STATE OF THE ART IN PREDICTING THE PROBABILISTIC RESPONSE OF PAVEMENT STRUCTURES

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Current design procedures that assume a homogeneous material do not account for the limited pavement distress that is experienced in the field; rather they are based on a catastrophic occurrence of distress. Variability of stress due to load variables and variability of pavement strength due to material characteristics are related in several recent developments, generally by making gross transformations between a statistical confidence level and satisfactory performance. Reliability techniques are described, and special attention is given to the Markov process. Reliability can be viewed as a probability that none of the following events occurs: (a) the number of overlays exceeds the allowed number; (b) the total cost of maintenance and overlays converted to net present value exceeds the allowed cost; and (c) a user delay penalty cost exceeds an allowed value.

•MATERIALS ENGINEERS recognize that the properties of the materials vary considerably from point to point in a specimen whether it is a steel bar or an asphalt surface. Figure 1, which is a continuous density profile for a crushed limestone base course, shows a typical dispersion of material properties in a pavement structure. Note that the density ranges in a random fashion from a low of 138 to a high of 147 lb/ft^3 (2210 to 2355 kg/m^3). The dispersion would be present for other material properties such as strength and modulus of elasticity, as well as for dimensions. Although these variations are recognized from a practical standpoint, current design procedures do not take this variation into account directly. Generally, design procedures assume a homogeneous material.

This type of approach assumes that distress is a catastrophic occurrence such that, when the stress exceeds a limiting value, a distress manifestation occurs. For example, the design premise for a static wheel load on a pavement is based on the assumption that if a limiting stress value is exceeded the entire pavement cracks. Experience and data from test roads indicate that this concept is contrary to what is observed in the field (1, 2, 3). Generally, pavement distress is experienced only over some percentage of the pavement area, and seldom does pavement distress appear throughout its length. Even on roads considered as a failure by engineers, the area of failure relative to the total area is small.

SIMPLE PROBABILITY TECHNIQUES

In recent years, several investigators and agencies have pointed to the need or attempted to take into account the stochastic process. The final report for NCHRP Project 1-10 pointed to the need for considering stochastic failure (4). In an independent study, Moavenzadeh and colleagues recognized the need and included stochastic processes in the development of the VESYS programs (5). The Texas Highway Department used stochastic processes in the development of its early versions of the flexible pavement system and rigid pavement system computer programs (6, 7).

The California Division of Highways (8) and the Asphalt Institute (9) in the development of their overlay design procedures considered variability of material properties by assigning a standard deviation to deflection values. McCullough in his overlay design procedure used probability analysis techniques to simulate the prediction of pave-

Figure 1. Continuous distribution of density of a crushed limestone base.

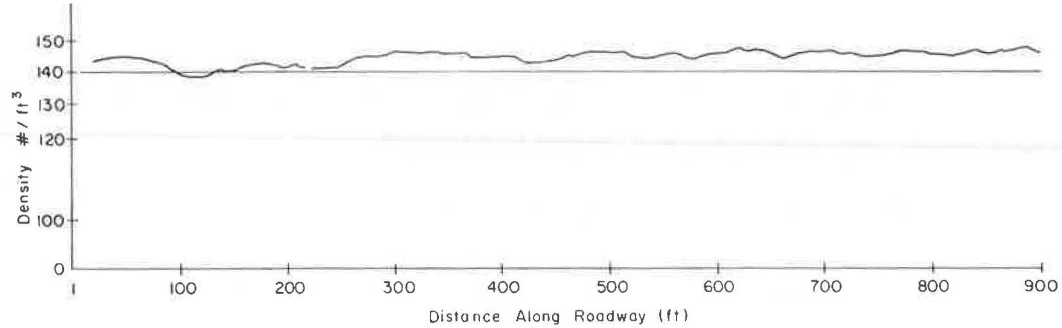
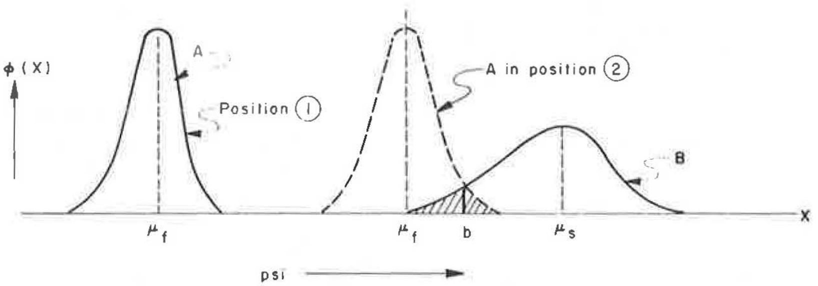


Figure 2. Probability of distress as a function of the dispersion of stress and strength density functions.



ment distress (10).

In the references, except for the VESYS program, probability techniques whether implied or stated were used in a design process to prevent the cracking (rupture) mode of distress. If the stress and strength variations in the pavement are characterized by normal distributions, then Figure 2 shows in a general fashion the fundamental hypothesis of these design methods. (A is the density function for stress and B the density function for strength in the specimen.)

If the dispersions of the stress and strength are clearly separated as is the case illustrated by position 1 in Figure 2 for the stress density function, then the material will perform satisfactorily without distress. The probability of distress in position 1 is zero. If the load, temperature, or moisture conditions change, then a shift of the stress density function to position 2 might be experienced; thus the probability of a failure assumes a finite value between 0 and 100 percent. Although the mean stress would still be less than the mean strength, a safe condition from a purely deterministic standpoint, distress will occur. Conceptually, the probability of distress may be expressed as the area beneath the intersection of curves A and B and is represented by the shaded area in the figure as follows:

$$P(D) = A \cap B \quad (1)$$

where P(D) = probability of distress. In Figure 2, the intersection of the two density functions may be designed as b on the psi axis. If the stress in a material is assumed to be independent of the strength, the probability of distress may be restated as follows:

$$P(D) = P(\text{stress} > b) + P(\text{strength} < b) \quad (2)$$

Each of the previously mentioned design methods takes variability into account by either quantifying equation 2 or making gross transformation between a statistical confidence level and satisfactory performance.

RELIABILITY TECHNIQUES

According to Darter and Hudson (11), "Reliability [of a pavement section] is the probability that the pavement system will perform its intended function over its design life and under the conditions (or environment) encountered during operation." They discussed the reliability of pavement design for a single performance period and touched on reliability considerations for pavements designed to be overlaid one or more times (11). The reliability and systems analysis of pavements of the second class are required to realistically simulate the long-term performance of a pavement structure.

Three approaches to simulation are reviewed on analytical grounds. Further empirical study may be needed to determine adequate approximating probability distributions required for implementation. The experimental needs and the particular sensitivity of reliability analyses on errors in the ad hoc distributions are briefly discussed.

SIMPLE RELIABILITY CALCULATION

Suppose a pavement is designed for k performance periods; i.e., it is designed to last for some time T assuming k - 1 overlays may be necessary. We are interested, then, in the probability that the life of the pavement exceeds T.

The following variables are used:

N_i = number of 18-kip (80-kN) loads before the ith overlay is required,
n = probability density function (pdf) of N_i, and
f_n = pdf of n.

Then the reliability is

$$P\left(\sum_{i=1}^{k} N_i > n\right) \quad (3)$$

where P( ) denotes probability.

The pdf of $\sum_{i=1}^{k} N_i$ is attainable by performing k - 1 convolution integrals (convolution via characteristic functions would probably be the most computationally efficient numerical approach):

$$f_{1,2}(N) = \int_0^{\infty} f_1(\xi)\, f_2(N - \xi)\, d\xi \quad (4)$$

where $N = N_1 + N_2$.

$$f_{1,2,\ldots,m}(M) = \int_0^\infty f_{1,\ldots,m-1}(\xi)\, f_m(M - \xi)\, d\xi \tag{5}$$

for m = 3, . . . , k - 1 where

$$M = \sum_{i=1}^{m} N_i$$

For convenience, $f_c = f_{1,2,\ldots,k}$.

Now a final convolution is required to obtain the pdf f_0 of $\sum_{i=1}^{k} N_i - n$.

Note that the pdf f_{-n} of -n is

$$f_{-n}(-n) = f_n(n)$$

Thus,

$$\begin{aligned} f_d(p) &= \int_0^\infty f_c(\xi)\, f_{-n}(p - \xi)\, d\xi \\ &= \int_0^\infty f_c(\xi)\, f_n(\xi - p)\, d\xi \\ &= \int_p^\infty f_c(\xi)\, f_n(\xi - p)\, d\xi \end{aligned} \tag{6}$$

Then, the reliability is simply

$$\int_0^\infty f_d(p)\, dp$$

Note that we have allowed for the possibility that the pavement may perform differently in different performance periods by treating f_i as different.

It is stated that n and N_1 are "believed to be" approximately lognormally distributed (1, pp. 35-36), from which we would suspect that the other N_i are also lognormal. Evidence for the normality of log N is stated to be available from fatigue tests, but further study is needed to verify that log n is normal.

The reliability analysis is particularly sensitive to errors in the distribution (12). Estimation of the probability of a rare event, abnormal failure, requires acceptable accuracy in the tails of the distributions; such accuracy generally requires larger samples than acceptable estimation of, say, the mean or variance.

The following example illustrates the problem. Suppose we have a sample size of 100. The mean estimate,

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

is based on the entire sample and is relatively insensitive to moderate variations in small subsets of the sample. If the population variance is σ^2, then the variance of $\bar{x}$ is $\sigma^2/100$.

The estimate of the upper 0.05 point, however, is the sixth largest point (since 95 percent of the data are less than or equal to this value). Thus, sampling variations among the few largest data are not moderated by an averaging process that weights all data equally.

Regarding the age at failure (13), "Available data suggest that the relative frequency histograms may be approximated by a normal distribution function." Because the distribution of the number of 18-kip (80-kN) loads actually applied in a given period is not definitely known, the statement is not inconsistent with the belief that the number of loads to failure is lognormal.

If we approximate the time periods T_i for $i = 1, 2, \ldots, k$ of the k performance periods by normal distributions with respective means and variances μ_i and σ_i^2 for $i = 1, 2, \ldots, k$, then the problem becomes very simple (3). The total lifetime of the pavement before the kth overlay is required is

$$T = \sum_{i=1}^{k} T_i \tag{7}$$

T, then, is normally distributed with a mean of

$$\mu = \sum_{i=1}^{k} \mu_i$$

and variance of

$$\sigma^2 = \sum_{i=1}^{k} \sigma_i^2$$

Then the probability that the life T exceeds the design life T_D is the reliability:

$$P(T > T_D) = P\left(\frac{T - \mu}{\sigma} > \frac{T_D - \mu}{\sigma}\right) \tag{8}$$

But $(T_D - \mu)/\sigma$ involves only constants, and $(T - \mu)/\sigma$ is normal with a mean of 0 and variance of 1. Thus, the reliability can be read directly from a table of values of the standard normal distribution function.

In the above, we have presupposed that the estimates to be used of μ and σ^2 involve sampling errors that are small compared to the standard deviation of T. This should be true, since the estimates $\hat{\mu}$ and $\hat{\sigma}$ are based on a sample, but T is a random variable whose variance is not reduced by averaging; e.g., T and $\hat{\mu}$ have respective variances σ^2 and σ^2/n where n is the sample size. Note that, if the sampling errors were not negligible, the problem would not simply involve computing the upper confidences of a t-statistic. The simplest solution in this case would be to replace μ and σ with $\hat{\mu}$ and $\hat{\sigma}$ in the above analysis to obtain an approximate reliability estimator. Alternatively,

$$p(T > T_D) = p(T - \hat{\mu} > T_D - \hat{\mu}) = p\left(\frac{T - \hat{\mu}}{\hat{\sigma}\sqrt{1 + \frac{1}{n}}} > \frac{T_D - \hat{\mu}}{\hat{\sigma}\sqrt{1 + \frac{1}{n}}}\right) \quad (9)$$

The variable on the left is a t-statistic, and the variable on the right does not have a known, simple distribution. Similarly,

$$p(T > T_D) = p\left(\frac{T - T_D}{\hat{\sigma}} > 0\right) \quad (10)$$

and the statistic does not have a t-distribution, because the numerator does not have mean 0, unless $E(T) = T_D$, in which case the reliability is 0.5.

This is not cause for alarm, inasmuch as the approximate reliability obtained by replacing μ and σ by $\hat{\mu}$ and $\hat{\sigma}$ should be acceptable unless $\hat{\mu}$ and $\hat{\sigma}$ have sampling errors that are not small compared to σ (the standard deviation of T), in which case the reliability calculation would be suspect anyway.

MARKOV CHAIN MODEL

McCullough and Hudson (4) discussed a clever approach for performing reliability analyses of single-performance pavements by using a Markov chain model. The four states of the pavement are

1. Normal aging,
2. Accelerated aging, "a state caused, for example, by the initiation of cracking or surface polishing,"
3. Maintenance, and
4. Failure.

The transition matrix is the matrix of probabilities of one-step transition among states. Transition matrices are suggested for four cases:

1. Standard operating procedures,
2. High maintenance activity, standard quality,
3. Standard maintenance activity, high quality, and
4. High maintenance activity, high quality.

The matrix for case 1 is

$$P_I = \begin{pmatrix} 0.95 & 0.05 & 0 & 0 \\ 0 & 0.40 & 0.20 & 0.40 \\ 0.60 & 0.30 & 0 & 0.10 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Thus, according to the first row the probability of remaining in state 1 in time step i, given that the pavement is in state 1 in step i - 1, is 0.95. The probability of passing from state 1 to states 2, 3, and 4 is 0.05, 0, and 0 respectively. In general, the one-step probability of passing from state i to state j is the (i, j) element of the matrix.

Then, the reliability is the probability of not reaching state 4 in the design lifetime. If ρ is the vector of probabilities of initial states and P is the transition matrix, it can be shown that ρP^n is the vector of state probabilities after n transitions. Then one minus the fourth element of ρP^n is the reliability if n/2 is the design life in years, since the time step is one-half year.

The model can be generalized to handle the multiple-performance-period case by altering the state space, as follows:

1. Normal aging, period 1,
2. Accelerated aging, period 1,
3. Minor maintenance, period 1,
4. First overlay,
5. Normal aging, period 2,
6. Accelerated aging, period 2,
7. Minor maintenance, period 2,
8. Second overlay, etc.

If the number of performance periods is k, then there are 4k states, the 4kth state being failure. If the design life is n/2 years, then the reliability is one minus the 4kth element of ρP^n.

In this approach, we have obviated estimation of the tails of distributions. However, moderate errors in the transition probabilities could cause large errors in the P^n matrix if n is large. Thus, accurate estimation of the transition probabilities is important. The possibility of drift in the P matrix as traffic patterns change is a difficult problem that should be considered.

High maintenance probabilities, especially over an extended period of time, are of practical interest because of their relationship to direct costs and user inconvenience. Moreover, the likelihood of frequent maintenance during key periods, e.g., just before expected overlay times, might signify the need for a decision policy alteration.

The desired probabilities are easily attainable from the model; the (4m - 1) th element of ρP^n is the probability of being in maintenance in the mth period at time step n.

Extended periods of high probabilities of being in a state associated with low serviceability are similarly of interest. Note that, by altering the transition probabilities to the maintenance and overlay states, we can study the sensitivity of direct maintenance and user delay costs and serviceability to rehabilitation policies. The sensitivity of the system to maintenance quality can be studied by varying the transition probabilities from the maintenance state. The sensitivity to overlay quality can be seen by varying the transition probabilities after the first overlay. Effects of random errors in the transition probabilities should also be studied via sensitivity analysis. It is clear that the Markov chain model yields much more information than a simple reliability calculation.

The undesirability of the uniform step size and allowance for a single state for an entire section is apparent. The disadvantages can be reduced by using a finer step size and a more detailed state space.

MARKOV PROCESS MODEL

We now consider a model in which time is continuous and the section is divided into several subsections of, say, 100 ft (30 m). For illustration, we consider the four states of the Markov chain model, but a state is associated with each subsection.

The Markov process model is suggested because of the extensive information that can be obtained from it; the policy and sensitivity analyses mentioned in conjunction with the Markov chain model can be carried out in more detail with the Markov process model.

The more sophisticated model, however, requires more extensive inputs, as is apparent from the discussion below; thus, the extra demands in experimentation to determine the inputs should be weighed against the advantages.

Because of the complexity of the model, digital simulation is suggested as a solution technique. Development of a computer program would be greatly facilitated by the use of a simulation language such as GASP (5). The elements of the simulation model are described below.

1. Randomly determine the initial state of each subsection as normal aging or accelerated aging.
2. For each section, randomly determine the time in the initial state and the state next to be occupied.
3. Upon transition of a section, randomly determine the time to remain in the new state and the state next to be occupied. After each transition, test whether it is time for an overlay by examining the states of all subsections and applying predetermined decision criteria; e.g., compare the percentage of subsections in the accelerated aging state to a threshold percentage.
4. If an overlay is indicated, randomly determine the time required for the overlay. After the overlay, repeat steps 1 through 3. Continue the entire process until the allowed number of overlays has been exceeded or the design life is completed.

The interrelations among events must be considered. For example, the transition of a subsection after an overlay could be a function of

1. The a priori probabilities of state transitions and distribution of the time in the state to be transferred from,
2. The states of nearby subsections, or
3. Convenient measures of the performance of the subsection before the overlay.

The following hypothetical list of events illustrates the basic information that is available from the simulation for analysis.

Time (months)	Event
0	Begin simulation; all subsections aging normally
20	Section 6 enters accelerated aging state
23	Section 23 enters accelerated aging state
26	Section 6 enters maintenance
26.05	Section 6 enters normal aging state ($1\frac{1}{2}$ days required for maintenance)
⋮	
128.7	Section 47 enters accelerated aging state; percentage of sections in accelerated aging state exceeds threshold value; begin overlay
⋮	

Thus, detailed statistics could be collected on things such as maintenance costs, maintenance time, and percentage of sections in accelerated aging at any given time.

As with the Markov chain model, sensitivity analyses should be performed to determine (a) the effects of varying the overlay and maintenance policies and (b) the effects of random errors in the empirically determined inputs.

A straightforward measure of reliability is the probability that the number of overlays exceeds the allowed number in the design life. Alternatively, reliability can be viewed as the probability that none of the following events occurs:

1. The number of overlays exceeds the allowed number,
2. Total cost of maintenance and overlays converted to net present value exceeds the allowed cost, and
3. A user-delay penalty cost exceeds an allowed value.

SUMMARY

The state of the art of predicting pavement probabilistic response may be summarized as follows:

1. Most of the early work and the currently implemented methods considering variability use simple probabilistic techniques to simulate performance;
2. Recent studies have been oriented toward approximating the complex multivariate distributions of material properties and pavement dimensions to achieve a stochastic prediction of performance; and
3. Future work should be directed along the thrust of item 2 considering that a pavement life will consist of the initial performance period plus numerous overlays.

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