APPLICATION OF STATISTICAL METHODS TO THE DESIGN OF PAVEMENT SYSTEMS

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This paper describes some of the applications of statistical or probabilistic methods to the design and analysis of pavement structures and discusses the theory on which they are founded. The major purpose for applying probabilistic methods to design of pavement systems is to help the engineer optimize design. The technology of statistical or probabilistic methods enables the engineer to directly consider the effect of many of the variabilities and uncertainties associated with the design, construction, and in-service life of pavements in the design process. Design adequacy or reliability or, conversely, the probability of distress can therefore be assessed to a much greater degree than without these concepts, and hence more optimal designs can be provided. Basic variabilities and uncertainties involved in the design, construction, and in-service life of pavements are described and shown to have significant effects on performance. Theory to estimate the probable fracture distress and the loss of serviceability of portland cement and concrete pavement due to repeated traffic loadings is presented and illustrated. A relationship between estimated probability of traffic-associated fracture distress and measured slab cracking is developed. The application of these techniques to design is illustrated by several examples. Some of the methods described have been implemented and have been shown to be practical and useful.

•APPLICATION of statistical or probabilistic methods to the design and rehabilitation of pavement systems is an essential step toward improving existing empirical procedures and developing mechanistic procedures. The purposes of this paper are to (a) describe why application of probabilistic methods is important and necessary, (b) outline the methodology for practical and useful application, and (c) provide example applications.

Probabilistic methods have been used extensively for several years in various areas such as structural engineering. The consideration of material variations, traffic load uncertainties, and reliability of a pavement structure was strongly advocated by participants at the FHWA-HRB Workshop on the Structural Design of Asphalt Concrete Pavement Systems in 1970 (1), and this need was listed as one of the 10 most pressing problems facing pavement engineers. Since the workshop, probabilistic methods have been applied to rigid pavement design and analysis by Kher and Darter (2), Darter (3), and Levey and Barenberg (4). Applications to flexible pavements were made by Darter and others (5, 6, 7), Moavenzadeh and others (8, 9, 10), and McManus and Barenberg (11). These efforts provided some significant results, but they represent only a beginning.

This paper discusses the basis for using probabilistic methods in design and rehabilitation and develops the theory and concepts for probabilistic stress-strength, fatigue, and serviceability applications to design and analysis of pavement systems.

BASIS FOR PROBABILISTIC DESIGN

Variabilities and Uncertainties

To those who have been closely associated with design, construction, and subsequent performance of pavements, the words variability and uncertainty have much significance. During design the engineer must estimate many input values from information that is usually very limited; available design procedures are in many respects inadequate. Many material property variations can be observed during construction, and construction deficiencies are also rather common. During the life of the pavement, many performance variations occur that are related to traffic loadings, climatic effects, maintenance procedures, and occurrence of distress along the pavement.

However, if these uncertainties and variations are identified and their magnitude is approximately quantified, they can be used in the design process to achieve more optimum designs. Many of the variabilities have been reported in the literature and therefore will not be repeated here (5, 13). Variations can be conveniently divided into three basic types: variability within a project, variability between assumed and actual design values, and variability due to the inadequacy of the design procedure to account for all necessary factors or to adjust for each factor in the correct manner.

Variation Within a Project

Factors that often vary within a construction project include aggregate gradation, thickness, moisture content, density, resilience, and strength. For example, strength variations in portland cement concrete (PCC) occur from point to point along a pavement slab and are caused by many factors including batching, mixing, transporting, placing, finishing, and curing. A frequency distribution of compressive core strength determined at 500-ft (150-m) intervals along one project is shown in Figure 1. A core thickness frequency distribution from the same project is shown in Figure 2. These are typical distributions, but some projects show much greater variability, particularly where inadequate concrete consolidation has occurred.

Variations in Assumed and Actual Design Values

Actual values often vary considerably from those predicted in the design phase. Factors that vary include actual versus design material resiliency and strength, actual versus predicted traffic loadings, and even actual versus predicted climatic conditions. Perhaps the most obviously uncertain design factor that must be estimated is the traffic loadings expected over the life of the pavement. Social, economic, and political factors cause much uncertainty in traffic forecasting. Probably the most dramatic difference between estimated and actual traffic has occurred for high-volume freeways. Pavement design for the New Jersey Turnpike was based on an estimate of 20 million vehicle applications, but actual 20-year counts showed more than 90 million applications. Similar situations have occurred on other highways such as the Dan Ryan Expressway in Chicago (26). Deacon and Lynch (14) estimated equivalent wheel loads for 20 locations in Kentucky by using a new method and compared the predictions with actual data measured over about 20 years. They concluded that the actual traffic will usually fall between one-half and twice the best estimate.

Variation Due to Inadequacy of Design Procedure

An interesting example of the inadequacy of design procedures can be obtained from the results of the AASHO Road Test for flexible or rigid pavements. The number of load applications to a terminal serviceability index of 2.0 was computed for several flexible sections by using the design equations developed at the road test and included in the AASHO Interim Guide (15). The computed load applications were then plotted versus the actual load applications (Figure 3) for flexible pavements. The scatter of data is indicative of the inability to predict performance with these empirical equations even under controlled testing conditions. To this uncertainty must be added uncertainties that are imposed when the design equations are used in other climates and soils and for mixed traffic loadings.

Statistical Analysis Units

As a pavement is subjected to traffic and environmental effects, various types of distress occur at seemingly random locations along the pavement. The variation along the project must be carefully estimated for the probabilistic approach to be meaningful. It is helpful to the analysis to consider the pavement as a series of short lengths or areas within which pavement properties are assumed to be homogeneous and the variability along the project is considered to be between these pavement lengths or areas. The size of these so-called statistical analysis units can vary depending on the purpose of the analysis from say 1-ft^2 (0.09-m^2) areas to 200-ft (61-m) pavement lengths. The smaller the area is, the greater the variability of mean values is, however.

Design Goal: Optimization

Choosing the best possible design for a given project situation, or optimizing, is a subject of vital importance to the pavement engineer and the essence of modern engineering practice. The increased need for better optimization arises from the scarcity of pavement funds, materials, and fuel on one hand and increasing public demand for better pavement performance and less traffic delay due to excessive maintenance activities on the other hand.

Probabilistic design applications help the engineer to achieve greater optimization in design in several ways. First, they permit direct consideration of variations and uncertainties. The applied safety factors are direct functions of the existing variations and uncertainties and, hence, increase or decrease depending on the amount of variations and uncertainties.

Second, probabilistic design procedures enable the engineer to predict occurrence of random distress. The amount of distress is directly related to the amount of required maintenance, which on pavements that carry moderate to high traffic volumes has significant effects on user delays.

Third, such design procedures provide the technology so that the engineer can better assess the reliability of designs and design pavements with different traffic volumes or functional requirements at different levels of reliability (e.g., farm-to-market road versus high-volume freeway).

Finally, use of the procedures optimizes facility and user costs. By designing pavements at the level of reliability best suited for each situation, the engineer minimizes total life cycle costs (including initial construction, maintenance, overlay, and user delay costs). This is particularly important, for example, for high-volume pavements where the consequence of failure is severe user delay due to required maintenance activities (12).

PROBABILISTIC STRESS-STRENGTH ANALYSIS AND DESIGN

Whenever the stress level in any portion of a pavement structure exceeds strength, a fracture occurs. The probability of fracture can be defined as

$$p_f = P(S > F) \tag{1}$$

where

P = probability of occurrence,

S = applied stress, and
F = strength.

Conversely the probability of no fracture, or the reliability R, can be defined as R = 1 - p_f. The magnitude of strength within a pavement structure is a random variable in that it varies from point to point. The magnitude of applied stress is also a random variable that depends in part on loading conditions from both climate (temperature and moisture) and traffic. Because both stress and strength are random variables, p_f can be expressed as

$$p_f = P(S > F) = P(d > 0) \tag{2}$$

where d = F - S. Therefore, f(d) is the difference density function of S and F. It is reasonable to assume, based on limited data, that the stress magnitude from heavy cargo vehicle loadings is approximately normally distributed (25); it has been fairly well established that the strength of many materials is approximately normally distributed. Because the level of p_f is relatively large for pavements, the effect of the exact shape of the distribution curve in the tail portion is less, and the assumption of normality appears reasonable. If F and S are normally distributed, d will also be normally distributed (Figure 4).

Bars are used above the expressions to represent their mean values:

$$\bar{d} = \bar{F} - \bar{S} \tag{3}$$

The standard deviation of d σ_d can be computed as follows:

$$\sigma_d = \sqrt{\sigma_S^2 + \sigma_F^2} \tag{4}$$

As shown in Figure 4, the probability of fracture p_f is given by the area to the left of 0.

$$p_f = P(d < 0) = P(-\infty < d < 0) = \int_{-\infty}^{0} f(d)\,d(d) \tag{5}$$

Reliability is the area to the right of 0 as shown.

Normal distribution tables can be used to calculate p_f or R. For example, consider a 7-in. (180-mm) PCC slab under a 9,000-lbf (40-kN) wheel load located at its edge. The mean tensile stress at the critical location in slab (based on Westergaard's edge equation) is $\bar{S}$ = 360 psi (2500 kPa). The mean flexural strength of the slab is $\bar{F}$ = 690 psi (4760 kPa).

$$\sigma_S = 48$$

$$\sigma_F = 104$$

Therefore,

$$d = 690 - 360 = 330$$

$$\sigma_d = \sqrt{48^2 + 104^2} = 115$$

The parameter d must now be transformed into a normal variate with a mean of zero and variance of one so that normal distribution tables can be used.

$$Z = \frac{d - \bar{d}}{\sigma_d} = \frac{0 - \bar{d}}{\sigma_d} = \frac{-330}{115} = -2.87$$

The area under the normal curve from $-\infty$ to -2.87 is 0.0021; therefore, p_f = 0.21 percent. An illustration of this area of failure is shown in Figure 5 where the actual distributions of the flexural strength and stress for a 7-in. (180-mm) slab are shown to overlap. This area of overlap is not the probability of failure but a function of the probability of failure. The figure also shows the stress distribution for a 9-in. (230-mm) slab where the probability of failure is very small. The other design inputs for this pavement section are given in Table 1.

In engineering terms, the probability of fracture in the PCC slab is 0.21 percent, or, given the large number of these slabs that would exist along a pavement project, about 0.21 percent of these 7-in. (180-mm) slabs would be expected to fracture if this stress level is applied to all. Note that fracture is, at most, only an initial hairline crack at the critical point in the slab. It is well-established that collapse load is at least twice the load calculated by the Westergaard expression. However, only a few load applications will cause the crack to widen and perhaps result in a serious failure condition as observed on in-service pavements and at the AASHO Road Test. The basic assumptions made in this analysis are that (a) stress and strength are normally distributed and (b) they are independent random variables.

Determination of the variations (or standard deviations) of S and F in real pavement structures is the next task. Again the example of a PCC pavement slab will be used for illustration. Stress is a function of several variables:

$$S = f(E, t, k, P, \ldots) \tag{6}$$

where

E = concrete modulus of elasticity,
t = slab thickness,
k = modulus of foundation support, and
P = traffic wheel load.

The flexural strength of the concrete is also a function of several parameters:

$$F = f(M, C, Q, \ldots) \tag{7}$$

where

M = materials used (quantity and type),
C = curing, and
Q = consolidation.

Figure 1. Variation of compressive strength of cores cut in 500-ft (150-m) intervals from a PCC slab.

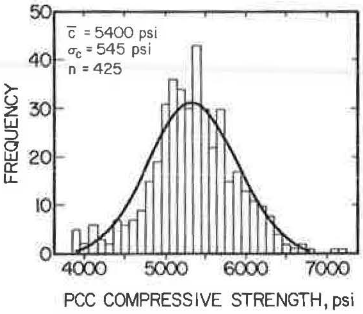


Figure 2. Variation of PCC slab thickness.

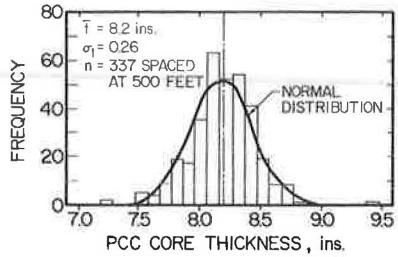


Figure 3. Actual versus predicted 18-kip (80-kN) single-axle load applications.

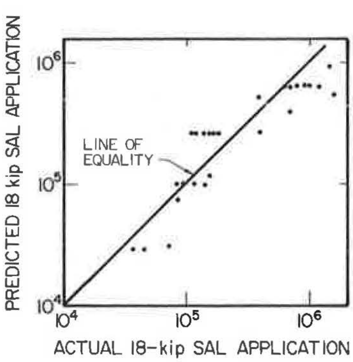


Figure 4. Distribution of d = F - S showing areas of failure probability and reliability.

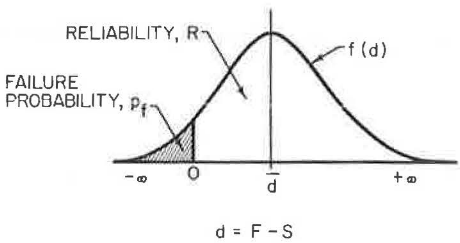


Figure 5. Distribution of flexural strength along a PCC slab and distributions of applied stresses estimated by using Westergaard's edge loading model.

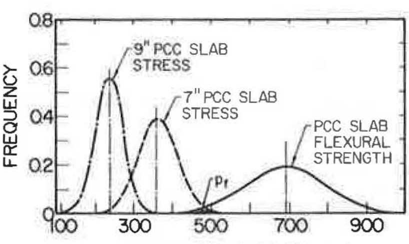


Table 1. Description of design inputs for a jointed PCC pavement.

		Expected Variation (percent)		
Design Factor	Mean	Low	Medium	High
Flexural strength, psi	690[a]	10	15	25[b]
Modulus of elasticity, psi	4.2×10^6	10	15	25[b]
Modulus of foundation support (top of subbase), lbf/in.^{3}				
k_c	108	20	35	50[b]
k_s	61			
Initial serviceability	4.5	—[c]		
Terminal serviceability	2.5			
Forecast 20-year 18-kip EAL in design lane	5×10^6			
Design equation variance of log W (error)	0		0.0353[d]	
Slab thickness, in.	Varies	—[e]		

Note: 1 psi = 6.9 kPa; 1 lbf/in.^{3} = 271 kN/m^3; 1 kip = 4.4 kN; 1 in. = 2.5 cm.
[a]Third point loading, 28-day curing.
[b]Coefficient of variation.
[c]Expected variation of 0.2 (low), 0.3 (medium), and 0.4 (standard deviation).
[d]Estimated from AASHO data (2).
[e]Expected variation of 0.2 (low), 0.3 (medium), and 0.5 (standard deviation).

The variation in S and F will, of course, depend on the variability of the factors that influence them. A linear first-order approximation may be used to determine the variance of S, for example, as a function of variations of the parameters E, t, k, and P according to the Westergaard stress model. This variance equation for S can be derived by using a Taylor's Series expansion of the function about its mean (5). The resulting expression, if we neglect moments greater than second order, is as follows assuming independence of the factors (for stress):

$$\sigma_S^2 \simeq \sum_{i=1}^{j} \left(\frac{\partial S}{\partial X_i}\right)^2 \sigma_{X_i}^2 \tag{8}$$

where

σ_S^2 = variance of S,
X_i = random variables included as parameters in S (i.e., E, t, k, P), and
$\sigma_{X_i}^2$ = variance of X_i.

The variance of S can then be determined as a function of the variations of E, t, k, P, and e, which is a random variable representing the inadequacies of the Westergaard equation to predict the true tensile stress in the slab.

$$\sigma_S^2 \simeq \left(\frac{\partial S}{\partial E}\right)^2 \sigma_E^2 + \left(\frac{\partial S}{\partial t}\right)^2 \sigma_t^2 + \left(\frac{\partial S}{\partial k}\right)^2 \sigma_k^2 + \left(\frac{\partial S}{\partial P}\right)^2 \sigma_P^2 + \sigma_e^2 \tag{9}$$

Equation 9 is called the variance equation of S and is given in full elsewhere (3). This technique has been used on several design models (2, 5, 7) and has compared well to simulation results (<±10 percent).

Determination of σ_E, σ_k, . . . , is an important task that requires special consideration. For example, consider the selection of the standard deviation of the foundation support modulus k. This parameter will vary from point to point along the grade. Predicting its mean value during the service life of the pavement will also be uncertain because of pumping and settlement conditions, and it will vary through the year depending on factors such as moisture and temperature. These variabilities can be estimated from past construction and performance data, if available, or from direct testing or experience, if necessary. These sources of variability must finally be combined to give an overall σ_k for use in design.

An application of these stress-strength concepts is presented by showing the effects of overloads on distress occurrence. Overloads in this example are considered as single-axle loads greater than 18 kips (80 kN). Again, the variations of material properties, thicknesses, and loads can be estimated, and the resulting variations in stress along the pavement can be predicted by using Westergaard's edge loading model (21) according to variance equation 9. In this example, the Westergaard edge loading model was selected and a variance expression was derived (3). The probability that tensile stress would exceed strength was calculated according to equation 5 for a 9-in. (230-mm) slab thickness. A plot of p_f versus axle load is shown in Figure 6. The upper curve represents a situation where material properties, thicknesses, and the like vary considerably along a project. The lower curve represents more uniform conditions (or better quality control) as given in Table 1. The type of fracture considered here is definitely not collapse fracture as discussed but is at most initial hairline cracking, which usually leads rapidly to wider cracks.

Results shown in Figure 6 indicate the relative effect of overloads on distress occurrence. For example, for an axle load of 24 kips (107 kN), the proportion of cracking as the load rolls along the pavement edge would vary from 0.03 to 1.8 percent compared to 0.002 to 0.49 percent for an 18-kip (80-kN) single-axle load. The effect of variations on distress is very evident from these analyses.

PROBABILISTIC FATIGUE ANALYSIS AND DESIGN

Fatigue Analysis

Repeated load applications cause several types of damage to pavement structures including fracture, permanent deformation or distortion, and disintegration. Basic theory is presented for damage due to fracture, but probabilistic concepts can also be extended to other distress types.

The basic approach shown in Figure 7 will be developed by using the PCC slab example previously described. First, the mean stress and its variation must be estimated from the means and variances of the parameters on which it depends as previously described (Figure 5). The mean strength $\bar{F}$ and standard deviation σ_F must also be estimated. Next a fracture fatigue curve must be obtained for the material under consideration such as the following for plain PCC (16):

$$\log_{10} N_{f_i} = a + b\left(\frac{S_i}{\bar{F}}\right) \tag{10}$$

where

N_{f_i} = load applications to fracture of a specimen in bending at i th load level,
a = 19.3,
b = -20.2, and
S_i = concrete tensile stress at critical location in slab due to i th load.

The variation of N_f will be a function of the variation of S, F, and other unknown parameters denoted by e′. A variance expression can be derived as follows:

$$\sigma^2_{\log N_f} \simeq \left(\frac{\partial \log N}{\partial S}\right)^2 \sigma_S^2 + \left(\frac{\partial \log N}{\partial F}\right)^2 \sigma_F^2 + \sigma^2_{e'} \tag{11}$$

A fatigue damage model must now be used to determine the effect on accumulated damage of varying wheel load magnitude. Miner's damage expression has been used in previous design and will be used here:

$$D = \sum_{i=1}^{m} \frac{n_i}{N_i} \tag{12}$$

where

D = total accumulated fatigue damage over pavement design life,
n_i = number of applied loads of i th magnitude, and
N_i = number of allowable traffic loads of i th magnitude at fracture.

D is not a constant for a pavement but varies depending on the randomness of n_i and N_i. There is a significant uncertainty in predicting n_i and variability in N_i. There are some data and theory to support the assumption that these variables are approximately lognormally distributed (5). Inasmuch as D is the sum of one or more ratios of n to N, its distribution will depend on the distribution of n and N. As a first-order approximation, it is assumed that D follows a lognormal distribution since both n and N are approximately lognormally distributed. Even though the quotient of two normally distributed random variables does not exactly follow a normal distribution, simulation and theory show that it is practically normally distributed. The total variation in D can be determined as

$$\sigma_D^2 \simeq \sum_{i=1}^{m} \left(\frac{\partial D}{\partial N_i}\right)^2 \sigma_{N_i}^2 + \sum_{i=1}^{m} \left(\frac{\partial D}{\partial n_i}\right)^2 \sigma_{n_i}^2 \qquad (13)$$

where

σ_D^2 = variance of accumulated damage,
$\sigma_{N_i}^2$ = variance in allowable fatigue applications to fracture (can be estimated from equation 11), and
$\sigma_{n_i}^2$ = variance in forecast traffic applications (5).

Finally, the distribution of fatigue damage D along a pavement can be obtained as shown in Figure 7. The probability of fatigue fracture p_{ff} is

$$p_{ff} = P(D > 1.0) = P(\log_{10} D > 0) \qquad (14)$$

If we assume that D is lognormally distributed, p_{ff} can be readily calculated as previously described. p_{ff} represents the probability that a slab will fracture because of fatigue, or for a long section of pavement it represents a proportion of length or area along the pavement that will fracture or crack because of traffic load bending stresses. Its relation to the cracking index will be discussed subsequently.

Example of Fatigue Damage Application

The theory and concepts for applying probabilistic methods to fatigue damage have been presented. This application is significant in interpreting the extent of fracture that may occur if a pavement is designed for $\bar{D} = 1.0$, as is the usual case. The deterministic approach leaves much to be desired in that no assessment of how much cracking will occur when D = 1.0 can be made.

This example uses AASHO Road Test rigid pavement data and a probabilistic distress analysis to show the correlation between the possibility of distress and the cracking index or linear feet of slab cracking per 1,000 ft^2 (93 m^2) of pavement. The procedure shown in Figure 7 is followed in the analysis of the various road test slabs.

1. Means and standard deviations of the pavement factors E, k, t, and P were determined from road test data (20), for all pavement sections that were subjected to single-axle loadings ranging from 3.5 to 9.5 in. (89 to 241 mm) in slab thickness for both reinforced and nonreinforced sections. For example, for section 523, loop 5, reinforced slab,

Factor	Value	Coefficient of Variation
$\bar{t}$, in.	6.5	0.05
$\bar{E}$, psi	6.25×10^6	0.15
$\bar{P}$, kips	11.2	0.10
$\bar{F}$, psi	790	
$\bar{k}$, lb/in.^{3}	108	0.35

In this example, $\bar{F}$ has a standard deviation of 61 psi (421 kPa). [k determined for loss of support of this slab due to severe pumping is 9 lbf/in.^{3} (2.4 MN/m^3).]

2. The Westergaard interior load stress model (21) is used to predict the mean maximum tensile stress of PCC slabs. Through use of the finite element program and the measured AASHO Road Test stresses, it is found that the Westergaard interior load model predicted reasonable maximum stresses that would occur from a load located about 20 in. (508 mm) from the pavement edge, which is the approximate mean of the outside wheel path loads at the road test. The loss of support for each slab due to pumping is considered through a reduction in the k-value by means of the correlation developed by McCullough and Yimpasert (22) between pumping index and eroded area and the correlation developed by Kher et al. (18) between eroded area and modified k-value. When pumping occurred, the modified k-value is used in the Westergaard interior stress calculation. The thermal shrinkage stress of the slabs is also considered by assuming an average temperature differential of 1 deg F/in. (0.219 deg C/cm) of slab throughout the loading period and by using the interior warping stress model developed by Westergaard (23) and Bradbury (24). Hence, the final stresses used in the fatigue damage analysis considered slab loss of support from pumping and a small thermal tensile warping stress.

As previously discussed, a variance equation was developed from the Westergaard interior stress model by using equation 9 so that the variation in stress due to variations in the design parameters E, k, t, and P could be estimated. For example,

$\bar{S}_{traffic}$ = 443 psi (3054 kPa), standard deviation = 60 psi (414 kPa);
$\bar{S}_{\substack{temp\\warp}}$ = 82 psi (565 kPa), 40-ft (12-m) joint spacing; and
$\bar{S}_{tot}$ = 525 psi (3620 kPa), critical tensile stress for interior loading.

3. The fatigue S-F:N curve determined by Kesler (16) for plain dry portland cement concrete is used as defined by equation 10. The variance of this equation as derived by equation 11 is

$$\sigma^2_{\log N_f} \simeq \frac{408\sigma_S^2}{F^2} + \frac{408\bar{S}^2\sigma_F^2}{F^4} \tag{15}$$

For example,

$\log \bar{N}_f = 19.3 - 20.2\ (S/F) = 5.8759$, or $\bar{N}_f$ = 751,450 load applications; and
$\sigma_{\log N_f} = 1.8514$ (from equation 15).

4. Miner's damage hypothesis is used to predict the average amount of fatigue damage for the AASHO Road Test slab and loading combination. The variance of D for each slab is computed by using equation 13, and the probability of distress is calculated according to equation 14.

$$\bar{D} = \sum \frac{n_1}{N_1} = \frac{828{,}000}{751{,}450} = 1.102$$

or

$$\log \bar{D} = 0.0421$$

$$\sigma_D^2 \simeq \frac{\sigma_{n_1}^2}{N_1^2} + \frac{N_1^2}{N_1^4}\sigma_{n_1}^2 = 22.06$$

where

$\sigma_{n_1}^2 = 0$ (i.e., no error in determining load applications applied at the road test),
$N_1 = 751{,}450$ load applications,
$n_1 = 828{,}000$ load applications to serviceability index of 2.5 (20), and
$\sigma_{N_1}^2 \simeq \frac{\sigma_{\log N}\,\bar{N}_1^2}{0.1886}$.

Therefore,

$$\sigma_{\log D} \simeq \frac{(0.4343)\,\sigma_D}{\bar{D}} = 1.8514$$

The probability of distress is estimated from the distribution of log D as follows:

$$z = \frac{0.0 - 0.0421}{1.8514} = -0.023$$

From normal distribution tables, the area from -0.023 to $+\infty$ is the probability of distress, which equals 0.51 or 51 percent.

5. The cracking index is determined for each slab, either at a serviceability index of 2.5 or at termination of the test at n = 1,114,000 load applications. The cracking index for this example section is 121 ft/1,000 ft^2 (403 m/1000 m^2), which corresponds to a probability of distress of 51 percent. A plot of probability of distress [or $P(D > 1)$] versus cracking index for the nonreinforced slabs is shown in Figure 8 and for reinforced slabs in Figure 9. A fair correlation exists but there is much scatter. For example, at a probability of distress of 50 percent, or when $\bar{D} = 1$, the cracking index is about 90 ft of cracking per 1,000 ft^2 (333 m/1000 m^2) for the nonreinforced slabs or an average of about 16 linear ft (4.9 m) of cracking per 15-ft (4.6-m) slab. This amount of cracking would have a significant effect on serviceability rating and also on maintenance requirements of the pavement. A cracking index of 90 would correspond to a serviceability rating of about 1.5 to 2.2 according to Figure 17-F in the road test report (20). From Figure 17-F in the AASHO Road Test report and Figure 9, a probability of fatigue fracture of 50 percent or $\bar{D} = 1$ corresponds to a terminal serviceability of approximately 1.5 for reinforced slabs. Therefore, it seems that, for the road test pavements at least, designing at $\bar{D} = 1$ may produce a pavement that will show extensive cracking before the design number of load applications is applied.

This probabilistic fatigue damage approach is of course simplified in several respects but has definite advantage for use in design over the deterministic approach. A plot of probability of distress versus slab thickness for given pavement design situations

Figure 6. Estimated probability of edge fracture versus mean single-axle load for a 9-in. (230-mm) PCC slab.

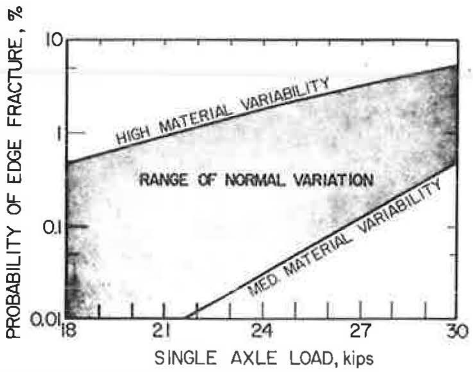


Figure 8. Cracking index versus estimated probability of fracture for AASHO Road Test nonreinforced rigid pavement sections.

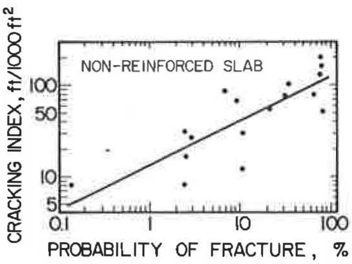


Figure 9. Cracking index versus estimated probability of fracture for AASHO Road Test reinforced rigid pavement sections.

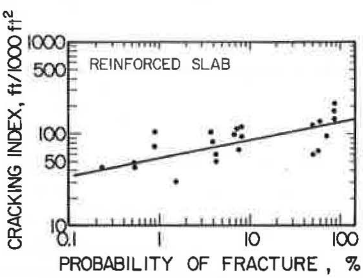


Figure 7. Approach for determining probability of fracture distress due to repeated traffic loadings.

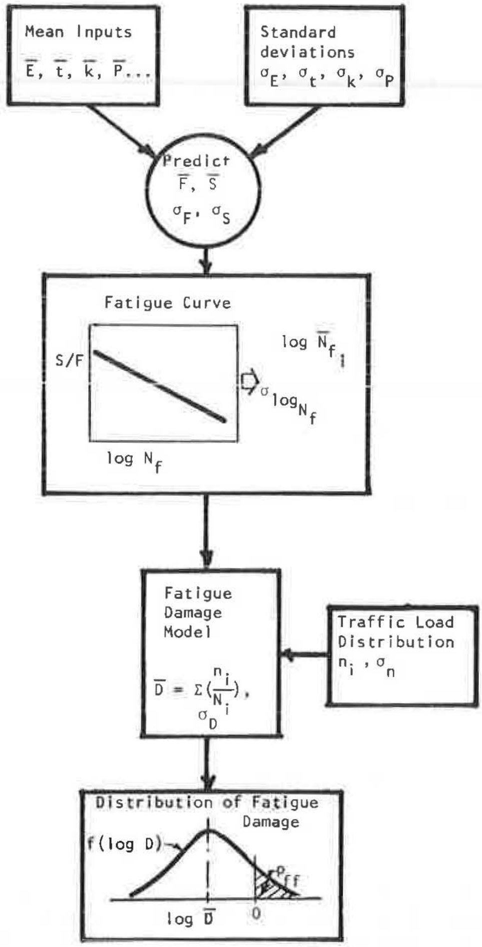
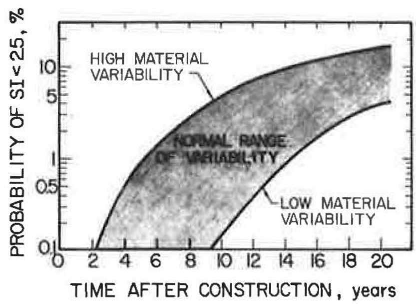


Figure 10. Years since construction versus estimated probability of the serviceability index being less than 2.5.



can be determined to assist in selecting the final design thickness. An estimate of the extent of cracking also provides information about the required maintenance.

PROBABILISTIC SERVICEABILITY ANALYSIS AND DESIGN

Serviceability Design Application

Probabilistic serviceability methods have been applied to four existing empirical design methods including the Texas flexible pavement design system (5, 6), the SAMP6 flexible pavement design system (17), AASHO rigid pavement design procedures (2), and the Texas rigid pavement design system (18).

The probability of distress is defined in terms of the probability of the serviceability level of a section of pavement dropping below a minimum acceptable level. If the loss of serviceability is predominantly due to traffic loadings, then

$$p_{sf} = P(w > W) \tag{16}$$

where

p_{sf} = probability that a pavement section will reach a minimum acceptance serviceability level during a specified design period,
w = forecast 18-kip (80-kN) equivalent single-axle load applications over design period, and
W = allowable 18-kip (80-kN) equivalent single-axle load applications at acceptable serviceability level (calculated from a predictive model based on the serviceability-performance concept).

It is important to realize that this definition considers only the detrimental effect of traffic loadings. In areas where much of the loss in serviceability is due to environmental conditions, the equation for W (if based on the AASHO Road Test results) must be modified to account for the additional detrimental effects.

Because the concept and theory concerning probabilistic serviceability analysis and design have been published in the references previously cited, only a few examples illustrating the application will be given.

Rigid Pavement Example

Consider the structural design of the pavement described in Table 1. Structural thickness of the PCC slab can be determined according to a commonly used deterministic design procedure such as AASHO Interim Guide (15). For the given foundation support, traffic, and other factors, jointed concrete slab thickness of 9.0 in. (230 mm) is required. The only applied safety factor is the reduction of working stress to three-fourths of the flexural strength.

An analysis is conducted by using probabilistic methods to analyze the performance and adequacy of this design. The variability of pavement serviceability level along the project as traffic loads are accumulated over time can be predicted by using the variance expression developed by Kher and Darter (2, equation 20). The variation of material properties along the pavement, possible design assumption errors, and design equation inadequacy must be estimated for the project as given in Table 1. If we assume a linear increase in traffic applications, the percentage of pavement area that will reach terminal serviceability at any time can be computed as follows.

The mean $\overline{W}$ for the jointed concrete pavement to reach terminal serviceability is 13.3×10^6 18-kip (80-kN) equivalent axle loads (EALs) based on the AASHO performance equation. The variance of log W due to a high level of variability in material properties, slab thickness, and lack of fit of the model is computed as

$$\sigma^2_{\log W} = 0.202$$

by using Kher and Darter's equation 20 (2). The percentage of pavement to reach terminal serviceability after 3×10^6 18-kip EALs is computed as

$$Z = \frac{\log_{10}(13.3 \times 10^6) - \log_{10}(3 \times 10^6)}{\sqrt{0.202}} = 1.44.$$

From normal distribution tables, the area between $-\infty$ and 1.44 is 0.075; therefore, p_{sf} = 7.5 percent. The 3×10^6 18-kip EALs are accumulated at 12 years assuming a linear increase of traffic with time. A plot of p_{sf} versus time in years since construction is shown in Figure 10. For example, at 10 years, the proportion of the project reaching a minimum acceptable serviceability level may range from about 0.1 to 5 percent, depending on the level of variability. At 20 years, the proportion reaching this level will be from 4 to 17 percent assuming only normal routine maintenance has been applied. How much pavement area can be allowed to reach this state of deterioration before major rehabilitation of the entire pavement is necessary depends on several factors such as traffic volume, available funds, and maintenance policies but may range between 5 to 30 percent.

Based on these examples, it is obvious that the traffic-load-associated distress shown in Figure 10 can be either reduced or increased by varying the thickness of the PCC slab (holding the subbase and subgrade constant) according to the AASHO design models. It is interesting to note that the original AASHO Road Test pavement sections still in service on I-80 in Illinois tend to confirm this conclusion after 16 years of service. There is less distress in the 11- and 12.5-in. (279- and 318-mm) reinforced slab sections than in the 8- and 9.5-in. (203- and 241-mm) sections. Hence, slab thicknesses that provide varying levels of traffic load design reliability can be determined. If environmental deterioration has a significant effect, the calculated reliability will be greater than the actual reliability. Also if other factors such as the slab joints show serious distress (i.e., poor joint design), the actual reliability will be less than the calculated reliability. Either the variance equation derived from the AASHO equations can be used directly or a nomograph developed by Kher and Darter (2) can be used to determine slab thickness at varying levels of $\bar{R}$ as shown in Figure 11.

Flexible Pavement Example

Similar examples can be developed for flexible pavements. The Texas flexible pavement system (FPS) considers variability in several parameters. The design system can provide designs at various levels of design reliability for traffic-related distress including complete life cycle costs. A summary of six pavement designs ranging in reliability from 50 to 99.99 percent is given in Table 2 for a recently constructed high-volume urban freeway.

The selection of the level of design reliability is an important task. An approximate estimate of the minimum level of reliability using the Texas FPS for various types of projects ranging from farm-to-market roads to urban freeways was determined based on the judgment of experienced pavement engineers. The selected design reliability increases with the function or type of highway pavement being designed, its urban or rural location, and the traffic volumes and equivalent load applications expected (5, 12, 19). Again, these reliability levels should not be considered as absolute values inasmuch as they are relative to the accuracy of the estimated variations of the design parameters and adequacy of the design equation.

Figure 11. Design reliability (for traffic loadings) versus required PCC slab thickness for a given subbase and subgrade.

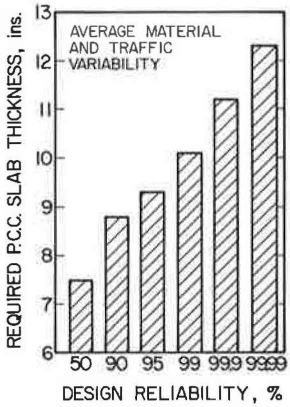


Table 2. Summary of optimum (total costs) designs for flexible pavement at various levels of reliability.

	Reliability Level (percent)					
Design Criteria	50	80	95	99	99.9	99.99
Initial cost	2.66	3.20	3.68	3.73	4.41	5.19
Routine maintenance	0.28	0.28	0.28	0.23	0.22	0.22
Overlay	0.00	0.00	0.00	0.27	0.54	0.50
User delay	0.00	0.00	0.00	0.29	0.16	0.24
Salvage	-0.32	-0.39	-0.49	-0.55	-0.69	-0.77
Total, $/SY	2.62	3.09	3.47	3.97	4.64	5.38
Thickness, in.						
Asphalt concrete	2.00	2.25	2.00	2.00	2.00	2.00
Black base	2.75	3.25	3.50	3.00	3.50	5.55
Crushed stone	6.00	8.00	12.00	14.00	18.00	18.00
Initial life, years	21.6	21.4	21.0	10.5	9.2	9.5
Overlay thickness, in.				1.3	2.3	2.3
Life of overlay, years				20.0	22.0	22.0

Note: 1 in. = 2.5 cm.

SUMMARY

Applying statistics or probabilistic methods to the design and analysis of pavements provides the technology for important applications not possible before. These include the following:

1. Provides the basis from which design optimization can be conducted—the degree of design adequacy can be balanced between facility costs and pavement user costs and effects;
2. Makes the design process sensitive and capable of adjusting for many of the uncertainties and variabilities associated with pavement design, construction, and performance;
3. Enables estimation of the amount of distress occurring along a pavement—reasonable correlation was found, for example, between the probability of fatigue fracture distress and cracking index for rigid pavements (the methodology may ultimately lead to prediction of maintenance requirements); and
4. Provides the capability to design at various levels of reliability—therefore, design adequacy can be estimated much better than ever before.

ACKNOWLEDGMENTS

This study was conducted at the Transportation Research Laboratory, University of Illinois. The author wishes to thank Stephen Alfield and Paul McNabb for their assistance with the analysis.

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