Lateral Buckling of Pony Truss Bridges

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At one time, low or pony truss bridges were popular for their economy and ease of construction. With the tremendous increase in commercial vehicle weights, especially after World War II, the load-carrying capacity of these bridges became suspect, and many were taken out of service or replaced by modern structures. An ultimate load test carried out by the Structural Research Section of the Ontario Ministry of Transportation and Communications on a pony truss bridge in 1969 indicated that these bridges possess an inherent strength that far exceeds the value predicted by elementary structural theories. The complex problem of lateral buckling of truss compression chords, which in the past has led to oversimplifying assumptions resulting in underestimation of the bridge strength, has been solved by a computer program based on a modified version of Bleich's method. The program, whose validity has been checked against experimental results, will provide bridge engineers with a better assessment of the load-carrying capacity of pony truss bridges than has been possible in the past. Since there are several hundred pony truss bridges in Ontario alone, it is economically important to determine the extent to which these bridges can usefully serve their purpose.

Low truss bridges that have horizontal wind bracings at the deck level only are usually referred to as pony truss bridges (Figure 1). Now that more efficient modes of construction are available and given the emphasis on aesthetics, steel pony truss bridges might not be constructed in the future. However, there are several hundred pony truss bridges in Ontario and possibly many thousands throughout North America. Most of these bridges are still serving as important traffic carriers on secondary highways and county roads. The increase in the volume of traffic and the weight of commercial vehicles makes it necessary to review the serviceability of the existing pony truss bridges by evaluating their load-carrying capacity. In the present economic environment, the structural strength of a bridge must be fully investigated before a decision is made on whether it should be replaced.

The weakest component of the pony truss bridge is usually the top chord of one of its trusses; for lateral stability these compression members depend on their own stiffnesses and the elastic restraints offered by the web members of the trusses.

This paper presents a method for investigating the lateral stability of pony truss bridges on a more rational basis than has been attempted in the past. The method is an iterative one that takes into account the secondary stresses caused by change in the geometry of the structure due to loading.

METHOD OF ANALYSIS

The problem of lateral instability of the compression chord of a pony truss is essentially that of a beam-column elastically restrained at discrete points. This problem has caught the attention of engineers since the turn of the last century (1, 2, 3, 4). It was Bleich (5, 7) who first solved the problem of stability of a beam-column that has varying sectional properties, is subjected to varying axial loads, and has discrete elastic supports with random spacings. The method is briefly outlined below.

For a pony truss (Figure 2a) with n number of panels, (n - 1) compatibility equations relating moments and beam stiffnesses and (n - 1) equilibrium equations relating moments, deflections, and elastic restraints offered by the transverse portals can be written for (n - 1) intermediate panel points. The very ends of the chord are assumed to be hinged. Thus, together with the assumed end conditions of the end moments being equal to zero, a total of 2n equations can be written from which the (n - 1) unknown moments and (n + 1) unknown deflections can be found for any stable condition of the top chord. For the top to be in an unstable condition, one of the unknown moments and deflections should have an infinite value. Such a condition would result in (n - 1) solutions of which the solution associated with the minimum load would give the critical buckling load for the truss.

Bleich's method ignores the fixity of the ends of the compression chord against rotation and does not account for the discontinuity in the direction of the compression chord, which is a common feature in North American pony truss bridges; the theory had to be modified to con-

Publication of this paper sponsored by Committee on Steel Bridges.

Figure 1. A typical pony truss bridge.

Figure 2. (a) European pony truss used in Bleich's thesis and (b) American pony truss with the numbering system used.

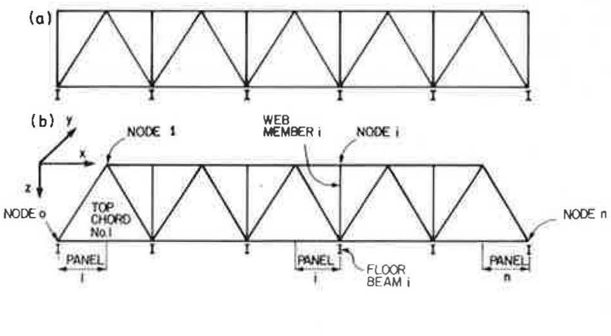


Figure 3. Deflected shape of transverse portal frame.

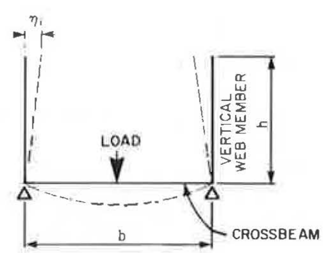


Figure 4. Deformation of a panel length of compression chord on the horizontal plane.

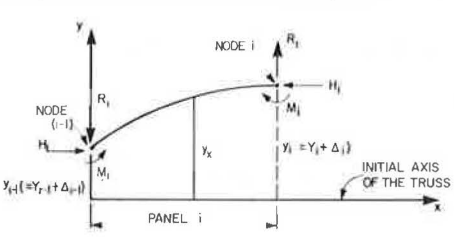
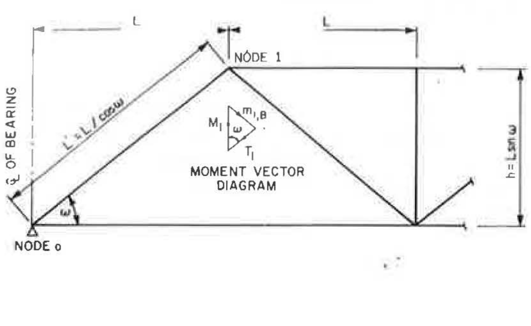


Figure 5. Moments at left shoulder node.



sider these aspects. Bleich's theory was also modified to include live load deflections of the panel points at the interface of the compression chord and the vertical members of the transverse portals. This modification enabled the solution to be obtained for any loading condition, and reduced the solution from that of stability to theory of second-order stresses. Smallest value of the applied load that caused the stress, anywhere along the chord, to exceed the yield stress of the material was regarded as the critical load for the chord.

Derivation of the modified set of equations, which were to be solved by using a computer, was based on the following assumptions.

1. The two trusses are of constant height, and their distance apart is also constant;
2. The lateral restraints offered by the diagonal web members are negligible;
3. The torsional rigidities of the compression chords, which are formed from open sections, are small and can be ignored;
4. The modulus of elasticity and yield stress of steel are constant;
5. Panel lengths are identical; and
6. Vertical members of the transverse portals have uniform flexural stiffness.

Although a solution based on the modified method of analysis can easily be formulated without any of those restrictions, the restrictions were included to avoid unnecessary generality. For brevity, many steps of derivation leading to the resulting equations are omitted. Detailed formulations can be found elsewhere (6).

Notation

The notation used in the equations is defined below.

A_i = function relating to panel i, as defined by equation 7;
a_i = cross-sectional area of the top chord of panel i;
B_i = function relating to panel i, as defined by equation 7;
2b = distance between the centerlines of the two trusses of the bridge;
D_i = lateral rigidity of the transverse portal frame at node i, defined as the force to cause a unit displacement of the top chord node;
$d_{i,m}$ = distance between nodes i and m, measured along x;
E = modulus of elasticity;
$F_{i,1}$, $F_{i,2}$, $F_{i,3}$ = transcendental functions defined by equation 9;
G = shear modulus;
H_i = axial force in chord member of panel i, in the direction of longitudinal axis of the truss;
h = height of the truss;
I_{bi} = moment of inertia of the floor beam at node i;
I_{hi} = moment of inertia of the portal column at node i (or equivalent inertia if the column has nonuniform moment of inertia);
I_i = moment of inertia of the compression chord of panel i, about Y-axis;
J_i = torsional inertia of the compression chord of panel i
K_i = torsional rigidity of the compression chord of panel i $= GJ_i$
L = length of truss panel;
L′ = length of the inclined end member;

M_i = lateral moment acting at node i on the compression chord;
M_x = lateral moment acting on the compression chord at a distance X from the left node of the panel under consideration;
$M_{i,A}$ = lateral moment acting at the left end of the compression chord of panel i;
$M_{i,B}$ = lateral moment acting at the right end of the compression chord of panel i;
n = number of panels in truss;
P = concentrated load acting on cross beam;
R_i = lateral force in top chord member at node i;
S_i = section modulus of the compression chord of node i;
T_1 = torsion in left inclined compression chord;
V_L = left-hand reaction of the truss in lateral direction;
V_R = right-hand reaction of the truss in lateral direction;
V_X = shear force, in Y-direction, at a distance X from the left node of the compression chord of panel i
X = distance of a point from the left node of the panel under consideration;
Y_i = lateral deflection of node i with respect to the specified initial position of the node;
y_i = total lateral deflection of node i with respect to the original axis of the truss;
$\left.\begin{matrix}\alpha_i\\ \beta_i\\ \gamma_i\end{matrix}\right\}$ = functions relating to panel i and defined by equation 5; and
Δ_i = initial lateral deflection of node i (that prior to the application of load), with respect to the original axis of the truss.

Lateral moment is the moment associated with displacements in the Y-direction, where two horizontal members meet at node i.

Characteristics of Portal Frames

The transverse portal frame, which provides the elastic restraint to the compression chord, is formed from two outstanding vertical members rigidly connected to floor beams, as shown in Figure 3. The rigidity of the portal frame is calculated on the assumption that the ends of the beam are restrained against relative vertical movement. Under loading, the top ends of the vertical, which restrain the movement of the compression chord, deflect inward. If at node i, Y_i is the deflection of the compression chord and η_i is the displacement of the node due to flexure of the floor beam, the resulting lateral force R_i is given by

$$R_i = D_i(Y_i - \eta_i) \tag{1}$$

where D_i is the rigidity of the idealized portal frame shown in Figure 3. It is given by the following relationship:

$$D_i = EI_{bi}/h^2\,[b + (hI_{bi}/3I_{hi})] \tag{2}$$

The value of the inward movement of the top of the portals can be obtained from elementary statics. For example, the value of the inward movement η_i for a portal loaded with a simple concentrated load P, as shown in Figure 3, is given by

$$\eta_i = (Phb^2/4EI_{bi}) \tag{3}$$

Compatibility Equation

A total of (n - 3) compatibility equations are obtained for nodes 2 through (n - 1) as follows:

The flexural behavior of the chord in horizontal plane, as shown in Figure 4, is given by

$$H_iY_x + M_{i-1} + M_i - H_iY_{i-1} + R_iX = EI''_{yx} \tag{4}$$

By introducing

$$\alpha_i^2 = (H_i/EI_i) \qquad \gamma_i^2 = (M_{i-1}/EI_i) \qquad \beta_i = (R_i/EI_i) \tag{5}$$

the solution of equation 4 is given by

$$Y_x = A_i \sin\alpha_i X + B_i \cos\alpha_i \times X + Y_{i-1} - (\gamma_i^2/\alpha_i^2) - (\beta_i X/\alpha_i^2) \tag{6}$$

where the expressions for A_i and B_i are found from the boundary conditions:

$$A_i = (M_i - M_{i-1} \times \cos\alpha_i L)/(H_i \times \sin\alpha_i L) \qquad B_i = (M_{i-1}/H_i) \tag{7}$$

The slope at a point is given by the first differential of the right side of equation 6 with respect to x.

By equating the slopes at the common node of two adjacent panels, the following typical compatibility equation is obtained.

$$\begin{aligned} &Y_{i-1} - 2Y_i + Y_{i+1} - (F_{i,2}/H_iF_{i,3})\,M_{i-1} - (F_{i-1}/H_iF_{i,3}) \\ &\quad + \{[G_{(i+1),1}/H_{i+1}F_{(i+1),3}]\}\,M_i - [G_{(i+1),2}/H_{i+1}G_{(i+1),3}]^{M_{i+1}} \\ &\quad = -\Delta_{i-1} + 2\Delta_i - \Delta_j + 1 \end{aligned} \tag{8}$$

where

$$\begin{aligned} F_{i,1} &= G_{i,1} = \sin\alpha_i L - \alpha_i L \cos\alpha_i L \\ F_{i,2} &= G_{i,2} = \alpha_i L - \sin\alpha_i L \\ F_{i,3} &= G_{i,3} = \sin\alpha_i L \end{aligned} \tag{9}$$

Equation 8 is not valid for nodes 1 and (n - 1), the shoulder nodes. On the assumption that both the torsion and moment act on the inclined member but that there is no resultant torsion in the horizontal members (Figure 5), the following compatibility equation is obtained for node 1, the left shoulder node.

$$\begin{aligned} &Y_1\,(1 + \cos^2\omega) - Y_2 - [(\cos^2\omega F_{1,2}/H_1F_{1,3})\,m_{1,A}] \\ &\quad + [(h^2\cos\omega/GJ) + (F_{1,1}\cos\omega/H_1F_{1,3}) - (G_{2,1}/H_2G_{2,3})]\,M_1 \\ &\quad - (G_{2,2}/H_2G_{2,3})\,M_2 = 0 \end{aligned} \tag{10}$$

Equation 10 can be easily adapted for node (n - 1).

Boundary Conditions

By equating the expression for slope at the left end of chord 1, obtained by differentiation of the right side of equation 6, to zero, the equation for the boundary condition is

$$Y_1 + \Delta_1 + (F_{1,1}/H_1F_{1,3})\,m_{1,A} + (F_{1,2}\cos\omega/H_1F_{1,3})\,M_1 = 0 \tag{11}$$

Similarly, an equation for the boundary condition at node n can be obtained.

Equilibrium of Lateral Forces

If we assume that there are no external forces acting on the compression chord, the only lateral forces are those resulting from the interaction of the compression chord and the vertical members of the portal.

Shear force is given by

$$V_x = -EI'''_Y \tag{12}$$

Differentiating equation 4 three times with respect to x and substituting it in equation 12 yield

$$V_L = [\alpha_1 L/\sin(\alpha_1 L)]\ [M_1 \cos\omega - m_{1,A}\cos(\alpha_1 L)/L] \qquad (13)$$

The equation for V_R can be similarly formed.

The reaction at node i, due to the interaction of the portal and the compression chord, is given by equation 1. From equations 1 and 3, the following equation for the equilibrium of forces can be obtained.

$$-V_L + \sum_{i=1}^{n} D_i (Y_i - \eta_i) + V_R = 0 \qquad (14)$$

Nonlinear Equilibrium Equations

Lateral forces at the interface of the compression chord and the portals change with the lateral deflection of the chord; so do the lateral moments caused by the axial forces. The equilibrium equations, which are necessarily nonlinear in nature, can be formed by equating the moments due to forces on one side of a node to the nodal moment. M_0 in terms of $M_{1,A}$ and $m_{1,A}$ is given by

$$M_0 = m_{1,A} \cos\omega + M_1 \sin^2\omega \qquad (15)$$

Taking the moment of forces to the left of node i and equating it to M_i yield

$$-M_i - M_0 - V_0 d_{i,1} + \sum_{n=1}^{i-1} (H_{m+1} - H_m)(Y_i - Y_m) + H_1 Y_i$$

$$+ \sum_{m=1}^{i-1} D_m (Y_m - \eta_m) d_{i,m} = 0 \qquad (16)$$

Substituting the expression for V_L and M_0 from equations 13 and 15 respectively and replacing y_i by $(Y_i + \Delta_i)$ from (n - 2) equilibrium equations, in terms of the unknown nodal displacements and moments, for nodes 2 through (n - 1).

SOLUTION

The equations thus formed are solved for the unknown displacements and moments at a given load level. From the nodal moments, the maximum moments and stresses between the nodes are calculated. If none of the members is stressed to the yield limit, the load is increased in steps, and the whole process of forming and solving the equations and calculating the maximum stress within the compression chord is repeated until the stress somewhere in the chord reaches the yield limit. This load is regarded as the maximum load that the truss can carry.

Description of Computer Program

Implementation of the method of analysis was only possible by using a computer. A computer program was developed for this purpose in standard ANSI FORTRAN (and partly in Assembly for the IBM 360 version); it is named LATBUK. The program is available on request from the Engineering Research and Development Branch of the Ontario Ministry of Transportation and Communications.

A step-by-step increment of the specified initial load to reach the lateral buckling load would have consumed a lot of computer time in most of the cases, especially when the initial load was a low guess. A search technique in which the solutions converge within 12 iterations almost irrespective of the initial load was used in the program. The technique consists basically of giving a large increment to the specified initial load and then iterating by either decreasing or increasing the load by an amount equal to half the previous step. The iterations are continued until the critical load within the accuracy of specified load increment is reached. The process is shown in Figure 6.

Validity of the Method of Analysis

For checking the validity of the program and the method of analysis, a Perspex model of a pony truss bridge with six panels and two transverse portals was constructed and tested for various loading conditions. The model was constructed from a 10-mm-thick (0.4-in) Perspex sheet. The dimensions and details of the model are shown in Figure 7. The model was used only to validate the prediction of the lateral deflections by the program.

The program assumes that the top chord is laterally supported only at the interface of the portal column and the top chord. This assumption, although valid in actual bridges, does not hold in the model, especially at the middle node where the diagonals join the chord. The truss was carved from a single sheet of Perspex. Because of the continuity of the members, the middle node was offered some lateral restraint by the inclined members, which at their lower ends were partly restrained against rotation through the portal and truss connection.

To account for the restraint at the middle node, a fictitious portal was placed at the center of the truss in the analysis by the program. The columns of the portal were given the stiffness offered by the two inclined members. One-quarter of the stiffness of the actual transverse beams was arbitrarily apportioned to the beam of the fictitious portal.

Some of the comparisons of the lateral deflections of the top chord as given by the model test and the program are shown in Figure 8. Given the fact that the model was not an exact idealization of a typical pony truss bridge for which the program is written, the program results compare well with experimental results.

The model test ensured that the program can correctly calculate the lateral deflections of the top chord for various loadings. The acid tests for the validity of the program were, however, provided by the test on an existing pony truss bridge and an accidental failure of another. The load test was carried out in 1969 on a bridge close to Exeter, located on the boundary of Perth and Middlesex in Ontario.

The Exeter Bridge consisted of two pony trusses spanning 15.2 m (50 ft). The trusses had eight panels each and were 4.6 m (15 ft) apart. A view of the bridge with the test loads is shown in Figure 9.

The program predicted a failure load of 721 kN (162 000 lbf) for the bridge. In the test the bridge failed through the buckling of one of the top chords under a load of 623 kN (140 000 lbf). The concrete blocks (44.5 kN or 10 000 lbf each) were placed off-center 152 mm (6 in) toward the instrumented truss. The equivalent central load is on the order of 667 kN (150 000 lbf). Furthermore, to ensure the safety of testing personnel, the blocks were always dropped from a height of 25 mm (1 in) to create an impact that would trigger the failure at the critical load. The actual capacity of the bridge was slightly more than 667 kN (150 000 lbf) for a central load position. Inasmuch as the calculated properties of the bridge could not have been exact, the correlation between the predicted failure load and the actual failure load seemed almost fortuitous. Later, anaother opportunity to test the validity of the program was provided by an accidental failure of the Holland Road Bridge at Thorold. This pony truss bridge failed on October 26, 1972, when a three-

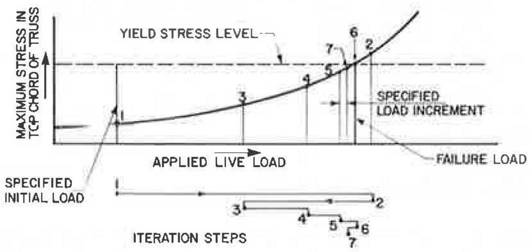


Figure 6. Search technique for fast convergence.

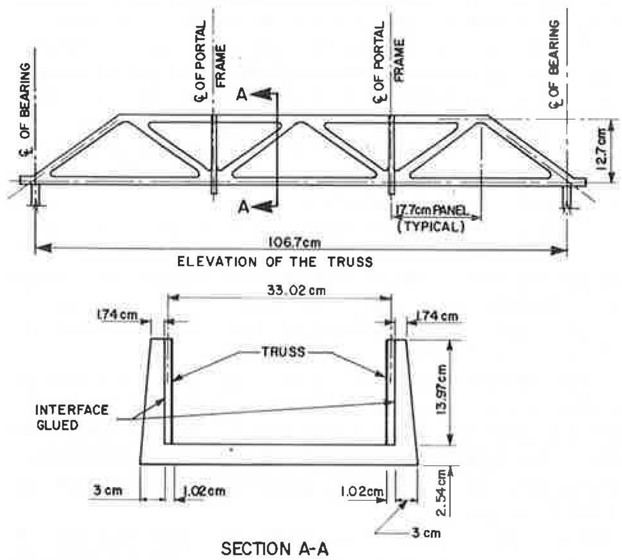


Figure 7. Details of the pony truss bridge model.

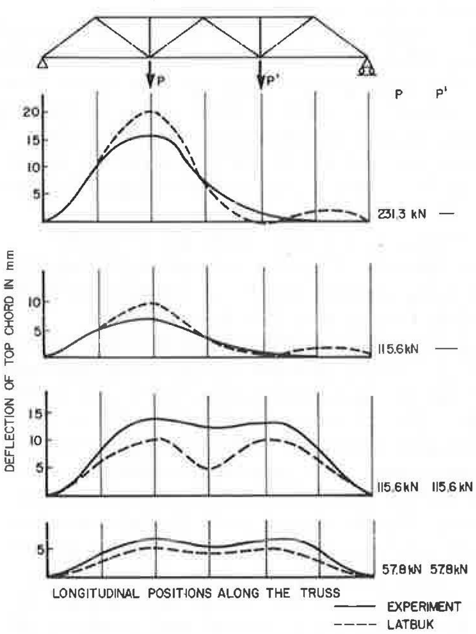


Figure 8. Lateral deflections of the top chord for truss shown in Figure 7.

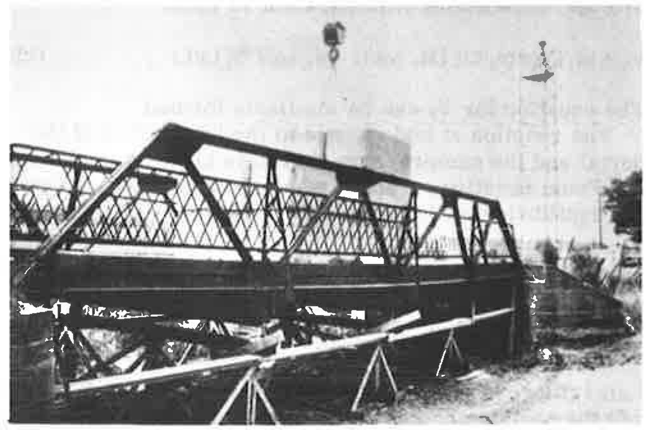


Figure 9. A pony truss bridge with test loads.

axle tandem truck passed over it. The truck was reported to weigh 20.7 Mg (457 000 lb). The bridge was analyzed by the program for various longitudinal positions of the vehicle. The smallest failure load of the vehicle given by the program was 200 kN (45 000 lbf). There seemed little doubt that the program can predict realistic values of the failure loads for the top chords of pony trusses.

CONCLUSIONS

The behavior of the compression chord of a pony truss cannot be predicted intuitively or by simple hand calculation. Based on the proposed method of analysis, which accounts for the interaction of the compression chord and the transverse portals and for the change of direction of the chord at the shoulder points, a computer program has been written. Comparison of the program results to those of model and full-scale tests has proved the validity of the method. The program can now be used to check the load-carrying capacity of the many existing pony truss bridges before their replacement is contemplated.

REFERENCES

1. E. Engesser. Die Sicherung offener Brucken gegen Ausknichen. Zentralblatt der Bauverwaltung, 1884, p. 415.
2. F. S. Jainski. La flexion des pieces comprimees. Annales des Ponts et Chausees, 2nd Part, 1894, p. 233.
3. H. Muller-Breslau. Die graphische Statik der Baukonstruktionen. Vol. II-2, A. Kroner, Leipzig, 1809.
4. H. Zimmermann. Die Knickfestigkeit der Druck gurte offener Brucken. W. Ernst und Sohn, Berlin, 1910.
5. F. Bleich. Die Knickfestigkeit elasticher stabverbindungen. Der Eisenbau, Vol. 10, 1919.
6. P. F. Csagoly, B. Bakht, and A. Ma. Lateral Buckling of Pony Truss Bridges. Ontario Ministry of Transportation and Communications, Research Rept. 199, 1975.
7. F. Bleich. Buckling Strength of Metal Structures. McGraw-Hill, New York, 1952.