Simplified Analysis of Change in Vehicle Momentum During Impact With a Breakaway Support

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The impact of an automobile with a breakaway support for a sign or luminaire can be divided into three phases. By using simplifying assumptions, one can determine the contribution to change in vehicle momentum (ΔMV) by each phase. The first phase involves the crushing of the automobile with negligible motion of the support. The second phase considers the contribution to ΔMV by base fracture. The third phase considers the contribution to ΔMV by the acceleration of the pole. The results of this analysis are that vehicle change in momentum can be approximated by an equation that has provided valuable insight into the effect of vehicle stiffness, breakaway force level, base fracture energy, pole inertial properties, and vehicle impact speed on ΔMV. This knowledge has facilitated the subsequent development of practical laboratory acceptance test criteria to promote safer sign and luminaire supports.

The performance of a breakaway support for a sign or luminaire is a function of the change in vehicle momentum (ΔMV) produced during impact with the support. The reason for this is that immediately after impact the velocity of an unrestrained occupant relative to the vehicle interior is about the same as the change in velocity of the vehicle. Current Federal Highway Administration (FHWA) and American Association of State Highway and Transportation Officials (AASHTO) criteria (1, 2) set a limit of 4890 N·s (1100 lbf·s) for acceptable $\Delta \overline{MV}$ in full-scale tests.

In order to develop practical laboratory acceptance test criteria, ENSCO conducted a study for FHWA that involved analysis, computer simulation, laboratory tests, full-scale tests, and the correlation of all results. This study has enabled the development of practical laboratory methods of testing breakaway supports to ensure their effectiveness. Owings, Cantor, and Adair, in a paper in this Record, give some of the background on laboratory acceptance testing. The simplified analysis of this paper forms part of that background.

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ANALYSIS OF IMPACT

The following simplified analysis provides a better understanding of the phenomenon of impact by an automobile with a sign or luminaire support. One major assumption in the analysis is that the support pole can be considered a rigid body. Other assumptions will be described when they are introduced. The essential validity of these assumptions, with regard to vehicle change in momentum, is confirmed by the good correlation of predicted results from this analysis with those of computer-simulated, laboratory, and full-scale tests as shown by Owings, Cantor, and Adair in a paper in this Record.

If one neglects tire-roadway forces, the momentum change experienced by a vehicle during impact is given by

$$\Delta MV = \int_0^{t_3} F_c \, dt \tag{1}$$

where

- t_3 = time to loss of contact between vehicle and fractured support and
- F_c = force exerted by the support on the vehicle.

To facilitate the evaluation of this impulse integral, the impact is divided into three phases. The first phase, from $t = 0$ to $t = t_1$, is characterized by the crushing of the automobile with negligible motion imparted to the support. The second phase, from $t = t_1$ to $t = t_2$, involves the contribution to ΔMV provided by the fracture of the base. The third phase, from $t = t_1$ to $t = t_3$, considers the contribution to ΔMV that is inherent in the momentum imparted to the support.

Figure 1 shows the geometry of impact; Figure 2 shows the three phases of impact. During the first phase of the impact, the work done in crushing the automobile is equal to the change in kinetic energy of the automobile and can be expressed as

$$\int_0^{X_1} F_c \, dx = 0.5M\,(V_0^2 - V_1^2) \tag{2}$$

where

X_1 = maximum crush deformation of the automobile,
M = mass of the impacting vehicle,
V_0 = impact speed, and
V_1 = speed at the end of the first phase of the impact.

Equation 2 can be rewritten as

$$\int_0^{X_1} F_c dx = M\Delta V_1 V_0/\beta \tag{3}$$

where

$\Delta V_1 \equiv V_0 - V_1$, and
$\beta \equiv [1 - (\Delta V_1/2V_0)]^{-1}$.

But the contribution to ΔMV during the first phase is simply

$$I_1 = \int_0^{t_1} F_c dt = M\Delta V_1 \tag{4}$$

Combining equations 3 and 4 yields

$$I_1 = \beta/V_0 \int_0^{X_1} F_c dx = \beta E_c/V_0 \tag{5}$$

where $E_c \equiv \int_0^{X_1} F_c dx$ = crush energy.

The value of β is always greater than unity. For moderate velocity change during the first phase, however, β is close to unity and may be regarded as more or less constant. For example, if V_0 = 32.2 km/h (20 mph) and if ΔV_1 = 9.7 km/h (6 mph), then β = 1.18. A representative value of β, for the range of acceptable impacts, is 1.1. Thus the contribution to ΔMV by the first phase can be considered proportional to the energy of vehicle crush (up to breakaway force level F_1) and inversely proportional to impact speed V_0. (Although β is treated as a constant for establishing the basic trends in the first phase of impact, the exact relationship of I_1 versus E_c can be determined by using the exact definition of β.)

For a linear force-deformation characteristic, E_c can be expressed as

$$E_c = 0.5(KX_1^2) = 0.5(F_1^2/K) \tag{6}$$

where

K = vehicle stiffness and
F_1 = breakaway force level of base.

In this case, equation 5 can be expressed as

$$I_1 = \beta F_1^2/2V_0 K \tag{7}$$

For a nonlinear force-deformation characteristic, a parameter ω can be introduced as defined in Figure 3. In this case,

$$E_c = 0.5(\omega K X_1^2) = 0.5[\omega(F_1^2/K)] \tag{8}$$

where

$K \equiv F_1/X_1$ = equivalent vehicle stiffness and
F_1 = breakaway force level of base.

Thus equation 5 can be expressed as

$$I_1 = \beta\omega F_1^2/2V_0 K \tag{9}$$

This illustrates how a large breakaway force level and low vehicle stiffness can increase the contribution to ΔMV during the first phase.

As shown in Figure 2, the contribution to ΔMV by the second phase (base fracture) is

$$I_2 = \int_{t_1}^{t_2} F_b dt \tag{10}$$

which can be rewritten as

$$I_2 = \int_{t_1}^{t_2} F_b d\delta/\dot{\delta} \tag{11}$$

where δ = base displacement relative to foundation. At the beginning of base separation ($t = t_1$), the displacement velocity $\dot{\delta}$ is zero and, from physical considerations, cannot change abruptly. (A step function change in $\dot{\delta}$ together with finite inertia of the support would require an infinite force.) After base separation (at $t = t_2$), $\dot{\delta}$ will be close to vehicle velocity V_2, which in turn is some large fraction of the initial impact speed V_0 (if one assumes a relatively small ΔV). This can be expressed as

$$\dot{\delta}(t_2) = \gamma V_0 \tag{12}$$

An average value for γ, over a range of acceptable impacts, is 0.8.

A reasonable form for $\dot{\delta}$, from $t = t_1$ to $t = t_2$, would be a linearly increasing function of time; that is,

$$\dot{\delta} = \gamma V_0 [(t - t_1)/(t_2 - t_1)], \quad t_1 \leq t \leq t_2 \tag{13}$$

or

$$\dot{\delta} = \gamma V_0 (\tau/\tau_m), \quad 0 \leq \tau \leq \tau_m \tag{14}$$

where

$\tau \equiv t - t_1$ and
$\tau_m = t_2 - t_1$.

Integrating equations 13 and 14 results in

$$\delta(\tau) = (\gamma V_0/2\tau_m)\tau^2 + C_1 \tag{15}$$

Because $\delta(0) = 0$, the integration constant $C_1 = 0$. Thus

$$\delta(\tau) = (\gamma V_0/2\tau_m)\tau^2, \quad 0 \leq \tau \leq \tau_m \tag{16}$$

At base separation ($\tau = \tau_m$), maximum displacement can be expressed as

$$\delta_m = (\gamma V_0/2)\tau_m \tag{17}$$

By using equations 16 and 17, one can express τ as

$$\tau = (2/\gamma V_0)\sqrt{[\delta_m \delta(\tau)]} \tag{18}$$

Substituting equations 17 and 18 into equations 13 and 14 yields

$$\dot{\delta}(\tau) = \gamma V_0 \sqrt{\delta(\tau)/\delta_m} \tag{19}$$

The integral I_2 of equation 11 can now be expressed in terms of displacements. Thus

$$I_2 = \sqrt{\delta_m}/\gamma V_0 \int_0^{\delta_m} F_b(\delta) d\delta / \sqrt{\delta} \qquad (20)$$

To evaluate equation 20 requires knowledge of the base breakaway characteristic $F_b(\delta)$. In general, $F_b(\delta)$ has a maximum value F_1 at $\delta = 0$, and the value of F_b decreases to zero when $\delta = \delta_m$. One reasonable form for the breakaway characteristic is a linearly decreasing function; that is,

$$F_b(\delta) = F_1 [1 - (\delta/\delta_m)], \quad 0 \leq \delta \leq \delta_m \qquad (21)$$

Substituting equation 21 into equation 20 and performing the integration yield finally

$$I_2 = (4/3)(F_1 \delta_m / \gamma V_0) \qquad (22)$$

Because, for this particular base characteristic, BFE = $0.5\ F_1\delta_m$, this is equivalent to

$$I_2 = 8/3\ [(BFE)/\gamma V_0] \qquad (23)$$

The result for I_2, as expressed in equation 23, can be shown to be not very sensitive to the form of the assumed base characteristic. For example, assume that the base had a characteristic that remains constant up to breakaway; that is,

$$F_b(\delta) = F_1, \quad 0 \leq \delta \leq \delta_m \qquad (24)$$

Then substituting this characteristic in equation 20 and performing the integration yield

$$I_2 = 2(BFE)/\gamma V_0 \qquad (25)$$

Thus the drastic change from a triangular to a square wave characteristic only changes the coefficient in equation 23 from 8/3 to 2, which is a change of only 25 percent in the momentum associated with the base fracture phase. Because the triangular characteristic is much closer to a typical base characteristic than the square wave is, equation 23 will be used for studying the contribution to ΔMV of base fracture.

Let us now turn to I_3, the contribution to ΔMV required for acceleration of the support. Figure 2 shows that

$$I_3 = \int_{t_1}^{t_3} (F_c - F_b) dt \qquad (26)$$

The force $F_c - F_b$ is the net force acting on the pole, at an approximate distance D_0 below the center of gravity, which accelerates the pole in both translation and rotation. The equations of motion are

$$F_c - F_b \equiv F_p = M_p \ddot{X}_{cg} \qquad (27)$$

and

$$F_p D_0 = I_p \ddot{\theta} \qquad (28)$$

where

M_p = mass of support and
I_p = moment of inertia of support about its center of gravity.

At time t_3 (when the support and automobile lose contact), the bottom of the support has attained a velocity that is denoted by V_3. From kinematics,

$$V_3 = \dot{X}_{cg} + D_0 \dot{\theta} \qquad (29)$$

From equations 27 and 26,

$$\dot{X}_{cg} = 1/M_p \int_{t_1}^{t_3} F_p dt = (1/M_p) I_3 \qquad (30)$$

From equations 28 and 24,

$$\dot{\theta} = D_0/I_p \int_{t_1}^{t_3} F_p dt = (D_0/I_p) I_3 \qquad (31)$$

Substituting equations 30 and 31 into equation 29 yields

$$V_3 = I_3\ [(1/M_p) + (D_0^2/I_p)] \qquad (32)$$

Because $I_p = M_p R^2$ where R = radius of gyration of the support about its center of gravity, equation 32 can be written as

$$V_3 = I_3\ [(R^2 + D_0^2)/M_p R^2] \qquad (33)$$

or

$$I_3 = M_p V_3 [R^2/(R^2 + D_0^2)] \qquad (34)$$

The velocity V_3 of the support bottom will be greater than the vehicle velocity V_2 at base separation because of the presence of some elasticity in vehicle crush and pole deformation. The vehicle velocity V_2 will be less than the initial impact velocity V_0 although, for an acceptable impact, V_2 will be fairly close to V_0. From empirical data on typical impacts with breakaway supports, it is estimated that V_3 can be expressed as

$$V_3 = \alpha V_0 \qquad (35)$$

where $\alpha \approx 1.1$. The actual value of α will vary depending on the elasticity of the impact, the severity of the impact, and the mass of the vehicle. However, the use of a constant, representative value for α has permitted the development of simplified predictive techniques for ΔMV, which have subsequently proved quite reliable.

Thus the final expression for I_3 becomes

$$I_3 = M_p \alpha V_0 [R^2/(R^2 + D_0^2)], \quad \alpha \approx 1.1 \qquad (36)$$

Unlike I_1 and I_2, the momentum change I_3 is proportional to initial impact speed V_0. The integral I_3 is also proportional to support mass M_p and to the support inertial ratio

$$r \equiv R^2/(R^2 + D_0^2) \qquad (37)$$

The value of r will generally lie between 0.25 and 0.5 as shown by the following. First, consider the case of a long, slender member of length L with uniform mass distribution. In this case, the radius of gyration is $R = L/\sqrt{12}$. $D_0 = 0.5L$, and the value of r becomes

$$r \triangleq R^2/(D_0^2 + R^2) = 0.25 \qquad (38)$$

Next, consider the mass to be distributed as two point masses, each 0.5M, located at the ends of the support. In this case, the moment of inertia is given by

$$I = 2(0.5M)(L/2)^2 = ML^2/4 \qquad (39)$$

Then R = L/2. Because D_0 is again equal to L/2, we have

$$r \equiv R^2/(R^2 + D_0^2) = 0.5 \tag{40}$$

The value of r ranges between 0.25 and 0.5 for cases ranging from uniform mass distribution to equally lumped masses at the end points.

Note that the support inertial ratio r inherently determines the ratio of support center of gravity velocity to base velocity as shown by the following. Eliminating I_3 between equations 34 and 31 yields

$$\dot{\theta} = D_0 V_3/(R^2 + D_0^2) \tag{41}$$

Substituting this into equation 29 and solving for $\dot{X}_{cg}$ yield

$$\dot{X}_{cg} = rV_3 \tag{42}$$

or

$$\dot{X}_{cg}/V_3 = r \tag{43}$$

Thus the support inertial ratio r determines the form of the support trajectory after breakaway in addition to affecting the inertial contribution to ΔMV. A lower value of r implies less motion of the support center of gravity relative to the base after breakaway.

In summary, the expression for the total momentum change is given by

$$\begin{aligned} \Delta MV &= I_1 + I_2 + I_3 \\ &\approx [\beta\omega(F_I^2/2KV_0)] + \left\{8/3[(BFE)/\gamma V_0]\right\} \\ &\quad + (M_p\alpha V_0)\,[R^2/(R^2 + D_0^2)] \end{aligned} \tag{44}$$

From equation 44, for a given vehicle and support, the relationship between momentum change and impact speed is of the form

$$\Delta MV \approx (a/V_0) + bV_0 \tag{45}$$

where

$a = (\beta\omega F_I^2/2K) + \{8/3[(BFE)/\lambda]\}$ and
$b = M_p\alpha[R^2(R^2 + D_0^2)]$.

Thus for high impact speeds, the inertia of the pole is the dominant term in producing momentum change. For low-speed impacts, the breakaway force F_1, the automobile stiffness K, and the base fracture energy are dominant.

Equation 45 allows us to estimate whether impact of a support is more severe at the high end of a given speed range or at the low end. Figure 4 shows two situations. In Figure 4a, the constants a and b are such that vehicle crush and base breakaway effects dominate and the impact is more severe at low speeds. (This is the case with most luminaires.) In Figure 4b, the inertial characteristics dominate and the impact is more severe at the higher speeds. (This is the case for certain massive sign supports.)

The constants a and b are recognized to be not truly constant because they involve parameters β, γ, and α, which are a function of certain velocities during the various phases of impact. However, for a range of breakaway support performance that includes that of any acceptable support, the variation in these "constants" does not produce significant errors in predicted ΔMV. Fairly good correlation has been obtained with results from computer-simulated, laboratory, and full-scale tests by using average or representative values for the parameters β, γ, and α. The selected values of these parameters have been 1.1 for β, 0.8 for γ, and 1.1 for α.

APPLICATION OF RESULTS

The results of this simplified analysis have been used to gain a better understanding of the impact phenomenon and to predict ΔMV for a given breakaway support and impacting vehicle. Comparison of these predicted results with those of computer-simulated, laboratory, and full-scale tests has shown generally good agreement. This agreement has confirmed the essential validity of equation 45 through the speed range of interest for breakaway support performance. This analytic tool permitted the development of rational and practical laboratory methods for testing breakaway supports as discussed by Owings, Cantor, and Adair in a paper in this Record.

Assume that a breakaway support must perform satisfactorily over a designated speed range of impacting vehicles. Then an examination of equation 45 reveals that the critical (worst) performance occurs either at the low impact speed V_L or at the high impact speed V_H. Thus a check of breakaway performance at both ends of the speed range is sufficient to validate support performance throughout the whole speed range. Furthermore, it will be shown that the results of one low-speed test can be used to predict the high-speed performance with reasonable accuracy (assuming repeatable vehicle crush and breakaway base characteristics). Thus one low-speed test can serve as a check of support performance over the entire speed range of interest.

Let the measured ΔMV at the low impact speed be designated as $(\Delta MV)_L$. Then from equation 45,

$$(\Delta MV)_L = (a/V_L) + bV_L \tag{46}$$

Designating the predicted ΔMV at the high speed as $(\Delta MV)_H$,

$$(\Delta MV)_H = (a/V_H) + bV_H \tag{47}$$

Solving for a in equation 46, and substituting the result into equation 47 yield

$$(\Delta MV)_H = [(V_L/V_H)(\Delta MV)_L] + b[V_H - (V_L^2/V_H)] \tag{48}$$

Recall that

$$b \triangleq M_p\alpha[R^2/(R^2 + D_0^2)] \tag{49}$$

and $\alpha \approx 1.1$. If the pole inertial parameters and the low-speed test results $(\Delta MV)_L$ are known, then equation 48 can be used to predict the high-speed performance of the breakaway system under evaluation.

The mass M_p and the center of mass D_0 can be easily determined by a weight measurement at each end of a horizontally aligned support. The radius of gyration R about the center of mass can be determined in several ways. One simple way is to suspend the support from either top or bottom and measure its natural period T for a small angle oscillation (<10 deg). Then

$$T = 2\pi\sqrt{I_1/gD_1M_p} \tag{50}$$

where

I_1 = moment of inertia of support about suspension point,
g = acceleration of gravity, and
D_1 = distance from suspension point to center of mass.

We have

$$I_1 = M_p(R^2 + D_1^2) \qquad (51)$$

Substituting equation 51 into equation 50 and solving for R yield

$$R = \sqrt{D_1\,[(gT^2/4\pi^2) - D_1]} \qquad (52)$$

The selected speed range for acceptable breakaway support performance is 32.2 to 96.6 km/h (20 to 60 mph). Then equation 48 becomes the following (1 m/s = 3.28 ft/s):

$$(\Delta MV)_H = 1/3\,(\Delta MV)_L + b(23.8\text{ m/s}) \qquad (53)$$

where

$b \equiv M_p \alpha r$ and
$\alpha \approx 1.1$.

As previously shown, the value of r for nearly all supports lies in the range of 0.25 to 0.5. If we use the larger value in equation 53, together with the largest acceptable value of 4890 N·s (1100 lbf·s) for $(\Delta MV)_L$, we obtain the following (1 N·s = 4.45 lbf·s and 1 m/s = 3.28 ft/s):

$$(\Delta MV)_H = (4890\text{ N·s}/3) + 0.55\,M_p\,(23.8\text{ m/s}) \qquad (54)$$

Setting $(\Delta MV)_H$ = 4890 N·s (1100 lbf·s) in equation 54 and solving for M_p yield 250 kg (550 lb or 17.1 slugs). Thus, if a support is less than 250 kg (550 lb) and has satisfied the acceptance criteria in a 32.2-km/h (20-mph) test, it will perform satisfactorily in a 96.6-km/h (60-mph) impact.

Figure 1. Impact geometry and definitions.

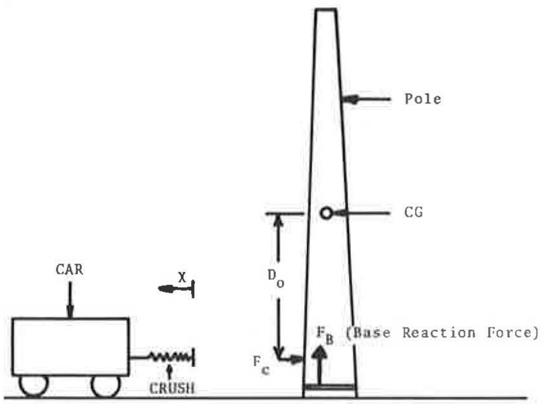
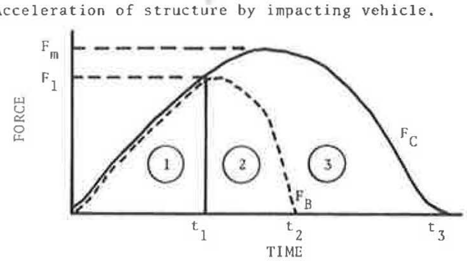


Figure 2. Separation of impact into phases.

PHASES OF IMPACT

1. Crushing of automobile with insignificant motion of structure being impacted.
2. Activation and completion of the breakaway failure mechanism.
3. Acceleration of structure by impacting vehicle.



EXPRESSION FOR MOMENTUM CHANGE

$$\Delta MV = \int_{t_1}^{t_3} F_c\,dt$$

$$= \int_0^{t_1} F_c\,dt + \int_{t_1}^{t_3} F_b\,dt + \int_{t_1}^{t_3} (F_c - F_b)\,dt$$

$$= \int_0^{t_1} F_c\,dt + \int_{t_1}^{t_2} F_b\,dt + \int_{t_1}^{t_3} (F_c - F_b)\,dt$$

$$\triangleq I_1 + I_2 + I_3$$

Figure 3. Vehicle crush characterization.

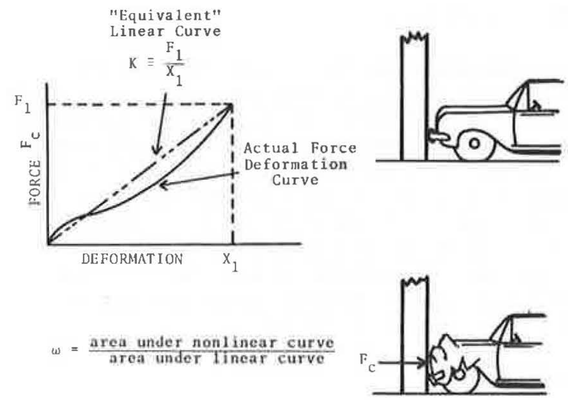
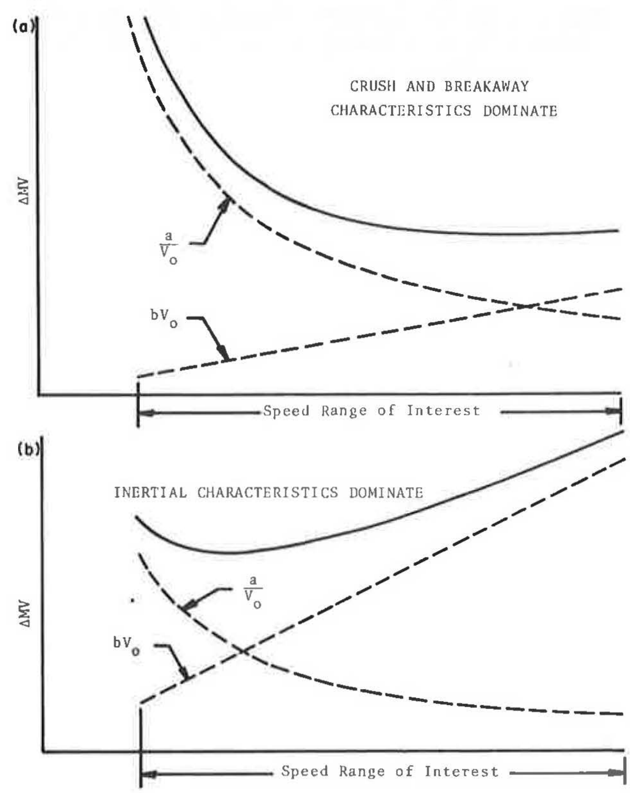


Figure 4. Characteristic variations of ΔMV with speed.



If, in the future, the allowable ΔMV is reduced to 3340 N·s (750 lbf·s) [which is listed as a desirable goal by the AASHTO specifications (2)], then the support mass that could cause excessive ΔMV in a high-speed impact would be reduced accordingly. Following the same procedure as above, we obtain 170 kg (375 lb or 11.6 slugs) as the value of support mass above which $(\Delta MV)_H$ may become excessive.

Because most current luminaire supports are less than 170 kg (375 lb), there is generally no problem associated with ΔMV produced in a 96.6-km/h (60-mph) impact with a luminaire support provided the measured ΔMV in the 32.2-km/h (20-mph) test is satisfactory. However, breakaway sign supports can be heavier and should be checked for $(\Delta MV)_H$ by using equation 51.

Thus the results of this simplified analysis have been very useful in guiding the development of practical laboratory acceptance test criteria for breakaway sign and luminaire supports. These criteria, and the studies leading to their development, are summarized by Owings, Cantor, and Adair in a paper in this Record.

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