# Derivation of Values of Time From Travel Demand Models 

Peter R. Stopher, Department of Environmental Engineering, Cornell University

Models based on the application of disaggregate behavioral theories and concepts to travel demand modeling are outlined, and problems associated with their application are discussed. The basic hypothesis of these models is stated, and the inference of values of time from mode choice and route choice models is seen to greatly depend on the accuracy and adequacy of the models. A number of methodological and conceptual problems are posed in the achievement of these objectives. A basic problem that demands attention is the determination of how the hypotheses on which the mathematical techniques are based relate to the hypotheses of choice behavior. An empirical analysis and evaluation of the logit, probit, and discriminant analysis techniques and their underlying mathematical assumptions have revealed the problem of determining a basis for comparing models from different statistical techniques. The importance is stressed of establishing statistical validity and confidence in the coefficients of the model variables and of ensuring that the interpretation of the coefficients is not made on the basis of extrapolating the results beyond the range of data. Comments are made on the behavioral interpretation of the coefficients, the time difference coefficient, alternative methods of dealing with user characteristics, and impurities relating to other differences among the modes or routes. The problems of the specification of the models for different treatments of trip segments are discussed, and operational problems associated with theoretical model structures for applying logit analysis to a multiple-choice situation are reviewed.

The basic rationale for deriving values of travel time from travel demand models lies in the assumption that such models reveal the preferences of travelers and therefore indicate the trade-offs among different trans portation system attributes. Specifically, if measures of both time and cost of travel by alternative modes, routes, or destinations are included in a model of travel demand, then the rate of substitution of time for money can be determined. However, the majority of travel demand models (21), developed in connection with major urban transportation studies, have been inadequate for inferring time values because of either lack of any system characteristics or the inclusion of only cost or only time.

In attempts to develop more accurate, more responsive forecasting models, a number of models have been developed recently, based on the use of explicit time and cost variables, particularly for the mode choice element of the travel decision process. The models, within this general approach, that appear most applicable for the derivation of travel time values are based on the applica-
tion of disaggregate behavioral theories and concepts to travel demand modeling. With one major exception (23), the models developed on this basis have been mode choice models (12, $14,15,19,27$ ). It is the basic model, typified by these, that is the primary concern of this paper.

## GENERAL STATEMENT OF MODELS

The basic hypothesis of these models is that potential travelers choose their modes or routes of travel by considering the relative efficacy of the available modes, scaled by the individual preference functions of the potential travelers. It is also assumed that the frequency distribution of probabilities of choice of any mode or route, over a total population, is symmetrical and asymptotically approaches zero for very large negative and very large positive values of the total preference function or stimulus (Figure 1). The distribution of cumulative probabilities therefore follows a sigmoid curve from zero at very large negative values of the total preference function to unity at very Iarge positive values (Figure $\overline{2}$ ).

The first operational problem that must be resolved, in building models that obey these theoretical statements, is to determine a mathematical function that behaves appropriately. A curve such as that shown in Figure 2 could be estimated by a piecewise linear procedure. However, such a procedure requires that arbitrary limits be set on each part of the linear relationship, and this is most likely to lead to a high degree of arbitrariness in the relationship determined. A number of nonlinear mathematical relationships do exist, however, that yield a symmetrical sigmoid curve or an approximation thereto. Among these are probit analysis (5), logit analysis (2), and discriminant analysis (6). The problem with apply ing any statistical technique to a hypothesized relationship is that the statistical technique may impose constraints or assumptions on the process being modeled. These constraints and assumptions may or may not be consistent with the underlying assumptions and hypotheses of the process being modeled. For instance, probit analysis requires that the probability distribution be a normal distribution; however, discriminant analysis assumes the probability distribution to comprise a combination of two overlapping normal distributions. A more

Figure 1.


Figure 2.

detailed discussion of these techniques is to be found later in this paper.

All of the mathematical techniques mentioned so far have in common the possibility of using a linear formulation for the preference function of an individual and provide estimates of the probability of an individual making a specific choice. In discriminant analysis, the discriminant function is assumed to be a linear function of user and system characteristics. In probit analysis, the probit, or upper limit of integration of the normal distribution, is similarly assumed to be a linear function, and the logit function may be assumed to be linear (although this is open to choice by the analyst). In general, then, the preference function for any of these techniques may be represented as
$F_{i}=\alpha_{o}+\sum_{\mathrm{t}=1}^{\mathrm{n}} \alpha_{\mathrm{t}} \mathrm{S}_{\mathrm{ti}}+\sum_{\mathrm{q}=1}^{\mathrm{m}} \beta_{\mathrm{q}} \mathrm{U}_{\mathrm{q} i}$
where

$$
\begin{aligned}
\mathrm{S}_{\mathrm{ti}}= & \mathrm{t} \text { th system characteristic of a travel mode } \\
& \text { for individual } \mathrm{i}, \\
\mathrm{U}_{\mathrm{q} \mathrm{i}} & =q \text { th user characteristic of individual } \mathrm{i}, \\
\mathrm{~F}_{\mathrm{i}} & =\text { preference function of individual } \mathrm{i}, \text { and } \\
\alpha \text { and } \beta & =\text { coefficients to be estimated. }
\end{aligned}
$$

In equation 1, the user's scaling of mode or route alternatives is assumed to be represented by the term $\sum_{\mathrm{q}=1}^{\mathrm{m}} \quad \beta_{\mathrm{q}} \mathrm{U}_{\mathrm{qi}}$. An alternative, which will be discussed later, would be to assume that the values of $\alpha$ are functions of $\mathrm{U}_{\mathrm{qi}}$ and that the values of $\beta$ are all zero.

In models of the general type of equation 1 , when $\mathrm{S}_{\mathrm{ti}}$ includes relative costs and times of travel for two modes or routes, it is possible to infer values for travel time
savings. This can be illustrated by taking a simple form of the function $F_{1}$ in equation 1 and by using cost and time differences for the relative measures:
$\mathrm{F}_{\mathrm{i}}=\alpha_{o}+\alpha_{1}\left(\mathrm{t}_{\mathrm{ki}}-\mathrm{t}_{\mathrm{mi}}\right)+\alpha_{2}\left(\mathrm{c}_{\mathrm{ki}}-\mathrm{c}_{\mathrm{mi}}\right)$
where

$$
\begin{aligned}
\mathrm{t}_{\mathrm{ki}} \text { and } \mathrm{t}_{\mathrm{mi}}= & \text { travel times by alternatives } \mathrm{k} \text { and } \mathrm{m} \text { for } \\
& \text { the } \mathrm{i} \text { th individual and } \\
\mathrm{c}_{\mathrm{ki}} \text { and } \mathrm{c}_{\mathrm{mi}}= & \text { travel costs by alternatives } \mathrm{k} \text { and } \mathrm{m} \text { for } \\
& \text { the } \mathrm{i} \text { th individual. }
\end{aligned}
$$

From equation 2, it is possible to infer a value of time from the rationale of investigating the changes in $F_{i}$ that result from a unit change in either the time or cost difference. Thus, a unit change in the time difference will cause a change of $\alpha_{1}$ units in $\mathrm{F}_{1}$. The same change in $F_{1}$ could be produced by a change of $\alpha_{1} / \alpha_{2}$ units of cost. Hence, a value of time may be inferred as $\alpha_{1} / \alpha_{2}$. For example, if $\alpha_{1}$ is 0.05 and $\alpha_{2}$ is 0.0125 , with costs measured in cents and times in minutes, then in this simple case, $1 / 4 \mathrm{~min}$ saved is equivalent to a 1 -cent additional cost outlay, or 1 hour saved is worth $\$ 2.40$.

Alternatively this function could be rewritten to give a combined cost and time difference for the evaluation of $\mathrm{F}_{1}$. Hence,
$\mathrm{F}_{\mathrm{i}}=\alpha_{\mathrm{o}}+\alpha_{2}\left[\left(\mathrm{c}_{\mathrm{ki}}-\mathrm{c}_{\mathrm{mi}}\right)+\left(\alpha_{1} / \alpha_{2}\right)\left(\mathrm{t}_{\mathrm{ki}}-\mathrm{t}_{\mathrm{mi}}\right)\right]$
In equation 3 , the factor $\alpha_{1} / \alpha_{2}$ may be regarded as the conversion factor to allow costs and times to be added together. It therefore represents the monetary value of a unit of time. Regardless of the addition of further variables in the formulation of $F_{1}$, this inference of a value of time may still be drawn.

Since the coefficients determined by model calibration are for the total sample population and are not specific to each individual, it can be stated that a unit change in the cost difference has, on the average, an equivalent effect to a change of $\alpha_{1} / \alpha_{2}$ units of time difference. This average equivalence is based on observed behavior of choices among modes and routes and results in the estimation of an average value of travel time savings. It is most unlikely that this average value of time will be the marginal value of time savings for any individual. The sample population used to build a model such as equation 2 will generally include three groups of travelers: those who (a) make trade-offs between costs and times, (b) choose a logical alternative that is both faster and cheaper, and (c) choose an illogical alternative that is slower and more expensive. Only the first of these groups provide useful information on positive time values, and these are marginal values only insofar as the actual available trade-offs allow. (For example, a person who gives up a possible cost saving to obtain a time saving will provide the analyst with an estimate of value of time that is less than or equal to his or her marginal value of time; however, the person who makes the reverse decision provides a value of time that is greater than or equal to his or her marginal value of time.)

The major issue here is whether the desire is to obtain marginal values of travel time savings or average values [this problem is raised by Harrison and Quarmby (7), but is not resolved]. The issue clearly depends on the uses to which the time values are to be put. Generally, time values are mainly used in travel demand models and in economic evaluation, as discussed by Reichman and Stopher in papers in this Record. When applied to travel demand models, an average value of travel time savings would probably be acceptable, provided that the mix of traders, logical choosers, and
illogical choosers was approximately the same as in the situation in which the time values were inferred. However, if the application called for estimation of travel behavior under circumstances that offered different trade-off opportunities, use of an average value of travel time savings would probably lead to erroneous predictions. Similarly, in applications to economic evaluation, newly created time savings form a major part of the benefits of a potential project. Under these circumstances, it would appear that marginal values of travel time savings should appropriately be used. Since comparisons of marginal and average values of travel time savings have not been made, one cannot assert how serious these problems are or state that average travel time values should not be used for evaluation or travel demand forecasting. However, it is clear that the average travel time values derived from choice models must be used circumspectly and with an understanding of the possible inappropriateness of these values.

However, note that the values of $\alpha_{1}$ and $\alpha_{2}$ will depend on the sufficiency of the specification of the model and also on the accuracy of measurement of the parameters in the model, the values of $\mathrm{S}_{\mathrm{ti}}$ and $\mathrm{U}_{\mathrm{qi}}$. Both measurement and specification errors will lead to erroneous values of $\alpha_{1}$ and $\alpha_{2}$ and, hence, to an incorrect value of travel time. Furthermore, the error variance of the ratio $\alpha_{1} / \alpha_{2}$ will be a complicated function of the error variances of $\alpha_{1}$ and $\alpha_{2}$, particularly if $\alpha_{1}$ and $\alpha_{2}$ cannot be assumed to be statistically independent.

To illustrate this problem, one may consider a simple (and possibly unrealistic) case in which $\alpha_{1}$ and $\alpha_{2}$ are assumed to be random, uncorrelated variates (i.e., the covariance of $\alpha_{1}$ and $\alpha_{2}$ is zero) and in which the ratio is assumed to be normally distributed. In such a case, the variance of the ratio $\alpha_{1} / \alpha_{2}$ is given approximately by [the variance when $\operatorname{cov}\left(\alpha_{1}, \alpha_{2}\right)$ is nonzero is given elsewhere (11, p. 232) ]:
$\mathrm{V}\left(\alpha_{1} / \alpha_{2}\right)=\left[\alpha_{2}^{2} \mathrm{~V}\left(\alpha_{1}\right)+\alpha_{1}^{2} \mathrm{~V}\left(\alpha_{2}\right)\right] / \alpha_{2}^{4}$
where $V\left(\alpha_{1}\right)$ and $V\left(\alpha_{2}\right)$ are the variances of $\alpha_{1}$ and $\alpha_{2}$. Using the previously assumed values of $\alpha_{1}$ and $\alpha_{2}$ and as suming the variances of the coefficients to be 0.00002 for $\alpha_{1}$ and 0.000002 for $\alpha_{2}$ give the variance of the ratio $\alpha_{1} / \alpha_{2}$ as $\mathrm{V}\left(\alpha_{1} / \alpha_{2}\right)=0.78$, approximately. Under the normal distribution assumption, one would obtain 95 percent confidence that the true value of travel time would lie between about $\$ 0.60$ and $\$ 4.20$. Clearly, such a range of values is excessive. Yet the error variances in $\alpha_{1}$ and $\alpha_{2}$ provide t-scores for the coefficients of the order of 8.5; these are clearly significant well beyond the 99.9 percent confidence point and define very narrow confidence limits for the coefficients. Hence, it is probable that the error variances of $\alpha_{1}$ and $\alpha_{2}$ will have to be much smaller for a significant value of the ratio $\alpha_{1} / \alpha_{2}$ than is needed for satisfactory fitting of the basic relationship of equation 1.

Two observations are in order here. First, it is not at all clear that the assumptions made in the above illustration are tenable. Rogers, Townsend, and Metcalf (18) show the value of travel time distributions for five values of time. In all cases, the distributions are skewed, thus placing doubts on the reasonableness of the normality assumption. Lianos and Rausser (13) showed that, provided $\mathrm{E}\left(\alpha_{1}\right) \neq 0$ and $\mathrm{E}\left(\alpha_{2}\right) \neq 0$, the underlying distribution of $\alpha_{1} / \alpha_{2}$ can be determined and will probably have derivable moments; this permits the computation of a variance but does not necessarily provide a basis for determining confidence intervals (11).

Second, note that, notwithstanding the extent of the theoretical errors of estimation of values of travel time savings, the actual estimates produced by the various
studies have been remarkably close. Most studies have provided estimates of commuter travel time values that range between 20 and 40 percent of the wage rate for most income groups and that represent dollar values of between $\$ 1.75$ and $\$ 3.50$ per hour of commuter travel time in most cases. These average travel time values clearly lie well within the confidence limits suggested by the above example. However, this may suggest that apparent systematic variations in travel time values with income are spurious and coincidental. Certainly, no investigator has explicitly reported thus far on any detailed analysis of the statistical significance of time value variations across income groups. This is clearly a potentially useful research topic that could yield significant information.

In summary, the inference of values of time from mode choice and route choice models greatly depends on the accuracy and adequacy of the models; as yet, this form of modeling is still in its infancy. It poses a number of problems that need to be addressed so that the time values resulting from this type of analysis will be less subject to question than they are at present. The remainder of this paper details a number of these problems and suggests some possible research that might lead to their successful solution.

## METHODOLOGICAL AND CONCEPTUAL PROBLEMS

## Mathematical Technique

As previously noted, the types of models that are most responsive to the inference of time values are probabilistic, disaggregate models that have been constructed by using probit, logit, or discriminant analysis. One of the major differences between this type of modeling process and the more conventional travel demand modeling is that the underlying basis of these probabilistic models is a hypothesis of the choice behavior of an individual. This results in more efficient use of data and tends to lead to the construction of more statistically reliable models. Furthermore, the disaggregate models have almost exclusively incorporated measures of both time and cost, thereby permitting an analysis of revealed trade-offs. [The primary instance of an aggregate model that included both times and costs is the Traffic Research Corporation model (9), but this used ratios and therefore does not supply a single estimate of an average value of travel time. 1 Since the primary purpose of this paper is to discuss how values of time have been derived from travel demand models and not how they could be derived, the derivation from aggregate models is not discussed in detail here. Suffice it to say that there appears to be no a priori reason why aggregate models should not be used as a basis for deriving values of travel time. However, it does seem likely that such models will be more seriously affected by statistical significance issues and the averaging effect on the derived values.

The basic problem is to determine how the hypotheses on which the mathematical techniques are based relate to the hypotheses of choice behavior. This problem has so far not been tackled in great depth within the context of travel choices. It is perhaps worthwhile to note some of the issues that are encountered in tackling this problem; these, in turn, form the basis of possible research to resolve the problem.

For the most part, the hypotheses on which each technique is based do not appear to be unreasonable but do differ significantly. Although hypotheses can be proposed on the choice process of an individual, information on the actual choice process is insufficient for clear judgments to be made regarding the appropriateness of such hypoth-
eses and the applicability of the mathematical techniques per se. Therefore, methods must be devised for comparing the results of the applications of the three techniques, and all the mathematical assumptions underlying the techniques must be fully investigated and evaluated against the observed properties of the individuals whose choices are being modeled.

An initial empirical analysis of this type has been attempted (22) and has clearly demonstrated some of the problems that arise in such an empirical task. The major problem that arises is determining a basis for comparing models derived from different statistical techniques. This problem, and some solutions to it, are discussed in detail elsewhere (22) and will not be repeated here. The results of that research suggest that discriminant analysis is somewhat inferior to probit or logit analysis, and that the latter two are statistically indistinguishable in performance. However, these results are based on restricted data sets and cannot be assumed to be generally applicable. The procedure used in that work (22) to compare the mathematical techniques does appear, however, to be useful for general application to other data sets. Additional development of comparative techniques would, however, be considerably beneficial for resolving the question of mathematical procedures for model building.

## Meaning of Coefficients

Since the probabilistic, disaggregate travel demand models are based on a hypothesis of choice behavior, attempting to place behavioral interpretations on the coefficients of the model variables (values of $\alpha$ and $\beta$ in equation 1) does not appear to be unreasonable. Effectively, the inference of a value of time from these models is such an interpretation. This specific interpretation will be treated in more detail in a later section of this paper.

Before any interpretative statements are made about the coefficients, the statistical validity and confidence in these coefficients must be established. Care must be taken to ensure that the interpretations are not made on the basis of extrapolating the results beyond the range of the data. Until knowledge of the entire modeling procedure is radically increased, extrapolations outside the range of data used for calibration must be fraught with dangers, particularly since confidence that the model accurately reflects the underlying choice process is currently unestablished.

Little can be said at present about the behavioral interpretations of the coefficients of transportation system attributes other than time and cost since the development of models that incorporate further terms is still a matter for future research. However, some researchers have attempted to include comfort or convenience indexes (12, 29). Interpretations of such coefficients will also de$\overline{\text { pend }}$ somewhat on the mathematical technique adopted for model building and on the solution to other problems dealt with later in this paper. Since general results have not yet been achieved for attributes other than time and cost, interpretations of other coefficients will not be discussed here. One of the major problems of concern is the inclusion of user characteristics in the model and the way in which they are included.

In many cases, the travel demand models discussed here have entered user characteristics as additional linear variables, as shown in equation 1. This is effectively a statement that the time difference coefficient is a linear function of income. The linear form of inclusion is effectively a behavioral assumption that the choice is based on system characteristics by themselves, with the addition of an individual bias. This bias is the
additive value of the user characteristics in the model. However, an alternative assumption is possible that, in many ways, appears to be more intuitively satisfying in terms of behavior. This assumption is that the weights (coefficients) attached to each of the system characteristics will depend on the user characteristics. In other words, an individual bias is assumed to exist on the importance of each system characteristic rather than on the final choice. This form of assumption can be represented mathematically as follows:
$\mathrm{y}=\alpha_{\mathrm{o}}+\sum_{\mathrm{i}=1}^{\mathrm{n}} \alpha_{\mathrm{t}}^{\prime} S_{\mathrm{t}}$
where
$\mathrm{a}_{\mathrm{t}}^{\prime}=\mathrm{f}\left(\mathrm{U}_{1}, \mathrm{U}_{2}, \ldots, \mathrm{U}_{\mathrm{m}}\right)$ and
$\mathrm{y}=$ general preference function, such as probit Y and $\log G(X)$.

In a few instances, alternative methods of dealing with user characteristics, notably income, have been attempted. A number of studies ( $4,8,12,25$ ) have entered income as a multiplicative term with time difference. The use of such a product term is a restricted version of the second assumption, discussed above, since it assumes that one coefficient is a function of one user characteristic. It is interesting to speculate why income should, in these applications, be considered to affect the weighting of time differences only, and not cost differences. The rationale appears to have been one of attempting to evaluate directly an income-dependent value of travel time savings. Inclusion of income in more than one variable may conceivably generate serious intercorrelations among the variables, thereby leading to poor estimations of the model coefficients. For elimination of this problem, data may be stratified by income, and separate models built for each of several income groups ( $8,19,22$ ). Significant differences occurred among all coefficients over the different income groups except in Hensher's data (8), where small stratum populations led to large standard deviations on the coefficients and relatively poor curve fitting.

Clearly, these two alternative assumptions have extensive implications on the meanings attributed to the coefficients of the system characteristics.

There is, however, one element of the model that is largely unaffected by such assumptions. This is the constant term $\alpha_{0}$. For interpretation of the meaning of the constant, a possible functional form may be considered. Let
$y=\alpha_{o}+\sum_{t=1}^{n}\left[\alpha_{t}\left(S_{t 1}-S_{t 2}\right)\right]$
and assume a logit model of the form
$P_{2}=e^{y} /\left(1+e^{y}\right)$
If choice depends on the differences in mode or route characteristics, then two modes or routes with the same system characteristics should yield a $50: 50$ split, i.e., $P_{2}=1 / 2$. Inspection of equation 7 shows that when $P_{2}=1 / 2$, $\mathrm{y}=0$; this will only occur with identical system characteristics if $\alpha_{0}$ is also zero. If $\alpha_{0}$ is not zero, then identical system characteristics give rise to a value of $P_{2}$, such as
$P_{2}=e^{\alpha_{0}} /\left(1+e^{\alpha_{0}}\right)$
Hence, the constant term $\alpha_{0}$ represents the bias for or against the second alternative (equation 8) on grounds
other than the specified system characteristics. It may be proposed, alternatively, that $y$ is given in terms of a linear function of both user and system characteristics:
$y=\alpha_{o}+\sum_{t=1}^{n}\left[\alpha_{t}\left(S_{11}-S_{t 2}\right)\right]=\sum_{\mathrm{q}=1}^{m} \beta_{\mathrm{q}} \mathrm{U}_{\mathrm{q}}$
In equation $9, \alpha_{0}$ has two possible interpretations. If user characteristics are fully specified, then $\alpha_{0}$ has the same interpretation as before. If user and system characteristics are both only partially specified, then $\alpha_{0}$ will represent the bias for or against an alternative compounded both of individual bias and the effects of nonspecified system characteristics. It is clear therefore that the constant term has no bearing on the value of time or on the meaning of any other coefficient.

However, incomplete specification of the model will affect the value of the coefficients of time and cost and also the constant term. Certain elements of comfort and convenience are most probably time dependent, e.g., standing may be acceptable for 5 min but is unlikely to be so for 30 min . If comfort and convenience variables are not specifically included, then some part of the choice variance associated with the time-dependent comfort and convenience attributes is likely to be included in the travel time coefficient. Since some of these comfort and convenience attributes will be likely to vary not only by mode but also by time of day and direction of travel, the inference of a single value of time from such incompletely specified models is of somewhat dubious value. The theoretical analysis of de Donnea (4) provides an additional reinforcement to this argument and clearly demonstrates that a true value of time can only be derived from a fully specified choice model.

These considerations of comfort and convenience will also apply to route choice models, in relation to timedependent attributes of comfort and convenience between alternative routes. Although past route choice derivations have explicitly assumed that only cost and time differences exist between toll roads and free roads, there are probably comfort differences also, some of which will be time dependent.

The danger of lack of specification in the choice models is that the value of travel time derived will include impurities relating to other differences between the modes or routes. The presence of these in the estimated value of travel time will then have serious and important implications regardless of whether the value is used in other demand models, or as part of an economic evaluation procedure.

There is some extensive controversy surrounding the idea of a true or pure value of travel time. It is generally accepted that different travel activities generate different utilities. For example, waiting probably has far less utility than walking, which, in turn, probably has less utility than riding in a vehicle (7, p. 3). However, waiting in a bus shelter probably has a different utility than waiting on a street corner or in a subway station. Thus, the utility of the time spent in an activity depends on the activity content of the time and the circumstances under which it is consumed. A number of studies have divided time by activity content $(14,29)$, but no studies have addressed the circumstances in which the activity is carried out. Furthermore, the segmentation of travel time does not address problems such as the difference between waiting for a vehicle transfer and waiting for a demand-actuated vehicle (e.g., taxi, clial-a-bus vehicle), where this is the sole mode of travel for a trip.

The existence of a pure value of time is a subject for philosophical debate. However, there are a number of parameters associated with the utility of travel time that should be explicitly recognized and taken account
of so that values of travel time can be derived that can be applied under different travel circumstances. Segmentation of travel time will partially achieve this. Quantification of convenience and comfort may provide some, or all, of the balance of the required information.

The initial research for this problem is to determine means of including, explicitly in the models, variables describing attributes such as comfort and convenience. This requires, first, the derivation of some form of mathematical expressions for various comfort attributes of travel modes and routes and, subsequently, the development of models that include these attributes, insofar as they are important to the decision-making process. Methods of marketing analysis and psychometric scaling techniques appear to hold out the greatest promise for proceeding toward this goal.

## Trip Segmentation

The majority of trips made in an urban area comprise several segments. Most commonly, transit trips comprise three segments: access to the transit facility, linehaul, and egress to the final destination. In relation to values of travel time, the problems that arise here are principally two: how to build a mode choice model for such a situation and the implications of this trip structure on the value of time.

This paper will not discuss at length the options for handling trip segments in mode choice models [a number of alternatives are discussed elsewhere (17)]; it will focus on the specification of the models for different treatments of trip segments. As discussed in the preceding section, problems arise mainly when the models are not fully specified. Trip segments may be handled by dividing up the times and costs between the segments, e.g., access, egress, and line haul, or by taking line-haul times and costs only or by using an average overall travel time and cost. Under each of these alternatives, with incomplete specification of the system attributes, different values of time will be obtained. Quarmby (14) found considerably different values of travel time for overall travel time and excess travel time (e.g., walking and waiting). Other researchers have similarly found different values for different pairs of modes (19), and work in Chicago has yielded different values of time for each of walking time, waiting time, and line-haul time.

Again, problems arising from alternative treatments of trip segments can be resolved by full specification of system variables. This includes not only measures of comfort and convenience but also complete specification with respect to the separate segments. A treatment using only line-haul system characteristics or only access and egress characteristics probably would yield inflated or deflated time values because of the lack of specificity.

## Measured and Perceived Mode Attributes

An important consideration in formulating behavioral mode choice models is the relation between objective and subjective estimations by the traveler of the system characteristics. Objective values are those values that are determined by engineering measurement, although subjective values are those values perceived by the (potential) traveler. The difference between objective and subjective values of, say, travel times or travel costs arises from two sources. One is inadequate information about, or experience with, alternative modes. With inadequate information or experience, people will, to make choices, fill in the necessary judgments subjectively. Obviously, this may bear little relation to objective reality but is nevertheless the basis on which
choices are made. Another is a bias that persists even with adequate knowledge of the alternatives. By definition, this bias is a stable preference function.

It is obvious that, for both predictive validity of the model and valuation of travel time differences, the former process is most critical since it may be assumed that any effects of a stable preference function may be resolved by a simple linear transformation. In fact, model calibration achieves this. The problem caused by lack of information is that a priori there is no way of knowing how these deviations from objectivity are distributed nor at what rate learning modifies the subjective values to make them approach objective ones. Ideally, if the distribution of subjective values around objective values of the system is normal, the errors will sum to zero. Alternatively, a consistent relationship may exist between subjective and objective values.

Since time values, inferred from behavioral mode choice models, are derived from the coefficients of time and costs in the models, the primary concern for accuracy of the time values will arise from the traveler's comparative knowledge of these two parameters. It is clear from the work of Watson (28) that research is needed to investigate the biases and patterns of random estimates of mode and route attributes. This may initially be undertaken by building travel choice models on the basis of objective measures of mode attributes and, subsequently, by investigating the unexplained variance. A large unexplained variance for the model would be indicative of a large, and therefore important, random es timation element in subjective values of mode attributes. It is not yet clear what research effort might then be needed to analyze and measure this random estimation element, assuming it is measurable.

## Variations in Value of Time

The discussion in this paper has referred to only one time value or to one that might vary according to the relative disutility of certain trip segments, and to assume that only one value of time exists seems implausible. Currently, several different studies have suggested that the time spent on the journey to work is valued at about one-quarter to one-half of the wage rate. On the other hand, vacation travel appears to be valued at between $1 / 2$ and $1 / 2$ times the wage rate (26). To hypothesize that the value of travel time will vary with trip purpose therefore seems reasonable. Such values can probably be obtained by studying travel choices for the various trip purposes and calibrating models to explain the choices. However, the traditional breakdown of trip purposes used in current transportation studies (3) may not necessarily be the ideal set to permit identification of the most pertinent travel time values. In carrying out studies of non-work-trip travel choices, consideration must first be given to hypotheses of variation in travel time values and model formulations as the basis for determining the most appropriate strata for trip pur pose.

The stability of time value with trip length or with time savings is also of concern. First, most of the probabilistic models developed so far use time and cost differences and have been calibrated on relatively short trips (usually $<1 \mathrm{~h}$ ). The hypothesis behind the use of differences is that time and money savings or expenditures are valued the same whether they are obtained on a 10 - or $60-$ min trip. There is good reason to suppose that this hypothesis does not hold for long trips and that the value of time and cost savings will be modified by the total outlay of time or money involved on a long trip. For example, Watson (29) found that travel time difference divided by total journey time was more effective
for his intercity study than simple travel time differences.

In addition, the values of time determined from present travel choice models have been determined for a relatively small range of time differences (generally, from 5 to 20 min ), and the stability of the values by time saving and the validity of extrapolating values to smaller or larger time savings beyond the observed range have been the subject of relatively little research (25). This problem requires further research but, by its nature, also requires a much larger data set than has generally been available in the past for probabilistic travel choice modeling. Extension of the range to smaller time savings may be potentially very troublesome, however. By the time one is considering values of time savings of less than 5 min , the time savings involved appear to rapidly approach the point at which they no longer affect travel decisions. Furthermore, reported time values are generally accurate only to the nearest 5 min and thus provide the analyst with insufficient information to investigate the effects of very small time savings. Hence, estimation of the value of time for small time savings is likely to be subject to considerable random variance.

## CONCLUSIONS

This paper outlines the basic methods of inferring values of travel time from travel choice models and discusses a number of the problems that arise in this application. In general, little research is in hand to determine the solutions to these problems.

Most of the problems discussed have as much bearing on the production of valid travel choice models as they have on the production of valid travel time values. As such, it appears that a major research effort on the building of probabilistic, disaggregate travel choice models is one of the possible ways to resolve the problems of travel time evaluation. Both the travel time values and travel choice models from this approach would be considerably useful to decision makers in evaluating alternative transportation plans.

## REFERENCES

1. M. Ben-Akiva. Structure of Travel Demand Models. M.I.T., Cambridge, Mass., Dec. 1972.
2. J. Berkson. Application of the Logistic Function to Bio-Assay. Journal of American Statistical Association, Vol. 39, 1944, pp. 357-365.
3. Manual of Procedures for a Home Interview Traffic Study. Bureau of Public Roads, 1954.
4. F. X. de Donnea. The Determinants of Transport Mode Choice in Dutch Cities. Rotterdam Univ. Press, Chapter 2, 1971.
5. D. J. Finney. Probit Analysis. Cambridge Univ. Press, England, 1964.
6. R. A. Fisher. The Use of Multiple Measurements in Taxonomic Problems. Annals of Eugenics, Vol. 7, No. 2, 1936, pp. 179-188.
7. A. J. Harrison and D. A. Quarmby. The Value of Time in Transport Planning: A Review. Paper presented at European Conference of Ministers of Transport, 6th Round Table, Paris, Oct. 29, 1969.
8. D. A. Hensher. The Value of Commuter Travel Time Savings: A Study of Land Modes. Paper prepared for Commonwealth Bureau of Roads, Melbourne, Australia, May 1971.
9. D. M. Hill and H. G. Von Cube. Development of a Model for Forecasting Travel Mode Choice in Urban Areas. HRB, Highway Research Record 38, 1963, pp. 78-96.
10. P. F. Inglis. A Multinomial Logit Model Split for
a Short Journey. McMaster Univ., Hamilton, Ontario, MS thesis, 1972.
11. M. G. Kendall and A. Stuart. The Advanced Theory of Statistics, 3rd Ed. C. Griffin, London, Vol. 1, 1969.
12. C. A. Lave. The Demand for Urban Mass Transportation. Review of Economics and Statistics, Vol. 52, No. 3, 1970, pp. 320-323.
13. T. P. Lianos and G. C. Rausser. Distribution of Parameters in a Distributed Lag Model. Journal, American Statistical Association, Vol. 67, No. 337, 1972, pp. 64-67.
14. D. A. Quarmby. Choice of Travel Mode for the Journey to Work: Some Findings. Journal of Transport Economics and Policy, Vol. 1, No. 3, 1967, pp. 273-314.
15. T. E. Lisco. The Value of Commuters' Travel Time: A Study in Urban Transportation. Univ. of Chicago, PhD thesis, 1967.
16. P. R. Rassam, R. Ellis, and J. C. Bennett. The N -Dimensional Logit Model: Development and Application. HRB, Highway Research Record 369, 1971, pp. 135-147.
17. S. Reichman and P. R. Stopher. Disaggregate Stochastic Models of Travel Mode Choice. HRB, Highway Research Record 369, 1971, pp. 91-103.
18. K. G. Rogers, G. M. Townsend, and A. E, Metcalf. Planning for the Work Journey. Local Government Operational Research Unit, Royal Institute of Public Administration, Reading, England, Rept. C67, April 1970.
19. P. R. Stopher. A Probability Model of Travel Mode Choice for the Work Journey. HRB, Highway Research Record 283, 1969, pp. 57-65.
20. P. R. Stopher. Transportation Analysis Methods. Cornell Univ., Ithaca, New York, 1971.
21. P. R. Stopher and T. H. Lisco. Modelling Travel Demand: A Disaggregate Behavioral ApproachIssues and Applications. Proc., Transportation Research Forum, 1970, pp. 195-2 14.
22. P. R. Stopher and J. O. Lavender. Disaggregate, Behavioral Travel Demand Models. Empirical Tests of Three Hypotheses. Proc., Transportation Research Forum, 1972.
23. H. Theil. A Multinomial Extension of the Linear Logit Model. International Economic Review, Vol. 10, No. 3, 1969, pp. 251-259.
24. T. C. Thomas and D. G. Haney. The Value of Tine fur Passenger Cars. Sianiura Research institute, Menlo Park, Calif., SRI Project 5074, Vols. 1 and 2, May 1967.
25. T. C. Thomas and G. I. Thompson. The Value of Time for Commuting Motorists as a Function of Their Income Level and Amount of Time Saved. HRB, Highway Research Record 314, 1970, pp. 1-19.
26. T. C. Thomas and G. I. Thompson. The Value of Time Saved by Trip Purpose. U.S. Bureau of Public Roads, Oct. 1970.
27. S. L. Warner. Stochastic Choice of Mode in Ur ban Travel: A Study in Binary Choice. Northwestern Univ. Press, Evanston, Ill., 1962.
28. P. L. Watson. Problems Associated With Time and Cost Data Used in Travel Choice Modeling and Valuation of Time. HRB, Highway Research Record 369, 1971, pp. 148-158.
29. P. L. Watson. The Value of Time and Behavioural Models of Modal Choice. Department of Economics, Univ, of Edinburgh, Scotland, PhD thesis, 1972.
