# A DETERMINISTIC TRAFFIC FLOW MODEL FOR THE TWO-REGIME APPROACH 

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#### Abstract

In recent years it has been shown that the relationship between flow and concentration is probably not continuous under maximum flow conditions. A previous paper (2) concerned with the evaluation of traffic flow models examined the steady-state equations derived from the generalized carfollowing model designed by Gazis, Herman, and Rothery. These macroscopic relationships were subjected to 45 data sets, in which most of the data were from freeway lanes, for both single- and two-regime models. From the results of these data sets, the deficiencies of the various models using the two-regime approach were identified and the need for investigating a new two-regime approach was stressed. This paper discusses the development of a new model at both the microscopic and macroscopic levels. The steady-state equation derived from the new model is analytically evaluated by using 45 data sets. The model, based on a new car-following sensitivity component, shows that the free-flow regime and the congestedflow regime are fairly well adapted to convexity and concavity properties respectively in a speed-concentration relationship. By using the analysis of driver performance as a sensitivity measurement, model parameters are defined and evaluated. In addition, the two flow regimes are incorporated by means of breakpoint evaluation procedures. In the light of two-regime phenomena the new steady-state formulation may be superior to the steadystate equations derived from the generalized car-following model, particularly in simplicity and clarity.


- EDIE (1) was the first to point out the possibility of discontinuity in the flow concentration curve under maximum flow conditions. He proposed two separate models. These models, which describe macroscopic relationships, are based on the convertibility models developed from the microscopic car-following model.

Because more and more road facilities are operating at near-capacity level, the importance of considering this discontinuity phenomenon is apparent. Furthermore, in a description of traffic behavior, this phenomenon will make the limitations more severe for hydrodynamic applications.

A previous paper (2) concerned with the evaluation of traffic flow models examined the macroscopic relationships derived from the generalized car-following model formulated by Gazis, Herman, and Rothery (3) as

$$
\begin{equation*}
\ddot{X}_{n+1}(t+T)=\alpha \frac{\left[\dot{X}_{n+1}(t+T)\right]^{n}}{\left[X_{n}(t)-X_{n+1}(t)\right]^{\rho}}\left[\dot{X}_{n}(t)-\dot{X}_{n+1}(t)\right] \tag{1}
\end{equation*}
$$

witere ine singie and úvuivie diots represent speeả and acceleration (deceleration) and

[^0]$\mathrm{X}_{\mathrm{n}}, \mathrm{X}_{\mathrm{n}+1}=$ positions of the leading car and the following car,
$\mathrm{T}=$ time lag of response to stimulus, and
$\mathrm{m}, \ell$, and $\alpha=$ some constant parameters.
Forty-five sets of speed-concentration measurements (2) were used to show the most appropriate speed-concentration relationships based on steady-state flow formulation obtained by integrating equation 1. These relationships were investigated in an m , $\ell$ matrix format to study the variability of those exponents of the sensitivity component that belong to equation 1 . The results were summarized in a two-dimensional $\mathrm{m}, \ell$ matrix for both single- and two-regime traffic flow models. By investigating singleregime models, other microscopic and macroscopic theories that can be reduced to the form of equation 1 can be evaluated. On the other hand, investigation of two-regime models stimulates speculation about the simplicity and clarity of the generalized carfollowing model for the two-regime approach.

The purpose of this paper is to develop and evaluate a simpler and more reliable traffic flow model than the generalized car-following model for two-regime traffic behavior.

This paper discusses the development of a new model at both the microscopic and macroscopic levels and presents on analytical evaluation of the new model by using actual traffic flow data.

The 45 data sets used in a previous work (2) are also the basis for the quantitative analysis performed in this paper. These data sets are described in detail elsewhere (2). The data sets can be separated into two groups: the first 32 data sets based on $1-\mathrm{min}$ time interval samples and the remaining 13 sets based on 5 -min samples. In both groups, the mean speed and mean concentration are calculated for each interval.

The sources of the first 32 data sets are given in Table 1. The remaining 13 data sets were taken on the Santa Monica Freeway, 11 (SM-12 to SM-22) on freeway lanes and two from a CD road and an on-ramp. The data selection procedure and other details concerning the data are described elsewhere (2).

The use of large amounts of actual data emphasizes the possibility that the new model could be applicable not only to traffic flow theory but also to planning new road facilities, road improvements, and freeway control projects.

After the deficiencies of two-regime traffic flow models based on equation 1 were identified, a new model was developed. Based on analysis of driver performance, this model shows how one can take into consideration the discontinuity phenomenon under peak flow conditions from gross aspects of traffic flow.

## MODEL DEVELOPMENT

Previous studies in car-following (1, 3, 4) note the following stimulus-response relationship:

$$
\begin{equation*}
\binom{\text { driver's }}{\text { response }}_{t+T}=\binom{\text { sensitivity }}{\text { factors }}_{t} \times\binom{\text { given }}{\text { stimulus }}_{t} \tag{2}
\end{equation*}
$$

Although attempts to improve the car-following theory have been made, two parts of this equation have not been modified. The first part of this relationship is the response of the following vehicle at time ( $t+T$ ) in terms of deceleration (or acceleration). This response is proportional to the second part, the stimulus. A given stimulus is described in terms of relative speed (between the pair of vehicles under consideration) at time $t$.

The third part of equation 2, the sensitivity component, has undergone several stages of refinement. It has had the following functions: constant factor, inversely proportional to spacing (headway distance), inversely proportional to spacing squared and proportional to the absolute speed of the following vehicle, and the generalized expression as shown in equation 1.

Because there are some difficulties in the generalized car-following model (equation 1 ), an attempt will be made to develop a new sensitivity component. The deficiencies of equation 1 are particularly emphasized for the two-regime approach and are summarized below.

1. It has a rather complicated sensitivity function, namely,

$$
\frac{\alpha\left[\dot{X}_{n+1}(t+T)\right]^{n}}{\left[X_{n}(t)-X_{n+1}(t)\right]^{q}}
$$

where $\alpha, l, m$ are arbitrary and are not subject to independent measurements (excluding particular $\mathrm{m}, \ell$ combinations) for the two-regime models.
2. When equation 1 is converted to steady-state flow equations, the boundary parameters, free-flow speed and jam concentration, are not always defined for the entire m, \& plane.
3. Nine steady-state flow equations can be derived from equation 1 for the entire m , \& plane, which increases the complexity of such macroscopic relationships.
4. It is difficult and perhaps impossible to make stability evaluation of the nonlinear model shown in equation 1 (excluding the case where $m=\ell=0$ ).

Herman and Potts (4) suggested that driver response to a given stimulus varies inversely with spacing and that there is a sensitivity function $\alpha_{0} / \mathrm{s}$, where s is the spacing between vehicles and $\alpha_{0}$ is a constant. Edie (1) proposed the introduction of the absolute vehicle speed into the sensitivity function and the square of the spacing $\left(\alpha \times u_{n+1}\right) / s^{2}$, where $u_{n+1}$ is the absolute speed of the following vehicle and $\alpha$ is a constant. Later, these two sensitivity functions were examined by Rothery et al. (5), who found that these two sensitivity components during the car-following mode improved the results obtained from the linear model, which has a constant sensitivity function. However, they indicated that there was no significant difference between the two functions that makes either of these sensitivity components superior to the other.

In addition, Pipe and Wojcik (6) used perceptual factors (rate of change of visual angle) to derive a car-following model. They demonstrated a sensitivity function of the form $\alpha / \mathrm{s}^{2}$. Finally, the previous paper (2) investigated two-regime models based on equation 1 and indicated that the $m$ value (the speed exponent belonging to equation 1) tends to be closer to 0 than to 1 in both regimes. Therefore, the search for a simple model was narrowed to those situations where the sensitivity function is only inversely proportional to the spacing in various degrees. In addition, the following criteria for the new sensitivity component were considered:

1. It should be capable of describing real-world traffic data; and
2. The steady-state flow equations derived from the car-following model should minimize the deficiencies of equation 1 stated and, therefore, should (a) be reasonably simple, (b) provide a complete definition of all traffic flow parameters (free-flow speed, jam concentration, and optimum parameters), and (c) describe differences between the free-flow regime and congested-flow regime on the basis of average car-following behavior.

These criteria led to the decision that a sensitivity function be a combination of a weighting factor and a reciprocal spacing function. In addition, macroscopic models freguently included the normalized enncantration component (k/k within the speodconcentration relationship (2). Based on the assumption of steady-state flow, this normalized concentration becomes equivalent to the ratio between jam spacing and spacing. This ratio is used in the proposed sensitivity function as a power of a crucial weighting factor.

Consequently, the following sensitivity function is proposed:

$$
\begin{equation*}
\alpha \times \frac{\mathrm{A}^{-s_{1 / a}}}{\mathrm{~s}^{2}} \tag{3}
\end{equation*}
$$

where
$\mathbf{s}$ and $\mathbf{s}_{\mathrm{j}}=$ spacing and jam spacing and
$\mathrm{A}=$ nondimensional weighting factor.

## MODEL SENSITIVITY

The microscopic form of the proposed model, which assumes that the ( $n+1$ )th vehicle will react in a stable way to any motion of the lead nth vehicle, is

$$
\begin{equation*}
a_{n+1}(t+T)=\alpha \times \frac{A^{-\frac{s_{j}}{s(t)}}}{s^{2}(t)}\left[u_{n}(t)-u_{n+1}(t)\right] \tag{4}
\end{equation*}
$$

where

$$
\begin{aligned}
\mathrm{a}_{\mathrm{n}+1} & =\text { deceleration or acceleration response at time } \mathrm{t}+\mathrm{T}, \\
\mathrm{u}_{\mathrm{n}}, \mathrm{u}_{\mathrm{n}+1} & =\text { speeds of the leading and following vehicles at time } \mathrm{t} \\
\mathrm{~T} & =\text { time lag of response to stimulus, and } \\
\alpha & =\text { a constant dependent on } \mathrm{A} \text { and } \mathrm{s}_{j} .
\end{aligned}
$$

The sensitivity term (equation 3) may not immediately indicate that the deceleration response increases inversely with spacing, which is required from a logical standpoint. Therefore, the following mathematical derivation is presented.

For convenience, a progressive ratio between the response and concentration is maintained (assuming a steady-state traffic stream) instead of an inverse ratio between the response and spacing. Thus, from equation 4, the required constraint will be

$$
\begin{equation*}
\frac{\mathrm{k}^{2}}{\mathrm{~A}^{\mathrm{k} / \mathrm{k}_{j}}}<\frac{(\mathrm{k}+\Delta \mathrm{k})^{2}}{\mathrm{~A}^{(\mathrm{k}+\Delta \mathrm{k}) / k_{j}}} \tag{5}
\end{equation*}
$$

for $\mathrm{k}=\frac{1}{\mathrm{~s}}, \mathrm{k}_{\mathrm{j}}=\frac{1}{\mathrm{~s}_{\mathrm{j}}}, 0 \leq \mathrm{k} \leq \mathrm{k}_{\mathrm{y}}, 0 \leq \Delta \mathrm{k} \leq \mathrm{k}_{\mathrm{j}}-\mathrm{k}$, or

$$
\mathrm{A}<\left(1+\frac{\Delta \mathrm{k}}{\mathrm{k}}\right)^{2 \mathrm{k}_{\mathrm{j}} / \Delta \mathrm{k}}
$$

By using boundary conditions, the maximum value of A in equation 5 for all k and $\Delta \mathrm{k}$ is

$$
\operatorname{Max} A=\lim _{\substack{\Delta k \rightarrow 0 \\ k \rightarrow k_{j}}}\left(1+\frac{\Delta k}{k}\right)^{2 k_{j} / \Delta k}=\lim _{\Delta k \rightarrow 0}\left[\left(1+\frac{1}{k_{j} / \Delta k}\right)^{k_{j} / \Delta k}\right]^{2}=e^{2}
$$

Therefore, the range of A values that will maintain the deceleration response inversely with spacing is

$$
\begin{equation*}
0<\mathrm{A}<\mathrm{e}^{2} \tag{6}
\end{equation*}
$$

If the time lag between response and stimulus and the variations in behavior from one driver to another are neglected, a steady-state equation can be derived from equation 4 since it is a perfect differential in $t$.

Thus, converting the symbols of equation 4 gives

$$
\frac{d u}{d \dot{d} t}=\frac{\alpha}{s^{2}} A^{-s_{j} / s} \frac{d s}{\dot{d t}}
$$

and integrating gives

$$
\mathrm{u}=\frac{\alpha \mathrm{k}_{\mathrm{s}}}{\ln A} A^{-\varepsilon / k_{\mathrm{j}}}+\mathrm{c}
$$

where $k=1 / \mathrm{s}$ and $\mathrm{k}_{\mathrm{j}}=1 / \mathrm{s}_{\mathrm{j}}$.
From boundary condition, i.e.,

$$
\begin{aligned}
& \mathrm{u} \rightarrow 0, \mathrm{k} \rightarrow \mathrm{k}_{\mathrm{j}}=\text { jam concentration }, \\
& \mathrm{k} \rightarrow 0, \mathrm{u} \rightarrow \mathrm{u}_{\mathrm{f}}=\text { free-flow speed }
\end{aligned}
$$

the following macroscopic model can be obtained [note that $\alpha=u_{\mathrm{p}} \mathrm{A} \times \ln \mathrm{A} / \mathrm{k}_{\mathrm{j}}(\mathrm{A}-1)$ ].

$$
\begin{equation*}
u=\frac{u_{f}}{A-1}\left(A^{1-k / k_{y}}-1\right) \tag{7}
\end{equation*}
$$

for $A \neq 1.0$. Note that, in the case of $A=1.0$, equation 4 becomes the same as the Pipe and Wojcik car-following model (6), which, in turn, is convertible to the Greenshields model (7) in steady-state flow. Therefore, an extension of the proposed model is

$$
\begin{equation*}
u=u_{f}\left(1-\frac{k}{k_{\mathrm{f}}}\right) \tag{8}
\end{equation*}
$$

for $\mathrm{A}=1.0$.
Using the steady-state flow equation, $q=u k$, where $q$ is the flow in vehicles per hour per lane, $u$ is the speed in $m p h(k m / h)$, and $k$ is the concentration in vehicles per mile (per km ) per lane, one can arrive at an equation satisfying optimum conditions, i.e., for $\mathrm{dq} / \mathrm{dk}=0$, as follows:

$$
A^{\left(x_{0} / x_{g}\right)-1}+\frac{k_{0}}{k_{d}} \ln A=1
$$

for $A \neq 1.0$ and

$$
\begin{equation*}
k_{o}=\frac{k_{j}}{2} \tag{9}
\end{equation*}
$$

for $A=1.0$. Furthermore,

$$
q_{\max }=u_{0} k_{0}
$$

where $u_{0}$ is obtained from equation 7 or 8 by substituting $k_{0}$ for $k$.
By drawing families of speed-concentration curves as functions of the weighting factor $A$ and holding $u_{f}$ and $k_{f}$ constant with reasonable values, one can determine that for $0<\mathrm{A}<1.0$ convex curves result, but, for $\mathrm{A}>1.0$, the curves are concave. As A goes to 1.0 from both sides, the concave and convex functions converge to a linear function and vice versa. Mathematically speaking,

$$
\mathrm{u}(\mathrm{k})_{\mathrm{k}}^{\prime \prime}<0 \quad \text { and } \quad \mathrm{u}(\mathrm{k})_{\mathrm{k}}^{\prime}<0
$$

for $0<\mathrm{A}<1.0$ and

$$
\begin{equation*}
\mathrm{u}(\mathrm{k})_{\mathrm{k}}^{\prime \prime}>0 \text { and } \mathrm{u}(\mathrm{k})_{\mathrm{k}}^{\prime}<0 \tag{10}
\end{equation*}
$$

for $A>1.0$, where the prime and double prime represent the first and second derivatives with respect to k .

Based on analysis of driver performance the correlation between microscopic and macroscopic traffic behavior will now be examined with respect to the sensitivity component in equation 3.

According to Michaels (8), one can have an approximate relationship between spacing and the minimum absolute relative speed ( $\min .|\dot{\mathbf{s}}|$ ) that can be detected for a given spacing. This relationship is shown in the upper right part of Figure 1. It is based on the mean value for the absolute threshold to angular velocity ( $\mathrm{d} \theta / \mathrm{dt}=6 \times 10^{-4} \mathrm{rad} / \mathrm{sec}$ ) and on the car-following model, which is based on the rate of change of visual angle as derived by Pipe and Wojcik (6).

The minimum absolute relative speed curve (Figure 1) includes the following points:

1. Small spacing, $s_{1,4}$, and small relative speed, $\left(\frac{d s}{d t}\right)_{1,2}$;
2. Large spacing, $\mathrm{s}_{2,3}$, and small relative speed, $\left(\frac{\mathrm{ds}}{\mathrm{dt}}\right)_{1,2}$;
3. Large spacing, $\mathrm{s}_{2,3}$, and large relative speed, $\left(\frac{\mathrm{ds}}{\mathrm{dt}}\right)_{3,4}$; and
4. Small spacing, $s_{1,4}$, and large relative speed, $\left(\frac{d s}{d t}\right)_{3,4}$.

Let the associated responses of the following vehicle be $a_{1}, a_{2}, a_{3}$, and $a_{4}$ to correspond with these points. If deceleration responses are considered, then the following relations between the responses can be obtained:
where $\mathrm{a}_{2} \rightarrow 0$.
By using the steady-state flow assumption, $\mathbf{s}_{2,3}$ and $\mathbf{s}_{1,4}$ can be applied to the freeflow and congested-flow regimes respectively.

Note that the only relation in equation 11 that is not trivial is between $a_{1}$ and $a_{3}$. Although the driver is able to detect (ds/dt) $3_{3,4}$ in large spacing, $s_{2,3}$, he still has time to either switch lanes or remove his foot from the accelerator. On the other hand, in small spacing, $s_{1,4}$, it is not likely that he will have these two choices, and, therefore, he will decelerate with higher magnitude.

In addition, the upper part of Figure 1 includes a chart of the proposed sensitivity function (equation 3) or of (s) versus the weighting factor A, where, for the $\alpha$ computation, $\mathrm{u}_{\mathrm{f}}$ and $\mathrm{k}_{\mathrm{g}}$ were taken as $60 \mathrm{mph}(95 \mathrm{~km} / \mathrm{h}$ ) and 220 vpm ( 138 vehicles $/ \mathrm{km}$ ) respectively.

By applying the results in equation 11 to the car-following equation, the relations between the sensitivity values corresponding to the four situations are similar to equation 11, i.e.,

$$
[\alpha f(s)]_{4}>[\alpha f(s)]_{1}>[\alpha f(s)]_{3}>[\alpha f(s)]_{2}
$$

where $[\alpha \mathrm{f}(\mathrm{s})]_{2} \rightarrow 0$.
The two sensitivity values corresponding to the congested-flow regime (points 1, 4) will yield values of $A$ in such a way that $A_{4}>A_{1}$; that is, because $a_{4}>a_{1}$, by applying the steady-state speed-concentration relationship in equations 7 and 8 and by using the mathematical properties indicated in equation 10, we see that as A increases the speed decreases (for the same value of concentration) and, obviously, that the speed is proportional to the response magnitude.

Figure 1 shows that the relation $A_{4}>A_{1}$ can be obtained only for $\mathrm{A}>1,0$, since one is concerned with the congested-flow regime where it can be assumed that $k / k_{j}>0.5$. For example, following the curve for $\mathrm{k} / \mathrm{k}_{\mathrm{d}}=0.6$ shows that the two requirements that $[\alpha f(\mathrm{~s})]_{4}>[\alpha f(\mathrm{~s})]_{1}$ and $\mathrm{A}_{4}>\mathrm{A}_{1}$ (for any two states on the curve) are fulfilled only for $\mathrm{A}>$ 1.0. Similarly, Figure 1 shows that $[\alpha f(s)]_{2} \rightarrow 0$ only for values of A less than 1.0 but that there are no limitations on the $A$ values for $[\alpha f(s)]_{3}$. Hence, A values for the freeflow regime should be $[0<\mathrm{A}<1.0 \cap 0<\mathrm{A}<\infty$ ], i.e., for $0<\mathrm{A}<1.0$. The validity of these results will be shown with real-world data.

In the lower part of Figure 1, the sensitivity component multiplied by spacing can be compared with the constant $\alpha_{0}$. Herman and Potts (4) used this constant in their reciprocal car-following model. Their results show that $\alpha_{0}$ ranges approximately from 18 to 30 in the Lincoln, Holland, and Queens Midtown Tunnels and on the General Motors test track. Thus for the congested-flow regime ( $k / k_{1}>0.5$ ), the A values are then approximately greater than 4.0 . That is, following the curve for $\mathrm{k} / \mathrm{k}_{\mathrm{g}}=0.6$ shows that $18 \leq \alpha f(s) \times s \leq 30$ can be obtained only for $A>4$, approximately. If one changes $u_{p}$ and $k_{j}$ for the $\alpha$ computation, the 4.0 value will be varied but will always be greater than 1.0 , another confirmation that, for the congested-flow regime, the A values should be greater than 1.0. Another result from the General Motors test track is that $\alpha_{0}=82.6$; this run involved high speed and violent maneuvering. Figure 1 shows that the 82.6 value results in $\mathrm{A}<1.0$ and $\mathrm{k} / \mathrm{k}_{\mathrm{j}}>0.75$. Therefore, for $\mathrm{A}<1.0$, high speed can be as sociated with free-flow regime, and for $k / k_{\mathrm{j}}>0.75$ violent maneuvering can be associated with small spacing at high speed.

Finally, both graphs in Figure 1 show greater fluctuation of the sonsitivity component when $0<A<1.0$ than when $A>1.0$. Inconsistency is greater in the free-flow regime than in the congested-flow regime; therefore, again, $0<\mathrm{A}<1.0$ best represents the free-flow regime, and $A>1.0$ best represents the congested-flow regime.

The reasonable range for the weighting factor is $0<A<e^{2}$ as was shown in equation 6. If we assume that a better fit to the congested-flow regime is obtained at A values
greater than $\mathrm{e}^{2}$, this range can be increased within a reasonable limit. If we assume that in very high concentration conditions the spacing is, for example, less than $40 \mathrm{ft}(12 \mathrm{~m})$ and the speed is less than $10 \mathrm{mph}(16 \mathrm{~km} / \mathrm{h})$, it is likely that drivers in a bumper-to-bumper situation will behave differently from those in congested traffic that does not stop. Therefore, from a very high concentration value $\mathrm{k}^{\prime}$, up to $\mathrm{k}_{\mathrm{y}}$, one may not consider the constraint of equation 5, i.e., for $k^{\prime} \leq k \leq k_{9}$. Thus, the range for A that is based on the derivation used in obtaining the previous range given in equation 6 can be increased as follows:

$$
\begin{equation*}
0<A \leq e^{2 k_{j} / k^{\prime}} \tag{12}
\end{equation*}
$$

and the determination of $\mathrm{k}^{\prime}$ from equation 12 , at the boundary condition, is

$$
\begin{equation*}
\mathrm{k}^{\prime}=\frac{2 \mathrm{k}_{\mathrm{J}}}{\ln \mathrm{~A}} \tag{13}
\end{equation*}
$$

where $k_{j}$ and A are associated with the congested-flow model.
In addition to the above analysis of deceleration situations, acceleration responses should be considered.

When the lead vehicle accelerates, the spacing increases and the response (if any) of the following vehicle should be positive with an increase in the acceleration magnitude. However, this interpretation results in a progressive ratio between the spacing and the acceleration response, which means that those situations could occur for $\mathrm{A}>$ $\exp \left(2 k_{\mathrm{s}} / \mathrm{k}^{\prime}\right)$ and for $\mathrm{A}=\exp \left(2 \mathrm{k}_{\mathrm{j}} / \mathrm{k}^{\prime}\right)$ where $\mathrm{k}>\mathrm{k}^{\prime}$ in a congested-flow regime. Therefore, when the above ranges of $A$ are adopted, the car-following rule cannot be applied in acceleration situations, particularly in tie free-flow regime.

The above analysis examines the car-following problem from the standpoint of driver performance when a steady-state stream of vehicles is assumed. However, more accurate consideration can be made for the car-following model in equation 4. It is possible to evaluate the stability of the nonlinear model in equation 4. Furthermore, the dynamic responses of the suggested nonlinear system in equation 4 result in reasonable values for spacing and relative speed, particularly for the two-regime traffic behavior (9).

## ANALYTICAL EVALUATION OF MACROSCOPIC DATA

The two-regime traffic flow models based on equation 4 and corresponding macroscopic equations 7 and 8 were calculated by using the 45 speed-concentration data sets. In addition, the two flow regimes were incorporated by means of concentration breakpoint procedure.

Based on the availability of a digital computer an optimization program was used. This program relies heavily on mean deviation, which has proved to be a good indicator of the appropriateness of the data. This mean deviation is definitely preferred to correlation coefficients, particularly in the nonlinear case. The statistical procedure is based on the linearization of the input data (speed and concentration); consequently, a linear regression model of the form $y=a x$ can be applied to determine the free-flow speed, $u_{f}$, in the equation

$$
\begin{equation*}
\mathrm{u}=\mathrm{u}_{\mathrm{t}} \mathrm{X} \tag{14}
\end{equation*}
$$

Table 1. Two-regime traffic-flow models.

| Location | Free-Flow Regime Model |  |  |  | Congested-Flow Regime Model |  |  |  | Two-Regime Model Characteristics |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  |
|  | A | $\mathrm{u}^{p}$ | $\mathrm{k}_{\mathrm{J}}$ | MD |  |  |  |  | A | $u_{r}$ | $k_{3}$ | MD | Qux | $\mathrm{u}_{0}$ | k 。 | point | $[k 1, k 2]$ |
| 1. Eisenhower at Harlem | 0.009 | 49.9 | 70 | 3.69 | 10.00 | 58.1 | 195 | 3.24 | 1,868 | 38.6 | 48.4 | 54 |  |
| 2. Holland Tunnel | 1.100 | 49.8 | 120 | 2.84 | 5.50 | 34.8 | 215 | 1.44 | 1,305 | 21.8 | 59.8 | $80^{\text {a }}$ | $k \rightarrow \mathrm{k}_{\mathrm{j}}$ |
| 3. Hollywood at Sunset | 1.400 | 50.6 | 225 | 5.71 | 12.00 | 58.2 | 205 | 5.88 | 1,966 | 37.6 | 52.0 | 52 |  |
| 4. Hollywood at Sunset | 0.090 | 50.5 | 140 | 3.12 | 12.00 | 58.3 | 205 | 2.13 | 2,116 | 44.0 | 48.0 |  | 48, 60 |
| 5. Hollywood at Hollywood | 0.090 | 46.4 | 190 | 4.16 | 12.00 | 61.0 | 230 | 3.47 | 2,117 | 42.3 | 50.0 |  | 50, 70 |
| 6. Hollywood at Hollywood | 0.001 | 45.6 | 75 | 3.30 | 10.00 | 68.8 | 200 | 2,30 | 2,102 | $3 \mathrm{B}$. | 55.2 | 66 |  |
| 7. Hollywood at Bronson | 0.001 | 43.8 | 55 | 2.64 | 11.00 | 56.1 | 230 | 3.63 | 1,555 | 37.5 | 41.4 | $50^{\text {a }}$ | k -0 |
| 8. Hollywood at Bronson | 0.090 | 54.1 | 95 | 3.22 | 19.00 | 107.1 | 175 | 5,62 | 2,025 | 40.5 | 50.0 | $50^{6}$ | $\mathbf{k} \rightarrow 0$ |
| 9. Hollywood at Fifield | 1.100 | 53.9 | 100 | 3.16 | 5.00 | 43.3 | 300 | 3.39 | 1,448 | 26.6 | 54.3 | 56 |  |
| 10. Hollywood at Fitield | 0.090 | 43.0 | 110 | 4.64 | 7.00 | 37.9 | 300 | 2,75 | 1,932 | 28.3 | 68.2 | $80^{2}$ | $\mathbf{k} \rightarrow \mathbf{k}_{\mathbf{i}}$ |
| 11. Hollywood at Franklin | 0.500 | 48.6 | 145 | 3.82 | 7.50 | 71.2 | 190 | 3.23 | 2,079 | 26.5 | 78.4 | $80^{*}$ | $\mathbf{k} \rightarrow \mathbf{k}_{\mathbf{q}}$ |
| 12. Hollywood at Franklin | 0.900 | 48.3 | 300 | 4.46 | 9.00 | 70.6 | 190 | 4.61 | 1,965 | 40.9 | 48.0 | 48 |  |
| 13. Pasadena at College East | 0.018 | 48.3 | 85 | 1.41 | 0.55 | 29.7 | 300 | 4,09 | 2,057 | 35.9 | 57.3 |  | 58, 62 |
| 14. Pasadena at College East | 0.110 | 53.4 | 105 | 1.60 | 8.00 | 48.1 | 260 | 2.65 | 2,217 | 34.5 | 64.3 |  | 68, 72 |
| 15. Pasadena at Castelar West | 0.110 | 49.8 | 110 | 2.76 | 7.00 | 40.3 | 300 | 1.90 | 1,860 | 28.5 | 67.0 |  | 68, 76 |
| 16. Pasadena at Castelar West | 1.700 | 49.8 | 220 | 1.44 | 7.00 | 48.7 | 260 | 1.12 | 2,285 | 28.6 | 80.0 |  | 68, 80 |
| 17. Fasadena at Castelar East | 0.130 | 12.9 | 85 | 2.21 | 9.00 | 59.3 | 220 | 1,22 | 1,650 | 32.1 | 51.1 |  | 40, 80 |
| 18. Pasadena at Castelar East | 2.000 | 49.3 | 215 | 3.00 | 7.00 | 46.7 | 300 | 2.84 | 2,009 | 30.4 | 66.0 | 66 |  |
| 19. Pasadena at Bishop West | 0.400 | 39.4 | 300 | 1.47 | 8.50 | 45.5 | 300 | 1,59 | 1,816 | 34.9 | 57.0 | 52 |  |
| 20. Pasadena at Bishop West | 0.800 | 55.8 | 185 | 1.59 | 8.00 | 55.9 | 235 | 2.58 | 1,912 | 30.8 | 62.0 | 42 |  |
| 21. Penn-Lincoln at Laurel | 0.380 | 44.1 | 300 | 4.92 | 9.00 | 62.5 | 205 | 4,47 | 2,487 | 37.7 | 66.0 | 66 |  |
| 22. Pern-Lincoln at Laurel | 0.080 | 48.3 | 105 | 6.07 | 7.50 | 55.3 | 225 | 2.61 | 2,101 | 32,1 | 65.5 |  | 62, 70 |
| 23. Pern-Lincoln at Braddock | 0.120 | 52.5 | 90 | 5.66 | 5.00 | 31.8 | 300 | 4,71 | 1,640 | 33.7 | 48.6 |  | 66, 74 |
| 24. Pern-Lincoln at Braddock | 0.080 | 54.8 | 80 | 4.10 | 8.00 | 54.0 | 245 | 3,50 | 1,822 | 36.4 | 49.9 |  | 58, 66 |
| 25. Penn-Lincoln at tunnel | 0.070 | 39.0 | 105 | 2.69 | 15.00 | 46.4 | 300 | 3.89 | 1,540 | 32.1 | 48.0 | 48 |  |
| 26. Penn-Lincoln at tunnel | 0.400 | 39.6 | 300 | 1.75 | 7.50 | 51.4 | 220 | 2.59 | 1,740 | 32.2 | 54.0 | 38 |  |
| 27. Virginia in lane 1 | 1.500 | 62.6 | 170 | 2.84 | 15.50 | 64.9 | 205 | 2.98 | 1,798 | 46.1 | 39.0 | 39 |  |
| 28. Virginia in lane 2 | 0.800 | 72.5 | 175 | 3.01 | 19.50 | 98.1 | 185 | 4.88 | 2,416 | 56.1 | 43.0 | 43 |  |
| 29. Munich-Salzburg in lane $1^{\text {b }}$ | 0.090 | 61.5 | 55 | 5.41 | 10.00 | 85.1 | 130 | 2.76 | 1,383 | 40.5 | 34.1 | 46 |  |
| 30. Munich-Salzibury in lane $2^{\text {b }}$ | 0.110 | 70.6 | 55 | 0.92 | 5.50 | 74.7 | 115 | 2,81 | 1,660 | 49.6 | 33.6 | 52 |  |
| 31. Munich-Salzburg in lane $1^{\text {c }}$ | 1.100 | 64.0 | 85 | 4.95 | 9.50 | 70.6 | 160 | 3.43 | 1,396 | 29.9 | 46.5 | 38 |  |
| 32. Munich-Salzburg in lane $2^{\circ}$ | 0.800 | 71.7 | 80 | 5.22 | 5.50 | 71.7 | 155 | 7.02 | 1,680 | 48.1 | 34.9 | 50 |  |

${ }^{\text {a }}$ Concentration breakpoint tends to approach either 0 or $k_{1}$ values as specified by the overlap interval,
${ }^{\text {b }}$ Northbound.
${ }^{\text {c }}$ Southbound

Figure 1. Sensitivity of the weighting factor A .


Figure 2. Typical minimization of the mean deviation in two regimes.

where $x=[1 /(A-1)]\left(A^{1-x / k J}-1\right)$ for $A \neq 1.0$ and $1-\left(k / k_{f}\right)$ for $A=1.0$.
The mean deviation is then determined from the sum of squares of the deviations of the data points from the considered model. Minimization of the mean deviation within certain concentration intervals is satisfactory, and no other criteria, such as those used in the single- and two-regime models based on the generalized car-following model (equation 1), are necessary. However, an upper limit is imposed on $\mathrm{k}_{\mathrm{y}}$ such that $k_{j} \leq 300 \mathrm{vpm}$ ( 188 vehicles $/ \mathrm{km}$ ) in order to restrict the time required for running the program.

From inspection of the plot of the data sets, an interval along the concentration axis is determined where it is likely that discontinuity or unstable flow will result. By using a concentration-incremented technique (i.e., within the concentration interval, the concentration range is increased by increments), the program covers this concentration interval separately for each regime. The search for the free-flow regime model considers data points from $\mathrm{k}=0$ to $\mathrm{k}=\mathrm{k} 1$ (lower bound) to $\mathrm{k}=\mathrm{k} 2$ (upper bound) by means of the concentration-incremented technique.

For the congested-flow regime, a similar search is performed by going backward from $\mathrm{k}=\mathrm{k}_{\mathrm{f}}$ to $\mathrm{k}=\mathrm{k} 2$ (in the first step) down to $\mathrm{k}=\mathrm{k} 1$. The optimization policy for finding a breakpoint within the concentration interval is

$$
\begin{equation*}
\min _{i}\left[\left(\min M D_{1}\right)_{F F R}+\left(\min M D_{1}\right)_{C F R}\right] \tag{15}
\end{equation*}
$$

for $\mathrm{i}=1,2, \ldots,(\mathrm{k} 2-\mathrm{k} 1) / \Delta \mathrm{k}$ where

```
FFR, CFR = free-flow regime and the congested-flow regime respectively, MD = mean deviation, i = each upper or lower bound, and \(\Delta \mathrm{k}=\) concentration-increment within the concentration interval (k1, k2).
```

After starting with initial values of $A$ and $k_{d}$, in each step the program holds A constant until the $k_{j}$ associated with minimum MD is found. Then $A$ is changed, by agiven increment, until the overall minimum MD is found for a particular $A, k_{j}$ (after several iterations).

Figure 2 shows part of the minimization procedure, for the two regimes, by using the Eisenhower Expressway data set. In that case, $\mathrm{k} 1=45 \mathrm{vpm}(28$ vehicles $/ \mathrm{km}), \mathrm{k} 2=$ $60 \mathrm{vpm}\left(25\right.$ vehicles $/ \mathrm{km}$ ), $\Delta \mathrm{k}=3 \mathrm{vpm}$ ( 1 vehicle $/ \mathrm{km}$ ), $\mathrm{i}_{\mathrm{ffR}}=1$ (data points considered up to $\mathrm{k}=45 \mathrm{vpm}$ ), and $\mathrm{i}_{\mathrm{CR}}=5$ (data points considered from $\mathrm{k}=60 \mathrm{vpm}$ to $\mathrm{k}_{\mathrm{f}}$ ). It can be seen that the minimum mean deviation is obtained in fewer steps and more smoothly in the congested-flow regime than in the free-flow regime. This figure illustrates a higher consistency in the congested-flow regime than in the free-flow regime.

## Results

As mentioned earlier, two groups of data sets were considered. By using a procedure similar to the one used previously (2), the results of the first group of data sets form a basis to estimate the results for the second group. Of the first group of data sets, 28 of 32 are from the center or left freeway lanes. Therefore, the conclusions drawn herein are particularly applicable to freeway facilities.

By running the data sets through the optimization procedure, three cases were identified:

1. Where an absolute breakpoint exists;
2. Where overlap exists, i.e., an unstable zone is created; and
3. Where the data points tend toward single-regime rather than two-regime phenomenon.

The criteria for determining each of the possible cases are as follows.

1. From inspection, a concentration interval is determined. If there is either a breakpoint or an overlap interval, it results within this concentration interval.
2. If an absolute minimum $\Sigma \mathrm{MD}$, based on equation 15 , within the considered interval is found, the program assumes that a breakpoint exists.
3. If either one of the neighboring concentration values to a breakpoint does not have the second minimum $\Sigma M D$, an overlap is determined between those concentration values associated with the first and second minimum $\Sigma$ MD.
4. If the absolute minimum $\Sigma$ MD belongs to either one of the concentration interval boundaries, then a trend toward a single regime exists.

The results of this investigation of two-regime models using the first 32 data sets are given in Table 1. It should be noted that $k_{d}$ of the free-flow model and $u_{p}$ of the congested-flow model are only parameters, but $u_{s}$ of FFR and $k_{d}$ of CFR are important traffic characteristics of the two-regime model.

Figure 3 shows different speed-concentration curves. These curves illustrate the above three cases. Also, on the upper right side of each curve the fluctuations of the mean deviations for the two regimes and for both regimes within the considered concentration interval (k1, k2) are shown.

In Figure 3 there are eight numbered curves. To associate each curve with the freeway location, the numbers on the curves are those given in Table 1. In curves 1, 27, and 28 there is a breakpoint, case 1 ; in curves 4, 24, and 17 there is an overlap that can be interpreted as an unstable zone, case 2; and in curves 8 and 11 a trend toward single regime exists, case 3. Among the eight curves in Figure 3, curve 8 shows a trend toward a single regime, with the use of only the congested-flow model, whereas curve 11 shows the same trend, but with the use of only the free-flow model. The most typical results among the 32 data sets are like curves 1 or 28 (breakpoint exists) and curve 4 (unstable zone exists). In addition, it is interesting to note that curves 27 and 28 are concerned with lane 1 and lane 2 of the same road facility.

Figure 4 shows the 32 data sets in an $A, k_{j}$ matrix format. These results confirm the following hypotheses:

1. As 25 out of 32 data sets suggest, the free-flow regime models had $A$ values of less than 1.0 ; and
2. As 31 out of 32 data sets suggest, in the congested-flow regime models the $A$ values were greater than 1.0.

However, it appears that A may have values greater than 1.0 in the free-flow regime when road facilities other than nonshoulder freeway lanes are considered, i.e., tunnel lanes and shoulder lanes. The results for such lanes are indicated in the A, $\mathrm{k}_{\mathrm{j}}$ matrix in Figure 4.

Additional 13 Data Sets
The results of the first group of 32 data sets formed the basis to estimate the parameter values for the second group of 13 data sets. That is, in the A, $\mathrm{k}_{\mathrm{y}}$ matrix the estimations were that the free-flow regime models associated with freeway lanes will be centered within the region of $0<\mathrm{A} \leq 0.2,50 \leq \mathrm{k}_{\mathrm{j}} \leq 120$, where the on-ramp and CDroad will be scattered above the line where $A=1.0$. In the congested-flow regime models the region of $5 \leq \mathrm{A} \leq 10,150<\mathrm{k}_{1}<300$ was estimated for all road facilities. The results of the Santa Monica data sets are given in Table 2 and shown in Figure 5. Generally, these results follow the estimations. However, the congested-flow regime models have somewhat lower A values for the second group of data sets (Figure 5).

From the overall 45 congested-flow regime models, the range of $\mathrm{k}^{\prime}$ that belongs to the constraint in equation 12 was determined. This range was 160 to 250 vpm ( 100 to 156 vehicles $/ \mathrm{km}$ ), which agrees with the analys is of driver performance mentioned above.

Figure 3. Typical and extreme speed-concentration output curves.


Table 2. Two-regime traffic-flow models ( 13 data sets).

| Station Number | Free-Flow Regime Model |  |  |  | Congested-Flow Regime Model |  |  |  | Two-Regime Model Characteristics |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  | k Break- | Overlap |
|  | A | $u_{r}$ | $\mathrm{k}_{\mathrm{J}}$ | MD |  |  |  |  | A | $\mathrm{He}_{\mathrm{P}}$ | $\mathrm{k}_{\mathrm{J}}$ | MD |  | $\mathrm{u}_{0}$ | $\mathrm{k}_{0}$ | point | [k1, k2] |
| SM-12 | 0.060 | 58.2 | 75 | 3.93 | 5.50 | 52.0 | 215 | 0.62 | 1,892 | 39.7 | 47.7 | 50 |  |
| SM-13 | 0.050 | 60.3 | 80 | 3.70 | 4.00 | 62.6 | 175 | 1.09 | 2,150 | 41.8 | 51.4 | 55 |  |
| SM-14 | 0.009 | 54.6 | 70 | 3.32 | 18,00 | 76.9 | 215 | 2.25 | 2,042 | 42.2 | 48.4 |  | 50, 60 |
| SM-15 | 0.060 | 58.4 | 80 | 2,61 | 5,00 | 64,0 | 175 | 0.60 | 2,027 | 39.9 | 50.8 | 60 |  |
| SM-16 | 0.030 | 60.4 | 65 | 3.30 | 21.00 | 77.4 | 200 | 0.99 | 1,861 | 43.6 | 42.7 | 50 |  |
| SM-17 | 0.090 | 62.1 | 85 | 2.65 | 22.50 | 69.4 | 230 | 1.00 | 2,155 | 40.9 | 52.7 |  | 55, 65 |
| SM-18 | 0.090 | 57.4 | 75 | 2.77 | 6,00 | 64.0 | 175 | 0.88 | 1,758 | 37.8 | 46.5 |  | 40, 60 |
| SM-19 | 0.080 | 61.7 | 80 | 2,90 | 5.00 | 59.8 | 185 | 1.35 | 2,052 | 41.0 | 50.0 | 60 |  |
| SM-20 | 0.009 | 52.9 | 70 | 3.33 | 7.00 | 74.9 | 170 | 1.80 | 1,978 | 40.9 | 48.4 |  | 40, 55 |
| SM-21 | 0.200 | 61.2 | 90 | 2.57 | 4.50 | 55,1 | 195 | 0.73 | 1,963 | 37.2 | 52.8 |  | 40, 60 |
| SM-22 | 0.400 | 62.1 | 90 | 3.38 | 4,00 | 72.5 | 150 | 1.06 | 1,892 | 34.8 | 49.7 |  | 45, 60 |
| LaBrea (on-ramp) | 4.200 | 47.4 | 250 | 2.33 | 0.50 | 34.1 | 200 | 1.06 | 2,023 | 20.1 | 100.4 | $80^{2}$ | $\mathrm{k} \rightarrow \mathrm{k}_{1}$ |
| Venice (CD-on) | 7.000 | 49.3 | 165 | 4.40 | 9.50 | 33.5 | 290 | 0.88 | 1,208 | 20.1 | 60.2 | $80^{3}$ | $\mathrm{k}-\mathrm{k}_{1}$ |

[^1]Figure 4. The two-regime models in $\mathrm{A}, \mathrm{k}_{\mathrm{j}}$ matrix ( 32 data sets).


Figure 5. The two-regime models in $\mathrm{A}, \mathrm{k}_{\mathrm{j}}$ matrix ( 13 data sets).


As mentioned earlier, the two flow regimes were incorporated by means of breakpoint evaluation procedures. The results of either the overlap intervals or breakpoints within a specified concentration range are given in Tables 1 and 2. Of the 45 data sets, 22 have a breakpoint and 16 have an overlap interval. The remaining seven data sets have a tendency toward single regime.

## Comparison With the Generalized Car-Following Model

The generalized car-following model (equation 1) and the corresponding macroscopic equations were evaluated earlier (2) by using the same 45 data sets analyzed here. However, instead of a breakpoint procedure, in the free-flow regime and in the congested-flow regime only data points with concentration values less than 60 vpm ( 38 vehicles $/ \mathrm{km}$ ) and more than $50 \mathrm{vpm}(31$ vehicles $/ \mathrm{km}$ ) respectively were included in analysis of equation 1. Therefore, to form a comparison basis, the proposed model was run along the same concentration ranges. In addition, because no criteria were imposed on the proposed model (excluding the upper limit on $\mathrm{k}_{\mathrm{j}}$ ), the comparison between the two approaches is based on consideration of only the minimum mean deviation models (2).

A comprehensive comparison of the proposed model and equation 1 is reported elsewhere (9). A summary of this comparison for the free-flow regime models follows:

1. Of the 45 data sets, 36 have lower MD when the proposed model is used than when equation 1 is used, and five data sets have the same MD; and
2. With regard to the preselected criteria (2), 43 and 45 of the data sets meet the $q_{m}$ and $u_{f}$ criteria when the proposed model is used, but only 25 and 39 of the data sets meet the $q_{w}$ and $u_{f}$ criteria when equation 1 is used.

The summary of the comparison for the congested-flow regime models is as follows:

1. Twenty-five data sets have lower MD when the proposed model is used than when equation 1 is used, and five data sets have the same MD; and
2. Twenty-six of the data sets meet the preselected $\mathrm{k}_{\mathrm{j}}$ criterion (2) when the proposed model is used, but only nine of the data sets meet the $\mathrm{k}_{\mathrm{j}}$ criterion when equation 1 is used.

## CONCLUDING REMARKS

This paper has developed a car-following model based on an analysis of driver performance from which relatively simplified macroscopic models can be derived. The comparison made in this paper is between the suggested model (equation 4) and the generalized car-following model (equation 1), which is found to be representative of previous microscopic and macroscopic theories.

Based on a criterion from previous approaches, a new sensitivity component of the stimulus-response relationship (equation 2) was suggested. Through analysis of driver performance, it becomes apparent that the suggested model is capable of describing both microscopic and macroscopic traffic behavior. The model is particularly useful when the two-regime phenomenon is considered for a single-lane traffic stream in a multilane environment. By using the 45 data sets, a comparison between the new and the generalized car-following models was performed, based on results obtained in the previous work (2). With respect to the two-regime phenomenon one can conclude that the proposed model is superior to the generalized car-following model, particularly in simplicity and clarity. The advantages of the proposed model can be summarized as follows:

1. Better actual data fit;
2. No need for criteria for $\mathrm{u}_{\mathrm{p}}, \mathrm{k}_{\mathrm{g}}$, and $\mathrm{q}_{\mathrm{q}}$ (excluding an upper limit on $\mathrm{k}_{\mathrm{j}}$ );
3. $\mathrm{u}_{\mathrm{p}}$ and $\mathrm{k}_{\mathrm{j}}$ (boundary characteristics) always defined;
4. Fewer arbitrary parameters in the car-following equation; and
5. Fewer basic macroscopic forms.

Besides the advantage of having a simpler nonlinear model, a stability analysis can be performed on the microscopic proposed model (9). Because drivers do not completely follow any deterministic behavior, the results of the stability analysis are valid at best only in some average sense. However, this analysis provides a method for understanding traffic behavior and the potential for modifying such behavior.

This paper has examined discontinuity, one of the most important characteristics of traffic behavior, under maximum flow conditions. Observations of the data sets suggest that discontinuity can result in an unstable traffic flow zone. It is realized, however, that the considered concentration breakpoint procedure is only one method for evaluating two-regime models. With regard to two-regime traffic behavior, perhaps a more reliable method is to consider the data points with respect to time (9). Such a method sheds light on traffic behavior, particularly under peak conditions. Because additional quantitative information about speed-flow-concentration relationships is required, particularly near the capacity level, future research should examine the traffic-flow behavior under these flow conditions. It is felt, however, that for the interpretation of the discontinuity phenomenon human factors as well as traffic-flow characteristics should be considered (10).

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[^0]:    *When this paper was written, Mr. Ceder was on leave from the Road Safety Center, Technion, Haifa, Israel. Publication of this paper sponsored by Committee on Traffic Flow Theory and Characteristics.

[^1]:    ${ }^{\text {a }}$ Concentration breakpoint tends to approach either 0 or $\mathrm{k}_{1}$ values as specified by the overlap interval.

