THEORY OF URBAN-HOUSEHOLD AUTOMOBILE-OWNERSHIP DECISIONS

Lawrence D. Burns, Thomas F. Golob, and Gregory C. Nicolaidis,
Transportation and Urban Analysis Department,
General Motors Research Laboratories, Warren, Michigan

Automobile-ownership behavior is modeled as a function of socioeconomic factors and the availability and levels of service of public transportation systems. Decision makers are assumed to maximize their individual and household utilities within budget and time constraints. The benefits of increased mobility are weighed against the loss in other consumption attributable to ownership of 1 or more automobiles. Variables specific to residential and activity-center locations include the attractiveness of destinations served by public transit, the attractiveness of those not served by public transit, and respective travel times by automobile and transit. Estimation equations are developed through the introduction of functional forms for utility components and random utility terms representing variances in perception and taste and omitted factors. Multinomial logit models are used to define the probability of homogeneous groups of households choosing to own a specific number of automobiles. Calibrations are performed by using data from home-interview surveys and network simulations in the Detroit metropolitan area. Results are encouraging. All coefficients representing partial equilibrium market parameters are signed correctly and are significantly different from 0 where expected, and goodness-of-fit measures indicate acceptable model descriptive power.

That an association exists among the number of automobiles available to an urban household, the total number of trips made by individual household members by various modes of transportation, the time of day of these trips, and their destinations (14, 28) is strongly supported by empirical evidence. That these empirical relationships are indicative of processes of the supply and demand of transportation services is axiomatic in well-known economic theories.

The research reported in this paper concerns the supply and demand relationships involved in household automobile-ownership decisions. These decisions are modeled in light of the accessibility of travel destinations by automobile and by alternative public transit modes, the service characteristics of these modes, and the income and demographic characteristics of the households themselves.

An earlier paper covering this research project proposed a theory for explaining automobile ownership in terms of generalized concepts of urban spatial-location factors (1). In this paper a model is developed from this theory for use as a tool in urban transportation planning procedures. First, theoretical considerations are addressed with the objective of accounting for the decisions of individual travelers within a framework of household-level automobile-ownership decisions. Second, the simplifying assumptions necessary before the theoretical model can be made operational with data from traditional transportation planning home-interview surveys and land use surveys are detailed. Third, estimation equations are formulated by postulating functional forms of model components and introducing stochastic terms.

Results are given from empirical calibrations of the model and initial tests of its hypotheses by using data from the transportation and land use study conducted in the Detroit metropolitan area (35). A multinomial logit model is employed to estimate the values of the coefficients for the postulated utility functions. The results are encour-
aging in that the estimated coefficient values of the model variables are in all cases correctly signed and have sufficiently small standard errors to reject the null hypotheses within traditional confidence limits. Traditional goodness-of-fit measures are at values that are quite acceptable for nonlinear estimation equations of the multinomial logit type. A new technique is introduced for assessing the goodness of fit of such nonlinear probabilistic choice models. The current model also looks good in terms of the various measures generated through use of this technique.

BACKGROUND

Forecasts of changes in automobile ownership resulting from changes in transportation systems or spatial activity patterns have been recognized as being important to the evaluation of costs and benefits for roadway or public transit investment levels. Most previous models developed for use in transportation planning have involved describing automobile ownership for spatially defined aggregations of households in the following terms: as a function of measures of residential density (10, 29, 30); as a function of family income (16, 29); and as a function of household socioeconomic and demographic characteristics (3, 22, 26, 31). The research efforts by Shindler and Ferreri (33) and by Dunphy (15) were the first known to us to explicitly incorporate transportation system characteristics into automobile-ownership forecasts. As discussed by Beckmann, Gustafson, and Golob (1), the current model differs significantly from these and other related efforts. The difference is primarily in terms of the use herein of an economic theory of decision-making behavior instead of the correlative relationships that characterize most other studies. Empirically, the definitions of accessibility and mobility also are different.

The current model, after it was further tested and refined, was judged to be an appropriate complement to traditional travel demand models. These traditional models usually are focused on the short-term demand and supply aspects of transportation such as choice of mode for a fixed destination trip, or, perhaps, choice of destination and time of trip given fixed residential location and automobile ownership.

Because this model is based on the use of individual households and individual travelers as the units of observation, the model is consistent with the class of short-term demand models referred to as disaggregate travel demand models (8, 34). Also, because the model is based on utility theory from microeconomics and welfare economics, it is intimately related to a series of models formulated by us and by others to explain various travel phenomena in terms of that theory (2, 9, 17, 18, 19, 21, 27).

BASIC THEORY

In Beckmann, Gustafson, and Golob (1), the decision-making behavior of households is postulated to be a result of individuals making trade-offs between the costs and benefits perceived to be associated with transportation-related alternatives. The idea that, if such household preferences are transitive and continuous, they can be represented by a numerical function called a utility function is well developed in economic theory (13).

The total utility to a household is defined here to be dependent on consumption of all goods, available leisure time, and travel to all destinations visited within a certain time period. The household increases its mobility with the purchase of an automobile but sacrifices other consumption; the decision to purchase is made when the utility to the household of the increased mobility exceeds the loss of utility of consuming other goods. A car is assumed to be a homogeneous good having a given fixed price. The extent to which an alternative to the car mode is available to satisfy the travel needs of the household is represented by the sets of destinations accessible by the alternative mode and by the travel times required to reach each destination. Assume that this alternative mode is the "best" public transit mode perceived to be available to a decision maker. Specifically, the utility to a household not owning an automobile is a function of income (representing all consumption), nontravel time, and trips to the set of destinations.
accessible by the modes of transportation that are alternatives to travel by personal automobile:

\[
U^0_i = U\left( y, T - \sum_{k \in D_0} r_{ik} X_{1k}, X_{1k} \right)
\]

where

- \( U^0_i \) is utility to a household without a car at location \( i \),
- \( y \) is total disposable income of the household,
- \( T \) is available household leisure time,
- \( r_{ik} \) is travel time from the household at location \( i \) to destination \( k \) by alternative mode,
- \( X_{1k} \) is number of trips the household makes to destination \( k \) by using alternative mode, and
- \( D_0 \) is set of all destinations accessible to the household by alternative mode.

Assume that when an automobile is available household members will make an insignificant number of trips by the alternative mode of transportation. Total utility for automobile-owning households then is specified as a function of the number of trips to destinations accessible by automobile:

\[
U^1_i = U\left( y - p, T - \sum_{k \in D_1} s_{1k} Z_{1k}, Z_{1k} \right)
\]

where

- \( U^1_i \) is utility to a household with a car,
- \( p \) is annual cost of owning and operating an automobile (assumed to be independent of the number of trips),
- \( s_{1k} \) is travel time for the household at location \( i \) to destination \( k \) by automobile,
- \( Z_{1k} \) is number of trips for the household to destination \( k \) by automobile, and
- \( D_1 \) is set of destinations accessible to the household by automobile.

The ownership of a single automobile is advantageous to the household whenever

\[
\text{Max } U^1_i > \text{Max } U^0_i \quad \text{for } Z_k \quad \text{and } X_k
\]

The household thus assesses the maximum utilities that can be derived from making the most out of travel by automobile or travel by alternative mode. It compares these utilities and then makes its decisions regarding automobile purchase. Changes in income, automobile or alternative mode costs, automobile or alternative mode travel times, or accessibilities call for reassessments. Note that travel times, costs, and accessibilities to all destinations, visited or not visited, are taken into account here because these factors cause readjustments in utility-maximizing travel patterns. Also note that residential location is a very important factor in this model because change in residential location dictates changes in each of the explanatory variables.

This theory makes explicit a proposed relationship between automobile-ownership decisions and transportation system characteristics. Specification of functional forms for the utilities and the introduction of proxy variables are necessary before the theory can be implemented on currently available household home-interview and transportation
system data. Also, to assess total automobile ownership, one must extend the theory to include decisions about additional automobiles. And, in both the single-car and multiple-car cases, one must consider the interactions of individual household members with respect to their travel needs and desires and their roles in automobile-ownership decisions.

MULTIPLE TRAVELERS AND MULTIPLE AUTOMOBILES

In looking at the increased household mobility due to the purchase of an automobile, the previously formulated theory considers the change in utility to the entire household as the result of the purchase. To properly assess the utility the household obtains from an automobile, one must consider the travel utilities of each individual trip maker in the household. Thus one must develop postulates about the way individual trip makers in the household interact in their usages of 1 or more family automobiles. Briefly, there are 3 postulates.

1. Each trip maker in the household maximizes his or her individual travel utility independently of the other trip makers in the household.
2. When a household purchases an automobile, 1 trip maker in the household is considered to have the exclusive use of this automobile. Thus the utility of 1 automobile to a household is reflected in the utility of the automobile to 1 trip maker in the household.
3. Trip makers who do not have the use of an automobile are indifferent about whether to use public transit (with greater travel times and limited accessibility) or postpone trips until the automobile becomes available. (For purposes of simplifying terminology, refer to travel by automobile when it becomes available as travel by alternative mode.)

Based on the 3 simplifying assumptions, the total utility of a household can be viewed as the aggregation of the travel utilities of individual trip makers within the household and the household-level residual-consumption term. The travel utility of each trip maker within the household is a function of his or her available leisure time and travel to all destinations visited within a specified time period.

In general, the set of destinations accessible by automobile differs from the set of destinations accessible by alternative mode of transportation. Also automobile travel times usually differ from the travel times of the alternative mode; walking and waiting times for public transit service are reflected in additional, perhaps more heavily weighted, time penalties for the alternative mode. Travel utilities of individual trip makers thus are dependent on whether the trip maker has the use of an automobile or whether he or she must rely on the alternative mode of transportation.

Specifically, the net travel utility to an individual $j$ from travel by the alternative mode of transportation is considered to be separable into 2 components: net leisure time and satisfaction of travel purposes (the household location subscripts $i$ are dropped here for simplification):

$$TU_j^o = U \left( T_j - \sum_{k \in D_0} r_k x_{kj}, x_{kj} \right)$$

where

- $TU_j^o$ = net utility to an individual $j$ from travel by alternative mode of transportation,
- $T_j$ = total leisure time of individual $j$, and
- $x_{kj}$ = number of trips individual $j$ makes to destination $k$ when relying on alternative mode.

Similarly, the net utility to an individual $j$ from travel by an automobile is:
TU_j^i = U(T_j - \sum_{k \in D_1} s_k Z_{kj}, Z_{kj}) \tag{5}

where

TU_j^i = net utility to an individual j from travel by exclusive use of an automobile,
and
Z_{kj} = number of trips individual j makes to destination k when using an automobile.

If next the assumption is made that the total utility to a household at location i is additive with respect to the utility from income available for other consumption and total household travel utility, then

U_{in}^n = \phi \left( y - \sum_{\ell=1}^n p_\ell \right) + \sum_{j=1}^n TU_j^i + \sum_{j=n+1}^m TU_j^o \tag{6}

where

\phi = functional form of the residual consumption contribution to utility, and
U_{in}^n = total utility of an m-trip-maker household at location i owning n automobiles.

Thus by substituting equations 4 and 5 in equation 6,

U_{in}^n = \phi \left( y - \sum_{\ell=1}^n p_\ell \right) + \sum_{j=1}^n U(T_j - \sum_{k \in D_0} r_k X_{1kj}, X_{1kj})
+ \sum_{j=n+1}^m U(T_j - \sum_{k \in D_1} s_k Z_{1kj}, Z_{1kj}) \tag{7}

This is the foundation for the model specifying the conditions under which automobile-ownership decisions are undertaken. To test the model hypotheses however, one must specify mathematical forms for the \phi and U functions and develop estimation equations.

FUNCTIONAL FORMS

A utility function that is logarithmic in terms of both time and trips was selected on the basis of its theoretical properties (such as its property of "diminishing marginal utility") and on the basis of its success in describing spatial interactions (19). This functional form also is related intimately to entropy formulations of trip distributions (41) and other transportation phenomena (42). Equations 4 and 5 then can be written respectively as follows (for simplicity the household location subscript has been dropped):

TU_j^0 = a_k \log \left( 1 + T_j - \sum_{k \in D_0} r_k X_{kj} \right) + \sum_{k \in D_0} a_k \log \left( 1 + X_{kj} \right) \tag{8}
\[TU_j^i = a_t \log \left(1 + T_j - \sum_{k \in D_1} s_k Z_{kj}\right) + \sum_{k \in D_1} a_k \log (1 + Z_{kj}) \]  

(9)

where

\(a_k\) = the value or attraction of destination \(k\), and  
\(a_t\) = the value of leisure time.

Each trip maker in the household maximizes his or her individual utility by adjusting number and distribution of trips. This maximization is performed independently of other members of the household in all matters not related to the availability of the household automobile or automobiles. In this process, the maximum household utility is not necessarily achieved.

Finding the maximizing solution for \(TU_j^i\) is facilitated through the introduction of a new set of variables:

\[\zeta_{kj} = 1 + X_{kj},\]
\[R^0_j = 1 + T_j + \sum_{k \in D_0} r_k,\]
\[A_0 = a_t + \sum_{k \in D_0} a_k\]

Maximizing equation 8 with respect to \(X_{kj}\) then implies

\[\frac{a_k}{1 + X_{kj}} = \frac{a_t r_k}{1 + T_j - \sum_{k \in D_0} r_k X_{kj}}\]

(10)

This equation may be solved for \(\zeta_{kj}\):

\[\zeta_{kj} = \frac{a_k R^0_j}{A_0 r_k}\]

(11)

Similarly, let

\[\eta_{kj} = 1 + Z_{kj},\]
\[R^1_j = 1 + T_j + \sum_{k \in D_1} s_k,\]
\[A_1 = a_t + \sum_{k \in D_1} a_k\]

(10)

(11)

(12)

(13)
Maximizing equation 5 with respect to $Z_{k1}$ and solving for $\eta_{k1}$ yield

$$\eta_{k1} = \frac{a_k R^1_k}{A_1 s_k} \quad (14)$$

Substituting $Z_{k1}$ and $\eta_{k1}$ into the utility equations 8 and 9 and simplifying yield

$$\begin{align*}
\max \left( T U^{i}_{X} \right) &= a_t \log \frac{a_t R^0_k}{A_0} + \sum_{k \in D_0} a_k \log \frac{a_k R^0_k}{A_0 s_k} \quad (15) \\
\max \left( T U^{i}_{Z} \right) &= a_t \log \frac{a_t R^1_k}{A_1} + \sum_{k \in D_1} a_k \log \frac{a_k R^1_k}{A_1 s_k} \quad (16)
\end{align*}$$

Distinguishing among the individual trip makers within a household is not within the scope of this theory. Thus

$$\begin{align*}
\max (T U^0) &= \max (T U^0) = \ldots = \max (T U^0) \quad (17) \\
\max (T U^1) &= \max (T U^0) = \ldots = \max (T U^0) \quad (18)
\end{align*}$$

The travel-utility-maximizing form of the utility function given in equation 7 for a household at location i then becomes

$$U^{i}_{t} = \varphi \left( y - \sum_{t=1}^{n} p_t \right) + n \max (T U^0) + (m - n) \max (T U^0) \quad (19)$$

where $\max (T U^1)$ and $\max (T U^0)$ are given by equations 15 and 16 and $p_t =\text{annual cost to a household of owning and operating the } t^{\text{th}}\text{ automobile.}$

Hypothesizing that the utility of all other consumption has a similar logarithmic "diminishing marginal utility" functional form, one can write equation 19 without any general terms as

$$U^{i}_{t} = b_o \log \left( y - \sum_{t=1}^{n} p_t \right) + b_t \left[ n \max (T U^0) + (m - n) \max (T U^0) \right] \quad (20)$$

where

- $b_o = \text{additive parameter adjusting the scale of the utility of residual consumption to that of overall utility, and}$
- $b_t = \text{additive parameter adjusting the scale of the utility of travel to that of overall utility.}$
CONDITIONAL AUTOMOBILE-OWNERSHIP DECISION

Assume that the maximum number of automobiles the household will own does not exceed the number of driver-aged trip makers in that household \( m \). Now whenever

\[
U^\eta_{is} > U^\theta_{is} \quad \text{for} \quad \theta \neq \eta, \; \theta = 0, 1, \ldots, m
\]

(21)

the household will purchase \( \eta \) automobiles. Specifying this condition in terms of the indirect utility function (equation 20) means

\[
0 < U^\eta_{is} - U^\theta_{is} = b_e \left[ \log \left( y - \sum_{t=1}^{\eta} p_t \right) - \log \left( y - \sum_{t=1}^{\theta} p_t \right) \right] \\
+ b_t \left[ (\eta - \theta) \left( \text{Max} \left( TU^\eta_i \right) - \text{Max} \left( TU^\theta_i \right) \right) \right]
\]

(22)

for \( \theta \neq \eta, \; \theta = 0, 1, \ldots, m \). Simplifying means

\[
0 < U^\eta_{is} - U^\theta_{is} = b_e \left[ \log \left( y - \sum_{t=1}^{\eta} p_t \right) - \log \left( y - \sum_{t=1}^{\theta} p_t \right) \right] \\
+ b_t (\eta - \theta) \left[ \text{Max} \left( TU^\eta_i \right) - \text{Max} \left( TU^\theta_i \right) \right]
\]

(23)

for \( \theta \neq \eta, \; \theta = 0, 1, \ldots, m \).

Substituting the indirect maximum travel utilities of equations 15 and 16 into this condition for ownership of \( \eta \) automobiles yields

\[
0 < U^\eta_{is} - U^\theta_{is} = b_e \left[ \log \left( y - \sum_{t=1}^{\eta} p_t \right) - \log \left( y - \sum_{t=1}^{\theta} p_t \right) \right] \\
+ b_t (\eta - \theta) \left[ a_t \log \frac{A_1}{A_0} - \log \frac{A_1 R^1}{A_0 R^0} \right] \\
+ \sum_{k \epsilon D_1} a_k \log \frac{A_1 R^1}{A_0 S_k} - \sum_{k \epsilon D_0} a_k \log \frac{A_0 R^0}{A_0 R^0}
\]

(24)

for \( \theta \neq \eta, \; \theta = 0, 1, \ldots, m \).

However, by using the definitions of \( A_1, A_0, R^1, \) and \( R^0 \) specified earlier, one can simplify equation 24:
\[ 0 < U_{is}^\eta - U_{is}^\theta = b_c \left[ \log \left( y - \sum_{t=1}^\eta p_t \right) - \log \left( y - \sum_{t=1}^\theta p_t \right) \right] \]

\[ + b_t (\eta - \theta) \left( A_0 \log \frac{A_0}{R^0} - A_1 \log \frac{A_1}{R^1} \right) \]

\[ + \sum_{k \in D_0} a_k \log \frac{r_k}{s_k} + \sum_{k \in D_1 - D_0} a_k \log \frac{a_k}{s_k} \]

for \( \theta \neq \eta, \theta = 0, 1, \ldots, m \). The set \( D_1 - D_0 \) is the set of destinations accessible by automobile but not accessible by transit; it is a simplified notation for the set represented by the intersection of \( D_1 \) with the complement of \( D_0 \).

Strictly speaking, \( A_1, A_0, R^1, \) and \( R^0 \) depend on household location \( i \). However, these terms are rather insensitive to exact location; they will be treated as constants in the initial tests of the model hypotheses given herein. Thus,

\[ 0 < U_{is}^\eta - U_{is}^\theta = \alpha + b_c \left[ \log \left( y - \sum_{t=1}^\eta p_t \right) - \log \left( y - \sum_{t=1}^\theta p_t \right) \right] \]

\[ + b_t (\eta - \theta) \left( \sum_{k \in D_0} a_k \log \frac{r_k}{s_k} \right) \]

\[ + b_t (\eta - \theta) \left( \sum_{k \in D_1 - D_0} a_k \log \frac{a_k}{s_k} \right) \]

for \( \theta \neq \eta, \theta = 0, 1, \ldots, m \) where \( \alpha = (\eta - \theta) \left( A_0 \log \frac{A_0}{R^0} - A_1 \log \frac{A_1}{R^1} \right) \).

The second term on the right side of equation 26 indicates the difference in utility to the household from consumption of all other goods when the household owns \( \theta \) automobiles compared to \( \eta \) automobiles. The third term on the right indicates the utility \((\eta - \theta)\) that trip makers in the household receive from travel time savings when traveling by automobile to destinations that can be reached by transit (destinations in set \( D_0 \)), and the fourth term on the right indicates the utility \((\eta - \theta)\) that trip makers receive from potential travel to destinations not accessible by public transit but accessible by automobile. The third and fourth terms together comprise the travel component.

**ESTIMATION EQUATIONS**

The model of automobile ownership developed in this paper is, of course, a simplification of reality. It does not take into account all of the differences in the tastes and perceptions of individual decision makers. Also data are not available to measure all of the stimuli that decision makers might consider important. Thus the values of the utility differences given in equation 26 are random variables across samples of households.

By letting \( \epsilon_i^n \) represent an unobserved random variable containing excluded decision factors and individual differences, one can specify the probabilistic utility function for an \( m \)-trip-maker household owning \( \eta \) automobiles as
The function generally can be scaled so that it can be written as the sum of the deterministic component and the random component \( \epsilon^\eta \):

\[
RU^\eta = U^\eta + \epsilon^\eta
\]

(28)

The probability that a sampled household finds ownership of \( \eta \) automobiles more advantageous than ownership of any other number of automobiles \( \theta \) can then be denoted by

\[
P^\eta = \text{Prob} [(U^\eta + \epsilon^\eta) > (U^\theta + \epsilon^\theta)]
\]

(29)

for \( \theta \neq \eta, \theta = 0, 1, \ldots, m \) where \( P^\eta \) = probability that the household will desire to own \( \eta \) automobiles. Equation 29 can then be rewritten as

\[
P^\eta = \text{Prob} [(\epsilon^\eta - \epsilon^\theta) < (U^\eta - U^\theta)]
\]

(30)

for \( \theta \neq \eta, \theta = 0, 1, \ldots, m \).

The problem of developing estimation equations for the current automobile-ownership model is now one of specifying a distribution for the probabilistic components in equation 30. The one chosen here is the reciprocal exponential distribution; random variables are assumed to be distributed independently as

\[
\text{Prob} (\epsilon^\theta \leq w) = \exp(-e^{-w})
\]

(31)

Probability equation 29 then takes the form

\[
P^\eta = \frac{\exp(U^\eta)}{\sum_{\theta=0}^{m} \exp(U^\theta)} = \frac{1}{\sum_{\theta=0}^{m} \exp(U^\theta - U^\eta)}
\]

(32)

This function is termed the conditional or multinomial logit function. It was first developed systematically by Gurland, Lee, and Dolan (20). A more general formulation in terms of choice behavior is provided by McFadden (24, 25). Other general developments are provided by Bloch and Watson (5), Bock (6), and Theil (37, 38). Applications of the functional form to transportation mode, destination, and trip-time choice were made by Rassam, Ellis, and Bennett (32) and Charles River Associates (9).

The probability that an \( m \)-trip-maker household decides to own \( \eta \) automobiles thus is specified by equation 32; the \( U^\eta - U^\theta \) utility differences for the alternative choices of \( \theta = 0, 1, \ldots, m, \theta \neq \eta \) are given by equation 26. By observing the actual choices made by households when they are faced with various values of the independent variables, one can make estimates of the constant \( \alpha \) and the \( b_\eta \) and \( b_\theta \) coefficients in equation 26 through the application of a maximum-likelihood procedure proposed by McFadden (24) and implemented by Manski (23). Statistics generated in this first empirical test of the model hypotheses are the subject of the remainder of this paper.
DATA

The data employed in the empirical test were obtained from individual household observations and network simulation results from the 1965 Detroit Transportation and Land Use Study (TALUS) (35). The variables to be used in the following analysis are defined as follows:

\[ y = \text{disposable income of the household (calculated by subtracting estimated taxes from respondents' reported gross income);} \]
\[ p_{it} = \text{annual cost to a household at location } i \text{ of owning and operating the } t \text{th automobile (assumed to be independent of the number of trips);} \]
\[ r_{ik} = \text{travel time from household at location } i \text{ to destination } k \text{ by public transit [generated from the 1965 Detroit transit network by using the Urban Transportation Planning System (UTPS) (40) including walking, waiting, transfer, and running times];} \]
\[ s_{ik} = \text{travel time from household at location } i \text{ to destination } k \text{ by automobile (taken directly from network simulation results generated in the TALUS study);} \]
\[ D_i^0 = \text{set of destinations accessible to an individual trip maker at location } i \text{ by public transit (a destination } k \text{ was considered accessible by transit if it could be reached within a specified period of time by transit);} \]
\[ D_i^1 = \text{set of destinations accessible to an individual trip maker at location } i \text{ when this trip maker has the exclusive use of an automobile (a destination } k \text{ was considered accessible by automobile if it could be reached within a specified period of time by automobile);} \]
\[ f = \text{number of automobiles a household owned in 1965; and} \]
\[ a_{ik} = \text{attraction of destination } k \text{ to a household at location } i. \]

Individual household observations were taken from the 4 percent survey of all households within the Detroit urbanized area, and the origin and destination subscripts (i and k) correspond to traffic analysis zones. The limit of the study spatial area is defined by the inclusion of all traffic analysis zones in TALUS that are located within the Detroit urbanized area as defined by the 1960 Census (34).

For the purposes of this initial test of the theory, households with the same number of driver-aged trip makers m were assumed to be homogeneous with respect to their automobile-buying behavior. The data were sorted into 3 sets:

1. Households with 1 driver-aged trip maker,
2. Households with 2 driver-aged trip makers, and
3. Households with 3 or more driver-aged trip makers.

Certain assumptions were made in preparing the data. These assumptions are the subjects of model-sensitivity analyses.

1. Automobile ownership cost \( p_{it} \) is constant for all automobiles and is independent (= \( p_i \)) of location i; an average figure of $1,000 (1965 prices) was selected on the basis of automobile cost data provided by Botzow (7).
2. A destination is a member of the set \( D_i^0 \) if it can be reached from i in 60 min by transit; a destination is a member of the set \( D_i^1 \) if it can be reached from i in 60 min by automobile; \( D_i^0 \subset D_i^1 \).
3. The attraction of a destination \( k \) is independent of the location of the household (= \( a_k \)) and is given by

\[ a_k = \frac{E_k + P_k}{\sum (E_i + P_i)} \]  
(33)
where

\[ E_k = \text{total employment at destination } k, \]  
\[ P_k = \text{total population residing at destination } k. \]

The summation in the denominator is over all traffic analysis zones in the Detroit urbanized area.

**EMPIRICAL RESULTS**

The multinomial logit estimation equation 32 for the utility differences of equation 26 was calibrated for each of the 3 population segments (1 trip maker, 2 trip makers, and 3 or more trip makers). Random samples of between 500 and 1,000 households in each segment were selected from the set of all households responding to TALUS (35). These samples were structured to obtain an approximately equal number of observed households that chose each of the alternatives; this structuring resulted in a higher sampling rate for 0-car households. Samples that were held out were employed for assessing goodness of fit of the models.

Two different forms of the model were calibrated for each of the 3 segments. In the simplest (choice-abstract) form, the utility scale weights \( b_0 \) and \( b_t \) are assumed to be independent of the choice alternatives. This is the usual assumption underlying use of multinomial logit functions in modeling travel-mode choice. It is the only form of the model applicable to the binomial 1-trip-maker case in which only the 2 choices of 0 car and 1 car are theorized to be relevant.

The second and more complicated form of the model for the 2-trip-maker and 3-or-more-trip-maker segments is developed by assuming that the \( b_c \) and \( b_t \) coefficients are specific to each sequential choice alternative. That is, it is proposed that the weights people place on consumption and travel components of utility may be different when they are evaluating the 0-car and 1-car choice alternatives than when they are evaluating the 1-car and 2-car alternatives and so forth. In the calibration of this form of the model, freedom thus is incorporated to estimate as many \( b_c \) and \( b_t \) coefficients as there are choice alternatives minus 1. Referring to these coefficients as \( b_i \) and \( b_i' \), where \( i \) denotes the choice between \( i \) and 0 automobiles, is convenient.

For purposes of clarifying the previous arguments, the following estimation equations can be written for the most general case of 3 or more trip makers in which 4 choices (0 car, 1 car, 2 cars, and 3 cars) are relevant (\( P_\eta \), probability that a household made up of \( m \) driver-aged trip makers will decide to own \( \eta \) automobiles, takes the form of equation 32 for \( \eta = 0 \)):

\[
U_0^0 - U_0^n = a_0 + b_0' \left[ \log (y - p) - \log y \right] + b_1 \left( \sum_{k \in D_0} a_k \log \frac{r_k}{s_k} + \sum_{k \in D_1-D_0} a_k \log \frac{a_k}{s_k} \right)
\]

\[
U_1^0 - U_0^n = a_0 + b_1' \left[ \log (y - 2p) - \log y \right] + b_2 \left[ 2 \left( \sum_{k \in D_0} a_k \log \frac{r_k}{s_k} \right) + \sum_{k \in D_1-D_0} a_k \log \frac{a_k}{s_k} \right]
\]

\[
U_2^0 - U_0^n = a_0 + b_2' \left[ \log (y - 3p) - \log y \right] + b_3 \left[ 3 \left( \sum_{k \in D_0} a_k \log \frac{r_k}{s_k} \right) + \sum_{k \in D_1-D_0} a_k \log \frac{a_k}{s_k} \right]
\]

(34)
Estimation equations for the other 2 population segments merely eliminate 1 or 2 of the utility differences in equation 34. The choice-abstract form of the model will have $b_2^i = b_2^x = b_1^x$ and $b_1^i = b_1^x = b_i$. The constant $a_0$ includes the constant $\alpha$ of equation 26 and the mean of the unobserved additive random term.

The results of the logit estimations for the 3 population segments for the choice-abstract form of the model are given in Table 1. The ratios are asymptotically distributed as t-statistics in a linear model by Theil (38) and thus can be used with large samples (as in the current case) to evaluate the probability that the coefficient estimates are in actuality 0. In the $-2 \log \lambda$ statistics for each model, $\lambda$ is the ratio of the initial likelihood for the model with all coefficients as 0 and the final (maximum) likelihood with the coefficients as listed. This statistic, the so-called likelihood ratio statistic, has an approximate $\chi^2$ distribution. Therefore, it can be used to test the joint hypothesis that the data are a result of processes inconsistent with the proposed theory (that is, the joint hypothesis: $b_0 = b_t = 0$).

The $-2 \log \lambda$ statistics in Table 1 indicate firm rejections of the joint hypothesis $b_0 = b_t = 0$ for all population segments. Moreover, the estimated coefficient values of the variables of the models derived from the theory are in all cases correctly signed, and the t-statistics indicate firm rejections of the hypothesis $b_0 = 0$ or $b_t = 0$ for all population segments. These are encouraging initial results for internal tests of the current automobile-ownership theory, but further goodness-of-fit assessments are called for.

The results of the logit calibrations for the choice-dependent form of the model were judged to be inferior to the results of the choice-abstract form. The criteria for this judgment were the $\chi^2$ statistics for the overall models and the t-statistics for the individual model coefficients. The $\chi^2$ statistics did not significantly increase in spite of the increase in the degrees of freedom in the choice-dependent form. The hypothesis that individual $b_1^1$ and $b_1^2$ values were insignificantly different from 0 was accepted for some coefficients at as high as the 10 percent confidence limit.

Sensitivity analyses were performed by using 3 different values of the annual automobile cost parameter $p$; the values were $750$, $1,000$, and $1,500$. The model based on the original $1,000 estimate was judged to be best on the basis of having the greatest t-statistic absolute values for the coefficients. The model based on the $750 estimate was only slightly different from the chosen model, and the model based on the $1,500 price was definitely inferior. Sensitivity analyses on other direct and definitional model parameters are within the realm of further research.

EVALUATION OF EMPIRICAL RESULTS

The statistics discussed in the preceding section provide insufficient information for assessing the goodness of fit of probabilistic or quantal choice models such as multinomial logit. The single overall measure of fit, the likelihood ratio test, is a rather insensitive test because of the questionable validity of the null hypothesis $b_0 = b_t = 0$. Also model significance generally becomes easier to obtain with this test as sample size increases.

<table>
<thead>
<tr>
<th>Table 1. Results of logit estimations.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population Segment (number of trip makers)</td>
</tr>
<tr>
<td>---------------------------------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3 or more</td>
</tr>
</tbody>
</table>

*3 degrees of freedom.
Other measures of goodness of fit have been developed that are comparable to the coefficient of determination $r^2$ in linear models. Use of the $r^2$ formula directly to measure proportion of variance explained is inappropriate because the dependent observations by definition lie on the asymptotes of the logit function. It can be shown that the maximum value of $r^2$ for continuous independent variables cannot be equal to 1.0 as it is in the continuous dependent variable case, and this maximum value cannot be determined deductively. More important, $r^2$ is a measure of linearity, and this is not a correct criterion for the logit model.

Two statistics have been developed that are attempted analogies to $r^2$. Both are referred to as "pseudo $r^2" and usually are denoted by $\rho^2$. The first, $\rho^2_1$, applied by Cragg (11), makes assumptions about the error term distribution resulting in the following mathematical expression:

$$\rho^2_1 = 1 - \exp \{-2[L^*(\hat{\theta}) - L^*(0)]/N\}$$  \hspace{1cm} (35)

where

$L^*(\hat{\theta})$ = value of log likelihood function for vector of estimated coefficients $\hat{\theta}$,
$L^*(0)$ = value of log likelihood function with all coefficients equal to 0, and
N = number of observations.

A second pseudo $r^2$ statistic, $\rho^2_2$, applied by McFadden (24) and discussed by Ben-Akiva (4), is equal to the ratio of explained log likelihood over total log likelihood and is expressed as follows $[L^*(\hat{\theta})$ and $L^*(0)$ are defined as they are in equation 35]:

$$\rho^2_2 = 1 - \frac{L^*(\hat{\theta})}{L^*(0)}$$  \hspace{1cm} (36)

Values of these pseudo $r^2$ statistics for the automobile-ownership models of Table 1 are tabulated as follows:

<table>
<thead>
<tr>
<th>Population Segment (number of trip makers)</th>
<th>$\rho^2_1$</th>
<th>$\rho^2_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.259</td>
<td>0.216</td>
</tr>
<tr>
<td>2</td>
<td>0.428</td>
<td>0.255</td>
</tr>
<tr>
<td>3</td>
<td>0.348</td>
<td>0.155</td>
</tr>
</tbody>
</table>

Four problems are associated with using either pseudo $r^2$ statistic ($\rho^2_1$ or $\rho^2_2$) as a measure of goodness of fit.

1. No well-developed distribution theory or associated statistic exists (such as an $F$-statistic) for either measure to permit an assessment of the statistical significance of the measure.
2. Neither statistic is comparable between models with different functional forms.
3. Maximum value for each statistic is not defined clearly.
4. Neither statistic provides an intuitive interpretation (comparable to percentage of variance explained) of goodness of fit.

In an effort to improve the understanding of the goodness of fit of probabilistic choice models, we propose a new technique. To describe this technique in detail and discuss
its purported strengths and weaknesses are not within the scope of this paper. A brief explanation is advanced, instead, and 1 example application is presented with the objective of providing insight into the performance of the models derived from the current automobile-ownership theory. Research publications directed to the specific subject of the goodness-of-fit evaluation technique are anticipated.

The proposed technique entails the formation of homogeneous groups of individual observations belonging to cells (classes) defined jointly by ranges of the independent (explanatory) variables of the model. Individuals in every cell, to an extent, we assumed to be faced with identical stimuli when evaluating their choice alternatives. Thus a hypothetical observation described by the mean values of the independent variables for all observations in a particular cell can be considered a representative observation for all the observations in this cell. The predicted probabilities for this particular representative observation then can be computed by using the estimated choice models. These predicted probabilities for each choice alternative finally can be compared to the probabilities given by the proportion of individuals in a particular cell that choose that alternative.

Figure 1 shows a typical scatter plot of predicted versus actual proportions of households owning no automobiles for a hold-out sample from the 1-trip-maker population segment. Each point represents a unique cell; the number corresponding to each point reflects the number of households in that cell. Ideally, all of these points should lie along the 45-deg line.

To compare actual and predicted proportions, $r^2$ (weighted by the number of households in each cell) is calculated for the best linear fit between these measures. Also the slope and intercept of this best linear fit are calculated to reveal any systematic biases in the model, and the significance of the difference between this best linear fit and the 45-deg line can be estimated through use of an F-statistic.

To determine the sensitivity of this approach to different ways of defining cells, actual versus predicted comparisons were made for various cell formation criteria.

Figure 1. Predicted probability versus actual probability for 1-trip-maker case.
Specifically, for the 1-trip-maker model, 10 completely different sets of cells were formulated by specifying different sets of ranges for the independent variables. The average value for the best fit $r^2$'s was $r^2_{best} = 0.81$, and the variance was $\sigma^2 = 0.016$. The relatively small variance in this statistic is an initial indication that the technique is rather insensitive to the manner in which the independent variables are divided in order to assign observations to cells as long as a degree of homogeneity is maintained within each cell and care is taken to create cells that have a similar number of observations.

The average slope and intercept of the best fit lines for the 10 sets of cells used are $\beta = 0.84$ and $\alpha = 0.076$ respectively; variances are $\sigma^2 = 0.016$ and $\sigma^2 = 0.007$ respectively. These average slope and intercept values suggest that the model predicts too high for small actual probabilities and too low for large actual probabilities. Thus there appears to be a systematic conservative prediction bias in the 1-trip-maker model. The relatively small variance for both of these statistics is further evidence that the proposed technique is rather insensitive to empirical issues such as the range of the variables defining the cells.

For the 1-trip-maker case overall, the technique reveals a relatively good correspondence between actual and predicted proportions. However, further work is required to determine exactly how good this correspondence is in a statistical sense. Assessing the goodness of fit in the manner presented here begins to give one insights into how well the logit model predicts disaggregate behavior. That the proposed technique is applicable only where large samples are available should be emphasized. Developing this procedure for choice settings with more than 2 alternatives and in a more statistically rigorous sense is within the realm of important further research.

**CONCLUSIONS AND DIRECTIONS FOR FURTHER RESEARCH**

Results from the initial tests of some hypotheses of the automobile-ownership theory are encouraging. The estimated coefficient values of the variables of the models derived from the theory are in all cases correctly signed and are significantly different from 0. Also traditional goodness-of-fit measures are at values that are acceptable for nonlinear estimation equations of the multinomial logit type. And a relatively good correspondence exists between predicted and actual probabilities for groups of hold-out observations in the 1-trip-maker segment model.

The current models developed from a specific theory of automobile-ownership decisions help to begin to identify causal mechanisms in urban-household travel behavior. However, much remains to be done before these models can be effectively applied in predicting automobile-ownership changes as results of transportation system changes. Sensitivity analyses are required for a number of variable definitions and assumptions of the models. In particular, the models need to be tested for different measures of destination attraction $a_n$, different accessibility cutoffs (used to define sets $D_0$ and $D_1$) and different urbanized area definitions. Improved understanding of how individual trip makers in the household interact in their uses of 1 or more family automobiles also must be sought. Finally, better understanding of the goodness of fit of probabilistic choice models, such as the multinomial logit model, must be obtained on both aggregate and individual observation levels before any of these types of models can be employed confidently in prediction. We hope that the goodness-of-fit technique proposed herein and the criticisms it encourages will lead toward a better understanding of the complex goodness-of-fit phenomena.

**ACKNOWLEDGMENTS**

We wish to thank Martin J. Beckmann, Technical University of Munich, and Richard L. Gustafson, formerly of General Motors Research Laboratories and currently an Oregon state representative, for their theoretical and methodological inputs. Appreciation also is extended to Larry L. Howson, formerly of General Motors Research Lab-
oratories and currently with General Motors Transportation Systems Division, for his invaluable computer assistance and to Wilfred W. Recker and K. S. Krishnan, General Motors Research Laboratories, for their criticism.

REFERENCES

23. C. P. Manski. Maximum Score Estimation of the Stochastic Utility Model of
DISCUSSION

Fred A. Reid, University of California, Berkeley

The paper by Burns, Golob, and Nicolaidis is a contribution to behavioral modeling and a utility theory of automobile ownership. Noteworthy points of the theory are expressing household choice in terms of the travel utilities of the individuals and maximizing budget and overall utility at the household level. Trip-frequency and destination-choice factors are included, although modal choice is considered only for adults in excess of the household automobile count. The theoretical development is valuable for identifying the construction of the utility function and the interplay of individuals in a household. However, if one is trying to develop accurate, manageable models in terms of available data, the theory seems too complex in relation to the number of parameters estimated.
(only 3 per population segment) and the restrictive assumptions necessary for calibration. The relation between trip level of service and attraction variables for all types of trips is completely specified by the theory rather than by allowing separate parameters to be estimated or different functional forms for alternate theories to be tested. For example, service variables have been found to best describe travel behavior entering as linear rather than logarithmic differentials in other studies.

The failure to characterize the individual attributes of the household work trip is a weakness when one considers the role these major trips play in ownership and because of the generality of the destination-attraction variable used. As only the percentage of total population and employment at a destination, this variable also does not distinguish non-work-trip ends as being unique to a household.

The assumption that owned automobiles are always used if they are available seems particularly restrictive. It leaves modal-choice alternatives to only a portion of the household members, weakening the advantage of the individual utility maximization.

The paper has shown some useful theoretical development, but I believe it has raised more questions than it has answered.

AUTHORS' CLOSURE

We are pleased that Reid viewed this paper as a contribution to behavioral travel-demand modeling in general and to the understanding of automobile demand specifically. We thank him for his comments. Comments on his discussion are warranted.

We are aware of the limitations of a rigid theoretical approach. However, we judge that attempting to assess causal mechanisms in urban household automobile-ownership decisions by merely studying ad hoc empirical correlations is a much less satisfactory alternative. Consequently, the development of an economic theory of decision-making behavior placed significant restrictions on the empirical study performed to test the hypotheses of the theory. The objective of establishing a valid causal model with sound theoretical underpinnings justified acceptance of these restrictions. Given that a pre-defined theory was rigidly followed, greater value can be placed on the significance of the empirical results because probabilities of uncovering random phenomena are minimized.

Admittedly, the existing theory assumes independence among household members in their trip-making behavior. As a result, the utility of additional automobiles to a household is not accurately modeled. Attempts to account for the interdependence of household members' trip-making behavior significantly complicates the model. These complications are not warranted in an initial study of automobile-ownership decisions. As noted in the original paper, however, such concerns, which were uncovered by following the theoretical approach, indicate important directions for further research.

The theoretical model can be adapted to include

1. Purpose-specific attraction measures;
2. Variances in the cost of automobile ownership due to residential location, household income, and type of automobile (first, second, or third household automobile); and
3. Travel times broken down into walking, waiting, and in-vehicle time components.

Including these features in the initial empirical study was prohibited by data availability. Also the final model allows assessment of impacts of improvements in public transportation or changes in automobile travel characteristics on automobile ownership. Improvements in public transportation are reflected in reduced travel times and increased service areas. Such changes can be incorporated directly into the model transit utility term, making assessment of the effects of public transportation improvements a relatively straightforward procedure. In a similar manner, the effects of increased or decreased automobile travel times can easily be reflected in the automobile travel utility terms.
We would like to emphasize again, as we did in our conclusion, that the objective of our research was to identify the structure of causal mechanisms in urban-household travel behavior. We are aware that, before the models resulting from this research can be effectively applied in predicting changes in automobile-ownership levels as results of transportation system changes, sensitivity analyses will be required to test further variable definitions and assumptions of the model. Specific areas of future research were covered in the paper.