Abridgment

Design of Freeway Entrance Ramps and Lane Drops by Using Semi-Markov Processes

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A continuous-time semi-Markov highway model can be used to simulate and design highway situations that require a merging maneuver. With such a model, mathematical manipulations of probability density functions are not required. A general continuous-time semi-Markov model can be used to simulate a number of merging situations by the substitution of appropriate values for the design parameters. The substitutions are made into established general formulas. The continuous-time semi-Markov process can then be analyzed with flow graph and Laplace transform techniques.

The design parameters included in the model are freeway vehicle headway distributions, lane volumes, lane running speeds, and a gap acceptance function that describes a driver's willingness to accept a given headway in an adjacent lane.

DESCRIPTION OF CONTINUOUS-TIME SEMI-MARKOV PROCESS

A continuous-time semi-Markov process is similar to the discrete-parameter Markov process, which has been used by others to model some freeway operations, but the continuous-time semi-Markov process requires two matrices to describe transitions:

1. A transition matrix, similar to that of the discrete Markov process, that defines the probability, p_{ij}, of making a transition from state i to another state j and
2. A holding time matrix that embodies the continuous probability density function, $f_{ij}(x)$, of the time or distance associated with making a transition from state i to state j.

The continuous-time semi-Markov process is used to model a multilane, unidirectional section of highway. In the simplest formulation, the states of the semi-Markov process represent lanes of a freeway. Transitions must always occur from one lane to an adjacent lane. The probabilities associated with a freeway driver making a transition from a given lane or state to any other lane or state are obtained by calculating the probability that the gap that is adjacent to the driver in question is equal to or greater than the driver's acceptance criterion. If the driver accepts this gap, the distance associated with making the transition, or changing lanes, is a random variable and is described by the holding time matrix. If the driver rejects the first gap, he or she must drive at a different speed from that of the vehicles in the adjacent lane and wait for another opportunity to change lanes. The next opportunity occurs when he or she is adjacent to to the next gap in the traffic stream. The waiting time from one merging opportunity to the next can be described in terms of the volume of traffic in the lane being merged into, which a merging vehicle must pass, or by the volume of traffic that must pass the merging vehicle.

An example of the merging process is shown in Figure 1. The figure shows a two-lane, one-way pavement with a vehicle in lane one attempting to merge into lane two. It is assumed in this case that the vehicles in lane two are traveling at a higher rate of speed than those in lane one. If the driver in lane one is constrained by a vehicle in front of him or her, as is normally the case in moderately heavy traffic, he or she must attempt to merge by moving into gap one. If the merging driver rejects gap one, a spacing G_1 separates him or her from gap two. The distance G_1 can be expressed as a function of the traffic that must pass the merging vehicle. It will be referred to as the differential distance and is measured in meters. The differential distance associated with rejecting a gap or making a transition into the same lane is a random variable with a distribution equal to the distribution of rejected gaps. The actual distance on the freeway that the vehicle in lane one must travel while waiting for the next opportunity to merge, a differential distance G_1 away, is dependent on the difference between the mean speeds of the vehicles in lanes one and two. If there is a speed differential between lanes, the differential distance G_1 between gaps can be converted to the downstream travel distance for vehicle one by applying the speed of vehicle one. If, for example, the lane one vehicle in Figure 1 rejects gap one, the time required

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to move into position to select gap two can be found by using the differential speed between lanes. This time multiplied by the speed, S_1, of the lane one vehicle gives the required downstream travel distance.

A flow graph of the process is shown in Figure 2. This flow graph can be reduced by using Mason's rule to yield the Laplace transform of the probability density function of the first passage time from lane one to lane two in terms of the lane differential distance. The Laplace transform of the differential distance probability density function, f(s), is

$$F(s) = [p_{12}f_{12}(s)]/[1 - p_{11}f_{11}(s)] \quad (1)$$

A variety of forms of gap density functions and gap acceptance functions could be substituted into equation 1 to find the probability density function of the differential distance.

Several methods can be used to represent a driver's willingness to accept a gap between vehicles during a merge maneuver including distributions of critical gaps and gap acceptance functions. The most convenient method of representing a driver's gap acceptance criterion for the continuous-time semi-Markov highway model is a gap acceptance function. The gap acceptance function is assumed to be stationary with respect to the distance from the lane termination point but may vary according to highway conditions and geographical location.

Weiss and Maradudin (1) showed that the probability of accepting a gap can be found by integrating the product of the gap density function and the gap acceptance function over the range of gaps. To obtain meaningful results from traffic delay calculations requires the assumption that vehicles occupy no space. The term gap is in such a case synonymous with spacing, and the former term is used throughout this paper. The probability, p, that any available gap will be accepted is therefore

$$p = \int_0^\infty p(x)f(x)dx \quad (2)$$

The probability of staying in lane one is 1 - p.

The distribution of rejected gaps is needed for the continuous-time semi-Markov model. The density function of rejected gaps is a conditional density function that gives the probability that any gap is less than or equal to a certain value, given that the gap is rejected. The probability that the length of a rejected gap is between x and x + Δx is therefore

$$p = [p(\text{gap between } x \text{ and } x + \Delta x) - (\text{gap between } x \text{ and } x + \Delta x \text{ accepted})]/p(\text{gap rejected}) \quad (3)$$

In terms of the gap acceptance function p(x), the gap probability density function f(x), and the probability of accepting a gap, the density function of rejected gaps, g(x), is

$$g(x) = [f(x) - f(x)p(x)]/(1 - p) \quad (4)$$

Equation 4 can be substituted for $f_{11}(x)$ in the semi-Markov process. Several sample problems are discussed elsewhere (2).

COMPARISON WITH ACTUAL FREEWAY CONDITIONS

Data were collected on the 404-m (1325-ft) auxiliary lane of I-475 in Toledo. This auxiliary lane was modeled by

Figure 1. Merging on a two-lane, unidirectional pavement.

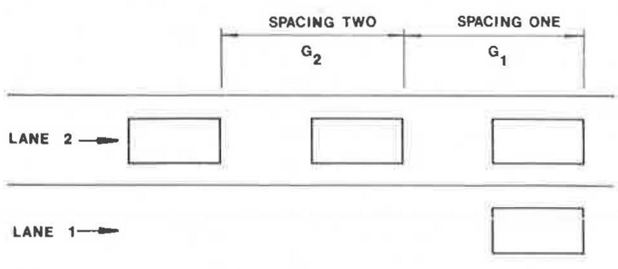


Figure 2. Entrance ramp situation.

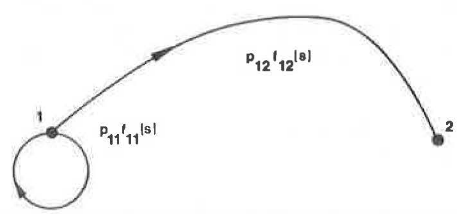


Figure 3. Comparison of observed gap acceptance characteristics and an assumed gap acceptance function.

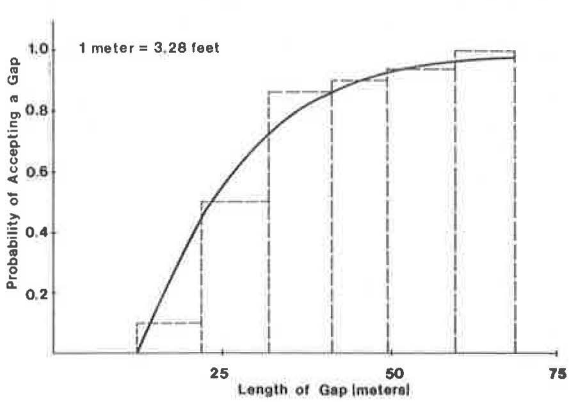
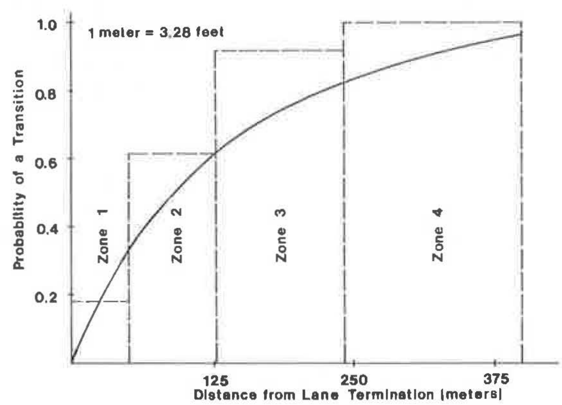


Figure 4. Comparison of results of continuous-time semi-Markov model and data collected at auxiliary lane site.



a continuous-time semi-Markov process.

The distribution of gaps during the study period was found to fit a negative exponential distribution. A mean volume of 1500 vehicles per hour and a minimum gap of 12.2 m (40 ft) were observed. A time mean speed of 82 km/h (51 mph) was measured in lane two, and the entrance ramp design was such that most entering vehicles attained a speed of approximately 74 km/h (46 mph) by the time they reached the auxiliary lane. The volume of entering and exiting traffic was low compared to the lane one volume.

A gap acceptance function was developed from motion picture data collected at the auxiliary lane site. During the study period, 123 accepted gaps and 62 rejected gaps were recorded. The gaps ranged from 12.2 to 64 m (40 to 210 ft). They were grouped in intervals of 9 m (30 ft) so that a gap acceptance function could be established. The probability that a gap in a given interval was accepted was calculated by dividing the total number of gaps in a particular time interval into the number of accepted gaps in that interval. The results are shown in Figure 3.

A gap acceptance function in the form

$$p(x) = 1 - \exp[-\lambda(x - T)] \quad x \geq T$$
$$= 0 \quad x < T \tag{5}$$

is suggested from the figure. A minimum gap, T, of approximately 12.2 m (40 ft) is suggested, with the parameter, λ, equal to 0.025. A gap acceptance function in this form and with the indicated parameters is compared in Figure 3 to the gap acceptance data collected at the site.

The auxiliary lane was modeled by using equation 5 and the negative exponential gap density function in the form

$$f(x) = a \exp[-a(x - N)] \quad N \leq x < \infty$$
$$= 0 \quad x < N \tag{6}$$

The constant 1/a is the mean gap and N is the minimum gap.

The distribution of rejected gaps was found by substituting into equation 4. Because the minimum accepted gap and the minimum observed gap between vehicles are taken to be equal, the result of this substitution is

$$g(x) = (a + \lambda)\exp[-(a + \lambda)(x - N)] \quad N \leq x < \infty$$
$$= 0 \quad x < N \tag{7}$$

The Laplace transform of g(x) is

$$L[g(x)] = (a + \lambda)/(s + a + \lambda)\exp(-Ns) \tag{8}$$

The distance required to merge into a gap was represented by the distribution of rejected gaps. Substitution of the known conditions gives the following result if q is substituted for 1 - p and c for a + λ and F(s) is the Laplace transform of the differential distance probability density function.

$$F(s) = pc/[s + c = cq \exp(-Ns)] \tag{9}$$

The inverse transform of equation 9 can be obtained by partial fraction expansion. It is an infinite series of exponential functions.

$$f(x) = pc \exp(-cx)u(x) + pcqc[(x - N)/1!] \exp[-c(x - N] u(x - N)$$
$$+ pc(qc)^2 [(x - 2N)^2/2!] \exp[-c(x - 2N)] u(x - 2N)$$
$$+ \ldots \quad x \geq 0$$
$$= 0 \quad x < 0 \tag{10}$$

where

$c = a + \lambda$,
p = probability of accepting a gap, and
N = size of the minimum gap.

The probability of making a transition for a number of differential distances was calculated by using equation 10 and the measured values of the required parameter. The differential distance was converted to the corresponding freeway distance by using an 8-km/h (5-mph) differential speed. The predicted cumulative transition probabilities were compared to corresponding probabilities calculated from observations of vehicle transitions at the auxiliary lane site. The observed probabilities were found by dividing the auxiliary lane into four zones of from 45.7 to 176.6 m (150 to 550 ft) long and calculating the probability for a vehicle merge into lane one within each zone. The cumulative observed probability of making a transition within a zone and the predicted probability of making transition are shown in Figure 4. Data given in Figure 4 show that the transition probabilities predicted by the semi-Markov model are similar to the observed data.

SUMMARY AND CONCLUSION

The continuous-time semi-Markov model can be used to find the distribution of the time spent by a driver waiting to emerge from an entrance ramp. This distribution can be used to evaluate the freeway entrance ramp designs. The effect of improved visibility on waiting time, for example, could be studied by using the waiting time distribution. Different gap acceptance functions can be used to reflect the effect of improved visibility on a typical driver's merging behavior.

The location of a warning that a freeway lane drop is imminent can also be studied by means of a continuous-time semi-Markov model. Design conditions can then be calculated. This proportion can be related to the level-of-service concept. Providing a warning at a point calculated to allow 95 percent of the vehicles to merge from the lane being terminated may, for example, be associated with level of service A. Further work is required before a definitive relation can be established between the output of a semi-Markov model and levels of service as those levels are currently defined.

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