A SYNTHESIZED TRIP-FORECASTING MODEL FOR SMALL- AND MEDIUM-SIZED URBAN AREAS

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Because of the high monetary and time costs associated with home-interview surveys for urban transportation studies, planning analysts have sought to model travel demand by using other data sources such as 1970 census work-trip data. The purpose of the research reported in this paper is to examine trip distribution functions that may be appropriate for estimating zone trip interchange in small- to medium-sized urban areas. Several functional forms of travel impedance were investigated. For the city sizes studied, model accuracy is shown to be relatively insensitive to the form of the travel impedance function. Analytical deductions are used to develop a calibration technique for a 2-way, constrained gravity model using the simple negative exponential function. Calibration of the model can be accomplished without using extensive origin-destination survey data. The model is tested by using data from actual studies, and an outline is suggested for calibrating the distribution model by using the 1970 census data.

•ORIGIN-DESTINATION (O-D) data collection is the most time consuming and costly part of any transportation study. Furthermore, the level of accuracy of O-D surveys is frequently so low that the interzone trip forecasts based on them are unreliable. This occurs because the number of dwelling units in a traffic zone is small and the number of traffic zones within the system is large; therefore, the trip exchange between 2 given zones is a rare attribute of the zone population, and a very high sampling rate is required to provide acceptably accurate O-D data. Long (6) has shown that, when a city with a population of 100,000 is divided into 200 traffic zones, the error of non-home-based interzone trip exchanges based on a 5 percent home-interview sampling rate could be as high as ±270 percent. The problem is compounded further if financial constraints force a sampling rate as low as 2 percent for small- or medium-sized urban areas (2).

The cost, time consumption, and inaccuracy of O-D surveys demonstrate the merit of exploring synthesized models that can be calibrated by using available information and, therefore, do not call for conducting a particular O-D survey. Given that O-D data have inherent inaccuracies and that given the inaccuracy inherent in O-D data and that synthesizing models are structured on the state of the art, it is not clear that the predictions based on an appropriate synthesizing model would be less reliable than those based on costly and time-consuming travel surveys. In fact, the ultimate goal of transportation science may be perceived as the reaching of a mature stage where models and not particular surveys are capable of providing sufficiently accurate predictions for decision making.

This paper presents a gravity-distribution model for small- and medium-sized urbanized areas that can be calibrated by using trip-end information. The suggested calibration method eliminates the need of having extensive home-interview O-D data for the distribution stage of the trip forecasting procedure. Furthermore, the paper outlines how use of this distribution model eliminates the need for conducting an O-D study for trip forecasting in small- and medium-sized cities. The elimination of O-D surveys from transportation studies is based on 3 premises.

1. Estimates of trip productions and trip attractions of home-based work person

Selection of Distribution Function

The principal problem in developing a distribution model is the selection of the form of $F(C_{ij})$ and the quantification of the parameters of this function. The appropriate functional form of the distribution function has been the subject of many research efforts (1, 7, 13, 14). Both simple and complex functions have been suggested. However, examination of the nature of the small- and medium-sized cities reveals that using a complex distribution function for such areas is not necessary.

Because of the limited destination opportunities available to travelers in a small- or medium-sized urban area, travelers usually do not face a real choice among equivalent but locationally different opportunities. Therefore, the cost of reaching an opportunity cannot be a major factor in selecting a given opportunity by a class of travelers. Furthermore, because of the insignificance of the travel cost in such areas, travelers may not differentiate meaningfully among the cost of reaching different opportunities. A study by Zaryouni (15) shows that the consideration of travel cost provided a model only 7 to 10 percent more predictive than a gravity model with no consideration of travel cost [$F(C_{ij}) = 1$] for Billings, Montana (population 60,000), and Decatur, Illinois (population 110,000). It may be concluded that the consideration of travel cost has a marginal effect on the predictivity of a gravity model compared to trip-end information for small- and medium-sized urban areas; therefore, the predictivity of a gravity model cannot be very sensitive with respect to the functional form of the distribution function. This logical conclusion has been supported empirically by the Zaryouni study (15). The study (15) demonstrated that the inverse power and the negative exponential distribution functions provide practically the same goodness of fit for the gravity model as the more complex gamma function does. In this study, the distribution model results were compared with actual trip tables developed from traditional transportation surveys. The principal measure used as the criterion for the predictivity of a gravity model was the relative deviation d defined here as

$$d = \sum_i \sum_j (T_{ij} - S_{ij})^2 / S_{ij} \tag{5}$$

where S_{ij} = interzonal trip volume from actual survey. The lower the d is, the better the goodness of fit and the more predictive the model will be. The parameter or parameters of each distribution function are determined by using an iterative procedure to minimize d.

The minimum deviation d obtained for 3 different distribution functions is given in Table 1. The table shows that the minimum deviation is practically the same for the 3 functions tested. Therefore, the negative exponential function that is a more appropriate function for calibration purposes is suggested for use in small- and medium-sized urban areas. Equation 1, then, becomes

$$T_{ij} = r_i s_j \exp(-\beta C_{ij}) \tag{6}$$

Model Calibration

The calibration problem is to estimate parameter β. One way to find β is to equate average travel cost predicted by the model $\overline{C}_m$ to actual travel cost measured from actual O-D data (4, 11). However, to do so, one would need actual average travel cost, which, in turn, would require a more extensive O-D survey. Instead, here, the value of β will be estimated by deriving a relationship between β and $\overline{C}_m$ that can be solved iteratively for β.

The average travel cost predicted by a 2-way, constrained gravity model can be written as:

$$\overline{C}_m = \frac{\sum_i \sum_j T_{ij} C_{ij}}{\sum_i \sum_j T_{ij}} = \frac{\sum_i \sum_j r_i s_j C_{ij} \exp(-\beta C_{ij})}{\sum_i \sum_j r_i s_j \exp(-\beta C_{ij})} \tag{7}$$

The product $r_i s_j$ is a function of the average travel cost of all trips that originated at i and the average travel cost of all trips that ended at j. Being a function of an overall average, the $r_i s_j$ dependency on a particular value of C_{ij} is not significant when the number of zones is relatively large (5). Therefore, $r_i s_j$ can be replaced by its expected value and can be taken outside the summation sign and cancelled from the numerator and the denominator. Equation 7 may then be written and reduced to

$$\overline{C}_m = \frac{\int_0^\infty C \exp(-\beta C) dC}{\int_0^\infty \exp(-\beta C) dc} = \frac{1}{\beta} \tag{8}$$

Equation 8 suggests a procedure for estimation of β. The value of β should be selected so that $\overline{C}_m$ equals $1/\beta$. This value of β can be obtained by using an iterative procedure. For each selected β, a trip table is computed by a 2-way, constrained gravity model. Then, the $\overline{C}_m$ associated with the selected β is computed from

$$\overline{C}_m = \frac{1}{T} \sum_i \sum_j C_{ij} T_{ij} \tag{9}$$

By using this procedure for a range of values of β, one can plot $\overline{C}_m$ against β. The intersection of the $\overline{C}_m$ curve and $1/\beta$ curve gives the optimum value of β. Figures 1 and 2 show the calibration method for the cases of Billings, Montana, and Decatur, Illinois. In these figures, the d associated with each value of β also is shown. The figures demonstrate that the value of β determined by the suggested method provides practically the minimum d for the gravity model.

A SYNTHESIZED MODEL FOR TRIP FORECASTING

This section, by using the result of the previous section, will demonstrate the possibility of trip forecasting without conducting an extensive O-D study. The aim is to suggest not a detailed procedure but a general outline. Published census tract data of 1970 provide many useful data other than those that will be mentioned. Indeed, other studies are being conducted to analyze the potential of census data for urban transportation planning (12). These research efforts are more detailed and often require significantly more manipulation and adjustment of the data. When this experience in estimating trip-generation rates from census studies is available this information should be exploited and, where appropriate, the more detailed procedures should be used.

The proposed synthesized model may be thought of in 4 parts:

1. Production of and attractions for home-based work (HBW) trips,
2. Interzone trip exchanges for HBW trips,
3. Assignment of HBW trips, and
4. Computation of peak-hour volume (PHV) and average daily traffic (ADT) for each major link.

Table 1. Minimum deviation and corresponding parameters for selected distribution functions.

Distribution Function	Billings, Montana			Decatur, Illinois		
	d	α	β	d	α	β
$F(C_{ij}) = C_{ij}^{-\alpha}$	37,026	-1.10		34,489	-1.0	
$F(C_{ij}) = \exp(-\beta C_{ij})$	36,504		0.11	34,305		0.09
$F(C_{ij}) = C_{ij}^{\alpha} \exp(-\beta C_{ij})$	36,474	-0.2	0.09	34,233	-0.4	0.06

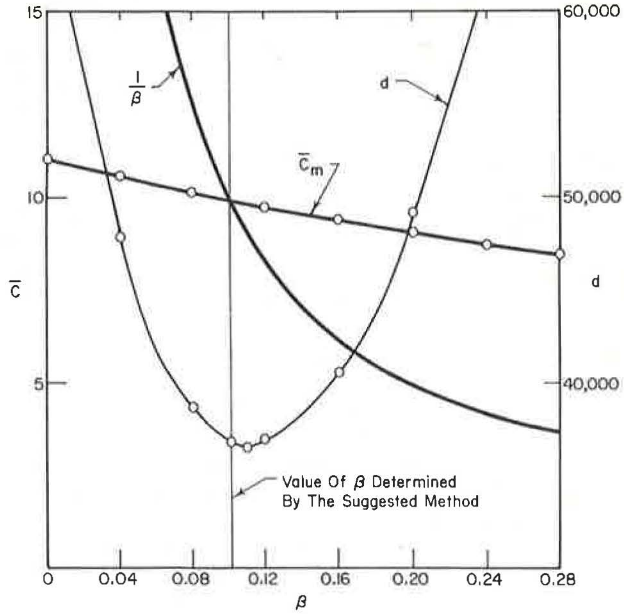

Figure 1. Model calibration relationships for Billings, Montana.

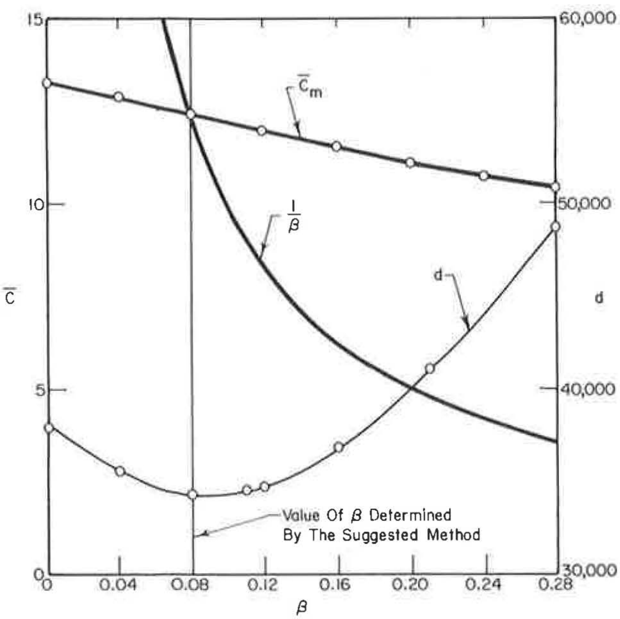

Figure 2. Model calibration relationships for Decatur, Illinois.

Production of and Attractions for Home-Based Work Trips

An HBW trip is defined as 1 trip/employee/day that originates from the place of residence and is destined for the place of work of each employee. Clearly, the total number of HBW trips for the whole study area is the same as the total number of employed people who live and work within the area.

Published census tract data provide the number of employed people who live in each census tract. This number is assumed to be the same as the number of trip productions for the HBW trips. Furthermore, census tract data of 1970 distinguish between those employees who work outside and those who work inside the standard metropolitan statistical area (SMSA). This distinction makes it possible to obtain the internal trip production for the HBW trips more accurately if the SMSA is selected as the study area. Also, in the census, the number of people employed in their residences is given for 15 industries. Therefore, stratification of the trip productions is possible.

If 1 HBW trip/employee is assumed, then the number of trip attractions for the work zones would be the number of employees in the work zone. Although the 1970 census recorded employee work address, the level of detail and accuracy is questionable. Employment location data may be supplemented, however, from other data sources such as state employment security records and the major employers. These government and private data sources provide an opportunity to stratify the attraction component of the HBW trips and to obtain information regarding work trips produced by employees outside the SMSA.

Because the production and attraction components of the HBW trips usually are not provided by the same source, the data components may not be consistent with each other. Some modification may be needed to make these components consistent. Rather than use the zone productions and attractions directly from these sources, one might be better advised to estimate the total number of HBW trips in the entire area first and then to compute the distribution of trip ends from the available data sources. Hence T equals the total number of people who live and work within the area and could be obtained now from existing data sources and later from lane use and economic forecasting. Then, if $(ER)_i$ is the total number of employed people who live at zone i and $(EW)_j$ is the total number of employed people who work at zone j, let

$$u_i = \frac{(ER)_i}{\sum_i (ER)_i} \tag{10}$$

$$v_j = \frac{(EW)_j}{\sum_j (EW)_j} \tag{11}$$

Then the production and attraction components of HBW trips become, respectively,

$$P_i = Tu_i \tag{12}$$

$$A_j = Tv_j \tag{13}$$

If the trip-end data base has been stratified according to industry, this procedure may be carried out for each industry separately.

Interzone Trip Exchanges for Home-Based Work Trips

Based on knowledge of the trip productions and attractions from the previous step, a 2-way, constrained gravity model with the negative exponential distribution function provides an interzone-trip-exchange model for the HBW trips:

$$F(C_{ij}) = \exp(-\beta C_{ij})$$

β is computed according to the previous section of this paper.

When the data have been stratified according to industries, a separate distribution choice for each industry should be followed. The results then must be added into a single interzone trip matrix for the next step.

Assignment of Home-Based Work Trips

Based on knowledge of interzone HBW trips from the previous step, one can determine the routes that travelers use when going from one zone to another and assign HBW trips to the street network on the basis of route choice. This step could be done by any existing assignment model, including the simple method of judgment.

Computing Peak-Hour Volumes and Average Daily Traffic for Each Major Link

Other researchers have examined the work trip to evaluate peak-hour and daily travel patterns (7, 10). Shunk, Grecco, and Anderson (10) have shown that a strong linear relationship exists between the HBW trips passing through a major street and the PHV and ADT of that major street. For link ℓ it may be written

$$(PHV)_\ell = K_1 \times (HBW)_\ell \tag{14}$$

$$(ADT)_\ell = K_2 \times (HBW)_\ell \tag{15}$$

or more accurately,

$$(PHV)_\ell = a + b(HBW)_\ell \tag{16}$$

$$(ADT)_\ell = a' + b'(HBW)_\ell \tag{17}$$

Some actual vehicle counts in major streets should be made (existing traffic maps may provide ADT information as well), and the PHV and ADT should be computed. Next, with the corresponding HBW from the previous step, K_1 and K_2 can be computed:

$$K_1 = \frac{1}{N} \sum_{\ell}^{N} \left(\frac{PHV}{HBW}\right)_\ell \tag{18}$$

$$K_2 = \frac{1}{N} \sum_{l}^{N} \left(\frac{ADT}{HBW}\right)_l \tag{19}$$

If the second more complex model is used, a, b, a′, and b′ can be derived by using an appropriate regression analysis. It should be noted that for each type of major street (freeway, arterials, and collectors) a different set of indexes preferably should be developed. Also the values of K_1 and K_2 or a, b, a′, and b′ derived for the present must be assumed to prevail in the future unless data forecasting a change are available.

SUMMARY AND CONCLUSION

The principal objective of this research was to evaluate trip-distribution models that can be calibrated with minimal travel-survey data and can be used to estimate travel patterns in small- and medium-sized urban areas. The 2-way, constrained gravity model was found to be relatively insensitive to the functional form of the distribution function for the cities studied. Therefore there is no need for oversophistication in the form of the distribution function. The negative exponential function is simple but not meaningfully less predictive than other complex functions. The calibration method for the model is based on analytical deductions. The primary advantage is that the interzone travel patterns can be estimated with limited travel-survey data.

The model is proposed to be used where trip-generation and trip-attraction estimates can be obtained primarily from census data. Although the validity of the distribution model has been tested, the total synthetic modeling approach still must be examined in an initial study.

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