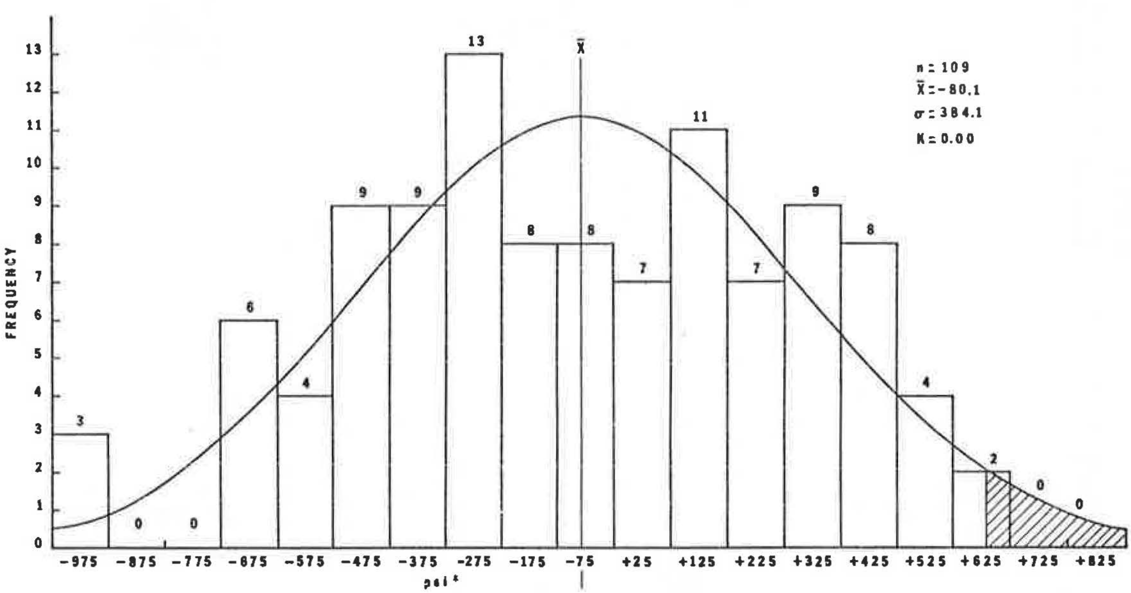


Figure 7. Frequency distribution of differences between values predicted by equation 4 and measured values.



the three equations were combined to provide an equation of more general application. This equation has the form log S_{28} = 2.9844 + 0.75 log S_e - 0.51 log m, which may be written as

$$S_{28} = 965 \frac{S_e^{0.75}}{m^{0.51}} \tag{4}$$

where

S_{28} = predicted 28-day compressive strength,
S_e = compressive strength of specimens tested at an early age and having a maturity m, and
m = degree hours of maturity at the time of the test.

Equation 4 is independent of the cement content of the concrete for three to six bags/yd^3 = (3.9 to 7.8 bags/m^3), and the early-strength specimens do not have to be tested at any particular maturity. Equation 4 can be solved by using the log scales on a slide rule, but, for frequent use, a nomograph similar to that shown in Figure 6 can be constructed.

This nomograph was used to compute predicted values for comparison with measured values. The distribution of the differences is shown in Figure 7. The standard deviation of the differences is about 384 psi (2600 kPa); there is a 95 percent probability that the true average strength exceeds the value of L.

Equations 1, 2, 3, and 4 illustrate types of mathematical relationships that can be used to predict minimum potential 28-day strength of concrete under particular conditions. Similar equations, with different parameters, can be developed for concrete mixtures with proportions and kinds of cement and aggregate other than those used in the mixtures reported here. The accuracy of the predictions will depend largely on the use of correct and uniform procedures in the making of the early and 28-day test specimens.

CONCLUSIONS

The findings of this experiment indicate the following:

1. Heat treatment or autogenous curing of concrete specimens that are more than about 20 hours old before early testing is not necessary and may complicate the derivation of equations predicting potential minimum 28-day strength. Testing cylinders after 1 to 3 days of curing at normal temperatures can effect savings in equipment expenditures and technical effort.
2. The use of sulfur mortar caps to prepare cylinders for test at early ages is not required. The use of cardboard or Celotex pressure pads for specimens expected to attain less than about 2,000 psi (13 800 kPa) can effect savings in material and equipment costs. When combined with testing without prior conditioning, the use of pressure pads greatly increases the practicality of the use of early compressive tests in the field as one means of quality determination.
3. When definite early test procedures are established and a series of laboratory tests have been performed, equations can be derived for predicting the minimum potential strength of concrete at 28 days. A suitable equation can predict minimum potential 28-day strength with sufficient accuracy to provide a valuable method for use in quality assurance systems.

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