

# MATHEMATICAL ANALYSIS OF PAVEMENTS WITH NONUNIFORM PAVING MATERIALS

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The variability of paving materials and the effects of such variations on the behavioral responses of pavement systems are evaluated. Methods for evaluating the effect of material variability on pavement system responses are presented and analyzed. Analyses are presented that evaluate the characteristic variations in components of pavement systems and methods of quantifying them. Factors such as the severity and location of variations are evaluated, and sensitivity analyses are performed to determine the relative and real effects of specific types, locations, and severities of the variations on pavement system responses. The paper discusses the relationship between the variability observed when discrete specimens of a material are tested and the variability of a continuum constructed from the same material. The importance of eliminating testing error and similar sources of error when the continuum is evaluated is emphasized. Monte Carlo simulation and other techniques are applied to obtain a pattern for pavement system responses on pavements with specific types of variations and loading conditions. Results show that the variability of the paving materials can have a profound effect on the responses of the pavement to load, especially to strain response, and that the location and extent of the variation are critical with respect to the response. Also, the Monte Carlo simulation technique for predicting the range of responses is limited unless a very large number of analyses are used in the simulation.

•LITTLE is known about the manner in which the nonuniformity of paving material properties and subgrade support affect the behavior and response of pavement systems to loads. Pavement systems do exhibit significant variations in response to loads, but the extent to which this variability is influenced by variations in material properties in different pavement components is not known (1, 2, 3, 4).

Laboratory test results from prepared specimens of paving materials show substantial variability in material properties. There are, however, several problems associated with using the variability from tests on discrete specimens to predict the variability of materials in pavement systems. One problem is that most test specimens are made up outside the pavement system. Thus, the variability observed in these test results may not reflect the true variability of the material in the pavement system. This is especially critical in the case of compaction because procedures for compacting materials in the pavement are completely different from those for compacting test specimens and most materials are highly sensitive to their compacted density.

Another problem in correlating the variability of test results from control test specimens and the material in the pavement is that the results from test specimens are influenced by the edge conditions of the specimen and by specimen size; the materials in the pavement are continuous. Because material in the pavement is continuous, there is probably a gradual transition in the material properties within the pavement system; the material in a test specimen is probably more uniform throughout the specimen but is affected by the specimen boundaries. Also, the variability of results from control specimens may be different from the corresponding variability of the material in the pavement because of the relative quantities of material involved. The response of the pavement system is influenced by properties of the material, but the pavement system includes a much greater volume of material than normally included in control speci-

mens. Thus, the area of influence that affects the response of the pavement must be considered in any analysis of pavement variability.

Laboratory testing is, of course, the only available method for evaluating material properties. Thus, the variability in test results from laboratory specimens must ultimately be reconciled with the variability of in situ materials or, more importantly, with variability in the responses of pavement systems to external stimuli. One method of reconciling these two factors is by inference, that is, by evaluating the effect of material variability on the response of pavement systems and by correlating calculated material variability from pavement responses with measured variability of control specimens. This, of course, requires models that can be used to evaluate the effects of material variability on pavement responses. With such a model, the effects of variability in the various pavement components can be evaluated. By using models capable of handling nonuniform material characteristics in the various pavement components, it will be possible to perform sensitivity analyses to establish the relative effects of changing levels of material variability on pavement response.

One problem with using mathematical models to evaluate these responses is that a data base must be developed for statistical analyses of the response of pavement systems. The problem is one of specifying the critical distributions of strong and weak areas through the pavement system so that the analyses will provide realistic indications of the worst and best possible conditions. If only static loads were applied, the probability of a load being applied at the most critical location with respect to pavement weaknesses could be remote. With moving loads, however, the likelihood of a load or loads being applied at the most sensitive locations becomes much greater. For example, if an area of weak subgrade support exists near the edge of a pavement, then it seems likely that the worst condition would be application of a load to the pavement directly over the location of the weak support, and there is a high probability that moving loads will pass over this critical location. Thus, not only must a statistical evaluation be made of the pavements, but the most likely worst conditions must also be identified and analyzed.

The purpose of this paper is to evaluate procedures for analyzing pavement systems with nonuniform paving material properties. Statistical procedures for assigning critical properties to the paving materials at specific locations are evaluated, the pavements are analyzed with the assigned properties, and the results are compared with what are considered to be the best and worst combinations of material properties for the pavement system. Analysis of the pavement system is done with a finite element model. Results of analyses of pavement systems with various levels of material variability are compared and evaluated.

## APPROACH

In an earlier paper, Levey and Barenberg (5) showed how numerical analysis models can be used to evaluate the effects of nonuniform paving materials on pavement responses. They demonstrated that the stresses, strains, and deformations of the pavement system were clearly affected by variability of the paving material and by the size and distribution of the variations within the pavement system. Their results clearly showed that a normal distribution of responses could be expected with paving materials having a normal distribution of properties.

They used a two-dimensional numerical model to evaluate a pavement system and concluded that the three-dimensional case could also be analyzed if sufficiently large high-speed computers were available or if models that represent the pavement system in three dimensions were more efficient (5). Inasmuch as computer capacity has remained nearly constant since that paper was prepared, a direct analysis of the three-dimensional pavement system with nonuniform material properties is still not practical.

Several existing models were evaluated to find the model best suited for this analysis. Included in the evaluation were the finite element model for the analysis of pavement slabs developed by Hudson and Matlock (6, 7) and the basic finite element models developed by Wilson (8). The model chosen for the study was a finite element model devel-

oped by Eberhardt (9) for analysis of two-layered slabs on a Winkler type of support. The Eberhardt model was chosen for several reasons. Because of time and resources, only one type of pavement could be analyzed. Inasmuch as rigid pavements have fewer variables with more clearly defined material properties, they were chosen for the evaluation. Also, finite element models were available to consider rigid pavements as a three-dimensional problem whereas the models applicable with flexible pavements more nearly represent a two-dimensional analysis of these systems. Thus, it was decided to use a finite element model that represents a continuously supported slab rather than a more generalized finite element model such as the model developed by Wilson (8) and modified by others. The above decisions reduced the choice of models to essentially the Hudson and Matlock model (6, 7) and the Eberhardt model (9). The Eberhardt model was selected because, when we verified the models, it gave stresses closer to those obtained with the elastic slab theory and it was capable of evaluating two-layered as well as single-layered slabs. Although two-layered slab systems were not evaluated in this study, the model can analyze such systems, and this capability permits evaluation of variable thicknesses of a subbase under the concrete slab.

The rectangular plate element shown in Figure 1 is used to represent the pavement slab. Each slab is made up of a number of such elements, and the loaded configuration of each element is defined in terms of the corner nodes of each element. Displacement of the nodal points is interpreted as actual displacement of corresponding points in the pavement system. The varying stress field in each element is defined by an equivalent set of discrete-element forces acting at the corner nodes. These fictitious element forces have no real physical counterpart but simply approximate the varying stress field in the real pavement system. The displacements of the corner nodes are described by three components: vertical displacement  $W$  ( $z$ -direction), rotation about the  $x$ -axis ( $\theta_x$ ), and rotation about the  $y$ -axis ( $\theta_y$ ). Figure 1 shows the positive displacement components. The procedure for developing the element stiffness matrix (9) was based on the classical theory of thin plates.<sup>1</sup> The final force-displacement equations for a plate element can be written as

$$\{F\}_e = (k) \{D\}_e \quad (1)$$

where  $(k)$  is a  $12 \times 12$  plate element stiffness matrix relating the nodal displacement vector  $\{D\}_e$  to the nodal force vector  $\{F\}_e$ .

#### SUBGRADE STIFFNESS MATRIX

The subgrade support is defined by a grid identical in size, pattern, and location to the grid used to define the slab elements. The two grid systems are aligned so that they coincide at all nodal points.

Energy principles were used to develop the force-deformation relationships for the subgrade support. The subgrade stiffness matrix was established by applying successive virtual unit displacements at each of the four nodal points of the plate elements, and the force-displacement equations for the plate and the subgrade element were obtained by equating the sum of the internal and external work to zero; i. e.,

$$\{F\}_e = [(k) + (k_s)] \{D\}_e \quad (2)$$

<sup>1</sup> Information on development of a finite element model was submitted with this paper as an appendix and is available in Xerox form at the cost of reproduction and handling. When ordering, refer to XS-64, Transportation Research Record 575.

where  $(k_s)$  is a 12\*12 stiffness matrix for the subgrade support element.

The global force-displacement equations for a finite element grid (Figure 2) can be written as

$$\{F\}_g = (K)_g \{D\}_g \quad (3)$$

where

$$\begin{aligned} \{F\}_g &= \text{vector containing all the global forces,} \\ \{D\}_g &= \text{vector containing all global displacement, and} \\ (K)_g &= \text{global stiffness matrix.} \end{aligned}$$

Equation 3 is solved for  $\{D\}_g$  and subsequently for strains  $(\epsilon_{xx}, \epsilon_{yy}, \epsilon_{xy})$  and stresses  $(\sigma_{xx}, \sigma_{yy}, \sigma_{xy})$ . Eberhardt (9) verified the finite model by comparing the results obtained by using this model with theoretical results from equations developed by Westergaard. Good agreements were obtained for both deformation and stresses.

In the model developed by Eberhardt, the stiffness of the pavement was assumed to be uniform, and stiffness values for all elements were assigned as a constant value. For this study, the computer program written by Eberhardt was modified so that specific values of the flexural rigidity  $D$  are assigned to each element so that the rigidity of the slab is varied in a random manner. Similarly, specified values for the subgrade support  $k$  are assigned for each subgrade element so the subgrade support can also be varied in a random manner.

#### INCORPORATION OF VARIABILITY

For isotropic slab materials assuming a state of plane stress, the 3\*3 matrix  $(D)$  represents Hooke's law for two-dimensional stress problems as follows:

$$(D) = \frac{Et^3}{12(1-\nu^2)} \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix} = D(N) \quad (4)$$

where

$$\begin{aligned} E &= \text{elastic modulus,} \\ t &= \text{plate thickness, and} \\ \nu &= \text{Poisson's ratio.} \end{aligned}$$

Thus, from equation 4,

$$(N) = \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{pmatrix}$$

and

$$D = \frac{Et^3}{12(1-\nu^2)} \quad (5)$$

The matrix (D) is used in formulating the plate element stiffness matrix (K).

The material variability is incorporated into this model by multiplying the matrix (D) described in equation 4 for each element by a random number R from a set that has a mean value of unity and a specified coefficient of variation. Then

$$(D') = RD(N) \quad (6)$$

where (D') is the modified (D) matrix. For the purpose of this paper R is assumed to reflect the variability in the modulus value E of the plate. However, as can be seen from equation 4, R can also be considered to reflect the variability in D, the flexural rigidity of the plate, which includes the variability in the plate thickness t as well as the variability in E.

#### GENERATION AND ASSIGNMENT OF RANDOM VARIABLES

A program that generates random numbers was described by Levey and Barenberg (5) and Levey (10). This program can produce sets of random numbers with normal distributions having specified means and standard deviations. This program was used to generate sets of R-values, which were assigned to the elements in a random pattern. Two methods were used to select the sets of random values used in the analysis.

One method of assignment was basically a Monte Carlo simulation procedure in which the values in the sets were generated and assigned to the model elements in a random manner and the system was analyzed with the randomly assigned values. The difficulty with this procedure is that it is not known whether arrangement of the randomly assigned values produced a pavement response that was better or worse than a mean response and to what extent. Thus, with this approach, it is necessary to make enough separate assignments and separate analyses so that the results can be analyzed statistically. A large number of independent runs of this type are required to develop the necessary background data for a statistically based analysis of the response characteristics of the pavement system. In the earlier work by Levey and Barenberg (5), 20 such independent analyses were made in the Monte Carlo simulation of the pavement response characteristics. Although this number appeared adequate, there is no sure way of determining the accuracy of the statistical parameter without developing larger data bases. Large data bases require substantial funds for computer costs and analyses.

Because of the high computer costs and the uncertainty when the Monte Carlo technique is used to establish the responses of pavement systems, an alternate method was used to determine the range of responses that could be expected from a pavement with a specified degree of nonuniformity of paving materials. The random number generator was used to generate sets of values for the specific property under investigation, and the sets were assigned to specific numbered elements of the pavement slab. Approximately 50 sets were generated, and the results were studied to determine which sets of values would likely produce the best and worst pavement responses. Figure 3 shows the numbered pavement elements and the location of loads near the corner and edge of the pavement. Figure 4 shows the values for modulus of elasticity of the slab material to produce the anticipated best and worst pavement responses for the edge loading condition. Figure 5 shows information for the subgrade support k. Both sets of values have a coefficient of variation of 30 percent. Notice that, for both the modulus of elasticity of the slab and the subgrade support, the values assigned to the four elements surrounding the loaded area are significantly higher or lower than the average values. Because all values were assigned to the elements in a random manner and the sets of values for the best and worst conditions were selected from a large number of randomly generated sets, results with these values should provide an excellent indication of the range of responses that could be expected if a full Monte Carlo simulation were made.

Because this latter approach required much less computer time than the full Monte

Figure 1. Rectangular plate element.

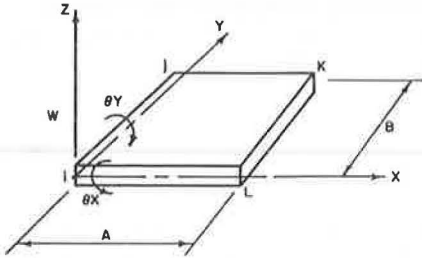


Figure 2. Finite element grid for assembling global stiffness matrix.

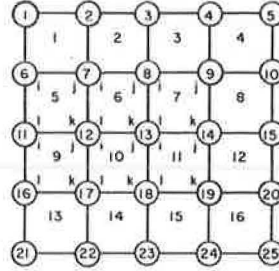


Figure 3. Numbered pavement elements and location of edge and corner loads.

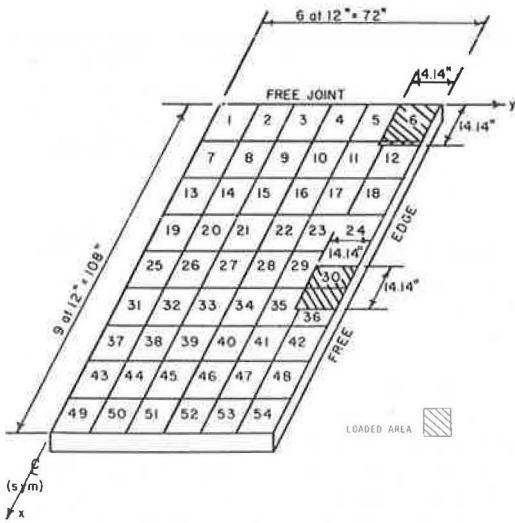


Figure 4. Pavement elements with the randomly assigned values for slab modulus.

4237	4413	5534	5385	4883	3541
4987	4121	5516	4760	4330	7888
3902	3913	3978	6976	3394	5789
5383	4186	2017	4452	3382	2761
4723	5048	5518	4555	4883	2503
4424	4379	5082	3780	2636	3842
2097	4440	3158	3963	3398	3966
5134	2294	4499	2719	3964	4061
2756	5566	5959	912	3761	4773
2505	4743	5564	3303	2942	1686
3551	2576	4785	5158	5669	5774
2528	4358	2960	3000	2444	427
5569	4451	4781	4207	3083	3261
5379	6109	2394	3002	4436	4630
4938	3388	3537	3220	3537	6461
4558	4102	4005	4194	5064	5218
4943	5422	4018	3350	3204	3622
4701	4206	2876	3385	4613	3677
4889	5780	4330	3168	2974	4163
3556	5229	4379	4179	4545	4260

Figure 5. Pavement elements showing the subgrade support values in lb/in.<sup>3</sup>.

106	110	139	135	122	86
125	103	137	119	108	197
97	98	99	175	88	145
135	105	50	111	95	69
118	126	138	114	122	63
111	110	127	94	66	96
52	111	79	99	85	99
128	57	115	68	99	102
69	139	149	23	94	119
63	119	139	83	74	42
89	64	120	129	142	144
63	109	74	75	76	61
139	111	119	105	77	81
11	135	153	60	75	111
123	85	88	81	88	161
116	114	103	100	105	127
124	136	100	84	80	91
130	118	105	72	85	92
122	145	108	79	74	104
89	131	110	105	114	107

Carlo simulation, it was used to study the effects of material variability and load position on the pavement response. A limited Monte Carlo simulation was also made to show the similarity and differences that can be obtained with the two procedures.

In these analyses only one parameter was varied at a time. Thus, when the effects of the slab modulus were evaluated, the subgrade was held uniform. Similarly, when the effects of variability of the subgrade were evaluated, the slab modulus was held uniform. The pavement system responses were evaluated for both edge and corner load conditions. All loads were 10,000-lb (4540-kg) wheel loads with a contact pressure of 50 psi (345 kPa) assumed to be uniformly distributed over the loaded area.

## RESULTS

The computer program used in the analysis provided all typical pavement responses such as deflections, stresses, and strains at all locations specified. However, only a limited amount of these data are presented here. Strain in the pavement slab was chosen as the parameter for presentation and analysis here because it represents a critical response in the performance of concrete pavements (4) and appeared to be the most sensitive to changes in paving material variability.

Figures 6 and 7 show the strains in the x- and y-directions for pavement systems with uniform paving materials under corner and edge loading conditions respectively. Note that for the corner loading condition the critical strains are not under the load. Similarly, for the edge loading condition the maximum strain in the x-direction ( $\epsilon_x$ ) occurs under the load, but the maximum strain in the y-direction ( $\epsilon_y$ ) occurs away from the loaded area, indicating a cantilever effect in the y-direction. Under the edge load, the strain in the x-direction  $\epsilon_x$  is significantly greater than the strain in the y-direction  $\epsilon_y$ . These results are consistent with results from closed form solutions such as the Westergaard model.

Figures 8 and 9 show the difference in  $\epsilon_x$  and  $\epsilon_y$  for the best and worst conditions under edge loading as the coefficient of variation for the modulus of elasticity of the slab material increases from 10 to 30 percent. Reasons for change in the shape of the curves between the best and worst conditions can be seen by studying the patterns of the assigned moduli values in Figure 4. In particular, the area of weakest material does not generally coincide with the points of maximum strain. Thus, as expected, when the variability in the moduli values increased, as with the higher coefficients of variation, the point of maximum strain tended to move toward the area of low bending resistance. A similar set of curves for the corner load condition is shown in Figures 10 and 11, and the same trends exist.

A Monte Carlo simulation of the responses under edge and corner loads was made, and the statistical parameters were calculated. Figures 12 and 13 show the differences obtained for  $\epsilon_x$  and  $\epsilon_y$  under edge loading by using the Monte Carlo simulation with 10 repetitions. The results from analyses with the designated best and worst conditions are also shown for comparison. There are significant differences in the range of values obtained by the two methods. Figure 14 shows a statistical summary of  $\epsilon_x$  obtained from the two approaches.

Because of the significant differences obtained when the Monte Carlo simulation techniques were used compared with the designated best and worst conditions, it is well to review again the differences in these two approaches.

In the Monte Carlo simulations, the appropriate moduli values were generated and assigned to numbered elements on a random basis. Then the system was solved and the responses of that system were recorded. A new set of random moduli values, with the appropriate mean and standard deviation, was then generated and assigned, and the system was again solved for responses. After a specified number of repetitions (in this case 10), pavement responses were analyzed to determine the statistical characteristics of these responses. Only 10 repetitions were made because of the computer costs involved for each solution and the limited funds available.

The designated best and worst conditions for the pavement responses were determined in the following manner. First, a number of sets of random values were gen-

Figure 6. Location and magnitude of strains in the x- and y-directions for a pavement with a corner load and uniform support.

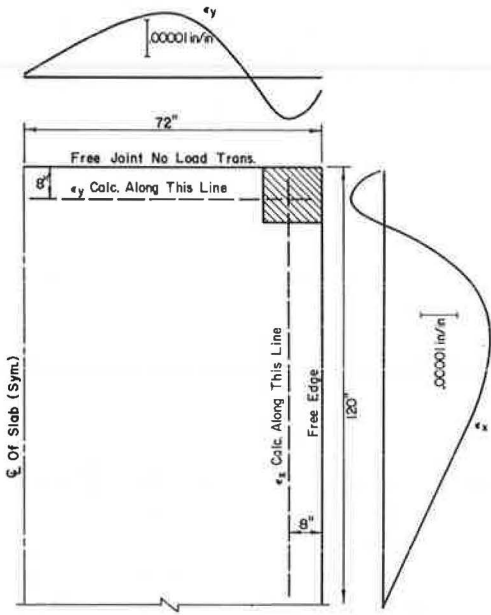


Figure 7. Location and magnitude of strains in the x- and y-directions for a pavement with an edge load and uniform support.

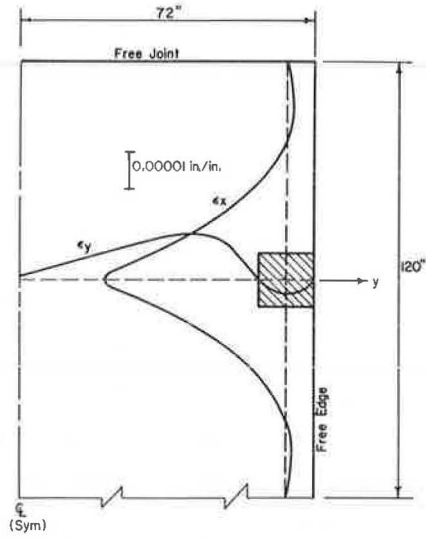


Figure 8. Effect of variability of slab modulus on the strains in x-direction due to edge loading.

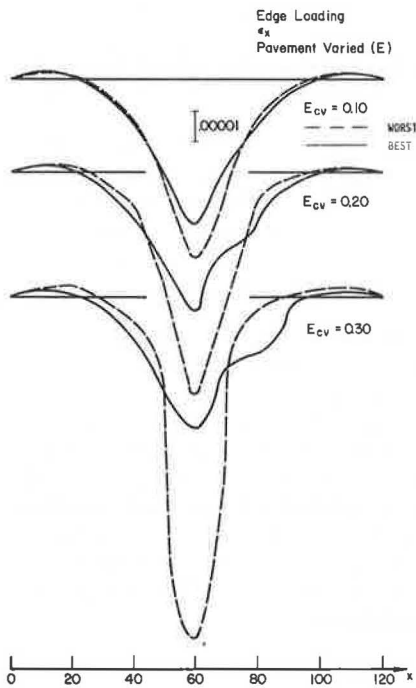


Figure 9. Effect of variability of slab modulus on the strains in y-direction due to edge loading.

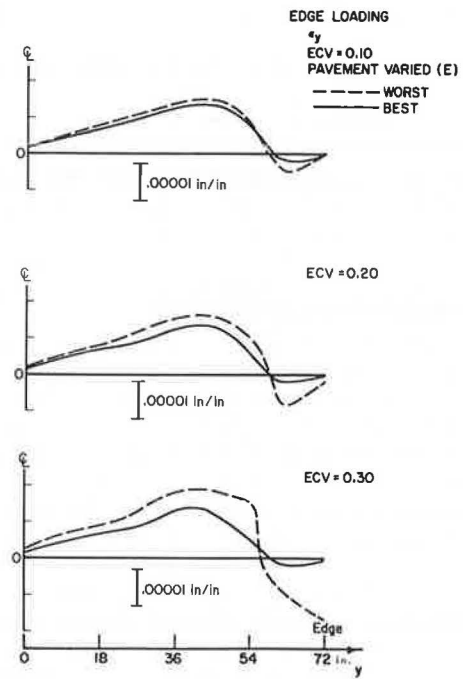




Figure 10. Effect of variability of slab modulus on strains in x-direction under corner loading.

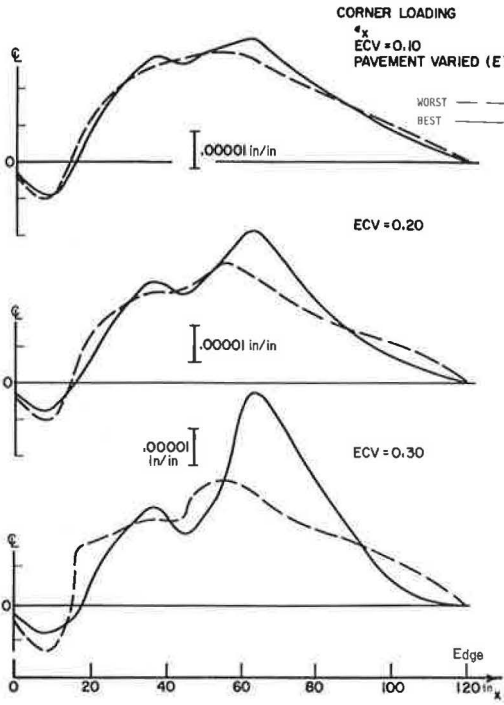


Figure 11. Effect of variability of slab modulus on strains in y-direction under corner loading.

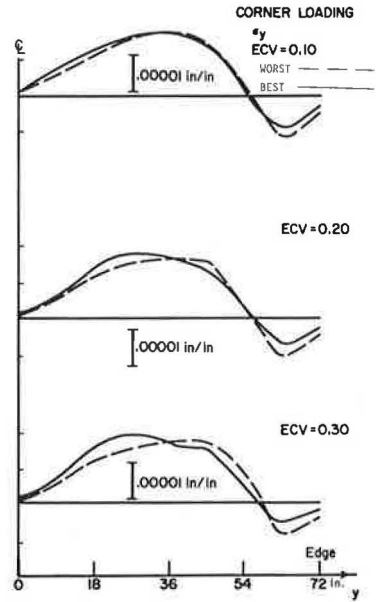


Figure 12. Variations in x-direction strains due to variations in the pavement.

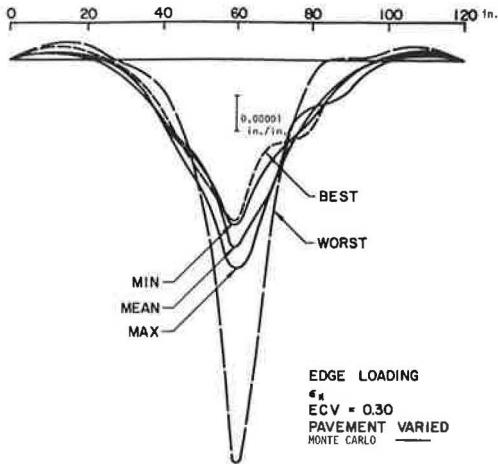


Figure 13. Variations in y-direction strains due to variations in the pavement.

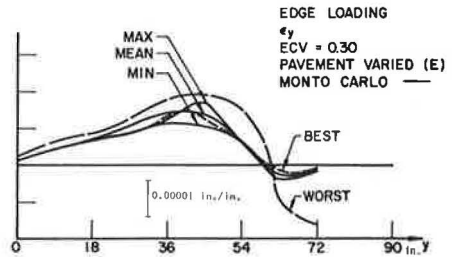


Figure 14. Comparison of ranges in maximum strains due to edge loading.

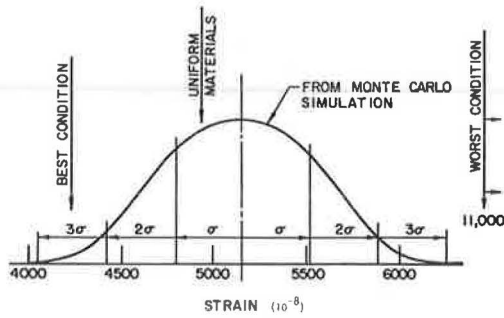


Figure 16. Variations in strains in x-direction due to variations in the subgrade for pavements under corner loads.

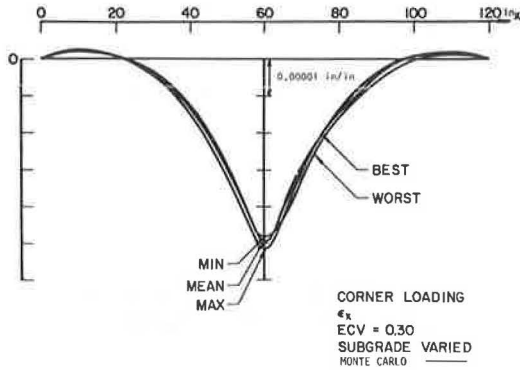


Figure 15. Variations in strains in y-direction due to variations in the subgrade for pavements under edge loads.

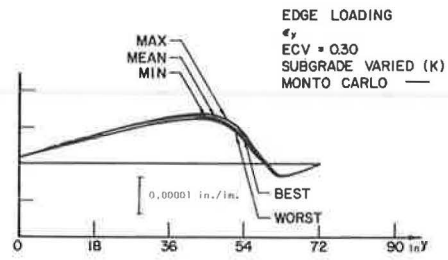
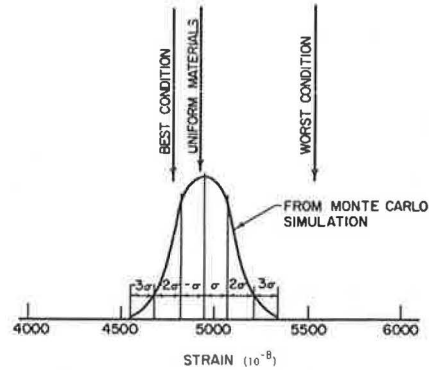


Figure 17. Ranges in maximum strains due to edge loading caused by variations in the subgrade.



erated and listed sequentially. These sets were then studied to find the one that had four values in critical relative positions that were significantly less than average. The values in the desired set were then shifted as a unit such that all values remained in the same relative position sequentially but that the four smallest values grouped together would be assigned to the four elements directly under or surrounding the loaded area. Similarly, the sets of generated values with the highest E-values grouped together were assigned to fall under and around the loaded area to produce the best condition.

It is believed that the above method of establishing the best and worst conditions is valid because all randomly generated values were allowed to remain in their same relative position, but the entire set was shifted so that the weakest and strongest regions of the pavement coincided with the critical loaded area. For pavements in normal service, the location of the strong and weak areas would, of course, remain fixed, but the loads could move so that eventually the load would occur over the strong or weak areas of the slab. Thus, the method used for determining the best and worst condition appears reasonable, and this information can be obtained for significantly less computer expense than with the Monte Carlo simulation.

The effects of varying the subgrade support while holding the modulus E of the paving material constant were not so dramatic as the effects of varying the moduli of the paving materials. A summary of the results using both methods of analysis is shown in Figures 15, 16, and 17. The low sensitivity of the pavement responses to varying

subgrade properties is surprising to the authors, especially since the total pavement deflection also proved to have a low sensitivity to the variability in the subgrade support.

Because time and money were limited, the effect of the size of the softer area of the subgrade on the pavement response was not evaluated. Variations in the subgrade for this analysis were assigned element by element. Because each element covers only  $1 \text{ ft}^2$  ( $0.09 \text{ m}^2$ ), a loss of support over an area of this size may not significantly affect the response of the pavement system. Then the worst condition was evaluated, the set of values with the lowest average  $k$  for the four elements around and under the loaded area was used. Again, the area of  $4 \text{ ft}^2$  ( $0.37 \text{ m}^2$ ) may not have been sufficiently large to seriously affect the response of the pavement system analyzed. Based on the known effect of changing the average  $k$  of the subgrade, it is expected that, as the extent of a weak area of support is increased, it will have increasingly greater impact on the pavement responses.

## CONCLUSION

This presentation and discussion show that nonuniform properties of paving materials have a significant effect on the behavioral response of the pavement systems analyzed. Further, the findings suggest that the Monte Carlo simulation may not be the most efficient method for determining the range of pavement responses due to nonuniform material properties. The Monte Carlo simulation technique is valuable in that it permits the calculation of statistical parameters of the pavement response. However, when the Monte Carlo simulation procedure is used, the sample size must be large enough to provide an adequate data base for a reliable statistical analysis of the problem. Results clearly show that a sample size of 10 is inadequate for such an analysis, but the sample size required to obtain a satisfactory simulation was not determined. Probably a combination of the two techniques used in this paper is the most reliable procedure.

It was interesting to note in the findings how the locations of the maximum strains changed with the different sets of randomly assigned properties for the paving materials. These results clearly show why cracks form at different locations in pavement systems even though theoretically the point of maximum stress or strain is at the same location for the different pavements. The phenomenon observed has long been assumed by paving engineers, but, to the authors' knowledge, this is the first time it has been demonstrated with analytical models.

The results clearly show that finite element models can be used to evaluate the effects of nonuniform paving materials on pavement behavior. Much work is still needed, however, to adequately characterize the specific levels of variability in the paving material and the level of nonuniformity that has a critical effect on pavement responses, and the relative severity of the nonuniformity on pavement performance. Such information is needed before models can be developed that will enable the engineer to make realistic trade-offs between better quality control and higher costs.

Work is still needed on the application of these procedures to flexible pavement systems. The problem involves selection of appropriate models and assigning the appropriate values to each element in the model. Associated with assigning of values is the question of the size of an area to represent an appropriate specimen size in the continuum, which will also reflect on the degree of variability in the system.

The procedures presented here can be used to resolve some of these questions, but there is still much work to be done before valid models will be available to make comprehensive risk analyses of pavement designs.

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