ANALYSIS OF ROAD PROFILES BY USE OF DIGITAL FILTERING

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This paper discusses the use of digital filtering to analyze measured road profiles. Applications of road roughness analysis are given. The discussion of the signal processing operators emphasizes considerations, including pitfalls, that are particularly important. The purpose of filtering is to isolate, for further analysis, the roughness of a given type, e.g., that with 10 to 30-ft (3 to 9-m) wavelengths. Such analysis might include the calculation of measures of overall or average roughness and the most severe roughness in a road section. Both artificial and real test cases are given to compare and demonstrate the capabilities of selected filters. A particular type of filter is recommended on the basis of the test cases. Digital filter design by specification of the squared-magnitude function is emphasized. Selected results from a pilot study in which comparisons are made between hot-mixed asphalt concrete and surface-treated pavements are given to demonstrate the application of the suggested filter and roughness measures.

•A DEVICE such as the surface dynamics (SD) profilometer can be used to measure a road profile, i.e., surface elevation versus distance along the road in both the right and left wheel paths (7, 8, 9). The road profiles themselves characterize the road roughness in that they contain information from which one can infer the nature and extent of the roughness.

Although plotted profiles are convenient for visual inspection and comparison of roads, other direct uses of measured profiles are limited because of the very large amount of data required to describe a road surface. Further use of the data is greatly facilitated by the calculation of a set of summary roughness measures. Such mathematical analyses as development of a regression model to predict serviceability (human panel evaluations of riding quality) in terms of pavement roughness require summary measures (5, 9, 10). The set should be small in number and be meaningful physically, e.g., from the standpoint of riding quality.

In this paper, road roughness is characterized based on the right and left profiles measured by an instrument such as the SD profilometer. The techniques to be discussed are by no means limited to analysis of the data from a particular measuring system; it is anticipated that they will be used in the future in conjunction with new systems, such as the noncontacting probe now being developed under contract to the federal government.

The potential for application of roughness analysis is great. Although it is not feasible to discuss all possibilities, for the purpose of illustration, new pavement evaluation, pavement feedback, study of effects of maintenance, and analyses of road user satisfaction are discussed as follows:

1. Roughness analysis based on measurements by the SD profilometer would be useful in evaluating new pavements. A future application might be the design of a set of acceptance criteria, including minimally acceptable levels of various classes of roughness, for new pavements. The decomposition of roughness into classes is discussed later in the paper.
2. Sophisticated roughness evaluation could be used for field evaluation of the roughness characteristics of different kinds of pavements. A comparison of roughness com-

ponents of bridges and nearby pavement and analysis of the short-wavelength roughness found near the ends of bridges would also be valuable, and comparisons could be made between the roughness components of samples of new and deteriorated pavements of a given type. The insights gained about predominant classes of roughness and roughness growth patterns in different kinds of pavements would be useful in evaluating construction practices.

3. Roughness analysis would allow quantitative comparisons between pavements immediately before and after maintenance. The objectives of this analysis include (a) determining whether a given type of maintenance is adequate and whether there are types of roughness the maintenance does not correct and (b) comparing different types of maintenance on the basis of improvement of different classes of roughness.

If the objective of the maintenance were to repair a few severely distressed places in a road section, overall roughness measures, such as a power spectrum, would not be adequate. Measures indicating the severity of the worst roughness in the section would be required. Because the power spectrum is not of central importance here, it will not be discussed in detail. Spectral analysis can be thought of as a method for computing a roughness measure for each of a finite set of wavelengths. Power spectra are discussed elsewhere (2, 8, 9, 10). Follow-up analyses at intervals of, say, 6 months would be useful in assessing the continuing effect of maintenance.

4. A study in which a regression model was developed to predict pavement serviceability in terms of power spectra of road profiles is discussed elsewhere (9, 10). In human-panel serviceability ratings, the fact that 89 percent of the road-to-road variation was explained in terms of the roughness measures (9, p. 25) proves that there is a close relationship between a human's evaluation of a road and roughness measures obtainable by signal processing of the road profile. Further work in this area could include a study of the relationships between serviceability and the individual components of roughness to determine which aspects people find most objectionable. This research would have application in the three areas previously discussed.

OBJECTIVES

Decomposition of the profile roughness on a frequency basis has been investigated and, as stated above, has been shown to be useful in explaining human-panel serviceability ratings. Although the power spectrum approach is effective and computationally efficient for computing overall roughness measures, digital filtering methods can be used to isolate the roughness within each of an arbitrary set of wavelength bands, and, subsequently, measures of local roughness, such as local root-mean-square (rms) amplitudes at a discrete set of points within the section, can be computed for each band. Thus, measures of overall roughness and of extreme roughness within the section can be computed.

Digital filtering here refers simply to the formation of a new road profile, y_i, $i = 1, \ldots, N$, from an input profile, x_i, $i = 1, \ldots, N$, by an operation such as

$$y_n = \sum_{i=0}^{r} K_i x_{n-i} - \sum_{i=1}^{s} L_i y_{n-i} \quad (1)$$

The convention $x_i = y_i = 0$ for $i < 0$ can be adopted to allow calculation of the first few points. The coefficients K_i and L_i are chosen according to the purpose to be achieved, e.g., to produce a profile y_i with all roughness removed except that with 10 to 30-ft (3 to 9-m) wavelengths. The passband is the band of frequencies or wavelengths of the information to be isolated. In this example, the passband is 10 to 30-ft (3 to 9-m) wavelengths or, equivalently, $1/30$ to $1/10$-cycle/ft (0.109 to 0.328-cycle/m) frequencies. A hypothetical sinusoid with a 10-ft (3-m) wavelength goes through one full cycle in 10 ft (3 m); thus, each foot (0.3 m) contains $1/10$ of a cycle, and the frequency is therefore

$\frac{1}{10}$ cycle/ft (0.328 cycle/m). Actually the cutoff at the edges of the passband is imperfect; some waves near the edges of the band, both within and without, are present in the filtered output at reduced amplitudes. The phrase sharpness of cutoff is used henceforth to refer to the extent of this phenomenon.

Plots of filtered and unfiltered road profiles are presented later in the paper to illustrate the isolation of roughness with wavelengths within a given band. A filter is called low pass, high pass, or band pass if, respectively, waves with low, high, or band-limited frequencies are to be isolated.

Since the profile in either wheel path is simply the surface elevation versus distance along the road, their pointwise difference is the elevation of one wheel path relative to the other versus distance. Changing elevations in one wheel path relative to the other cause vehicle roll, and are, therefore, related to ride quality. The analysis discussed above can be applied to the pointwise-difference profile to study transverse roughness.

Note that a difference in the vertical reference levels for the right and left profiles would introduce a spurious zero-frequency component into the difference profile (2). Similarly, a zero frequency component would be present if the pavement were sloped for drainage purposes; however, this is of no concern, since the zero-frequency component, which is not interpretable as roughness, would be removed by filtering.

PITFALLS

Caution must be used in selecting a digital filtering method. The following pitfalls must be kept in mind:

1. Digital filters tend to smooth the amplitudes of the input profiles over several $\frac{1}{2}$ cycles. If an extremely narrow-band filter with sharp cutoff characteristics were used, the smoothing might be so great that the cycle-to-cycle amplitude variations in the filtered output, which are of interest, could become less than measurement error. If this were the case, calculation of the distribution of roughness within a section would clearly be futile.
2. Filters introduce phase shifts that vary with frequency, thus distorting the profile. A phase shift can be thought of as a spatial translation. Thus, a frequency-dependent phase shift translates surface deformations of different wavelengths relative to each other. An approach such as presented elsewhere (6, p. 41) can be used to eliminate this problem. If the profile is filtered forward and then the output profile is filtered backward, the phase shifts of the two operations cancel, thus yielding a filtered output free of phase shift. The forward filter should be extended beyond the data record to allow transients to die out. The double filter has sharper cutoff characteristics but produces somewhat more amplitude-smoothing distortion than a single filter. The double filter was used in all test cases discussed in this paper.
3. Road profiles are known generally to have amplitudes that increase sharply as wavelength increases. Thus, if the roughness in a band of wavelengths (λ_l, λ_u) is to be isolated, there must be a reasonably sharp cutoff at the long-wavelength edge of the passband to avoid overshadowing of the roughness within the passband by longer waves.
4. Some attention must be paid to the filter end effects at the first and last of the data record. If zeroes are extended beyond the data (to allow calculation of the first few points in both the forward and backward filtering operations), then the profile is discontinuous at the endpoints. If the terminal ordinates are extended, then the first derivatives are discontinuous at the endpoints. In either case, spurious waves are introduced near the endpoints by the filter.

The simplest solution is to exclude a short interval at the beginning and end of the record after filtering. For the filters for which test cases are presented in this paper, one cycle of the longest wave in the passband of interest has been found to be adequate. Alternatively, a nonlinear extension, chosen to be continuous at the endpoints and decay to zero within a reasonably short distance beyond the endpoints, could be used. Of course, the frequency composition of the extension should be considered relative to that near the end of the data record. The end effects may be unimportant if only overall rms values are calculated but can have significant effects in the upper tail of the distribution of local rms values.

CHOICE OF FILTER

The low-pass digital filter specified by the squared gain is as follows:

$$G^2(\omega) = \frac{1}{1 + [\tan^{2n}(\omega T/2)/\tan^{2n}(\omega_c T/2)]} \tag{2}$$

where

$G(\omega)$ = gain at frequency ω;
ω = frequency in radians per unit change in the independent variable, wavelength = $2\pi/\omega$;
T = step size of the independent variable; and
ω_c = cutoff frequency.

Equation 2 has been found to be very effective. The gain $G(\omega)$ can be thought of as the fraction of the amplitude of a steady sinusoid of a given frequency retained after the filtering process. Ideally, the gain would be unity in the passband and zero elsewhere; realistically, we know that a rectangular gain versus frequency function can only be approximated.

As the integer n, referred to as the order of the filter, increases, the sharpness of the cutoff and the number of terms required both increase; in equation 1, r = s = n. The system output for the squared-gain function for the low-pass filter is shown in Figure 1, and plots of these filters are shown in Figure 2. The phase shift of the filter is not shown, since, as is discussed in the preceding section, the shift can be eliminated by a double-filtering operation.

It is possible to design a filter of any order of this type with any positive value of ω_c less than the Nyquist frequency of π/T radians. Thus, for example, if one wanted to look at the part of the road roughness with wavelengths of 10 ft (3 m) or more, one would select 10 ft = 2π radians/ω_c or $\omega_c = 2\pi/10$ radians/ft = $\frac{1}{10}$ cycle/ft (0.328 cycle/m). When ω_c is chosen, the coefficients (K_i and L_i in equation 1) required for using the filter can be derived. The derivation from equation 2 of the coefficients is lengthy. Thus, to allow further discussion of ride quality applications, we recommend another source (4) for the mathematical background, which will not be repeated here.

It was computationally efficient to use differences between low-pass filtered outputs to isolate the irregularities in a contiguous set of passbands. This is verified by comparing the number of filtering operations and the number of terms per filter of either the low-pass or band-pass type to achieve the same objective.

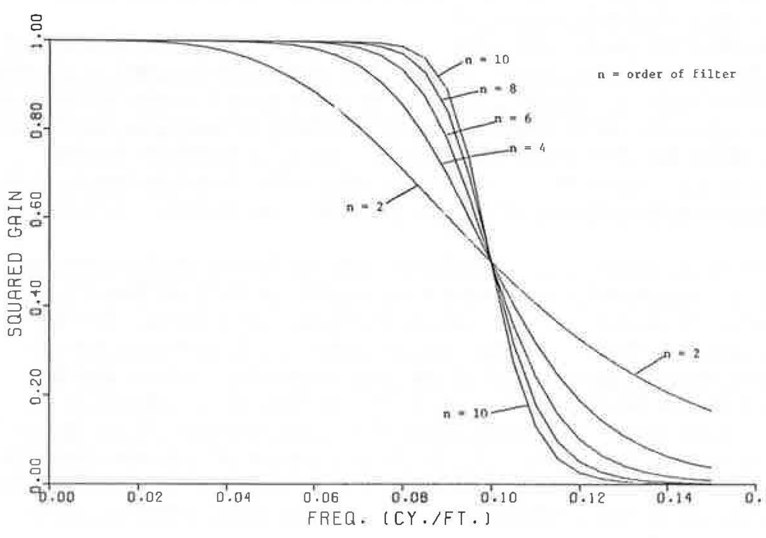
The following are considered to be desirable characteristics of the tangent form of the squared-magnitude-specified filter:

1. Having relatively sharp cutoff for a given order (i.e., a relatively low order is required to achieve a given frequency resolution) and being recursive, the filter is computationally efficient. A filter is recursive if at least one y term is included in the sum on the right of equation 1. Thus, a recursive filter uses previously computed filtered values in subsequent filtering operations. Recursive filters are generally more efficient than nonrecursive filters.
2. Gain versus frequency function is monotone; there are no spurious ripples, as with the Chebyshev filter, a well-known filter mentioned here only for comparison.
3. Flat-topped gain versus frequency function (Figure 1), as contrasted to that of, say, the curve-topped digital resonator, yields results that are somewhat more easily interpreted physically.

Figure 1. System output for squared-gain function for low-pass filter.

FREQUENCY (cy./ft.)	WAVELENGTH (ft.)	ORDER 2	ORDER 4	ORDER 6	ORDER 8	ORDER 10
0.	----	1.00000E+00	1.00000E+00	1.00000E+00	1.00000E+00	1.00000E+00
5.00000E-03	2.00000E+02	9.99994E-01	1.00000E+00	1.00000E+00	1.00000E+00	1.00000E+00
1.00000E-02	1.00000E+02	9.99900E-01	1.00000E+00	1.00000E+00	1.00000E+00	1.00000E+00
1.50000E-02	6.66667E+01	9.99496E-01	1.00000E+00	1.00000E+00	1.00000E+00	1.00000E+00
2.00000E-02	5.00000E+01	9.98408E-01	9.99997E-01	1.00000E+00	1.00000E+00	1.00000E+00
2.50000E-02	4.00000E+01	9.96122E-01	9.99985E-01	1.00000E+00	1.00000E+00	1.00000E+00
3.00000E-02	3.33333E+01	9.91992E-01	9.99935E-01	9.99999E-01	1.00000E+00	1.00000E+00
3.50000E-02	2.85714E+01	9.85262E-01	9.99776E-01	9.99997E-01	1.00000E+00	1.00000E+00
4.00000E-02	2.50000E+01	9.75114E-01	9.99349E-01	9.99983E-01	1.00000E+00	1.00000E+00
4.50000E-02	2.22222E+01	9.60719E-01	9.98331E-01	9.99932E-01	9.99997E-01	1.00000E+00
5.00000E-02	2.00000E+01	9.41328E-01	9.96130E-01	9.99758E-01	9.99985E-01	9.99999E-01
5.50000E-02	1.81818E+01	9.16361E-01	9.91738E-01	9.99240E-01	9.99931E-01	9.99994E-01
6.00000E-02	1.66667E+01	8.85507E-01	9.83557E-01	9.97843E-01	9.99721E-01	9.99964E-01
6.50000E-02	1.53846E+01	8.48803E-01	9.69246E-01	9.94380E-01	9.98994E-01	9.99821E-01
7.00000E-02	1.42857E+01	8.06678E-01	9.45686E-01	9.86423E-01	9.96712E-01	9.99210E-01
7.50000E-02	1.33333E+01	7.59936E-01	9.09262E-01	9.69439E-01	9.90140E-01	9.96864E-01
8.00000E-02	1.25000E+01	7.09693E-01	8.56655E-01	9.35937E-01	9.72763E-01	9.88676E-01
8.50000E-02	1.17647E+01	6.57256E-01	7.86202E-01	8.75803E-01	9.31142E-01	9.67869E-01
9.00000E-02	1.11111E+01	6.03995E-01	6.99365E-01	7.80128E-01	8.44033E-01	8.91938E-01
9.50000E-02	1.05263E+01	5.51202E-01	6.01342E-01	6.49443E-01	6.94686E-01	7.36460E-01
1.00000E-01	1.00000E+01	5.00000E-01	5.00000E-01	5.00000E-01	5.00000E-01	5.00000E-01
1.05000E-01	9.52381E+00	4.51271E-01	4.03459E-01	3.57414E-01	3.13858E-01	2.73353E-01
1.10000E-01	9.09091E+00	4.05642E-01	3.17774E-01	2.41215E-01	1.78281E-01	1.28976E-01
1.15000E-01	8.69565E+00	3.63495E-01	2.45927E-01	1.57005E-01	9.61361E-02	5.72625E-02
1.20000E-01	8.33333E+00	3.24998E-01	1.88193E-01	1.00409E-01	5.09997E-02	2.52222E-02
1.25000E-01	8.00000E+00	2.90155E-01	1.43163E-01	6.39300E-02	2.71584E-02	1.12824E-02
1.30000E-01	7.69231E+00	2.58844E-01	1.08712E-01	4.08574E-02	1.46590E-02	5.16887E-03
1.35000E-01	7.40741E+00	2.30866E-01	8.26510E-02	2.63319E-02	8.05223E-03	2.43068E-03
1.40000E-01	7.14286E+00	2.05968E-01	6.30436E-02	1.71541E-02	4.50694E-03	1.17299E-03
1.45000E-01	6.89655E+00	1.83878E-01	4.83107E-02	1.13079E-02	2.57027E-03	5.80255E-04
1.50000E-01	6.66667E+00	1.64319E-01	3.72236E-02	7.54484E-03	1.49258E-03	2.93836E-04

Figure 2. Squared gains of filters specified by tangent form of squared-magnitude approximating function.



4. Characteristics of the filter can be changed tremendously by varying the order.

Although several other methods, including the digital resonator, certain nonrecursive filters, and moving Fourier transforms, were considered, extensive comparisons among flat-topped recursive filters were not made. In view of characteristics 1 and 4 above and the test results, it is doubtful whether such an investigation would uncover a filter with significant practical advantages over the tangent filter.

ARTIFICIAL TEST CASES

In the following test cases, we will refer only to the tangent form of the squared-magnitude low-pass filter as applied doubly to remove phase shift.

Figures 3, 4, and 5 show the amplitude-smoothing effect for case 1. The artificial test case, zeroes followed by ½ cycle of a sine wave followed by zeroes, is almost unrealistically conducive to filter distortion because of the sharply discontinuous derivatives and the abrupt amplitude changes from 0 to 1 to 0 in successive ½ cycles. This artificial profile, however, resembles a smooth road with an isolated bump and is more realistic than, say, a step function. The difficulties presented and the realism were the reasons for the choice of this test case. Units are intentionally omitted in the figures to emphasize the fact that artificial data are being used. The wavelengths of the signals are given in feet (meters) for convenience and illustration.

The cutoff of the sixth-order filter is at 10 ft (3 m) (Figure 1); therefore the 11.33, 14, and 20-ft (3.45, 4, and 6-m) signals are all in the passband. Thus, the difference between the filtered and unfiltered profiles should be interpreted as distortion introduced by the filter.

The fact that the distortion decreases as the signal's wavelength moves away from the edge of the passband illustrates why extremely narrow-band filters are not adequate: There is no area within the passband sufficiently far from the edges so that excessive distortion is not encountered.

The sharpness of the cutoff is actually less than the gain versus frequency function would indicate when the amplitude variation is great, as given in Table 1.

One might ask whether the results would differ greatly if the sampling rate were decreased or the cutoff frequency were changed; the two questions are really the same: Does changing the number of points per cycle at the cutoff frequency change the results significantly? The test case discussed above was run on 14 and 20-ft (4 and 6-m) wavelength signals with a step size of 2 ft (0.6 m).

The rms values centered at the peak after filtering for the 2-in. (50.8-mm) and 2-ft (0.6-m) cases are, respectively, 0.4492 and 0.4429 for the 14-ft (4-m) signal and 0.4861 and 0.4825 for the 20-ft (6-m) signal. Each rms value is taken over 1 cycle at the wavelength of the signal. The corresponding rms value of a perfectly distortion-free filtered profile would be 0.5 (the value for the unfiltered profile). Thus, a decrease in the sampling rate by a factor of 12 increases the distortion only in the third decimal place. The differences are considered trivial since they are less than the expected measurement errors.

Figures 6, 7, and 8 reveal somewhat improved performance in the less severe test case 2, zeroes followed by 4 cycles of a sine wave with amplitude varying from ½ cycle to ½ cycle. Tables 2, 3, and 4 give the results for test case 2 for the fourth-, sixth-, and tenth-order filters. There is surprisingly little variation from order to order. It can be seen from examining the gain at the frequency of sine wave and the various error measures that, as the order increases, the errors do not decrease as quickly as the gain approaches unity. This is because the amplitude smoothing is exacerbated as the cutoff sharpness increases; i.e., there is a trade-off between distortion due to the nonrectangular gain function and distortion due to amplitude smoothing. The sixth-order filter is recommended as a reasonable compromise considering both distortion and computational efficiency.

It is felt that the rms and the maxima of the errors in the local rms values are adequately small in these test cases chosen to be conducive to filter distortion and to justify use of the sixth-order filter for computing measures of local roughness versus frequency. The test cases with real data in the following section provide further and more convincing evidence.

TEST CASES USING ROAD PROFILES

Although the artificial test cases are useful, real cases such as those discussed below are the ultimate tests. The following points are made in this section:

Figure 3. Filtered and unfiltered profiles for case 1, 11.33-ft (3.45-m) wavelength signal, sixth-order filter.

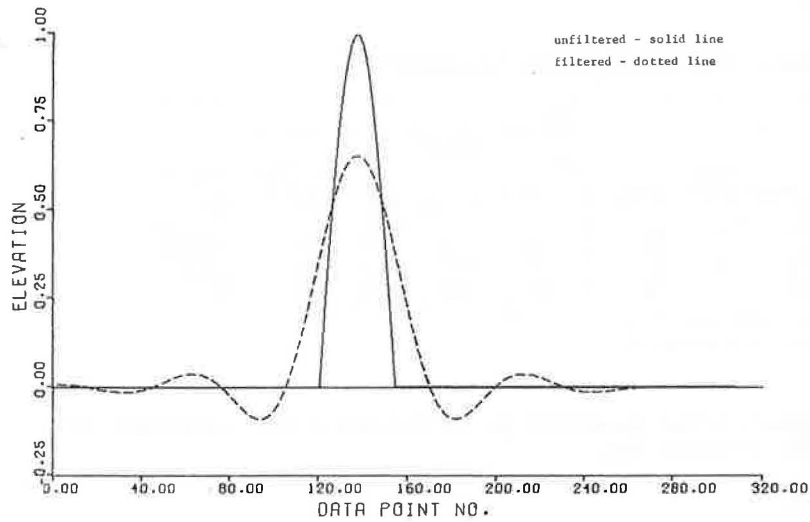


Figure 4. Filtered and unfiltered profiles for case 1, 14-ft (4-m) wavelength signal, sixth-order filter.

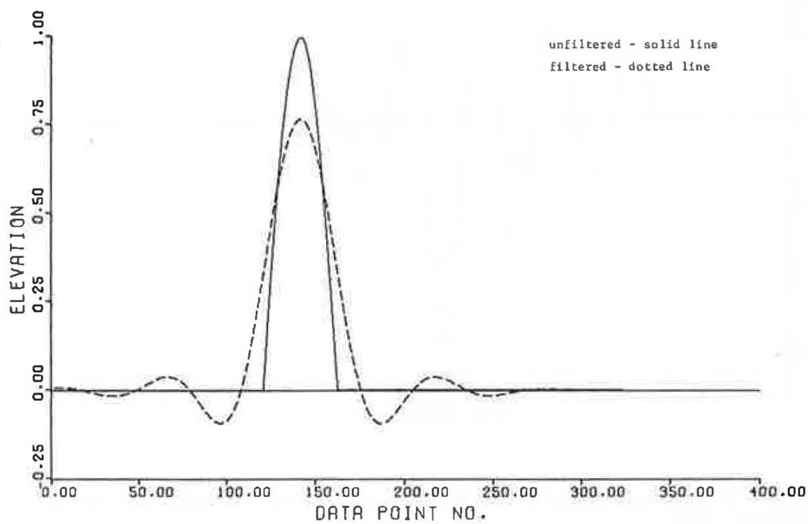


Figure 5. Filtered and unfiltered profiles for case 1, 20-ft (6-m) wavelength signal, sixth-order filter.

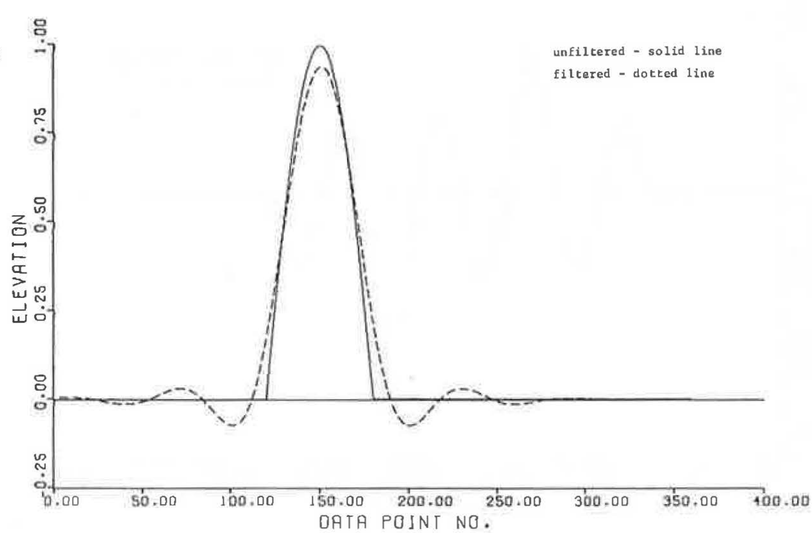


Table 1. Results for test case 1, sixth-order filter.

			Amplitude					
			Unfiltered		Filtered			
Frequency (cycle/ft)	Wave-length (ft)	Points/ Cycle	At Peak	rms at Peak[a]	At Peak	rms at Peak	At ½ Cycle Past Last Peak	Gain at Frequency of Sine Wave
0.500	2	12	1	0.5	0.125	0.122	0.116	0.0313×10^{-7}
0.250	4	24	1	0.5	0.252	0.231	0.187	0.0158×10^{-3}
0.167	6	36	1	0.5	0.374	0.310	0.183	0.0021
0.125	8	48	1	0.5	0.487	0.363	0.117	0.0639
0.100	10	60	1	0.5	0.591	0.400	0.027	0.5000

Note: 1 cycle/ft = 3.2 cycles/m. 1 ft = 0.3 m.

[a]Taken over 1 cycle at the wavelength of the signal.

Figure 6. Filtered and unfiltered profiles for case 2, 11.33-ft (3.45-m) wavelength signal, sixth-order filter.

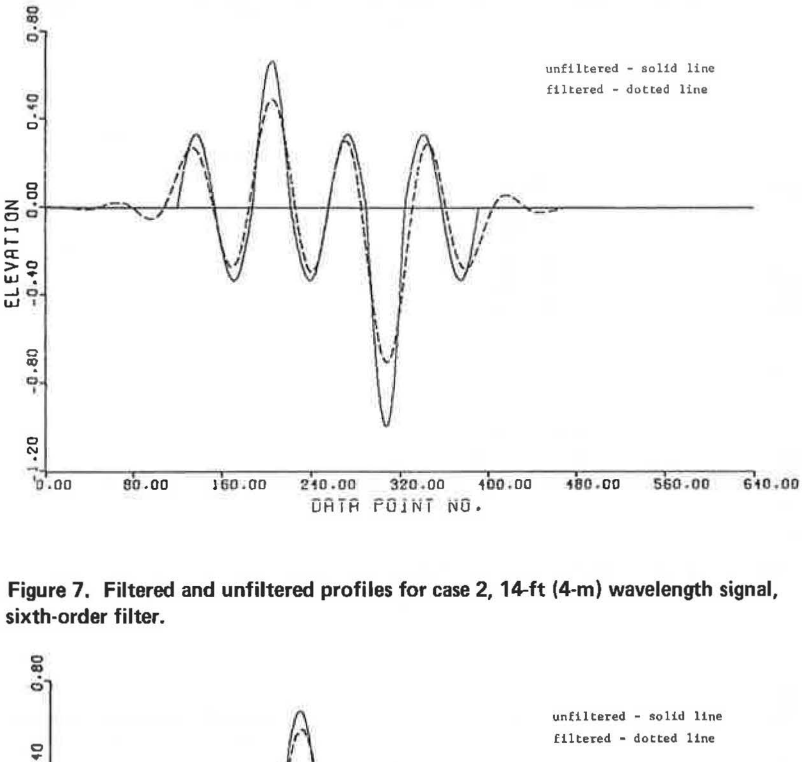


Figure 7. Filtered and unfiltered profiles for case 2, 14-ft (4-m) wavelength signal, sixth-order filter.

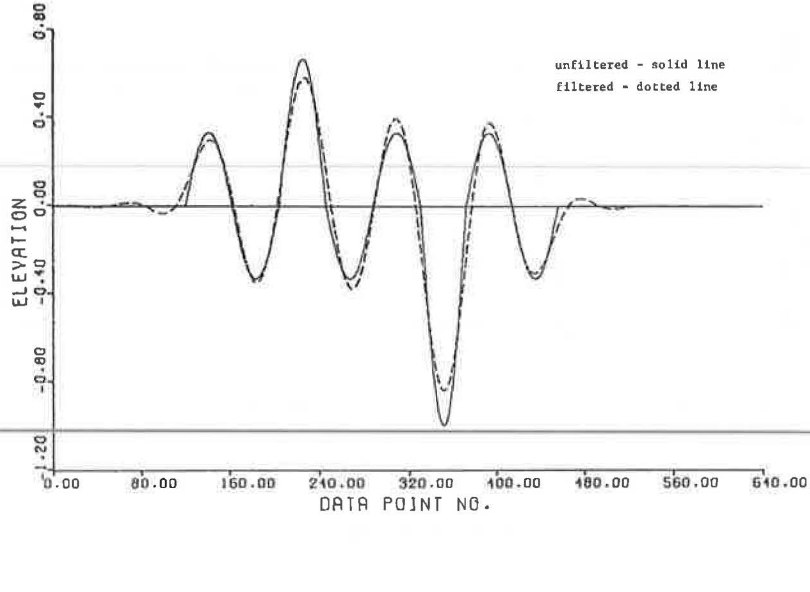


Figure 8. Filtered and unfiltered profiles for case 2, 20-ft (6-m) wavelength signal, sixth-order filter.

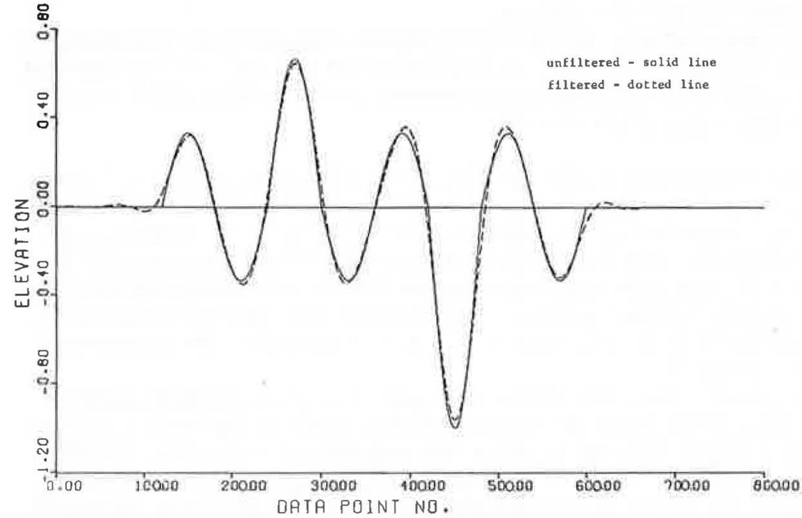


Table 2. Results for test case 2, fourth-order filter.

Item	Wave-length (ft)	Points/ Cycle	Filtered Amplitude of ½ Cycles at Peaks 1	2	3	4	5	6	7	8	Error rms[a,b]	Error Maxi-mum[b]	Filtered Amplitude at ½ Cycle Past Last Peak	Gain at Frequency of Sine Wave
Frequency, cycle/ft														
0.088	11.33	68	0.236	0.244	0.464	0.257	0.259	0.681	0.250	0.240	0.078	0.132	0.015	0.732
0.079	12.67	76	0.271	0.298	0.529	0.320	0.329	0.766	0.314	0.278	0.047	0.088	0.029	0.869
0.071	14.00	84	0.289	0.329	0.568	0.346	0.363	0.823	0.352	0.294	0.029	0.059	0.029	0.937
0.060	16.67	100	0.305	0.351	0.611	0.349	0.372	0.896	0.374	0.305	0.014	0.029	0.021	0.984
0.050	20.00	120	0.319	0.345	0.643	0.341	0.355	0.953	0.357	0.318	0.007	0.014	0.011	0.996
Amplitude of input time series			0.333	0.333	0.667	0.333	0.333	1.0	0.333	0.333				

Note: 1 cycle/ft = 3.2 cycles/m. 1 ft = 0.3 m.
[a]Points from ½ cycle before to ½ cycle after sine waves were used in rms error calculation. [b]Moving values taken over 1 cycle at wavelength of signal.

Table 3. Results for test case 2, sixth-order filter.

Item	Wave-length (ft)	Points/ Cycle	Filtered Amplitude of ½ Cycles at Peaks 1	2	3	4	5	6	7	8	Error rms[a,b]	Error Maxi-mum[b]	Filtered Amplitude at ½ Cycle Past Last Peak	Gain at Frequency of Sine Wave
Frequency, cycle/ft														
0.088	11.33	68	0.257	0.268	0.492	0.295	0.294	0.705	0.274	0.264	0.063	0.114	0.036	0.818
0.079	12.67	76	0.288	0.321	0.557	0.363	0.372	0.791	0.343	0.301	0.034	0.071	0.045	0.945
0.071	14.00	84	0.299	0.348	0.584	0.375	0.396	0.840	0.378	0.308	0.022	0.046	0.034	0.983
0.060	16.67	100	0.305	0.365	0.617	0.352	0.384	0.904	0.392	0.302	0.012	0.025	0.021	0.998
0.050	20.00	120	0.321	0.347	0.648	0.340	0.354	0.960	0.359	0.318	0.006	0.013	0.011	1.000
Amplitude of input time series			0.333	0.333	0.667	0.333	0.333	1.000	0.333	0.333				

Note: 1 cycle/ft = 3.2 cycles/m. 1 ft = 0.3 m.
[a]Points ½ cycle before to ½ cycle after sine waves were used in rms error calculation. [b]Moving values taken over 1 cycle at wavelength of signal.

Table 4. Results for test case 2, tenth-order filter.

Item	Wave-length (ft)	Points/ Cycle	Filtered Amplitude of ½ Cycles at Peaks 1	2	3	4	5	6	7	8	Error rms[a,b]	Error Maxi-mum[b]	Filtered Amplitude at ½ Cycle Past Last Peak	Gain at Frequency of Sine Wave
Frequency, cycle/ft														
0.088	11.33	68	0.281	0.293	0.518	0.327	0.323	0.721	0.291	0.280	0.054	0.104	0.046	0.925
0.079	12.67	76	0.295	0.330	0.574	0.396	0.403	0.802	0.353	0.309	0.030	0.066	0.053	0.991
0.071	14.00	84	0.302	0.350	0.581	0.368	0.410	0.837	0.384	0.311	0.023	0.047	0.034	0.999
0.060	16.67	100	0.296	0.370	0.608	0.342	0.381	0.893	0.400	0.287	0.014	0.028	0.018	1.000
0.050	20.00	120	0.315	0.336	0.640	0.330	0.347	0.949	0.350	0.312	0.009	0.017	0.014	1.000
Amplitude of input time series			0.333	0.333	0.667	0.333	0.333	1.000	0.333	0.333				

Note: 1 cycle/ft = 3.2 cycles/m. 1 ft = 0.3 m.
[a]Points ½ cycle before to ½ cycle after sine waves were used in rms error calculation. [b]Moving values taken over 1 cycle at wavelength of signal.

1. Plots of filtered and unfiltered profiles demonstrate directly the capability of the methods to isolate a specified type of roughness.
2. Pilot-study results demonstrate the use of roughness measures computed from filtered profiles in comparing new hot-mixed asphalt concrete and new surface-treated roads in Texas. This is important because the measures can be used to infer important differences between two types of pavements.

An asphalt surface-treated section on the Old San Antonio Road near Bryan, Texas, was chosen to illustrate the sixth-order, low-pass filter's performance with real data. The two-lane road is very rough; the serviceability index (9, 10) is 1.7. Swelling clay distress is known to be present. The filtered and unfiltered profiles are shown in Figures 9 and 10. In Figure 10, the difference between the low-pass filtered outputs was used to simulate the band-pass filtered profile. The desired information seems to be portrayed extremely well in the filtered profiles. Note, for example, the pronounced long waves in Figure 10, frame 2.

Although the high-frequency waves are shown in Figure 9 to be accurately isolated by the filter, there are occasional curious-looking results, such as the large v in Figure 9, frame 1, at about a 335-ft (102-m) position that appears as a shorter, smaller amplitude v in the filtered profile. The v in the raw profile is apparently interpreted by the filter as a long wave not in the passband with short waves (composing the small v) in the band. Generally, because of the presence of many waves superimposed, visual filtering to see what the digital filter should and should not respond to is difficult. The step size for the test cases using road profiles is 0.169 ft (0.05 m) [about 2 in. (50.8 mm)] for each wheel path.

Finally, as an example of an application of filtering, we consider selected results from a pilot study in which new (less than 1-year-old) hot-mixed asphalt concrete pavements were compared to the less expensive surface-treated pavements. All sections were either new or rebuilt except for one of the hot-mixed sections, which had an overlay on an existing structure. The two samples contained 10 hot-mixed and 11 surface-treated sections; each 1,200-ft (366-m) section was from a different project. The projects are distributed over the state of Texas.

The mean serviceability indexes for the surface-treated and hot-mixed samples are 3.5 and 3.9 respectively. The ranges for the two samples are 2.7 to 4.1 and 3.5 to 4.5 respectively. Further information on the sections is given elsewhere (3). Because of the limited nature of the study, the results are presented as illustration, not as conclusive findings.

For (the filtered output for) a given passband, calculations are done for each of the two samples as follows:

1. Recursively compute the local rms amplitude for the right, left, and pointwise difference profiles at each step throughout the section. The rms values are taken over 1 cycle at the longest wavelength in the passband.
2. Compute the 50th, 75th, 90th, 95th, and 99th percentile points of each of the three sets of local roughness measures. The q th percentile point is the value greater than or equal to exactly q percent of the set of rms amplitudes in question (compare with 1). The 50th percentile amplitude, being greater than exactly half of the moving rms values, is the median of the local amplitudes. Thus, the 50th percentile value is the average roughness in the road section. The 95th percentile is greater than exactly 95 percent of the local rms amplitudes. Thus, the local roughness is worse than the 95th percentile value only 5 percent of the time, and the 95th percentile value indicates how bad the most severe roughness in the section is.
3. Average the corresponding right and left percentile amplitudes yielding a set of measures of longitudinal roughness. The pointwise difference percentile amplitudes are measures of transverse roughness.
4. Compute the sample mean and the standard deviation of the mean for each roughness measure, e.g., the 75th percentile amplitude for longitudinal roughness.

Calculations 1, 2, and 3 were done for each section separately.

Figure 9. Filtered and unfiltered profiles for Old San Antonio Road, 0 to 10-ft (0 to 0.3-m) wavelength signal, sixth-order signal.

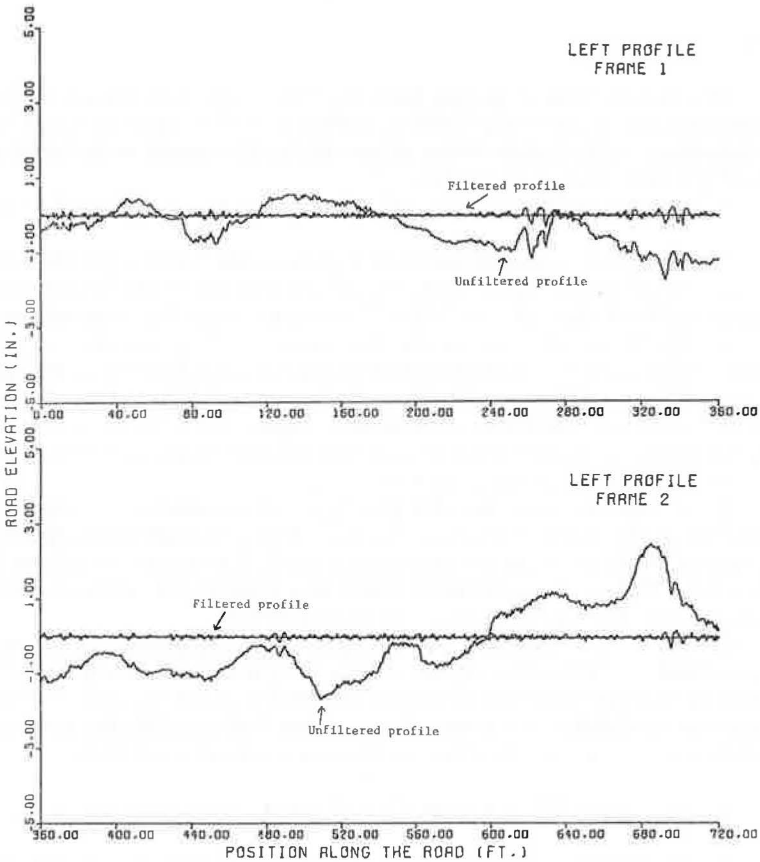
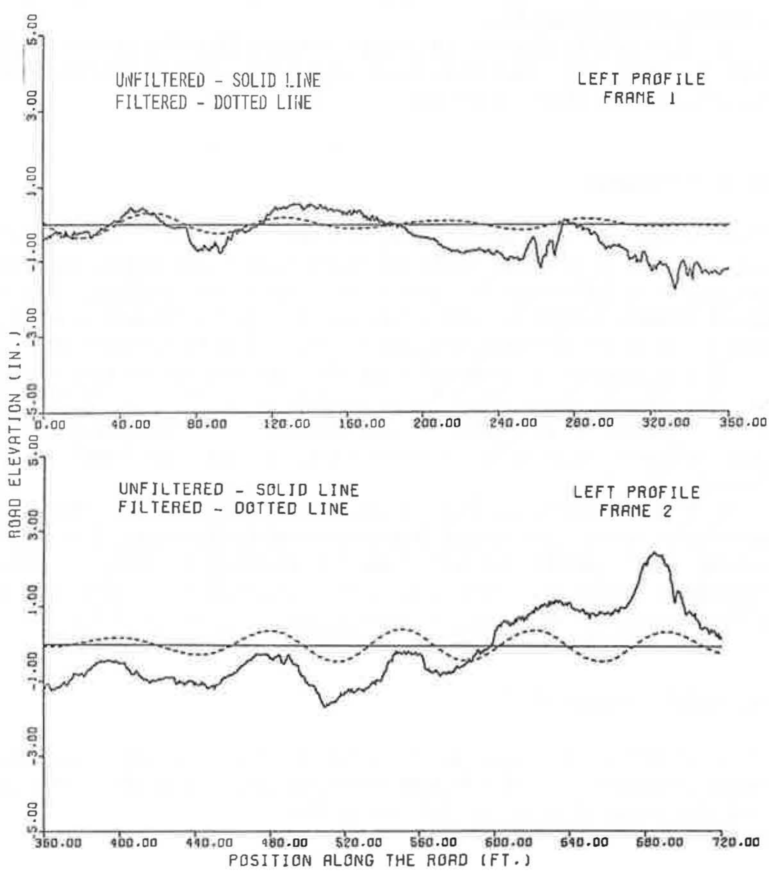


Figure 10. Filtered and unfiltered profiles for Old San Antonio Road, 60 to 100-ft (18 to 30.5-m) wavelength signal, sixth-order signal.



The means and ±1 sigma bars for the means are shown in Figures 11 and 12. Comparisons are made graphically, rather than through the more conventional t- and F-statistics, to facilitate clear illustration of several relationships that can be studied through this type of analysis.

From Figure 11, we note the following facts about the 0 to 1-ft (0 to 0.3-m) passband:

1. Roughness amplitudes are significantly larger for the surface-treated roads.
2. Confidence bars indicate that the sample of surface-treated roads (S) is much more diverse than the sample of hot-mixed asphalt concrete roads (H).
3. Both samples are more diverse at the high percentile points than at the 50th percentile points. This is probably due in part to larger sampling errors in the high percentile amplitudes. Further study is needed to determine the sampling distributions of the various roughness measures. Thus, the variation in the road sections with respect to the presence of a few severe places is greater than the variation with respect to the average roughness in a section.
4. Difference between the two types of pavements is much greater at the high percentile points than at the low points. Thus, the surface-treated roads have a much greater tendency than the hot-mixed asphalt concrete roads to have a few very severe bumps. The surface-treated roads are also worse with respect to overall section roughness, but the difference is not so great.
5. For either sample, the transverse amplitudes are larger than the longitudinal amplitudes. This was expected; for very short wavelengths, a transverse rms amplitude is analogous to the standard deviation of the difference between two independent random variables; however, the right or left amplitudes are analogous to the standard deviation of one or the other of the two random variables.

From Figure 12, we note the following facts about the 27 to 81-ft (8 to 25-m) passband:

1. Confidence bars overlap, indicating that the two types of pavement (S and H) do not differ significantly.
2. Longitudinal amplitudes are greater than the cross amplitudes for the longer waves. This was expected, since the right and left profile fluctuations are positively correlated for the long waves.

CONCLUSIONS

The recursive filter designed by the tangent form of the squared-magnitude approximating function is recommended for use in analyzing road profiles. Although there is surprisingly small order-to-order variation in the artificial test cases chosen to study local transient effects, the sixth-order filter is recommended because it has acceptably sharp cutoff characteristics and is computationally efficient.

The distortion introduced near the edges of the filter passband when the input road profile varies significantly in amplitude from ½ cycle to ½ cycle must be kept in mind. It is probably futile to expect to estimate, with high accuracy, the local amplitudes of the surface irregularities within a very narrow passband, e.g., 30 to 33 ft (9 to 10 m) in wavelength.

It is felt, however, that the artificial test cases, which were chosen to be highly conducive to filter-induced distortion and to illustrate the types of transient effects to be expected, justify the use of digital filtering to compute measures of local amplitude versus wavelength. Probably more important is the fact that the chosen sixth-order filter gave realistic results when applied to road profiles.

ACKNOWLEDGMENT

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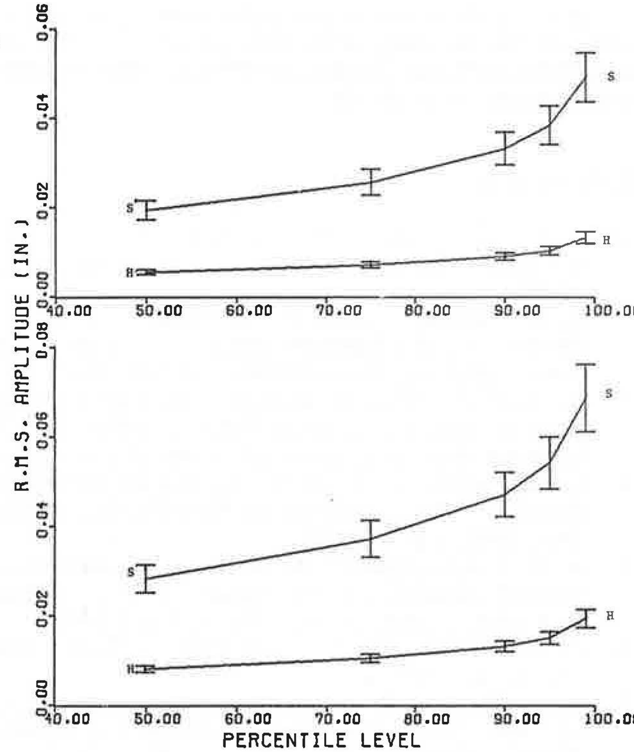

Figure 11. Root-mean-square amplitude versus percentile level longitudinal roughness for 0 to 1-ft (0 to 0.3-m) passband.

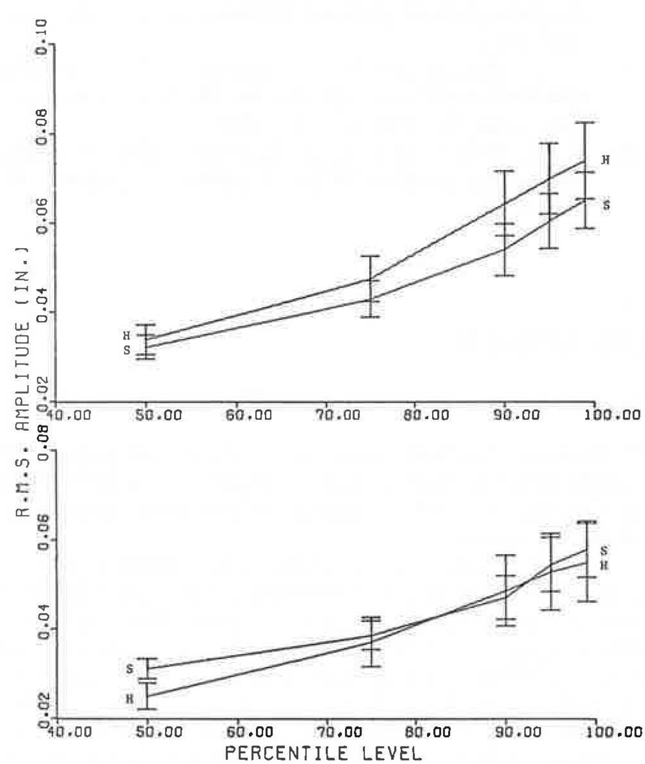

Figure 12. Root-mean-square amplitude versus percentile level longitudinal roughness for 27 to 81-ft (8 to 25-m) passband.

The contents of this paper reflect the views of the author, who is responsible for the facts and the accuracy of the data presented. The contents do not necessarily reflect the official views or policies of FHWA. This report does not constitute a standard, specification, or regulation.

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DISCUSSION

B. E. Quinn, Mechanical Engineering School, Purdue University

Williamson has demonstrated a way of comparing pavement profiles by filtering various wavelengths from the profile in question and then by comparing the filtered results. In some respects this is similar to pavement spectrum analysis with which I have had some experience.

If an analysis is to be made of the extent to which various wavelengths are found in a pavement profile, the question about what filtering technique to use inevitably arises. The problems of filtering highway profiles provide more of a challenge than certain other types of data because an enormous range of wavelengths exists in a pavement profile, where wavelengths from a few feet (meters) to several hundred feet (meters) may exist together. The problem is further complicated because the long wavelengths usually have a much greater amplitude than the shorter wavelengths. I have observed approximately five orders of magnitude difference in the wavelengths that exist in certain pavement profiles.

The existence of this large variation in amplitudes complicates the filtering process which, as Williamson shows, is never perfect. The mathematical filters never exclude all of the side-band frequencies as his filter characteristics so clearly show. One complication arises because a very small percentage of a side-band frequency having an amplitude several orders of magnitude larger than the frequency of interest may introduce into the filtered results an effect having the same order of magnitude as the band of wavelengths under consideration.

I prefer making a pavement roughness spectral analysis, but in either case this filtering problem is encountered because spectral analysis can usually not be made on pavement elevation profiles when some type of preliminary data processing procedure has not been applied to the profile.

I have experimented with different filters to remove the long pavement wavelengths. A variety of results could be obtained depending on the filtering technique used. Moreover, each filtering technique produced its own distortion in the filtered results thus causing additional problems. The question can therefore be asked about what type of filtering process is appropriate when pavement profiles are studied. This may depend on the use that is to be made of the filtered results; in this case the dynamic tire forces in passenger vehicles were to be predicted. A variety of pavement roughness spectra could be obtained as inputs to the mathematical model depending on the filtering procedure used for processing the elevation measurements. In this case, the question was resolved by measuring the dynamic tire forces experimentally and by computing the spectral density of the dynamic tire forces from the force-time histories. These records did not contain the enormous variation in frequency nor the corresponding large variation in amplitude that existed in the pavement profiles. As a result, the filtering procedure was not as critical for these data as it was for the profile. The power spectrum of the dynamic tire forces was then computed by using a mathematical model in which the pavement roughness spectrum was included as the input and in which the vehicle characteristics were represented.

The pavement roughness spectrum when used with the vehicle characteristics did not give a dynamic tire force spectrum that agreed particularly well with the spectral analysis of the dynamic tire force data obtained experimentally. This was particularly true in the high-frequency region because the elevation measurements used to define the pavement profile were not accurate enough when used with this type of analysis to predict the vehicle behavior. This was true because the vehicle was very sensitive to pavement excitation at the wheel-hop frequency. No filtering could improve the original accuracy of the elevation measurements, and only experimental data revealed the limits to the usefulness of the frequency analysis of the pavement profiles.

It is possible to make frequency analyses of road profiles by using many different types of filters. A basic problem that must be resolved however is that of determining which wavelengths are significant in the profile and to what degree of accuracy they must be determined.

In my opinion, the evaluation of a pavement profile must be related to the vehicle that will be using the pavement and to the velocity at which the vehicle will be operated. Objective criteria can then be obtained that make it possible to determine whether a pavement is either good or bad.

DISCUSSION

Arthur D. Brickman, Department of Mechanical Engineering, Pennsylvania State University

Williamson is to be commended for describing a detailed mathematical investigation most lucidly and for showing how data analysis can reduce an apparently endless random profile measurement to a few simple graphs. In developing and testing the analysis scheme, however, he has made some decisions that bear further justification:

1. A reverse filtering procedure is applied to the input profile to eliminate phase-shift effects from the filtered profile. Since we are concerned with relatively narrow passband filtering in which the passed signal components all have about the same lag and since we subsequently look only at the amplitude characteristics of the filtered profile, why is this special procedure necessary? If the input profiles are measured by the GM profilometer, they already contain some inherent phase-shift effects at the longer wavelength components because of the on-board filter typically used in profile taping.

2. Although the ½ sine pulses used as test inputs for the digital filter do emphasize its limits of accuracy, these signals are difficult to associate with typical roughness singularities that appear on actual profile records. It would be instructive to see what the filter response would be to more realistic inputs such as a modified single step (slab edge), ramp (bridge approach), and versine (single smooth bump). Comparative filter response results for such inputs have been observed in other investigations including those of the GM profilometer (11).

3. The end result of processing a roughness profile by using Williamson's method is a graph that shows the cumulative distribution of roughness amplitudes to be expected in a given range of roughness wavelengths for a given road. Thus, a complete description of the roughness by this method involves a family of curves covering all the wavelength bands of interest. What advantages does this system for describing road roughness have over existing methods (12) based on analog filtering and amplitude counting?

Part of the analysis system described by Williamson involves computation of a transverse roughness characteristic based on the difference between the right and left wheel path profiles. This presumably relates to a possible roll type of excitation of the traversing vehicle due to road roughness. An interesting extension of this would be to find another type of difference based on the instantaneous difference between the profile as experienced by the front and rear wheels. It would appear that the digital techniques used in the investigation could be readily adapted to obtaining and analyzing this wheelbase roughness. If so, the complete characterization of a road profile could include longitudinal, transverse, and fore-and-aft values corresponding to the heave, roll, and pitch modes of possible vehicle vibration. Such a roughness description would be extremely useful for determining road user satisfaction as mentioned by Williamson.

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AUTHOR'S CLOSURE

I appreciate the perceptive comments made by both reviewers. Quinn has added some valuable discussion from his experience about practical considerations in the signal processing of road profiles and related time series. His discussion of the problems that can be caused by amplitudes that change sharply with frequency, for example, is an excellent supplement to the brief comments made on this subject in the paper. Because of this and other pitfalls discussed, I strongly recommend side-by-side comparison of measured and filtered profiles for a representative set of cases to verify that a given filtering scheme does not introduce excessive distortion.

Brickman has raised some valid questions for discussion. My responses correspond to his specific points as follows:

1. If only band-pass filters with narrow passbands were to be used and if only amplitude characteristics were of interest, then the phase shift would probably not be important. It is true also that the analog filters on the GM profilometer induce a phase shift in the longer waves. For the vehicle speed of 20 mph (32 km/h) and the two filter selections most commonly used in our work, the phase shifts begin at about 30 and 60 ft (9 and 18 m) in wavelength. There are some applications, however, for which the location of specific short waves might be important, such as the study of roughness on the approaches and at the ends of bridges and the study of roughness associated with joints in concrete pavements. Furthermore, the convenience of examining measured and filtered profiles plotted in the same figure requires that there not be a phase-shift difference between the two profiles. This type of visual analysis, which was used in the paper to demonstrate filter characteristics, can also be valuable in studies of pavement properties such as those mentioned above. It may appear that the requirement of double filtering is an excessive computational burden. A double filter, however, has much sharper cutoff characteristics than the single filter on which it is based; the gain at a given frequency of the double filter is the square of the gain of the single filter. Thus, to achieve a certain degree of frequency resolution, the order required for a double filter is lower than the order that would be required if the filter were to be applied only once. This is significant because the number of terms required increases linearly with the order, but the incremental frequency resolution decreases with order. In addition, even though a phase shift is present for longer waves, the accuracy of the analysis is not improved by the introduction of phase shifts for all waves. Although in many applications the phase-shift effect may not be serious, it is a source of distortion, and since it is easily eliminated, there seems to be no reason to include it.

2. Artificial profiles were used because sufficiently realistic and difficult test cases with a known answer could be associated and provide meaningful comparisons. It is felt that the ½ sine waves, which resemble an isolated bump on an otherwise smooth road, and the cases with 4 sine waves with varying amplitudes served this purpose. Additionally, these test cases involving only a single frequency were convenient for studying the relationship between filter distortion and nearness of frequency to the edge of the passband.

Real road profiles provide more realistic tests than any artificial profile could provide. This is the reason for the inclusion of plots showing both the measured and the filtered profiles and the discussion of the adequacy of the filter on the basis of the plots. Brickman has suggested some interesting possibilities for further testing with artifical data, however. The smooth bump in particular is similar to, but more realistic than, the ½ sine wave. Even if it were composed only of a part of a single sine wave, however, the smooth bump would involve other frequencies. Consider

$$f(x) = 1 + \sin(x) \quad \text{if } -90 \text{ deg} < x < 270 \text{ deg}$$

$$= 0 \quad \text{if otherwise} \qquad (3)$$

The unit step, which is added to achieve continuity, introduces frequency components other than that of the sine wave. Thus, if only for the purpose of studying the relationship between distortion and nearness of the frequency to the edge of the passband, the test cases that were used have some value.

3. Amplitude frequency distribution (AFD) (1) is apparently closely related to another method (12). I am in basic agreement with the AFD approach, since it provides an effective means for evaluating the amplitude variation as well as the average or overall roughness in a road section. The statistical approach I used is essentially the same as that used in the AFD method except that the distribution of a local roughness

measure, the rms of the filtered elevations over 1 cycle at the longest wavelength in the passband, is investigated instead of the distribution of the peaks in the filtered profile. The local roughness measures were used because the local rms values have smaller errors than the filtered values at the peaks (Tables 1 and 2). In Table 2, for example, the maximum error in the moving rms values is considerably smaller in each case than the error in the filtered value at the largest peak. Although these results were obtained with artificial data, the same effects should be expected in real cases because of the nature of the amplitude-smoothing effect. Although I prefer the use of local roughness measures for the reasons state above, the AFD approach is a natural and valid means of characterizing the roughness.

Brickman is right that the wheelbase roughness is important because of the special type of motion it induces in passing vehicles. This could perhaps be handled by including in the analysis a filter passband whose wavelengths spanned the range of values of twice the wheelbases of standard automobiles.