A STUDY OF TRAFFIC PERFORMANCE MODELS UNDER AN INCIDENT CONDITION

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When an incident occurs on a roadway, traffic performance is usually evaluated by means of either the shock wave analysis or the queuing analysis. In this paper, a comparison of these two approaches is given under the assumption that there exists a unique flow-density relationship. It is shown that the two methods of evaluating the performance give the same result if the traffic density is not time dependent.

•TRAFFIC PERFORMANCE under various conditions has been studied for many years. This is an important area not only because of theoretical interests, but also for a better understanding of traffic behavior whereby a good system design and control strategies can be achieved.

In the case of a roadway incident (or accident), two well-known approaches are available to evaluate traffic performance, namely, the shock wave analysis (1, 4) and the queuing analysis. Then a question may arise: How would the results derived from one method be different from those derived from the other? The purpose of this paper is to answer this question by comparing the two approaches from two aspects:

1. Discharge time—the time required to discharge the stored vehicles after the incident is removed, and
2. Total delay—the increment of the total travel time (TTT) due to the incident, i.e., the difference between TTT under the incident case and TTT under the normal case (no incident).

In this paper, it is assumed that a unique flow-density relationship (a q-k diagram) exists. For a given blockage time (the duration that the traffic is blocked by the incident), the discharge time and the total delay will be estimated.

RELATIONSHIP BETWEEN BLOCKAGE TIME AND DISCHARGE TIME

Whenever a freeway incident occurs, traffic conditions are changed because of a reduction in capacity. If the reduced capacity is less than the flow, then the region upstream of the incident location becomes congested, while the traffic situation downstream is improved because of the lower traffic flow. The traffic situations at different stages are given in Table 1. [Note that the normal flow $q(t)$ may be a function of time t. Therefore, the traffic flows before and after an incident may be different.] Figure 1a shows a q-k diagram, and Figure 1b shows a shock wave diagram, which is valid only for constant flow condition. A more realistic case can be shown by letting the flow be a function of time t. Therefore, a generalized shock wave diagram is shown in Figure 1c, where

$L(t)$ = front edge of the shock wave at time t,
$s_1(t)$ = shock wave speed at time t,

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Figure 1. Traffic conditions under incident case: (a) q-k, (b) shock wave with incident occurring at 1 = 0 and constant flow, and (c) shock wave with incident occurring at t = 0 and time-dependent flow.

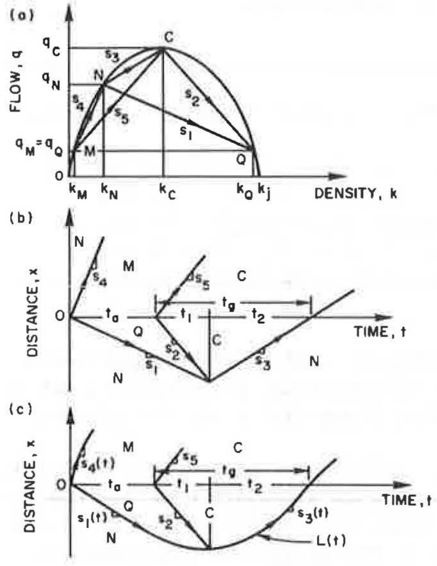

Figure 2. (a) Queuing diagram and (b) shock wave diagram with an incident occurring at t = 0.

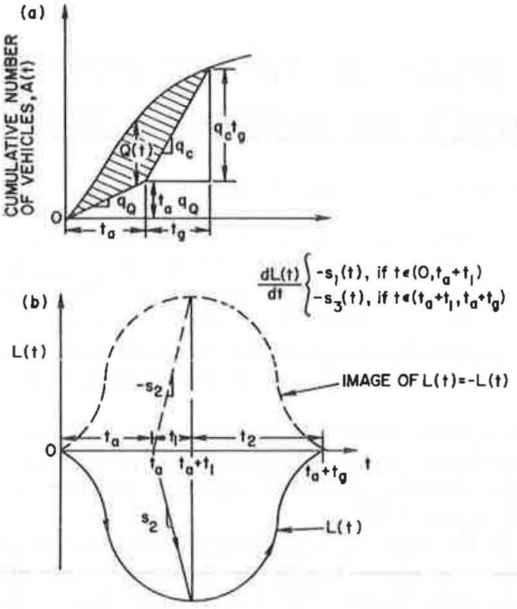

Table 1. Traffic conditions when incident occurs (3).

Events	Upstream	Downstream
Before incident, N	Normal flow, $q(t)$	Normal flow, $q(t)$
Incident occurs, Q	Queueing flow, q_Q	Metered flow, q_M
Incident removed, C	Capacity flow, q_c	Capacity flow, q_c
Traffic recovered, N	Normal flow, $q(t)$	Normal flow, $q(t)$

q_c = capacity flow,
k_c = capacity density,
q_Q = queuing flow,
k_Q = queuing density,
$q(t)$ = normal flow at time t, and
$k(t)$ = normal density at time t.

Clearly, according to the above definitions, for $L(t) < 0$,

$$\frac{dL(t)}{dt} = \begin{cases} s_1(t) = [q(t) - q_Q]/[k(t) - k_Q], & \text{if } t \in [0, t_a + t_1] \\ s_3(t) = [q_c - q(t)]/[k_c - k(t)], & \text{if } t \in [t_a + t_1,\ t_a + t_g] \end{cases}$$

where

t_a = blockage time and
t_g = discharge time = $t_1 + t_2$.

Note that the shock wave speeds $s_1(t)$ and $s_3(t)$ are time dependent if the traffic flow $q(t)$ is; s_2 is independent of time if normal capacity and reduced capacity are constant.

This statement is certainly true if the unique relationship between flow q and density k is assumed.

Now, according to Figure 2b, and based on the image of the shock wave diagram with $-L(t) > 0$ (represented by the dashed line), it follows that

$$-L(t_a + t_1) = -\int_0^{t_a + t_1} s_1(t)\, dt \tag{1}$$

$$= -s_2\, t_1 \tag{2}$$

$$= \int_{t_a + t_1}^{t_a + t_g} s_3(t)\, dt \tag{3}$$

By using equations 1 and 2,

$$\int_0^{t_a + t_1} \frac{q_Q - q(t)}{k_Q - k(t)}\, dt = \frac{q_c\, t_1}{k_c - k_Q} - \frac{q_Q\, t_1}{k_c - k_Q} \tag{4}$$

And equations 1 and 3 imply that

$$\int_0^{t_a + t_g} -s_1(t)\, dt = \int_{t_a + t_1}^{t_a + t_g} [s_3(t) - s_1(t)]\, dt \tag{5}$$

Equations 4 and 5 will give the solution for t_g, if t_a is known. Now, if there is little change in k(t) with respect to time, i.e., $k(t) \approx k$ (some constant), then equation 4 implies

$$\frac{q_Q \times (t_a + t_1)}{k_Q - k} - \frac{A(t_a + t_1)}{k_Q - k} \approx \frac{q_c \times t_1}{k_c - k_Q} - \frac{q_Q\, t_1}{k_c - k_Q}$$

and equation 5 implies

$$A(t_a + t_g) \approx \frac{q_Q(t_a + t_1)\,(k_c - k)}{k_Q - k} - \frac{A(t_a + t_1)\,(k_c - k_Q)}{k_Q - k} + q_c t_2$$

where

$$A(t) = \int_0^t q(r)\, dr \text{ and}$$

$$A(0) = 0.$$

By using these two equations and eliminating the term $A(t_a + t_1)$, it follows that

$$A(t_a + t_g) \approx q_c t_g + q_Q t_a$$

or

$$\int_0^{t_a} [q(t) - q_Q] \, dt \approx \int_{t_a}^{t_a + t_g} [q_c - q(t)] \, dt \tag{6}$$

Figure 2a shows that the queue length at time t is

$$Q(t) = \begin{cases} \int_0^t [q(r) - q_Q] \, dr, & t \in [0, t_a] \\ Q(t_a) - \int_{t_a}^t [q_c - q(r)] \, dr, & t \in [t_a, t_a + t_g] \end{cases} \tag{7}$$

Because $Q(t_a + t_g) = 0$, equation 7 implies that

$$\int_0^{t_a} [q(r) - q_Q] \, dr = \int_{t_a}^{t_a + t_g} [q_c - q(r)] \, dr$$

The same relationship between t_a and t_g is obtained as in equation 6.

RELATIONSHIP BETWEEN BLOCKAGE TIME AND DELAY

The formula used to calculate the total travel time is

$$\text{TTT} = \text{density} \times \text{length} \times \text{time}$$

Therefore, when the shock wave diagram is drawn in a time-distance space (Figure 2b), the formula can be rewritten as

$$\text{TTT} = \text{density} \times \text{area under the image of } L(t)$$

[The image of $L(t)$ is shown by a dashed line in Figure 2b, which is equal to $-L(t)$.]

Because the delay is the difference between TTT under the incident case and TTT under the normal case, it is legitimate to write the delay in the form

$$D = \int_0^{t_a + t_g} [-L(t) \times \Delta K(t)] \, dt \tag{8}$$

where $\Delta k(t) = k_Q - k(t)$ in the congested region Q and $k_C - k(t)$ in the discharge region C (Figure 2b). From Figure 2b,

$$-L(t) = \begin{cases} \int_0^t [-s_1(r)]\, dr, & t \in [0, t_a + t_1] \\ -s_2 t_1 - \int_{t_a + t_1}^t s_3(r)\, dr, & t \in [t_a + t_1, t_a + t_g] \end{cases}$$

$$D = \int_0^{t_a + t_1} [k_Q - k(t)] \int_0^t [-s_1(r)]\, dr\, dt + \tfrac{1}{2}(-s_2 t_1)(k_C - k_Q) t_1$$

$$- \int_{t_a + t_1}^{t_a + t_g} [k_C - k(t)] \times \left[s_2 t_1 + \int_{t_a + t_1}^t s_3(r)\, dr \right] dt \tag{9}$$

Again, if $k(t) \approx k$, then it becomes

$$D = \int_0^{t_a + t_g} \int_0^t [q(r) - q_Q]\, dr\, dt - \tfrac{1}{2}(q_C - q_Q) t_1^2$$

$$- t_1 t_2 \frac{q_C - q_Q}{k_C - k_Q}(k_C - k) - \int_{t_a + t_1}^{t_a + t_g} \int_{t_a + t_1}^t [q_C - q(r)]\, dr\, dt$$

$$= \int_0^{t_a + t_g} A(t)\, dt - \tfrac{1}{2} q_Q t_a^2 - \tfrac{1}{2} q_C t_g^2 - q_Q t_a t_g \tag{10}$$

According to Figure 2a, if the delay is computed by using the simple queuing diagram, then

$$D = \int_0^{t_a + t_g} Q(t)\, dt$$

$$= \int_0^{t_a + t_g} [A(t) - U(t)]\, dt$$

where

$$U(t) = \begin{cases} q_Q \times t, & t \in [0, t_a] \\ q_Q t_a + (t - t_a) q_c, & t \in [t_a, t_a + t_g] \end{cases}$$

$$D = \int_0^{t_a + t_g} A(t)\, dt - \tfrac{1}{2} q_Q t_a^2 - \tfrac{1}{2} q_c t_g^2 - q_Q t_a t_g$$

which is the same relation obtained by using shock wave analysis.

DISCUSSION AND SUMMARY

Two methods have been proposed to compute both the duration of time to discharge stored vehicles and the delay to passing motorists. These two methods (shock wave analysis and queuing analysis) lead to the same results if the density is not dependent on time. In particular, this is the case if the flow rate $q(t)$ varies slowly during the time period $(t_a + t_g)$, and the unique relationship between $q(t)$ and $k(t)$ can be assumed. This result can easily be generalized by letting both flow and density be functions of distance. Furthermore, this result is valid even if $q(t)$ is a random process. In this case, the equivalence of these two methods holds for every realization of $q(t)$, if $k(t)$ is independent of time.

When $k(t)$ is time dependent, these two methods may yield different results. It seems that shock wave analysis has more physical meaning. On the other hand, if queuing analysis is used, computational effort is saved and, in some cases, the delay can be evaluated even when the traffic is stochastic; e.g., if traffic counts form a Poisson process and the duration of the blockage time is exponentially distributed, then the delay can be estimated by using the model developed by Loulou (2). The time-dependent case has not been done in the paper. However, if the explicit form of $k(t)$ is known, numerical comparison can easily be made by using the equations derived in this paper.

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