# FATIGUE ANALYSIS FROM STRAIN GAUGE DATA AND PROBABILITY ANALYSIS 

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#### Abstract

This report presents a rational approach for determining remaining fatigue life of a bridge. A methodology was developed to determine fatigue damage from a probability analysis of traffic data by reconstituting or synthesizing the load (traffic) history of bridges. A mechanical seratch gauge was used to obtain a short period of stress history of bridge members on the Central Bridge over the Ohio River at Cincinnati. Stress histories deduced from the strain gauge records were used to evaluate fatigue damage to the bridge. The remaining life of the bridge obtained by these two methods was then compared.


- THE Central Bridge over the Ohio River between Newport, Kentucky, and Cincinnati, Ohio, was completed in 1891 and in 1972-73 was considered to be in danger of fatigue failure. A series of investigations was undertaken to determine the likelihood of failure and to estimate the time of probable failure. During the investigation, a methodology was developed to determine fatigue damage from a probability analysis of traffic data by reconstituting or synthesizing the load (traffic) history of bridges. Strain gauge data obtained with Prewitt scratch gauges and SR-4 resistivity gauges were used to evaluate fatigue damage incurred by the Central Bridge.

Repeated stressing of metals above certain limits induces intercrystalline and intracrystalline dislocations and cleavages and eventually cracks that propagate to failure. This internal damage is insidiously cumulative and irreversible. This phenomenon was recognized as early as 1829 and was termed fatigue as early as 1839 (1). From the beginning of fatigue testing (Wholer, 1858-1870), results have been reported as $S-N, S-\log N$, or $\log S-\log N$ curves, where $N$ is the number of repetitions of stress $S$. One purpose of fatigue testing was to find the endurance or fatigue limit (i.e., $\mathrm{f}_{\mathrm{e}}$ ) and thereby to establish the design or working stress.

To plot $\mathrm{S}-\mathrm{N}$ graphs required that many specimens be tested at several stress levels, each in simple, repetitive cycling. About 1910, compound loading tests evolved. The linear summation of cycle ratios is believed to have originated with A. Palmgren in 1924. In this country, it was proposed by B. F. Langer in 1937, although credit is often given to M. A. Miner in 1945. This hypothesis suggested that the fractional fatigue damage in a specimen caused by $N$ repetitions of a stress $S$ is the ratio of the number of those repetitions to the number of repetitions at the same stress level that would cause failure (determined from other specimens). Inherent in this notion is the fact that fractional damages are additive and that the totality of fractions cannot exceed one. It is therefore possible, on this premise, to predict remaining fatigue life from $\mathrm{S}-\mathrm{N}$ envelopes and to do so in terms of compound stressings. Unfortunately, the simplicity implied here is perhaps unreal. Indeed, the variability attending fatigue tests may introduce incertitudes that may otherwise limit the summation of damage increments (fractions) to a value less than 1 , perhaps 0.80 . Some commentaries have suggested fail-safe values of 0.30 . More complete reviews of fatigue technology are available elsewhere ( $2,3,4$ ).

Many bridges built more than 50 years ago were designed to resist fatigue due to the then-standard loads. Legal allowable gross weights of trucks have increased more than fourfold in 20 years. The possibility of fatigue failure becoming imminent demanded investigation and analysis. The catastrophic failure of the bridge at Point

Pleasant, West Virginia (5), and the necessary subsequent retirement of the C\&O Bridge (US-25) at Covington, Kentucky, are conspicuous events in engineering history and both examples of delimited service life.

## PROBABILITY ANALYSIS OF TRAFFIC DATA

## Basic Fatigue Equation

In addition to stresses due to the dead load DL and live load LL, stresses due to wind loading WL and temperature changes TC must be considered if they significantly affect the stress level. The fatigue F of a bridge member due to one repetition of a particular loading combination LC can be computed from

$$
\begin{equation*}
F=f(L C) \times f\left(S_{L \mathrm{C}}\right) \times f\left(F_{\mathrm{LC}}\right) \tag{1}
\end{equation*}
$$

where $f(L C)=I \times f(L L+W L+T C)+f(D L) . f\left(S_{L C}\right)$ is a function for transforming the total equivalent load of the loading combination to the corresponding stress level in the structural member and the fatigue damage in the member due to the stress induced by one repetition of LC.

Equation 1 is a generalized relationship for computing the fatigue damage of a bridge member due to a single repetition of a particular loading combination. The total fatigue damage $F_{t}$ in a design period includes the cumulative fatigue contributions of all loading combinations placed on the structure, or

$$
\begin{equation*}
F_{t}=365 \sum_{\text {all years }} \sum_{\text {all LC }} A A D T \times P_{\mathrm{Lc}} \times F \tag{2}
\end{equation*}
$$

where $P_{\mathrm{Lc}}$ is the probability of any loading combination LCoccurring on the bridge section.

## Vehicle Loading Distributions

Because many bridge spans are very long, the load cannot be designated simply as that for a single vehicle or series of axle trains. A long span, for example, could hold several large combination trucks at one time if both lanes were completely loaded. All of these vehicles must be considered as contributing to fatigue. The occurrence of such a fully loaded bridge span is rare. The probability of a lesser number of vehicles occurring on the span at the same time is of course much greater. Therefore, probabilities of each of the loading possibilities must be determined. Because of the extreme length of many bridge spans and because stresses in members vary as the load moves along the span, gross vehicle weight was chosen in this study as the smallest unit weight to be considered.

## Loading Probabilities

One-Directional Probabilities
When a single vehicle passes a designated point on a highway, the probability that this vehicle is of vehicle classification is given by $P_{1}$, the frequency of vehicle type $i$ in the total traffic stream. The probability that $n$ consecutive vehicles traveling in the same direction past a point are type $i$ is given by

$$
\begin{equation*}
P_{n 1}=P_{1}^{n} \tag{3}
\end{equation*}
$$

Equation 3 can be modified to give the probability that these vehicles will pass the point of interest within a specified time interval $t$ :

$$
\begin{equation*}
P_{n!G}=P_{1}^{n} P_{G(t)}^{(n-1)} \tag{4}
\end{equation*}
$$

where $P_{G}(t)$ is the probability of a gap being of average length $G(t)$. Gap length probabilities required in equation 4 were developed previously for specific bridges spanning the Ohio River in Kentucky (6). Final probability curves were developed by recording actual vehicle gap lengths (in seconds) and then converting the gap distributions from units of time to units of length by considering the average vehicle spot speeds at these locations.

If we assume that gap distances are equal, the average gap length for vehicles within the critical length of roadway $L$ is found to be

$$
\begin{equation*}
G(t)=\left(L-n_{1} V L_{1}\right) /\left(n_{1}-1 / 2\right) \tag{5}
\end{equation*}
$$

where $\mathrm{VL}_{1}$ is the average length of vehicle type i (Table 1). The average gap for mixed traffic (Figure 1) in one direction is found from

$$
\begin{equation*}
G_{\text {mix }}=\left(L-\sum_{\text {all } i} n_{1} V L_{1}\right) / \sum_{\text {all } i}\left(n_{1}-1 / 2\right) \tag{6}
\end{equation*}
$$

where

$$
\sum_{\text {all i }} n_{1} V L_{1}<L
$$

Because of the large number of variable combinations, we restricted the vehicle classification to the following three vehicle types:

| Vehicle Type | Code |
| :--- | :--- |
| Automobiles | $\mathrm{i}=1$ |
| Single-unit trucks | $\mathrm{i}=2$ |
| Combination trucks | $\mathrm{i}=3$ |

In this classification, automobiles include four-tired, single-unit trucks, and singleunit trucks include buses. For this vehicle classification system, the probability of any one-directional, mixed vehicle grouping occurring in the critical length L is given by

$$
\begin{equation*}
P_{n_{1}, n_{2}, n_{3}}=\frac{\left(n_{1}+n_{2}+n_{3}\right)!}{n_{1}!\times n_{2}!\times n_{3}!} \times \frac{\Pi\left(P_{1} P_{6}\right)^{n_{1}}}{\sum_{\text {all } i} n_{1} P_{6} / \sum_{\text {all } i} n_{1}} \tag{7}
\end{equation*}
$$

where $P_{G}$ is the probability of an average gap of $G_{m 1 \times}$ occurring.

## Two-Directional Probabilities

Equations 3 through 7 concern the probabilities of the occurrence of various vehicle groupings on a specified length of highway for only one direction of travel. However, vehicle loadings in both traffic streams contribute to the fatigue of a bridge member. A previous study (6) of Ohio River bridges indicated that the effects of direction of travel on parameters such as vehicle spot speed, traffic volume, percentage of each vehicle type, gross vehicle weight, axle weight, vehicle length, and axle spacings were not statistically influenced at the 10 percent level of significance; i.e., the directional flows are essentially the same in composition and operational characteristics.

Because previously derived probabilities (7) relative to a particular point did not consider the parameter time, they were not instantaneous probabilities. To obtain instantaneous probabilities, necessary when more than one lane is to be considered concurrently, requires that an assumption be made concerning the acceptable distance D within which the effects of vehicle placements are considered as equal. $D= \pm 50 \mathrm{ft}$ ( 15 m ) was thought reasonable because this would be less than 2 sec in most cases. These values are maximum; i.e., at least 50 percent of the time the error would be less than a second. This assumption was adapted to the procedure by developing the instantaneous probability that a vehicle is present within this time limit. Based on the ratio of the total time (in seconds) that this length $D$ contains a vehicle to the total number of seconds in the day, this probability is found to be

$$
\begin{equation*}
P_{D}=A A D T \times D / 255,640 \mathrm{SP} \tag{8}
\end{equation*}
$$

where SP is the average spot speed.
The traffic composition probabilities for $r$ lanes of a one-directional highway can be found from

$$
\begin{equation*}
P R=\left[\prod_{\text {all } r}\left(P_{0} P_{n_{i}, n_{2}, n_{3}, r}\right] / P_{0}\right. \tag{9}
\end{equation*}
$$

Corresponding probabilities for two-lane, two-directional traffic can then be computed from

$$
\begin{equation*}
\overline{\mathbf{P}}=\mathbf{P}_{\mathrm{D}} \prod_{\mathrm{r}=1}^{\mathrm{r}=2} \mathrm{P}_{\mathrm{n}_{1}, \mathrm{n}_{2}, \mathrm{n}_{3, r}} \tag{10}
\end{equation*}
$$

Although these probabilities are based on numerous assumptions, the fact that traffic operation is continuous requires such assumptions. Any such probability derivation must be made with similar qualitative assumptions, although the quantitized criteria are subject to reevaluation based on actual traffic and loading studies at the particular point under consideration. Here, the number of vehicle loading combinations to be considered by these probability equations increases rapidly as the length of roadway under study increases.

## Use of Probability Equations

Before final loading distributions can be developed, traffic data must be analyzed to find the frequency of occurrence of each vehicle grouping. Based on these frequencies (probabilities), the total number of repetitions for a particular vehicle grouping ( $\mathrm{N}_{\mathrm{n}_{1}, \mathrm{n}_{2}, \mathrm{n}_{3}}$ ) during an analysis period of $Y$ years can be computed from

$$
\begin{equation*}
\mathrm{N}_{\mathrm{n}_{1, \mathrm{n}_{2}, \mathrm{n}_{3}}}=365 \sum_{\text {all years }} \text { AADT } \times \overline{\mathbf{P}} \tag{11}
\end{equation*}
$$

The total number of vehicle groups TOT to be analyzed by equation 11 during the minimum time period for an r-lane highway is obtained from

$$
\begin{equation*}
\text { TOT }=\left[\operatorname{malli}\left(\mathrm{MN}_{1}+1\right)\right]^{T} \tag{12}
\end{equation*}
$$

where $M N_{1}=L / V_{1}=$ maximum number of vehicles of type $i$ that can occur in length $L$ at one time. When the stress level falls below the endurance limit of the member being analyzed, the computational routine presented in equation 12 is terminated.

## Gross Load Distribution

Associated with each loading configuration is the probability distribution of the gross weight of that particular loading condition. To derive such a probability requires a knowledge of the parameters mentioned previously:

1. Total number of repetitions of each possible loading configuration $\mathrm{N}_{\mathrm{n}_{1}, \mathrm{n}_{2}, \mathrm{n}_{3}}$ during each year,
2. Probability $P_{n_{1}, n_{2}, n_{3}}$ of the occurrence of each loading condition in the length under consideration for each year, and
3. Individual gross vehicle load probability distribution $\mathrm{PL}_{1}$ for each vehicle type considered in the fatigue analysis.

The basic procedure considers all possible loading combinations for each gross vehicle load interval of $\mathrm{GL}_{1}$ for each vehicle of each type found in the loading configuration. The total gross loading probability distribution having $q$ intervals can be found by combining the individual gross bridge loading distributions corresponding to the individual loading configurations ( $\mathrm{P}_{\mathrm{GL} \mathrm{ijq}_{\mathrm{i}}}$ ) by the following:

$$
\begin{equation*}
\mathrm{P}_{\mathrm{TL}_{q}}=\sum_{\text {all LC }} \mathrm{P}_{\mathrm{GL}_{\mathrm{ijq}}} \mathrm{P}_{\mathrm{n}_{1}, \mathrm{n}_{2}, \mathrm{n3}} \tag{13}
\end{equation*}
$$

The $P_{G L}$ terms can be developed for a particular loading distribution from

$$
\begin{align*}
\mathrm{P}_{\mathrm{GL}_{\mathrm{ijq}_{q}}}= & \mathrm{P}_{\mathrm{GL}_{1,1, \mathrm{q}}} \times \mathrm{P}_{\mathrm{GL}_{1,2, \mathrm{q}}} \times \ldots \times \mathrm{P}_{\mathrm{GL}_{1}, \mathrm{~N}_{1}, \mathrm{q}} \times \mathrm{P}_{\mathrm{GL}_{2,1,9}} \times \mathrm{P}_{\mathrm{GL}_{2,2, \mathrm{q}}} \times \ldots \times \mathrm{P}_{\mathrm{GL}_{2}, \mathrm{~N}_{2}, \mathrm{q}} \\
& \times \mathrm{P}_{\mathrm{GL}_{3,1,9}} \times \mathrm{P}_{\mathrm{GL}_{3,2, \mathrm{q}}} \times \ldots \times \mathrm{P}_{\mathrm{GL}_{3, \mathrm{~N} 3,9}} \tag{14}
\end{align*}
$$

where $P_{G L i j q}$ is the probability that the $j$ th vehicle of type $i$ is in the $q$ th weight group.

The gross load level Q is computed from

$$
\begin{equation*}
\mathrm{Q}=\sum_{\text {all } \mathrm{i} \text { all } \mathrm{q}} \mathrm{~W}_{\mathrm{t}} / \mathrm{K}_{1}+\mathrm{K}_{2} \tag{15}
\end{equation*}
$$

where
$\mathrm{K}_{1}$ and $\mathrm{K}_{2}=$ constants used to obtain a reasonable set of gross load intervald and $W_{1 q}=$ mean of the $q$ th weight interval for the $i$ th vehicle type.

Equivalent Load Distributions
The influence (stress) in the member due to a particular load depends not only on the magnitude of the load combination but also on the relative positioning of the load and member. After due consideration, we modified the loading trains positioned in the span to an equivalent single load placed at the same position and in the same configuration as the design vehicle.

Consider loading configuration A shown in Figure 2. Alternative loadings containing the same vehicle types but distributed differently are shown as conditions B and C. The probability of the occurrence of each of these conditions in the span is identical. Because of the assumed random distribution of the vehicles over the length for any particular loading distribution, the loading conditions shown in Figure 2 can be modified to obtain the equivalent continuous loading distributions shown in Figure 3. Superpositioning of all three of these conditions results in the uniform continuous loading distribution shown in Figure 4.

The loading conditions shown in Figures 2, 3, and 4 illustrate equivalent loading distributions. Modification of these loading systems or combinations to an equivalent uniform loading for the design vehicle positioning and configuration is done by

$$
\begin{equation*}
\mathrm{LC}_{\varepsilon}=\mathrm{f}(\mathrm{LC}) \tag{16}
\end{equation*}
$$

where $f(L C)$ is the load equivalency function relating these loads. Modifying the individual load distributions $\mathrm{P}_{\mathrm{GLijq}}$ allows the final equivalent loading distribution to be determined for input into the fatigue analysis presented later.

Specified loads can be simulated at different points on the span of a particular bridge. The stress induced in the critical member by the load placed at each of these positions can be computed. The magnitude of the loads at the critical point of the span corresponding to these stresses can then be computed. Based on this knowledge, ratios of the equivalent loads at the critical point to the load at different positions can be determined. A plot of such points-load ratio versus position of load in the critical length-is then made (Figure 5). A line of best fit is obtained either statistically or visually. This curve is the desired function $f(L C)$. The determination of this curve for numerous members of the same bridge and for a number of bridges should provide the data required for developing a generalized relationship for $f(L C)$.

## FATIGUE ANALYSIS

Table 1. Average vehicle lengths used in fatigue analysis (7).

|  | Average Vehicle <br> Length (ft) | Vehicle Type | Average Vehicle <br> Length (ft) |
| :--- | :--- | :--- | :--- |
| Automobile | 19 | C-5A | 48 |
| SU-2A-4T | 21 | C-6A | 52 |
| SU-2A-6T | 24 | Automobile | 20 |
| SU-3A | 28 | Single unit | 25 |
| C-3A | 45 | Combination | 47 |
| C-4A | 48 |  |  |

Note: $1 \mathrm{ft}=0,3 \mathrm{~m}$.

Figure 1. Vehicle distribution on a two-lane, two-directional highway.


Figure 2. Vehicular loading conditions.


Figure 3. Equivalent vehicular loading conditions.


1. The influence of differential stresses resulting from the same gross load but different vehicular axle spacings (i.e., the same gross load but different equivalent rectangular load) is negligible (Figures 6 and 7). If significant stress differentials are observed, some simple parameter (such as number of axles or total vehicle length) should be used to resolve these differences. The methodology used here compromises these extremes. Instead of combining all vehicles into a single classification, we chose three vehicle classes (automobiles, single-unit trucks, and combination trucks).
2. Critical bridge members were designed such that the stress due to the dead load plus live load was at a specified level (e.g., 55 percent of the yield stress).

Figure 4. Equivalent uniform loading condition.


Figure 5. Effect of placement of load on force transmitted to structural member.


Figure 6. Bending movement due to combination four-axle vehicles.


Figure 7. Errors in computations for combination four-axle vehicles.

3. Stress in a structural member is approximately proportional to the load transferred to the member for all stress levels below the proportional limit.

## Input Parameters

The following input parameters are believed to be minimal:

1. Actual design stress,
2. Dead load,
3. Vehicular live load (this requires a knowledge of the axle loads and configuration of the design vehicle),
4. Critical member section, and
5. A relationship between a measure of rusting and the time elapsed since the bridge was constructed.

These parameters, except the rusting relationship, are readily obtainable from design calculations. The degree of rusting of a member at a specific time might be available from periodical maintenance studies and observations. It should be emphasized that all input values must represent those of the particular bridge member under study.

The dead load of a bridge structure may change from time to time. Loss of section due to rusting will result in decreased weight; any overlays on the bridge deck will increase the dead load. If the fatigue analysis includes the time variable, no problems will arise because these weight changes can be considered.

## Load-Stress Relationship

Based on these assumptions, generalized equations can be developed relating stress to loading conditions. Immediately after erection of the bridge, the actual designed stress of a particular bridge member can be found from

$$
\begin{equation*}
\mathrm{S}_{\mathrm{d}}=(\mathrm{LL} \times \mathrm{I}+\mathrm{DL}) / \mathrm{Z} \tag{17}
\end{equation*}
$$

where Z is the cross-sectional area of the structural member in question.
If we assume that the percentage of section lost due to corrosion of a member is some function of time $f_{r}(y)$, equation 17 can be modified such that the design stress for a particular year can be computed from

$$
\begin{equation*}
S_{d}(y)=[L L(y)+I(y)+D L(y)] / Z\left[1-\sum_{l}^{y} f_{r}(y)\right] \tag{18}
\end{equation*}
$$

## Load-Stress Curve

Under the assumption of a linear relationship, points on the load-stress curve can be obtained as follows:

1. The origin of the load-stress axis (zero stress, zero load);
2. Stress due to dead load,

$$
\begin{equation*}
S_{D L}(y)=D L / Z\left[1-\sum_{\text {all } y} f_{r}(y)\right] \tag{19}
\end{equation*}
$$

3. When maximum single load that can be carried by the member before yielding will occur,

$$
\begin{equation*}
L C(y)=Z\left[1-\sum_{\text {all } y} f_{r}(y)\right] \times S(y) \times I(y) \tag{20}
\end{equation*}
$$

4. Minimum fatigue-producing load,

$$
\begin{equation*}
L C_{E L}=Z\left[1-\sum_{\text {all } y} f_{r}(y)\right] \times f_{e}(y) \times I(y) \tag{21}
\end{equation*}
$$

where $f_{e}$ is the endurance limit of the material.

## Cumulative Stress Distributions

Development of stress distributions $\mathrm{S}_{\mathrm{TL}_{q}}$ from load distributions $\mathbf{P}_{\mathrm{TL}}$ is done by multiplying the frequencies of each loading interval in the load distribution by the unit stress for the mean load of the loading interval. This unit stress is obtained by substituting the midvalue of the loading interval into either equation 17 or 18 . The results are in terms of a double array, i.e., the intermediate stress values are in the form of a discrete set of stress repetitions ( $\mathrm{R}_{\mathrm{TL} \mathrm{q}_{\mathrm{q}}}$ ) corresponding to a specified discrete stress distribution, or

$$
\begin{equation*}
S_{T L_{Q}}=\sum_{\text {all } q} R_{T_{\mathrm{q}}} P_{\mathrm{TL}_{q}} \tag{22}
\end{equation*}
$$

The choice of stress intervals in this distribution depends on the accuracy of the input data, the total stress range, and the desired output accuracy.

## Transformation of Stress Distributions Into Fatigue History

Load-fatigue relationships include the intermediary computations of stress. This was necessary because similar vehicle loadings result in different stress levels for different members of the same bridge. These situations occur because of (a) different levels and ratios of dead load to live load, (b) different impact values, and (c) the wide variety of structural frames.

Fatigue ( $\mathrm{S}-\mathrm{N}$ ) Curves
The basic inputs required are

1. Ultimate strength $f_{u}$,
2. Yield strength $f_{y}$,
3. Endurance limit $f_{e}$, and
4. Number of repetitions $\mathrm{N}_{e}$ associated with the endurance limit.

Also, the following basic assumptions were made:

1. The $S-\log \mathrm{N}$ curve passes through the point for one repetition of the maximum
stress (stress in the member when subjected to a maximum load),
2. The endurance limit is equal to one-half of the yield strength,
3. The member does not suffer damage by an unlimited number of stress repetitions below the endurance limit,
4. A finite number of stress repetitions $\mathrm{N}_{\mathrm{e}}$ are required at the endurance limit before the member will fail, and
5. The slope of the $S-\log N$ curve between $N_{1}\left(\right.$ at $\left.f_{u}\right)$ and $N_{e}$ (at $f_{e}$ ) is constant, and the slope of the $S-\log \mathrm{N}$ curve between $\mathrm{N}_{\mathrm{e}}$ and $\mathrm{N}>\mathrm{N}_{\mathrm{e}}$ is zero.

The applicability of the assumption concerning the linearity of the $\mathrm{S}-\log \mathrm{N}$ curve depends on the type of material used. Most steels now used in bridge construction have relationships approaching linearity. If this assumption cannot be considered applicable, the fatigue-stress relationships presented in the equations derived below should be modified.

## Fatigue Factors

Consider a typical, idealized $S-\log \mathrm{N}$ curve. The slope m of this curve in the fatigue range $N_{1}$ to $N_{e}$ is

$$
\begin{equation*}
m=-\left(f_{u}-f_{e}\right) / \log N_{e} \tag{23}
\end{equation*}
$$

The generalized $\mathrm{S}-\mathrm{log} \mathrm{N}$ curve equation can then be obtained by substituting the above parameters into the generalized form of a linear equation so that

$$
\begin{equation*}
S_{1}=\left[\left(f_{u}-f_{e}\right) \log N_{l} / \log N_{e}\right]+f_{u} \tag{24}
\end{equation*}
$$

where $N_{1}$ is the number of repetitions at the $S_{1}$ stress level causing fatigue. Rearranging equation 24 so that the dependent variable is in terms of the number of stress repetitions gives the $\mathrm{S}-\log \mathrm{N}$ relationship as

$$
\begin{equation*}
\log N_{1}=\left(f_{u}-S_{1}\right) \log N_{e} /\left(f_{u}-f_{e}\right) \tag{25}
\end{equation*}
$$

Comparing the $\mathrm{N}_{1}$ values to a base value of $\mathrm{N}_{\mathrm{e}}$ allows equivalent fatigue factors corresponding to differential stress levels to be computed. If we designate this equivalency factor as the equivalent bridge loading EBL, the equivalent number of endurance limit stressings required to fatigue a member to the same extent as one repetition of a $S_{1}$ stress is found from

$$
\begin{equation*}
E \mathrm{EL}_{1}=\mathrm{N}_{\mathrm{e}} \times 10^{\log N_{e}\left(S_{1}-f_{\mathrm{w}}\right) /\left(f_{\mathrm{f}}-f_{e}\right)} \tag{26}
\end{equation*}
$$

To computerize EBL calculations requires that another parameter be quantitized since discrete rather than continuous distributions are used as input. In addition to the input parameters previously designated, some measure of the discontinuity of these distributions must be developed. The relationship between the number of repetitions required for fatigue at a particular stress level has been found to be a geometric relationship (7). The normal form of this equation is

$$
\begin{equation*}
E B L_{1}=N_{e} B^{\left(S_{1}-f_{u}\right) / S I} \tag{27}
\end{equation*}
$$

where B is a constant and SI is the stress interval. The value of B for a particular material is dependent on the ultimate strength, the yield strength, and either the stress interval of the input data or the total number of stress intervals (8). If the stress interval is specified, then B can be found from

$$
\begin{equation*}
B=\log ^{-1}\left[S I \log N_{e} /\left(f_{u}-f_{e}\right)\right] \tag{28}
\end{equation*}
$$

If the total number of stress intervals is known, the constant can be computed from

$$
\begin{equation*}
B=\log ^{-1}\left(\log N_{0} / q\right) \tag{29}
\end{equation*}
$$

Substituting into equation 27 yields

$$
\begin{equation*}
E B L_{1}=N_{e} \log ^{-1}\left[S I \log N_{e} /\left(f_{u}-f_{e}\right)\right]^{\left(S_{1}-f_{u}\right) / S I} \tag{30}
\end{equation*}
$$

and

$$
\begin{equation*}
E \mathrm{EBL}_{1}=N_{e} \log ^{-1}\left[\left(\log N_{e} / q\right)^{q\left(S_{1}-f_{u}\right) /\left(f_{u}-f_{e}\right)}\right] \tag{31}
\end{equation*}
$$

## METHODOLOGY FOR PROBABILITY ANALYSIS OF FATIGUE

The total fatigue of a member for a specified time period is found by summing the fatigue contributions from all individual loading systems on the bridge during the time interval. The generalized procedure for obtaining this total fatigue contribution is as follows:

1. Determine the probability of the number of repetitions of each vehicle loading configuration occurring during the study period;
2. Transform the vehicle loading distribution generated in step 1 into a corresponding distribution of stresses in structural members;
3. Determine the appropriate fatigue ( $\mathrm{S}-\mathrm{N}$ ) curve for the member based on available design criteria;
4. Determine the equivalent bridge loading contribution due to the application of one stress in each stress interval;
5. Multiply the number of stress applications in each stress interval by the corresponding EBL factor; and
6. Sum the EBLs over all stress groups, and compare the total to the maximum safe value $\mathrm{N}_{\mathrm{e}}$.

Formulating steps 5 and 6 as an equation gives the percentage of fatigue life PFL used during a design period of $Y$ years as

$$
\begin{equation*}
\mathrm{PFL}=100 \sum_{\text {all } 1} \sum_{\text {all } \mathrm{y}} \mathrm{~N}_{1 \mathrm{y}} \mathrm{EBL}_{1 y} / \mathrm{N}_{\mathrm{e}} \tag{32}
\end{equation*}
$$

where $N_{1 y}$ is the number of stress repetitions of the 1 th stress level using the bridge during the $y$ th year and $\mathrm{EBL}_{1 y}$ is the corresponding fatigue equivalency factor.

Most simply, past traffic trends may be assumed to be indicative of future traffic characteristics. Because various loading distributions from past traffic studies for a bridge are necessarily discrete, extending these parameters into the future is unreasonably tedious. Instead, it is recommended that a new traffic parameter be developed, average EBL per vehicle AEBL. This value is obtained for each time interval by dividing the total number of EBLs by the total number of vehicles. This ratio can then be plotted as a function of year to obtain AEBL over the design period.

The remaining parameter necessary for the development of the fatigue analysis is the AADT curve as a function of time. The portion of the curve representing the time from the bridge erection date to the time of the analysis is available from past traffic data.

When curves representing these parameters have been plotted, they are extrapolated into future years. Expected $\mathrm{EBL}^{\prime}$ s accumulated in any particular year are then found from

$$
\begin{equation*}
\operatorname{EBL}(\mathrm{y})=365 \times \operatorname{AADT}(\mathrm{y}) \times \operatorname{AEBL}(\mathrm{y}) \tag{33}
\end{equation*}
$$

The total number of EBLs accumulated from the present time to the end of year Y can be computed from

$$
\begin{equation*}
\text { TEBL }=\sum_{\text {all } y} \operatorname{EBL}(\mathrm{y}) \tag{34}
\end{equation*}
$$

## STRAIN GAUGE ANALYSIS

Scratch Gauges
On April 18, 1972, scratch gauges were placed on four members of Central Bridge. Two additional gauges were attached on April 26. Gauges were placed on the following paired I-bars:

1. April 18, D14L3L2-3, D14L3L2-4, U14L6L'5-3, and U14L6L'5-4; and
2. April 26, U15L'5L'4-3 and U15L'5L'4-4.

The bars selected for instrumentation had the maximum loss of section according to a previous study $(9,10)$. Gauges were $48-\mathrm{in}$. $(1.22-\mathrm{m})$, temperature-compensating Prewitt scratch gauges. The operation and use of those gauges were reported previously ( 9,10 ).

Gauges $\overline{w e r e}$ attached to the I-bars with C-clamps. Threads of the clamps were soldered to provide a more permanent attachment. Restraining straps made of aluminum foil were placed at $1-\mathrm{ft}(0.3-\mathrm{m})$ intervals along the gauge to prevent possible buckling, which might induce errors in the records. The gauges were then covered with plastic to provide protection. Two gauge targets showed no record; one indicated two complete rotations and could not be read. This accounted for the differences in total number of days of record noted in the results.

Scratch gauges were monitored for approximately $4 \frac{1}{2}$ months. Data collected from the discs are given in Table 2. These data were analyzed by the equivalent-bridgeload criterion and a Goodman diagram to determine fatigue damage. In EBL calculations, it was assumed that loading was constant (at the current rate) and that corrosion occurred linearly throughout the life of the bridge. Differences in stresses on parallel bars were also determined.

To calculate stresses given in Table 3, the following equation was used:

Table 2. Number of events per stress level.

| Live Load Stress (psi) | Bar |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | D14L3L2-3 | D14L3L2-4 | U14L6L'5-3 | U14L6L'5-4 | U15L'5L'4-3 | U15L'5L'4-4 |
| $<200$ | 324 | 338 | 278 | 513 | 323 | 44 |
| 200 | 597 | 502 | 462 | 830 | 543 | 91 |
| 400 | 1,009 | 530 | 367 | 779 | 338 | 129 |
| 600 | 742 | 313 | 267 | 448 | 157 | 132 |
| 800 | 99 | 60 | 103 | 97 | 69 | 32 |
| 1,000 | 33 | 25 | 45 | 29 | 35 | 31 |
| 1,200 | 13 | 9 | 44 | 21 | 26 | 11 |
| 1,400 |  | 4 | 9 | 3 | 9 | 4 |
| 1,600 | 3 | 1 | 6 | 2 | 5 | 1 |
| 1,800 |  |  | 3 |  | 2 | 2 |
| 2,000 | 1 |  | 4 |  | 1 |  |
| 2,200 |  | 1 |  |  | 1 |  |
| 2,400 |  |  | 1 |  | 2 |  |
| 2,600 |  |  |  |  |  |  |
| 2,800 |  |  |  |  | 1 |  |
| Total events | 2,821 | 1,783 | 1,589 | 2,722 | 1,507 | 467 |
| Total time, days | 129 | 91 | 69 | 129 | 121 | 83 |
| Average stress, psi | 491 | 433 | 489 | 425 | 423 | 570 |
| Dead load stress, psi | 14,180 | 14,180 | 14,260 | 14,260 | 14,260 | 14,260 |
| Percentage of original section remaining | 78 | 85 | 78 | NA | 77 | 85 |

Note: $1 \mathrm{psi}=6.9 \mathrm{kPa}$.

Table 3. EBL ( $\mathrm{DL}+\mathrm{LL}$ ) with loss of section considered.

| Total Stress (psi) | Bar |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | D14L3L2-3 | D14L3L2-4 | U14L6L'5-3 | U15L'5L'4-3 | U15L'5L'4-4 |
| 16,750 |  | 372 |  |  | 48 |
| 17,000 |  | 602 |  |  | 109 |
| 17,250 |  | 689 |  |  | 168 |
| 17,500 |  | 438 |  |  | 185 |
| 17,750 |  | 92 |  |  | 49 |
| 18,000 |  | 42 |  |  | 35 |
| 18,250 | 586 | 16 | 503 |  | 20 |
| 18,500 | 1,164 | 8 | 901 | 630 | 8 |
| 18,750 | 2,159 | 2 | 785 | 1,162 | 2 |
| 19,000 | 1.729 |  | 622 | 787 | 4 |
| 19,250 | 249 | 2 | 259 | 395 |  |
| 19,500 | 90 |  | 122 | 187 |  |
| 19,750 | 26 |  | 87 | 70 |  |
| 20,000 |  |  | 29 | 85 |  |
| 20,250 | 10 |  | 21 | 32 |  |
| 20,500 |  |  | 11 | 19 |  |
| 2.0,750 | 4 |  | 17 | 8 |  |
| 21,000 |  |  |  | 5 |  |
| 21.250 |  |  | 5 | 5 |  |
| 21:500 |  |  |  | 10 |  |
| 21,750 |  |  |  |  |  |
| 22.000 |  |  |  | 6 |  |
| Total | 6.017 | 2,263 | 3,362 | 3,401 | 628 |
| Damage. percent per year | 0.85 | 0.45 | 0.89 | 0.48 | 0.14 |

Note: $1 \mathrm{psi}=6.9 \mathrm{kPa}$

$$
\begin{equation*}
S_{T}=100\left(S_{D L}+S_{L L}\right) / C \tag{35}
\end{equation*}
$$

where

$$
\begin{aligned}
S_{T} & =\text { total stress, } \\
S_{0 L} & =\text { dead load stress from Table 2, }
\end{aligned}
$$

$\mathrm{S}_{\mathrm{LL}}=$ live load stress from Table 2, and
$\mathrm{C}=$ percentage of section remaining from Table 2.
The equivalent bridge load factor was calculated from equation 34. The number of cycles of each load was found from

$$
\begin{equation*}
\mathrm{N}^{\prime}=\mathrm{N} \times \mathrm{EBL} \tag{36}
\end{equation*}
$$

where
$\mathrm{N}^{\prime}=$ number of equivalent loads corresponding to total stress level $\mathrm{S}_{\mathrm{T}}$ and
$\mathrm{N}=$ number of events from Table 2 for live load stress level $\mathrm{S}_{\mathrm{L}}$ corresponding to total stress level $\mathrm{S}_{\mathrm{T}}$.

The yearly damage caused by the recorded loads was found from

$$
\begin{equation*}
\mathrm{d}=365 \Sigma \mathrm{~N}^{\prime} / \mathrm{N}_{\mathrm{o}} \mathrm{t} \tag{37}
\end{equation*}
$$

where
d = percentage of damage per year caused by recorded loads and
$t=$ elapsed time of record, in days.
Values used in making EBL calculations were

1. Ultimate strength of steel $\left(f_{u}\right)=60 \mathrm{ksi}(414 \mathrm{MPa})$,
2. Endurance limit of steel $\left(\mathrm{f}_{\mathrm{e}}\right)=16.5 \mathrm{ksi}(114 \mathrm{MPa})$, and
3. Events to failure at endurance limit $\left(\mathrm{N}_{\mathrm{o}}\right)=2,000,000$.

From data given in Table 3, it was apparent that damage caused by the recorded loads was significant when the EBL criterion was used. The most critical member noted in the analysis was U14L6L'5-3, which showed a yearly loss of service life of 0.89 percent. This would yield a service life of 112 years if damage remained constant over the life of the bridge. If we assume that corrosion occurs uniformly over the life of the bridge, the loss of fatigue life that has occurred can be computed. When damage was computed in this way, it was found that 30 percent of the service life had been used. Another computation was made that extended present conditions into the future; this showed that the bridge had 40 years of remaining service life if corrosion continued to increase at the same uniform rate previously considered. In these calculations, wind and temperature loadings were not considered. These loads could have considerable effect on the service life of the bridge.

The maximum damage stress ( $\mathrm{S}_{\mathrm{DL}}=18.5 \mathrm{ksi}$ or 127 MPa and $\mathrm{S}_{\mathrm{LL}}=3.6 \mathrm{ksi}$ or 25 MPa of U15L'5L'4-3) was plotted on a Goodman diagram to show its relationship to the endurance limit. It was noted that the stress is well within the safe limits according to that criterion. Because of wind and temperature loadings and age and condition of the steel, the more conservative EBL criterion is probably more appropriate for this situation.

Comparisons (Figure 8) were also made of scratch gauge data to determine what percentage of the load was being carried by each of the paired parallel bars. Differences in stresses are apparent for all pairs. These differences are prominent at low stresses but also occur at higher stress levels. These differences do not appear on the figures at the higher stress levels because of the low percentage of events at those stresses. The cause of the differences in stress in the members cannot readily be identified but several possibilities are apparent:

1. There may be loose pin connections in the I-bars,
2. The strain gauges may not have been placed on sections of equal areas, or
3. The strain gauges might not have been exactly parallel.

SR-4 Resistivity Strain Gauges
On August 23, 1972, SR-4 resistivity strain gauges were placed on bars D14L3L2-3 and D14L3L2-4. The gauges were placed parallel to each other on a normal section of the I-bar so that any differences in recorded strain could be attributed to differences in stresses on those members.

A simultaneous record was made of strain in each bar. These data were then used in a least squares analysis to obtain equations relating stress in one bar to that in the companion bar. Channels of the recorder were then reversed, and the least squares analysis was rerun. An average equation was then computed so that any differences in recorder channels would be eliminated. The equations and their plots are shown in Figure 9.

Differences in stresses in the instrumented, paired members were relatively small. These differences could be attributed to any of the causes mentioned earlier regarding differences in scratch gauge data.

## General

According to the equivalent-bridge-load criterion and data obtained from the scratch gauges, there is noticeable fatigue damage occurring in corroded I-bars of the Central Bridge. Although the calculations of remaining service life in the bars are vague, they do show that possible danger exists.

It was also found that strains in parallel members were nearly equal. Some differences were recorded, but this was more than likely due to gauge locations and recording differences rather than actual differences in strains in the bars themselves. The only large differences in recorded strains were for bars U15L'5L'4-3 and U15L'5L'4-4. In those cases, there were also large differences in numbers of events per day and in percentage of events per load increment, so it is possible that errors in the records for these bars may be present.

## PROBABILITY ANALYSIS

## Input Data

A computer program was developed to calculate loss of fatigue life by using probability analysis. All traffic data used in this analysis came from papers by Lynch (6, 7). Input data are given in Table 4.

All computations covered a period of 81 years (from 1891 to 1992). When corrosion was taken into account, the section was considered normal in 1891 but advanced to a 23 percent loss of section by 1972. Both uniform and parabolic aging (due to corrosion) were considered (Figure 10). Wind and temperature stresses were not considered because of the difficulty in measuring such stresses accurately.

## Results and Discussion

Eight computer runs using different loads, considerations of corrosion, and endurance limits were made. Results of these runs are given in Table 5. In runs 1, 2, and 3, loss of section due to corrosion was not considered; it was found that very little damage resulted even when all vehicle classes were considered at their maximum recorded weight (Figure 11) and all recorded AADTs were doubled. All other runs took corrosion

Figure 8. Cumulative percentage of vehicles versus live load stress.


Figure 9. Stress in D14L3L2-3 versus stress in D14L3L2-4.


Table 4. Input data for probability analysis.

| Data | Item | Value | Data | Item | Value |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Vehicle | Percentage of total traffic |  | Material | Yield strength, psi | 33,000 |
|  | Cars | 91.4 |  | Ultimate strength, |  |
|  | Trucks | 7.3 |  | psi | 60,000 |
|  | Combination trucks | 1.3 |  | Endurance limit | As indicated |
|  | Average length, ft |  |  | Events to failure at |  |
|  | Cars | 20 |  | endurance limit | 2,000,000 |
|  | Trucks | 25 | Bridge | Length of span, ft | 254 |
|  | Combination trucks | 47 |  | Width of span, ft | 23 |
|  | Average spot speed, mph | 28.2 |  | Design load, 1bt/ft ${ }^{2}$ | 75 |
|  | Average weights | $-{ }^{\text {a }}$ | Critical member | Dead load stress, |  |
|  | AADT | - |  | psi | 14,260 |
|  | Gap probabilities | - |  | Design live load stress, psi | 5,950 |

[^0]Figure 10. Percentage of section remaining versus year for the critical member on Central Bridge.


Figure 11. Cumulative percentage of vehicles versus gross weight for all vehicle types on Central Bridge.


Table 5. Life estimates from probability analysis.

| Run | Percentage of Life Used | Age (years) | Calender <br> Year | Gross <br> Vehicle <br> Weight $\left(\right.$ percentile) ${ }^{\pi}$ | Percentage of Section Loss Due to Corrosion | Endurance <br> Limit <br> (psi) | DL Stress at Calendar Year Shown (psi) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0 | 81 | 1972 | 50th | 0 | 16,500 |  |
| 2 | 0 | 81 | 1972 | 90th | 0 | 16,500 |  |
| 3 | 5 | 81 | 1972 | 100 th $^{\text {b }}$ | 0 | 16,500 |  |
| 4 | $100^{+}$ | 25 | 1916 | 50 th | 23, linear | 15,000 | 14,780 |
| 5 | $100^{+}$ | 45 | 1936 | 50th | 23, linear | 16,000 | 16,060 |
| 6 | $100^{+}$ | 55 | 1946 | 50th | 23, Iinear | 17,000 | 16,820 |
| 7 | $100^{+}$ | 66 | 1957 | 50th | 23, parabolic | 17,000 | 16,800 |
| 8 | $100^{+}$ | 68 | 1959 | 50th | 23, linear | 18,000 | 17,580 |

Note: For ADT, see Figure 13 ( 7 ). $1 \mathrm{psi}=6.9 \mathrm{kPa}$.
${ }^{\text {a }}$ See Figure 12. $\quad{ }^{\mathrm{b}}$ Maximum recorded loading from 1968 weighings (7).

Figure 12. Gap versus probability of occurrence for all vehicle types on Central Bridge.


Figure 13. AADT versus year for all vehicle types on Central Bridge.

into account. These runs considered loading at the 50th percentile level and AADTs as recorded (Figure 12); variables were endurance limit and type of corrosion aging. From these results, it became obvious that the most important factor is the range between the dead load stress and the endurance limit of the steel. Also, the relationship assumed between loss of section and time, as seen from runs 6 and 7, greatly affects the duration of the range. Small changes in the assumed endurance limit caused great changes in the calculated service life of the bridge member. Inasmuch as failure was predicted in all runs where corrosion was considered, it appears that some assumptions regarding the severely corroded members in the Central Bridge are too extreme.

Failure was predicted when the dead load stress in the member reached a value near that of the endurance limit; thereafter, all vehicles crossing the bridge became damaging loads. However, fatigue damage is a function of dynamic (live load) and static stresses, and the Goodman diagram tends to moderate the damage attributable to the live loads in similar situations. Inasmuch as wind and temperature stresses have not been considered in these analyses, the original condition of the steel is not known, and because the effects of aging on the steel are not known (at this time), the calculations are somewhat overly conservative in assessing fatigue damage.

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[^0]:    Note: $1 \mathrm{ft}=0.3 \mathrm{~m} ; 1 \mathrm{mph}=1.6 \mathrm{~km} / \mathrm{h} ; 1 \mathrm{psi}=6.9 \mathrm{kPa} ; 1 \mathrm{lbf} / \mathrm{ft}^{2}=48 \mathrm{~Pa}$.
    ${ }^{\text {a See Figure 11, }}{ }^{\text {b }}$ See Figure 12. 'See Figure 13

