

VARIABILITY OF INITIAL SERVICEABILITY AS A PAVEMENT DESIGN PARAMETER

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The need to be able to design for a specific level of reliability and account for the inherent variations in the different design factors has been recognized in many areas of design. Basic stochastic design theory was applied to the structural subsystem of the Texas flexible pavement design system by Darter and Hudson, and this allowed the designer to determine the reliability of a specific design or to design for a specific level of reliability. So that this added feature of the system could be used, estimates of the variations associated with the different design factors were needed. A study was made to quantify the variations associated with the initial serviceability index of flexible pavements. The study was designed to obtain better estimates of expected average values and the variations of this design factor. A total of 21 newly constructed pavements were measured by using the surface dynamics profilometer. Most of the models used in the flexible pavement system are empirical equations based on data collected from test sections and in-service pavements. There is a certain lack of fit associated with the equations, and this lack of fit has to be estimated to determine the variance of a design. The lack of fit associated with the performance equation used in the flexible pavement system is believed to be quite large because of wide variations in traffic, environmental conditions, and materials in Texas. A method for determining the lack of fit of the performance equation is also presented.

• **CONSIDERATION** of pavement design as a stochastic process is important to the development of more rational design methods. This was selected as one of the most important research needs in the field of pavement research by the Workshop on the Structural Design of Asphalt-Concrete Pavement Systems in 1970 (1):

So that designers can better evaluate the reliability of a particular design, it is necessary to develop a procedure that will predict variations in the pavement system response due to statistical variations in the input variables, such as load, environment, pavement geometry, and material properties including the effects of construction and testing variables. As part of this research, it will be necessary to include a significance study to determine the relative effect on the system response of variations in the different input variables.

Kher (14) and Kher and Darter (15) did the initial research on developing concepts of pavement reliability. Their work has been a basis for the extensions to theory presented in this paper. Other basic concepts are presented by Darter, Hudson, and Haas (16).

The latest development in applying stochastic design concepts to pavement design systems was done by Darter and Hudson (2). In this work, stochastic design concepts

were applied to the structural submodel of the Texas flexible pavement design system (FPS).

The Texas FPS is a computerized working system containing several submodels. Stochastic design theory has been applied to one of these submodels, the structural subsystem. Basically, the structural subsystem consists of three models: (a) the performance model, (b) the deflection model, and (c) the traffic model, which combined prediction of the performance or serviceability loss with time of a pavement design. Darter and Hudson (2) applied basic stochastic design theory to the structural submodel enabling the designer to evaluate the reliability of the design.

Darter and Hudson (2) state the purpose of applying stochastic concepts to pavement design as follows:

The underlying reason for formulating a probability-based design procedure is to make the design process responsive to the actual existing variabilities and uncertainties associated with the design, construction, and performance of flexible pavement.

To be able to evaluate the reliability of a design or to design for a certain reliability level, one must predict the variations of the design outputs due to the variations in the design inputs.

In addition, we discuss some of the basic concepts developed by Darter and Hudson (2) and the lack of fit of the performance equation in the FPS along with the results of a study that was done on quantifying means and variations associated with the initial serviceability index, one of the input factors to the FPS.

RELIABILITY OF PAVEMENT DESIGN

Darter and Hudson (2) defined the reliability R of a pavement design as the probability that the allowable load applications of the pavement-subgrade system N will exceed the traffic loads to be applied n . This is compatible with the statement that reliability is the probability that the serviceability level of pavement will not fall below the minimum acceptable level before the performance period is over:

$$R = P(N > n) \quad (1)$$

Both allowable applications N and traffic loads to be applied n depend on many factors. Allowable load applications N depend on such factors as pavement thickness T , material properties P , and environment E :

$$N = f(T, P, E, \dots) \quad (2)$$

The estimate of the number of traffic loads that will be applied to a pavement during a certain time period depends on such factors as average daily traffic A , percentage of trucks T , axle load distribution L , and equivalency factors F :

$$n = f(A, T, L, F, \dots) \quad (3)$$

Results from the AASHO Road Test indicate that both N and n are approximately log-normally distributed (2, 3). The quantity n is also believed to be approximately log-normal, although further research is needed in this area (2). If lognormal distribution

is assumed and statistical theory is applied, the following relationship gives an upper confidence for design value for $\log \bar{N}$ that includes a safety margin for unpredictable variations in N and n (2):

$$\overline{(\log N)} = \overline{\log n} + Z_R \sqrt{S_{\log N}^2 + S_{\log n}^2} \quad (4)$$

where

$$\begin{aligned} \overline{(\log N)} &= \text{design value of the average of the log of number of 18-kip (80-kN) single-axle load applications to be used for design at level of reliability } R, \\ \overline{\log n} &= \text{average of the log of the traffic forecast of 18-kip (80-kN) single-axle load applications,} \\ Z_R &= \text{standardized normal deviate from normal distribution tables with a mean of zero and a variance of one for given level of reliability,} \\ S_{\log N} &= \text{standard deviation of } \log N, \text{ and} \\ S_{\log n} &= \text{standard deviation of } \log n. \end{aligned}$$

This reliability function can be used either to design a pavement for a specific reliability level or to analyze the reliability of a given pavement design.

Reliability was defined as the probability that N exceeds n :

$$R = P[(\log N - \log n) > 0] \quad (5)$$

or

$$R = P(D > 0) \quad (6)$$

where $D = \log N - \log n$.

The function $f(D)$ is called the difference density function of $\log N$ and $\log n$, and since $\log N$ and $\log n$ are both normally distributed, D will be normally distributed also. Function $f(D)$ is shown in Figure 1. If bars are used above the expressions to represent their mean value, the mean value of the difference can be written as $\bar{D} = \overline{\log N} - \overline{\log n}$.

The variance of the difference between two functions is given as the sum of the variances of the two functions. Hence the standard deviation of D , S_D , can be computed by the following equation:

$$S_D = \sqrt{S_{\log N}^2 + S_{\log n}^2} \quad (7)$$

where

$$\begin{aligned} S_{\log N}^2 &= \text{variance of } \log N, \text{ and} \\ S_{\log n}^2 &= \text{variance of } \log n. \end{aligned}$$

As shown in Figure 1, the reliability is given by the area to the right of zero:

$$R = P(D > 0) = \int_0^{\infty} f(D) dD \quad (8)$$

or

$$R = P[0 < (\log N - \log n) < \infty] = P(0 < D < \infty) \quad (9)$$

The relationship between D and the standardized normal variable Z is given by

$$Z = \frac{D - \bar{D}}{S_D} \quad (10)$$

for

$$D = 0, \quad Z = Z_0 = -\frac{\bar{D}}{S_D} = -\frac{\overline{\log N} - \overline{\log n}}{\sqrt{S_{\log N}^2 + S_{\log n}^2}} \quad (11)$$

and for

$$D = \infty, \quad Z = Z_\infty = \infty \quad (12)$$

By evaluating Z for the limits of D , equation 9 can now be written as

$$R = P(Z_0 < Z < Z_\infty) \quad (13)$$

and the reliability can be easily determined by means of the normal distribution table.

Estimates of the average values and the variance associated with $\log N$ and $\log n$ must be determined if a specific level of reliability is to be designed for. Since both N and n depend on several factors, the total variances of each depend on the variance of each of the several factors involved. This study was aimed at developing a technique to determine the variation in $\log N$ caused by the lack of fit of the performance equation and at collecting field data to determine average values and the variance of the initial serviceability index, which is one of the inputs to the equation for predicting $\log N$.

TEXAS FLEXIBLE PAVEMENT DESIGN SYSTEM

The Texas FPS was developed for the Texas State Department of Highways and Public Transportation. The FPS resulted from 7 years of work to apply AASHTO Road Test results to the Texas design system (4). The FPS has been computerized to provide an output of feasible pavement designs sorted by increasing total cost. The primary purpose of FPS is to provide the designer with a means to systematically and efficiently investigate various pavement design options (5).

The FPS consists of several submodels:

1. The structural subsystem consists of a traffic, a deflection (or a material-pavement characterization), and a performance model;
2. The safety subsystem is restricted to the skid resistance of the pavement surface;
3. The user-delay model calculates the cost to the user in case of overlay;
4. The economic subsystem calculates the total cost of the project through its design

life, accounting for initial cost, overlay cost, routine maintenance cost, user cost, and salvage value; and

5. The overlay design model calculates the necessary overlays for rehabilitation of the pavement (6).

Stochastic design theory has been applied to the structural subsystem of the FPS only. This paper also discusses that part of the structural subsystem that includes the performance model and the variance associated with the different design inputs to the model.

Performance Model

The performance equation is based on the present serviceability concept developed at the AASHO Road Test (7), and it predicts the loss in serviceability depending on deflection of the pavement structure, the number of load applications, temperature, and foundation movements due to swelling clay. The effect of swelling clay is not considered in this study. The performance equation for the Texas FPS is as follows:

$$\log N = \log Q + \log \alpha - 2 \log SCI - \log B + 6.0 \quad (14)$$

where

N = number of predicted equivalent 18-kip (80-kN) single-axle load applications the pavement can take for one performance period;

$Q = \sqrt{5 - P_2} - \sqrt{5 - P_1}$, the function of serviceability loss;

P_1 = estimated initial serviceability index for the type pavement considered;

P_2 = minimum acceptable serviceability level;

α = temperature parameter, estimated from previous weather data;

SCI = surface curvature index, calculated by the deflection equation using estimated strength coefficients and thicknesses for the different layers; and

B = regression coefficient.

Equation 14 was derived by using data from the AASHO Road Test (8). $\log N$ is assumed to be normally distributed, and it has a certain variance associated with it at the design stage.

In its present form, there are four input variables in the performance equation, three of which are considered to be random. The three random variables are the initial serviceability index, the temperature parameter, and the surface curvature index. The fourth input, the minimum acceptable serviceability level, is a design constant, and, although it can be changed from one design to another, it cannot be considered a random variable.

Variance Model of Performance Equation

Since $\log N$ is assumed to be normally distributed, the function is uniquely defined by its mean and standard deviation. There are different methods available for determining the variance of a variable that is a function of several random variables. The method adopted in the FPS is called the partial derivative method, and it was selected because of the complexity of the equation and its relative ease and accuracy in adaptation. The random variables that are considered to influence the variance of $\log N$ are P_1 , α , and SCI . There is also an associated lack-of-fit error in the performance model itself, which contributes to the variation of $\log N$. When the partial deviative method is used, the variance of $\log N$ is given as

$$S_{\log N}^2 = \left(\frac{\partial \log N}{\partial P_1} \right)^2 S_{P_1}^2 + \left(\frac{\partial \log N}{\partial \alpha} \right)^2 S_{\alpha}^2 + \left(\frac{\partial \log N}{\partial \text{SCI}} \right)^2 S_{\text{SCI}}^2 + S_{\text{or}}^2 \quad (15)$$

where

- $S_{\log N}^2$ = total variance associated with log N,
 $S_{P_1}^2$ = variance of the initial serviceability index,
 S_{α}^2 = variance of the temperature parameter α ,
 S_{SCI}^2 = variance of SCI of the pavement-subgrade system, and
 S_{or}^2 = variance associated with the lack of fit of the performance equation.

Darter and Hudson (2) presented approximate estimates of these variances, but some of the estimates were crude because of limited data. The input factor for which there had been the least amount of data gathered was the initial serviceability index. A study was performed in which one of the goals was to obtain a better estimate of the variance of this input factor. The results from the study are described later.

Part of the variation associated with the performance equation is caused by the so-called lack of fit of the equation. The conventional way to quantify lack-of-fit error is to analyze repeat measurements or measurements taken on exact, similar specimens. In evaluating the lack-of-fit error of the performance equation, this method constitutes a problem because of the inability of the pavement engineer to build exactly similar pavement sections. Therefore, a different approach had to be taken, and a method by which the lack-of-fit error of the performance equation can be evaluated is presented in the following sections.

LACK OF FIT OF PERFORMANCE EQUATION

The performance equation used in the Texas FPS is an empirical equation based on data collected from in-service pavements and test sections. Part of the variation in the performance equation is caused by lack of fit. The lack of fit represents the inability of the design model to predict exactly the results when actual average values of all the design parameters are known. The basic reasons for lack of fit are as follows:

1. The model does not contain the proper design factors or
2. The design factors used are not in proper combination within the model.

Evidence from several studies that the residual variance of log N [the variance of log (actual) minus log (predicted)] is on the order of 0.0025 to 0.0453 is given elsewhere (2, pp. 91-99). Darter and Hudson collected data on six flexible pavement projects in Texas and estimated the number of load applications carried since the pavements were opened to traffic by using the performance equation and then compared these figures with actual applied load applications. Two of these projects had untreated base materials, and the difference between predicted and actual applied loads was relatively small. In the four other projects the base materials were treated with lime, cement, or asphalt. For all these projects, the performance equation predicted a much higher number of load applications than were actually applied. The performance equation is very sensitive to the SCI of the pavement-subgrade system because of its exponent. Pavements with treated base materials result in a stiff structure with low SCI values, causing log N to be large. In addition, it appears, based on the limited amount of data, that the lack of fit associated with the performance equation is larger for pavements having treated base materials than for those with untreated materials. The following concepts are outlined by Kher and Darter et al. (2, 14, 15, 16).

Model for Determining Lack of Fit

Several representative flexible pavement projects will have to be selected throughout Texas to quantify the lack of fit of the performance equation. The projects must be in their first performance period (must not have been overlaid), and the following data will have to be collected for separate 1,200-ft (366-m) or 0.2-mile (0.3-km) sections:

1. The present serviceability index, measured with the surface dynamics profilometer or Mays road meter,
2. The surface curvature index, measured with the Dynaflect, and
3. The number of 18-kip (80-kN) equivalent load applications that have passed over the section since construction.

For every project, the initial serviceability index should be estimated. The results from the study presented later in this paper should be used for estimating initial serviceability where measurements have not been taken. The temperature parameter is also needed. The average temperature parameter has been estimated for each highway department district headquarters in Texas (8), and these estimates should be used.

When the serviceability index, surface curvature index, and the estimated initial serviceability and temperature coefficient have been measured, the total number of 18-kip (80-kN) single-axle load applications can be calculated by using the following equation:

$$\log M = \log (\sqrt{5 - \text{PSI}} - \sqrt{5 - \text{PI}}) + \log \alpha - 2 \log \text{SCI} - \log B + 6.0 \quad (16)$$

where

- M = estimate of total number of 18-kip (80-kN) single-axle load applications that have passed over the section since it was opened to traffic,
 PSI = measured present serviceability index,
 PI = estimated initial serviceability index for that particular type of pavement,
 SCI = surface curvature index measured with the Dynaflect,
 α = estimated temperature parameter, and
 B = regression coefficient.

Log M should not be confused with log N, which is an estimate of the total number of 18-kip (80-kN) equivalent single-axle loads for a project at the design stage at which all the design inputs have been estimated.

The certain variation associated with log M is caused by estimation error of PI and α , measurement error of PSI and SCI, within-section variance of SCI, and error due to lack of fit of the equation. This can be written as

$$\begin{aligned} \text{Var} [\log M] = & \text{Var} [\log (\sqrt{5 - \text{measured PSI}} - \sqrt{5 - \text{PI}})] \\ & + \text{Var} [\log \alpha] + \text{Var} [2 \log \text{SCI}] + \text{Var} [\text{LOF}] \end{aligned} \quad (17)$$

Equation 16 is the same as the performance equation explained previously, except that the inputs differ from those used at the design stage. Some of the inputs to equation 16 are measured in the field, thus reducing some of the estimation (or pure) error introduced when all the parameters have to be estimated. The error introduced by lack of fit is the same in both cases, and by solving equation 17 for the lack-of-fit term, the lack of fit of the performance equation at the design stage should be obtained.

The estimated load applications obtained from equation 16 should then be compared with the actually applied 18-kip (80-kN) single-axle loads m, which can be obtained from the Planning Survey Division, Texas State Department of Highways and Public

Transportation. Most likely there will be a difference (error) between the two figures. Both $\log M$ and $\log m$ are assumed to be normally distributed, and the error will therefore be normally distributed and have a variation associated with it. The variance of error is due to the variance of $\log M$ plus the variance of $\log m$ and can be written as

$$\text{Var} [\log M - \log m] = \text{Var} [\log M] + \text{Var} [\log m] \quad (18)$$

or

$$\text{Var} [\log M] = \text{Var} [\log M - \log m] - \text{Var} [\log m] \quad (19)$$

There are now two equations for the variance of $\log M$, which are set to be equal, and the equation can be solved for the variance due to the lack of fit as follows:

$$\begin{aligned} \text{Var} [\text{LOF}] = \text{Var} [\log M - \log m] - \text{Var} [\log m] - \text{Var} [\log \alpha] \\ - \text{Var} [2 \log \text{SCI}] - \text{Var} [\log (\sqrt{5 - \text{measured PSI}} \\ - \sqrt{5 - \text{PI}})] \end{aligned} \quad (20)$$

Data for Quantifying Lack of Fit

Many data must be collected to quantify the lack-of-fit error associated with the performance equation in the FPS, and a large-scale study would be required to account for all the possible sources of variation in the data. The type and amount of data needed for solving equation 20 will be collected through the pavement feedback data system (PFDS) currently being developed and initiated in Texas.

STUDY OF INITIAL SERVICEABILITY INDEX OF FLEXIBLE PAVEMENTS

The initial serviceability index (SI) of a pavement section is a function of the road profile; the smoother the road is, the higher the serviceability index will be. The index is a measure of the pavement's smoothness or riding quality immediately after construction. Since it is not possible to build a perfectly smooth pavement, the serviceability index is always below the ultimate level of 5.0.

Currently, the initial serviceability index is only indirectly controlled in construction. Specification control for roughness usually includes a criterion of approximately $\frac{1}{8}$ in. in 10 ft (3.2 mm in 3.1 m). The pavement surface may often be within this specification criterion but have a relatively low serviceability index because of longer wavelengths, which affect vehicles moving at high speeds. Designers, therefore, have to estimate an expected average value for initial serviceability index in design. The estimates currently being used are based on AASHO Road Test overlays, and one of the goals of this study, therefore, was to obtain better estimates of average serviceability index expected for newly constructed flexible pavements.

The initial serviceability index is an important design parameter because it designates the starting point for the performance curve of a pavement design. If a pavement is built with a lower serviceability index than that assumed in design, the pavement might reach its terminal level before expected; on the contrary, if it is built smoother than assumed, it might last longer than expected. This concept is shown in Figure 2, where T_L is the predicted service life for a low estimate of the initial serviceability

index, T_A is the actual service life, and T_H is the predicted service life for a high estimate of the initial serviceability index (Figure 2 is not drawn to scale).

The serviceability index is measured over a specific length of pavement, usually 1,200 ft (366 m). Serviceability might vary from section to section along a paving job and also among different pavement projects. The serviceability index has previously been assumed to be a normally distributed variable, and one of the goals of this study was to test this assumption. Perhaps more important than the distribution is the magnitude of the variation associated with the various design parameters. Another goal for this study, therefore, was to quantify the variation associated with the serviceability index of newly constructed pavements.

Therefore, the main goals set for the study were as follows:

1. To obtain reasonable estimates of expected values of initial serviceability index for use in future design calculations,
2. To test the assumption that the initial serviceability index is normally distributed, and
3. To estimate the variance of initial serviceability index so that the variation of this design factor can be accounted for in the FPS.

In addition to these goals, factors influencing the initial smoothness of a pavement were to be studied to the extent that such influence is explained by the obtained data.

Experimental Design

One of the first questions that arose when this study was initiated was, How many data are required to give reasonable estimates of average values and the variance associated with the initial serviceability index? Assuming the serviceability index is normally distributed, the number of projects required to obtain an adequate estimate of the average serviceability level can be calculated at a given confidence level, given an estimate of the variance associated with the serviceability index. Similar studies conducted elsewhere in the United States were consulted to obtain a reasonable estimate of between-project variation of the initial serviceability index.

In a study in Utah, the initial serviceability index was measured on 76 sections, each 0.2 mile (0.3 km) long and each located on a different project (9). The standard deviation of the serviceability index was calculated to be 0.2. Because only one section was measured on each project, it was impossible to break the variation of the data into between-project and within-project variation, and the calculated variation therefore represents the total variation of the serviceability level on new flexible pavements in Utah.

In a similar study in Minnesota, the serviceability index was measured on 38 newly constructed flexible pavement sections, each 0.2 mile (0.3 km) long and each located on a different project (10). The standard deviation of the serviceability level was calculated to be 0.28, and, as in the Utah study, it represents the total variation of the serviceability index. Some of the difference in variation of the data collected in the two studies might be due to different types of measuring equipment and greatly unequal sample sizes used.

Based on the results from Utah and Minnesota, an 0.2 SI unit was taken as a reasonable first estimate of the standard deviation of average initial serviceability index of the project on flexible pavements in Texas. The number of projects required in the study was then calculated to be 10 with 90 percent confidence that the obtained mean serviceability index would be within the limits of ± 0.1 SI unit of the population mean. Because of the relatively large difference in design and construction methods between hot-mixed asphalt concrete pavements and surface-treated pavements, it was decided that these two types should be analyzed separately: Data were collected from at least 20 projects, 10 hot-mixed and 10 surface-treated projects.

Equipment and Measurements

The surface dynamics profilometer (11) was used for measuring the surface profile. The data were first stored on analog tape. They were then processed on an analog-to-digital computer at the Texas State Department of Highways and Public Transportation and stored on magnetic digital tapes (12). These tapes were then placed on the CDC 6600 computer at the University of Texas at Austin, and the serviceability index computer program was run on the data to obtain serviceability index readings for individual consecutive 1,200-ft (366-m) pavement sections. The surface dynamics profilometer is quite repeatable and stable against change in vehicle characteristics, but in this study, to allow for error, all projects were run twice. Nonrepeatability was found on only a few sections on two of the projects; this was due to minor mechanical or electrical problems with the profilometer. When the deviation between two runs exceeded 0.4 SI unit on any 1,200-ft (366-m) section, the section was excluded from the analysis. The surface dynamics profilometer is quite sensitive to changes in road alignment and profile. During the data collection process, notes were taken about the roadway geometry, and sections having sharp curves or sharp changes in profile were also excluded from the analysis. Because of these limitations, the number of useful sections within different projects varied, and this is the reason the results from only a few sections on some projects could be used in the final analysis of the data.

Normality Assumptions

The normal distribution is a convenient distribution to work with in stochastic design, and the initial serviceability index has previously been assumed to be normally distributed without too many data to support the assumption. For a test of this assumption, the serviceability values from the 10 hot-mixed asphalt projects were put in one group, and those from the 11 surface-treated projects in another group. The total number of 1,200-ft (366-m) sections measured on the hot-mixed projects was 113, ranging from 3 sections on project 2 to 17 sections on project 7. On the surface-treated pavements, a total of 144 SI values were obtained, ranging from 7 measured sections on project 7 to 17 sections on project 6. The data are given in Table 1.

It should be noted that the samples include several sections from each of a random sample of projects, and not, strictly, a random sample of pavement sections. The effect should be to broaden the peaks of the distributions, since each sample includes data clustered around means of subsamples (a subsample being the data for one project).

Figures 3 and 4 show the histograms of the serviceability index for the hot-mixed projects and the surface-treated projects respectively. A visual analysis of the distributions to test the hypothesis that the samples come from normally distributed populations may be aided by three statistical tests available: (a) the chi-square goodness-of-fit test to judge the normality assumption; (b) the skewness test to determine whether the data are significantly skewed to one or the other side; and (c) the kurtosis test to show whether the distribution is too peaked or too flat-topped (13).

The distributions were tested at a 5 percent confidence level, and neither of the two distributions could be rejected for any of the tests except for the hot-mixed distribution, which had to be rejected for the kurtosis test, indicating a too flat-topped distribution. However, the distribution was also tested at the 1 percent confidence level at which it could not be rejected for any of the three tests. As a result of these tests, it can be concluded that the assumption that the initial serviceability index comes from a normally distributed population is valid.

Hot-Mixed Asphalt Concrete Projects

In the analysis of 10 projects, 113 sections 1,200 ft (366 m) long were included, but, because the projects were of different lengths and some sections had to be omitted because of the abrupt roadway geometry or the presence of bridges, the number of sections

Figure 1. Difference density function.

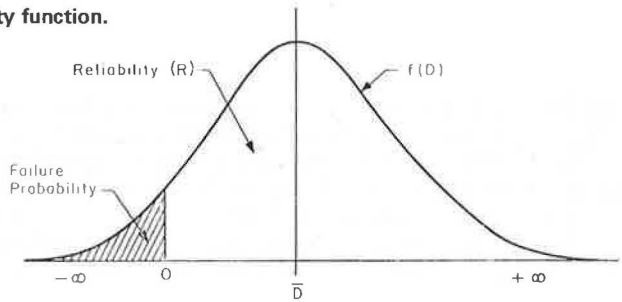


Figure 2. Influence of initial serviceability index on pavement performance.

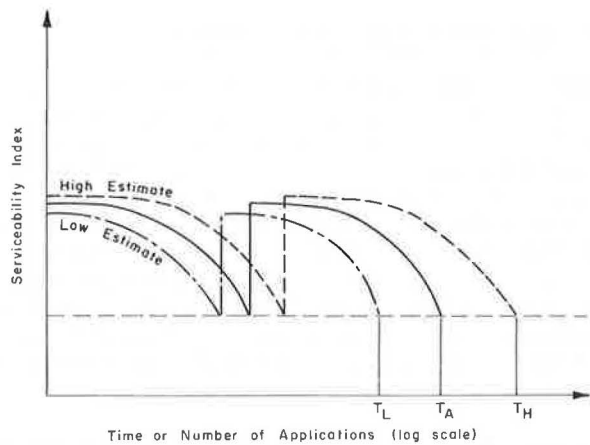
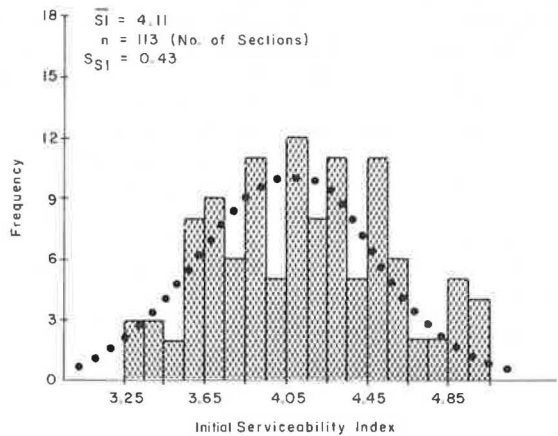


Table 1. Measured initial serviceability index on hot-mixed asphalt concrete pavement and surface-treated pavement.

Section	Hot-Mixed Asphalt Concrete Pavement Project										Surface-Treated Pavement Project										
	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	11
1	4.00	4.15	4.15	3.60	3.70	4.00	4.65	4.50	3.35	4.60	3.05	3.70	4.05	3.80	3.55	3.75	3.45	3.75	3.25	3.70	3.80
2	5.00	4.25	3.85	3.25	4.00	3.60	4.30	3.90	3.95	4.45	3.30	3.55	4.00	2.85	4.00	3.65	3.25	3.40	3.50	3.15	3.85
3	4.85	4.05	4.20	3.55	3.65	3.80	4.45	4.50	4.35	4.50	3.35	3.85	3.95	3.50	3.20	3.80	3.10	3.30	2.80	3.55	3.50
4	4.45	4.10	3.60	3.80	4.15	4.50	4.30	4.15	4.70	3.95	3.95	3.50	3.85	3.40	3.35	3.80	3.25	3.70	3.30	3.55	3.55
5	4.60	3.95	3.85	4.60	4.25	4.95	3.70	3.65	4.85	3.10	4.00	3.75	3.25	3.00	4.40	3.70	3.90	3.80	3.70	3.65	
6	5.00	3.90	3.80	4.80	4.10	4.05	3.85	4.10	4.50	3.15	3.80	3.80	3.45	3.20	3.60	3.15	4.05	3.30	3.70	3.35	
7	4.30	3.65	3.55	4.90	3.55	4.20	4.10	4.10	4.30	3.20	3.90	3.75	3.50	3.50	3.65	3.50	3.65	3.85	4.00	3.75	
8	4.40	4.05	3.55	3.90	3.90	4.25	4.45	3.60	4.45	3.75	4.10	3.55	3.70	2.80	3.55	4.15	4.00	3.95	3.25		
9	4.25	3.75	3.25	4.55	3.50	4.35	4.05	4.20	4.20	5.00	3.30	3.55	3.45	3.00	3.25	3.85	3.80	3.70	3.90	3.30	
10	4.10	3.65	3.50		3.80	3.90	3.40	3.70	3.05	3.70	3.85	3.70	3.85	3.35	3.55	3.55	3.55	3.15	3.10	3.70	
11		3.70	3.85		4.15	4.75	4.25	4.45	3.50	3.40	3.30	3.40	3.30	3.75	3.30	4.05	3.60	3.90		3.65	
12		3.85	3.30		3.90	3.80	4.85	3.55	3.55	3.90	3.60	3.15	3.65				4.60	3.45		3.35	
13		3.40			4.05	4.25	4.40	3.80	3.65	3.70	3.20	4.05						4.05		3.90	
14		4.40			4.10	4.20	4.85	3.70		3.90	3.25	3.65								3.70	
15						3.90	2.90	3.80		3.85	3.30	3.70								3.80	
16						4.55	3.20	3.85		3.20	3.85									3.90	
17						4.55															
Mean	4.51	4.15	3.90	3.52	4.21	3.92	4.33	4.15	3.94	4.54	3.35	3.65	3.83	3.50	3.29	3.79	3.29	3.79	3.54	3.63	3.59

Figure 3. Hot-mixed asphalt concrete pavement sections.



within each project varied. The average initial serviceability index of all the sections was calculated to be 4.11; however, the project mean ranged from a low of 3.52 to a high of 4.54. The standard deviation of the project average was calculated to be 0.3103, which is somewhat higher than was assumed. A summary of data for the 10 projects is given in Table 2. A one-way analysis of variance was run on the data, and the results are given in Table 3. The between-project mean square was calculated to be 1.1979, and when tested with the F-test it revealed significant variation between the projects at the 1 percent level.

Surface-Treated Pavement Projects

Table 2 gives a summary for the 11 surface-treated projects; a total of 144 sections 1,200 ft (366 m) long were measured, and the mean serviceability index was calculated to be 3.59. The project means ranged from a low of 3.29 to a high of 3.83, and the standard deviation of the project means was 0.20. A one-way analysis of variance was run on the data, and the results from this analysis are given in Table 3. The within-project mean square was calculated to be 0.0821, and the between-project mean square to be 0.4744. Both the within- and between-project mean squares were considerably lower than for the hot-mixed asphalt concrete projects, indicating a more uniform surface smoothness for surface-treated roads. The F-test showed the variation between the projects to be significant at the 1 percent level, and it was decided to consider some of the factors that might affect the initial serviceability index.

Factors Influencing the Initial Serviceability Index

Quite a variety of factors or combinations of factors can influence the smoothness of a new flexible pavement: type of design, laydown machine, roller equipment, experience of equipment operators, and type of control. A considerable effort would have to be spent if all possible factors were to be defined, characterized, and quantified. Since this was not one of the main goals of the study, it was not done. The only factor that was studied was type of design since these data were readily available at the Texas State Department of Highways and Public Transportation. On the surface-treated pavements, little correlation could be found between type of design and the initial serviceability index. However, on the hot-mixed asphalt concrete pavements, an accounting could be made for about 50 percent of the variation in the serviceability index from the number of asphalt material lifts or passes with the laydown machine and total thickness of asphalt materials.

CONCLUSIONS AND RECOMMENDATIONS

1. The initial serviceability index varies, depending on whether the pavement has a hot-mixed asphalt concrete surface or whether it has been surface-treated. Surface-treated pavements are generally rougher than hot-mixed asphalt concrete pavements.
2. The analysis of variance showed that there is a significant variation in the initial serviceability index of new flexible pavements. The variation could be broken into between-project variation and within-project variation. The within-project variation was about the same for both hot-mixed and surface-treated pavements. The between-project variance, however, was much larger for hot-mixed asphalt concrete pavements than for surface-treated pavements, indicating that surface-treated pavements are more uniform. This can probably be credited to less variation in design parameters (number of layers and thicknesses) for the surface-treated projects.
3. Serviceability readings collected on newly constructed pavements indicate a smoother surface as the number of passes with the laydown machine increases. It is therefore recommended that designers consider the trade-off between increase in cost

Figure 4. Surface-treated pavement sections.

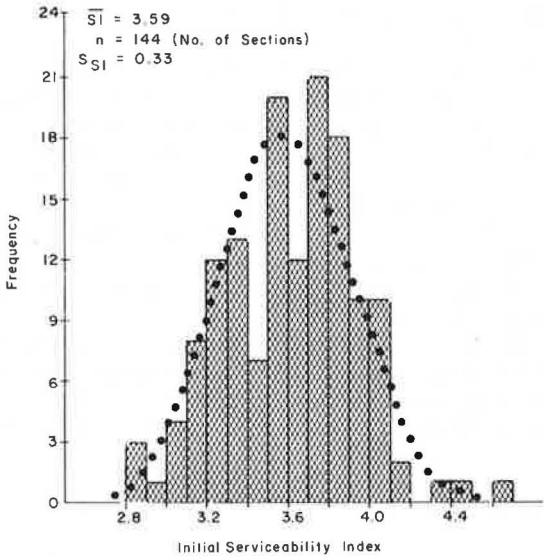


Table 2. Summary of data for 10 hot-mixed asphalt concrete and 11 surface-treated pavement projects.

Pavement Project	Project Number	Number of Sections	Project Mean	Standard Deviation	Coefficient of Variation
Hot-mixed asphalt concrete	1	10	4.51	0.34	7.61
	2	3	4.15	0.10	2.41
	3	14	3.90	0.26	6.78
	4	12	3.52	0.20	5.69
	5	9	4.21	0.50	11.80
	6	14	3.92	0.24	6.12
	7	17	4.33	0.31	7.26
	8	9	4.15	0.30	7.25
	9	11	3.94	0.33	8.31
	10	14	4.54	0.32	7.07
Surface-treated	1	12	3.35	0.28	8.49
	2	17	3.65	0.30	8.10
	3	13	3.83	0.22	5.77
	4	14	3.50	0.30	8.53
	5	15	3.29	0.27	8.09
	6	17	3.79	0.22	5.74
	7	7	3.29	0.32	9.69
	8	12	3.79	0.35	9.35
	9	13	3.54	0.38	10.62
	10	10	3.64	0.31	8.47
	11	14	3.59	0.21	5.93

Table 3. Analysis of variance of initial serviceability index on 10 hot-mixed asphalt concrete and 11 surface-treated pavement projects.

Pavement Project	Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F-Ratio
Hot-mixed asphalt concrete	Between projects	10.7812	9	1.1979	12.5032*
	Within projects	9.8683	103	0.0959	
	Total	20.6495	112		
Surface-treated	Between projects	4.7439	10	0.4744	5.7799*
	Within projects	10.9161	133	0.0821	
	Total	15.6600	143		

*Significant variation.

resulting from more passes with the laydown machine and the sacrifice in final smoothness of the surface with fewer passes.

4. A method for quantifying the lack of fit of the performance equation in the FPS is shown in equation 20. There now exists a need for collecting the data necessary for determining this error. Immediate needs are as follows: a detailed experimental design and selection of projects from which the data should be collected. Data should be collected from across the state, and the most convenient way to obtain these data would probably be through the PFDS.

5. Among the types of data needed for quantifying lack-of-fit error of the performance equation are deflection measurements. The surface curvature index needs to be quantified more accurately for different types of pavement structures, and an experimental design needs to be performed so that the necessary data needed for quantification can be obtained from data gathered for the lack-of-fit study.

6. Most types of data collected on pavements are stochastic in nature. To ensure that adequate data are collected through the PFDS currently being initiated in Texas, experimental designs and sampling plans should be set up.

ACKNOWLEDGMENT

This investigation was conducted at the Center for Highway Research, University of Texas at Austin. We wish to thank the sponsors, the Texas State Department of Highways and Public Transportation and the Federal Highway Administration, U.S. Department of Transportation. This work is an extension of basic work (2) done by Kher and Darter at the University of Texas at Austin.

The contents of this paper reflect the views of the authors, who are responsible for the facts and the accuracy of the data presented. The contents do not necessarily reflect the official views or policies of the Federal Highway Administration. This report does not constitute a standard, specification, or regulation.

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DISCUSSION

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Holsen and Hudson have done an excellent job of bringing forward some concepts of probabilistic design and lack-of-fit error inherent in empirical models. The first half of the paper deals with the uncertainties and variabilities of the Texas FPS model. Most of this material has been published previously (2, 15, 17) with the exception of the discussion involving the lack of fit of the performance equation. Lack of fit or inadequacy of this model (and, in general, of all statistically derived predictive models) is such an important concept in improving and developing design methods that a few additional comments seem appropriate.

For the sake of illustration, consider Figure 5 in which experimental data points are given by (X_1, Y_1) , (X_2, Y_2) , ..., (X_n, Y_n) . For a given value of X , say X_1 , there is a difference between the value Y_1 and the corresponding predicted value \hat{Y}_1 as determined from the regression curve. This difference is denoted by Δ_1 , which is referred to as a deviation, error, or residual and may be positive, negative, or zero.

A fitted regression line derived from a set of data by using various assumptions about its functional form cannot accurately reproduce observed data exactly. There is always some variation between the predicted values and the observed values.

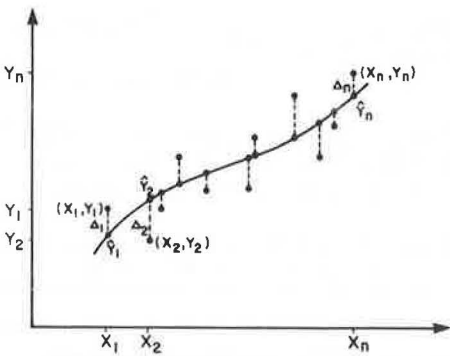
A measure of the total scatter of data about the regression line is called residual mean square s^2 and is given by the equation

$$s^2 = \frac{SS_R}{n - 2} \quad (21)$$

where

$n - 2$ = degrees of freedom, and
 SS_R = sum of squared residuals, for which

Figure 5. Regression line fitted to experimental data.



$$SS_R = \sum_{i=1}^{i=n} = \sum_{i=1}^{i=n} (Y_i - \hat{Y}_i)^2$$

(22)

SS_R is a prime indicator of how useful the regression line is as a predictor of data or the trend hidden or implied in the data.

In a carefully controlled experiment or a road test, total scatter of data about the regression line is generated by

1. Repeatability errors of measuring equipment (or testing errors),

2. Replicate errors due to differences within test sections or test samples, and

3. Model errors due to the model having an inadequate number of terms or incorrect combination of existing terms.

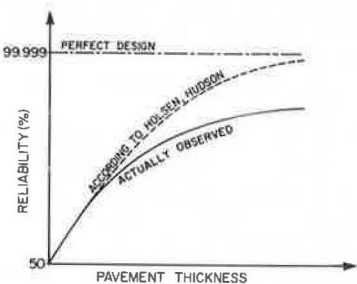
Items 1 and 2 give rise to a portion of total scatter called pure error; item 3 gives rise to the remaining portion called lack-of-fit error.

Pure error is basically the difference between the measured responses of replicate samples or test sections. It is caused by the inherent and apparently uncontrollable variabilities in parameters of the experiment, differences between other factors not controlled or not known to have an effect, testing variations, and equipment variations. Equipment measuring different parameters in a road test do not often reconfirm the observed readings and give different readings under different operating conditions. In addition, replicate sections of identical dimensions and materials often do not show the same performance. To substantiate the argument, the values of replicate and total errors for the flexible pavement experiment at the AASHTO Road Test (3) are given below:

Item	Mean	
	Total	Replicate
Residual Δ serviceability index (weighted)	0.46	0.46
Residual Δ load applications (weighted)	0.23	0.17

Lack-of-fit error is defined as a systematic bias or inadequacy, apart from the pure error, of the model to simulate the response or the trend implied in the experimental

Figure 6. Field-observed thickness versus reliability relationship as compared with that according to Holsen and Hudson.



data. If all the necessary terms affecting the response are not included in the model or if these terms are not in proper combinations, a considerable lack-of-fit error will result.

An additional source of error associated with empirical models occurs when the model is used for prediction under conditions different from the conditions for which it was developed. An example of this is the use of the predictive equations developed at the AASHO Road Test, which was conducted in different environments, soils, and traffic conditions. This error can be considered an additional lack-of-fit error for the purpose of this discussion.

The method used to obtain the best fit curve gives rise to a part of the lack-of-fit error especially when the curve is based on individual judgment rather than on a statistical procedure such as least squares analysis. In certain cases, however, a combination of individual judgment and statistical procedure has to be used to obtain a curve that best serves the purpose. This is especially true when the magnitudes of one or two observations are so much different from the rest of the data that they are weighting or biasing the least squared regression line because of their high or low magnitudes. In such cases, individual judgment has to be exercised to find the reasons for such unusual observations and also to ignore these unusual observations if the purpose of the regression is best served by such action.

Particularly, in regard to the Holsen and Hudson paper, equation 20 given to obtain the lack-of-fit error is incorrect since it assumes that the entire variability in response is attributed to lack-of-fit error. A term needs to be subtracted from the right side of this equation to account for the pure error so that a true value of the lack-of-fit can be obtained. It is suggested that the experiment, when designed to quantify the lack-of-fit error, should contain several replicate sections and repeat measurements so that the pure error can be quantified and accounted for.

It has been assumed in the paper that the model given by equation 16 is the correct performance prediction model. This assumption should not be blindly accepted but should rather be tentatively entertained. Since the mere existence of a lack-of-fit error in a model is due to inadequate terms (parameters) or an erroneous functional form of the equation, we cannot simply blindly assign the entire scatter of data to a black box called lack-of-fit error. Rather, in cases where lack-of-fit error is substantial, we should investigate the specific factors that bias the response. Additional terms, for example, would produce a better correlation and thus reduce the magnitude of this error since this error actually results in greater overall pavement uncertainty and cost.

Since various factors affect pavement performance, additional terms added to the model in the paper will surely reduce the lack-of-fit error. An additional factor that characterizes the environmental effect more appropriately (in this model only a temperature parameter has been used), an additional subgrade characterization factor, or an additional factor to characterize the pavement strength more appropriately than SCI alone are parameters worth considering to reduce the lack-of-fit error in the model. There has been an effort in Texas to achieve a better characterization of subgrade through the use of a swelling clay model. However, in this paper, this model has been ignored for stochastic analysis. It is suggested that the experiment, when designed to quantify the lack-of-fit error, should contain several sections on swelling clay foundations. The analysis conducted in this way will yield a smaller lack-of-fit error.

In fact, the inability of the model to adequately characterize the pavement strength becomes fairly obvious in the authors' description of the actual performance observed on pavements in Texas where stabilized base and subbase materials were used. Even when measured SCIs of these pavements were used in the model to predict the number of load applications to be carried by these pavements, the authors state,

For all these projects, the performance equation predicted a much higher number of load applications than were actually applied. . . . It appears, based on the limited amount of data, that the lack of fit associated with the performance equation is larger for pavements having treated base materials than for those with untreated materials.

This probably occurred because there were no stabilized bases included in the factorial experiment of the AASHO Road Test, from which the empirical equations were developed.

The performance model as described in the paper is based on the AASHO Road Test data, and therefore certain assumptions have been made to make use of the road test data in the development of this model. An obvious assumption is the correlation of Benkelman beam deflections measured at the AASHO Road Test to the Dynaflect deflections used in development of this model. Because of these assumptions and the fact that data developed in one set of environmental conditions and by one set of measuring procedures are being used to develop a model to be used in an entirely different set of environmental conditions, it is especially important that the model itself be scrutinized for additional factors instead of pooling all the differences in the unexplainable category of the lack-of-fit error.

If all or most of the additional terms needed in modeling pavement performance in Texas are included in the Texas model, the lack-of-fit error will be reduced to nearly zero. Since the lowest value that the total scatter error can be reduced to is the pure error, the only variability observed after that will be due to the inherent variability in material properties and other parameters of a pavement. These variabilities can then be reduced by better quality control of material properties, layer thicknesses, and compaction of layers.

The lack-of-fit error actually justifies additional research to investigate additional factors that might affect pavement performance. The magnitude of this error has a significant effect on pavement design. The greater the error (or uncertainty) in the model is, the greater the safety factor required for having adequate confidence in the design will be, and hence the greater the cost of such a design will be. For this reason, a large lack-of-fit error associated with an empirical model justifies extensive research to develop a more rational and complete design model.

Several assumptions are implicit in the mathematical expressions derived in the paper for the application of probabilistic theory in pavement design. These assumptions, scattered throughout the text of the paper, are statistically controversial, and an effort must be made to achieve a better understanding and a validation or modification of them. Some of the more important assumptions are pointed out in the following sections.

Reliability R of a pavement design is defined as the probability that the allowable load applications N of the pavement-subgrade system will exceed the traffic loads n to be applied:

$$R = P(N > n) \quad (1)$$

Holsen and Hudson further stated that "the assumption is compatible with the statement that reliability is the probability that the serviceability level of pavement will not fall below the minimum acceptable level before the performance period is over." The two definitions of reliability are only compatible when most of the loss in serviceability of the pavement is due to traffic loadings. In certain areas where service climatic conditions exist, much pavement deterioration will be due to climate, and hence this definition is not adequate.

It is assumed that the total residuals, including lack of fit and pure error, are all normally distributed random variables. This assumption may not be unreasonable when the data are fairly centered around the regression line but will be unreasonable if certain conditions result in extreme values as, for example, in the case of treated bases mentioned in the paper.

Holsen and Hudson state that

If lognormal distribution is assumed and statistical theory is applied, the following relationship gives an upper confidence for design value for $\log \bar{N}$ that includes a safety margin for unpredictable variations in N and n :

$$\overline{(\log N)} = \overline{\log n} + Z_R \sqrt{S^2 \log N + S^2 \log n} \quad (4)$$

It is fairly obvious from equation 4 that a design value for $\log N$, called $\overline{(\log N)}$, based on the confidence level, is being determined and used to predict pavement thickness requirements (for example, if actual traffic $n = 5$ million, $S^2 \log N + S^2 \log n = 0.35$, and confidence level is 95 percent, N will be equal to 47 million; i.e., pavement will be designed for 47 million applications). This approach, although it will end up with beefed-up thicknesses, has drawbacks as discussed below.

Greater thicknesses is by no means an answer to obtaining higher reliability. There are examples on North American highways where extremely high pavement thicknesses have been used, but a considerable distress has still been observed. In fact, greater thicknesses of materials are a complete waste after a certain pavement structure thickness is reached. As shown in Figure 6, reliability increases as thickness increases up to a certain point, and beyond that, any increase in reliability can only be achieved by better materials or quality control rather than by increased thickness (i.e., thicknesses will be unable to compensate for poor materials or poor quality control). The approach used in the paper (dotted line in Figure 6) seems contrary to this fact since it assumes that any (or even the highest) reliability can be achieved if the pavement thickness is

increased in accordance with the calculated $\overline{\log N}$ value.

Since the performance model is based on AASHO data that were observed for slightly more than 1 million applications, designing a pavement thickness for millions of applications (to obtain beefed-up thicknesses) by using the extrapolation of an equation that is only based on 1 million applications needs further validation. Statistically, reliance on such a prediction would be doubtful.

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AUTHORS' CLOSURE

We wish to thank the discussants for their review of the paper. They have presented some valuable suggestions for future research in this study area, and we think that it is appropriate to give specific responses to some of their comments. We are sorry the reviewers think the paper is redundant, but we believed it necessary that a brief, referenced literature review of concepts already developed in applying stochastic design to highway pavements be presented to provide the reader with an understanding of the continuation of theory presented in the paper.

We fully agree with the comments regarding the importance of differentiating between pure and lack-of-fit errors and the suggestions for estimating pure error in future experiments. However, we think that the pure error term is represented in equation 20: The third, fourth, and fifth terms on the right side of the equation constitute the pure error variance in the estimation, since they represent that part of the error that could be reduced by obtaining replicate measurements.

The comments regarding the importance of quantifying the sources of the lack-of-fit terms are appropriate, since this is the only method to reduce or eliminate the bias from the prediction. Although such an investigation was not undertaken in the study reported here, the mere inclusion of a lack-of-fit term implies the need for either additional terms or other combinations of the terms given.

It is true that application of research results in other geographical areas than the

ones in which the experimental work was done can be dangerous. Climatological differences are taken into account at least to some extent, however, by the temperature parameter included in the performance equation. Until another study of the size of the AASHO Road Test is performed, however, we think that it is better to use the results obtained there, with the intention of adding terms to reduce lack of fit when possible, than to be without a performance equation altogether.

It is true that repeated traffic loadings are only one contributing factor in pavement failure. The use of the accumulated number of load applications as an independent variable for the purposes of making probabilistic statements, however, does not imply that there is a cause-and-effect relationship among the chance event, failure, and loading.

It is stated in the paper and discussion that the bias in the prediction is larger in the case of treated bases. This means that the residual does not have a mean of zero, or near zero, in these cases. But the normality of a random variable (here, the residuals) does not depend on its mean being small; it depends only on the nature of the random distribution. In addition, Kher and Darter are certainly right in saying that, beyond a point, there are diminishing benefits in making a pavement thicker, but they never imply that such is the case. In fact, the design methods for increasing the expected life of a pavement are not discussed in the paper.

In summary, we would like to mention that the other major goal of the paper is to point out the information on serviceability that has been developed, and this is one example of how the individual variance elements can be evaluated.