Design Traffic Loads on the Lions' Gate Bridge

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During the first major renovation of the Lions' Gate Bridge joining Vancouver, British Columbia, to its northern suburbs, there was evidence of deterioration due to the corrosive sea atmosphere. Because traffic loads have increased, both in volume and in mass, since the bridge was built, a set of analytical equations and a computer simulation were developed to estimate the vehicle traffic load on the bridge. The analytical formulation handles the critical vehicle load as a function of the bridge and approach lengths, the number of lanes, the percentage of cars, buses, and trucks in each lane, the number and severity of stoppages, the weight distribution of trucks, and the return period for the critical load. The simulation includes additional factors such as the trickling of vehicles past a stoppage, the time of day and day of the week of the stoppage, the location on the bridge or approach of the stoppage, the stopped lane or lanes, and the duration of the stoppage. The application of these two approaches using traffic data observed on the Lions' Gate Bridge gave practically the same vehicle load per unit length. The resulting design loads were considerably less than those suggested by Ivy and coworkers or Asplund and guite similar to those used in the original analysis.

The Lions' Gate Bridge is an 830-m, three-lane suspension bridge connecting Vancouver, British Columbia, to its northern residential and commercial suburbs and carries approximately 60 000 vehicles per day. The bridge spans a navigational channel 365 m wide and is 1524 m long. The center span is 472.4 m and the two side spans are each 187 m long. A 915-m concrete roadway having three 2.9-m-wide traffic lanes forms the causeway from the Vancouver city center to the bridge. The northern ramp of the suspension bridge is a series of standard truss sections supported on steel columns. Built in 1937-38 by a private land developer, the bridge has been subjected to a corrosive saltwater atmosphere for 37 years and increasingly heavier vehicle loads, particularly of buses and trucks. This study was conducted to confirm the validity of the original traffic loading and to set the legal load limit for heavy vehicles using the crossing.

The validity of the original traffic loading was rather important because the loads were considerably less than

any suggested in the available literature, such as the American Association of State Highway Officials (1) or the Canadian Standards Association (2) loads. Further review (3, 4) indicated that no general formula or analytical procedures existed to help designers set the traffic loads on long span bridges. The commercial interests using the bridge for trucking were particularly concerned about the legal load limit because the only other bridge crossing is approximately 8 km to the east.

Vehicle traffic was simulated in the study for two reasons:

- 1. Early attempts at an analytical solution were not encouraging, and it was fairly certain that an acceptable answer could be obtained by simulation; and
- 2. The results of the simulation could be used to check an analytical solution if one could be found.

The development of the computer simulation, the simulator, the analytical solution, and a comparison of the results of the two methods are described below.

TRAFFIC LOADING PARAMETERS

The aim of the study was to estimate the maximum traffic load on any general loaded length of bridge with a given return period. The selection of simulation variables was based on the criteria of importance and availability of data. For the simulation, information was needed on

- 1. The vehicle-weight and length;
- 2. The bridge—location of stoppage, number of lanes blocked, and direction of center lane;
- 3. Stoppage—type of event, time to clear event, and time of stoppage by hour and day; and
- 4. Traffic flow—vehicle mix, maximum vehicle flow by hour and day, spacing of vehicles when stopped and when moving, and speed of moving lanes.

Some of this information was collected by the British Columbia highway department and some by the bridge patrol. Some was supplied through traffic counts (6), and some was estimated.

The distribution of vehicle weights on the bridge was

a critical component in the study. The weight of cars and buses was set at 1360 and 13 600 kg respectively. Truck weights were found to follow a gamma distribution having a mean of 0.7 registered gross vehicle weight (GVW). The traffic weight was a function of the length of vehicles and their spacing. The length of buses and trucks with a GVW of more than 5400 kg was set at 12 m (5), and all others were set at 5 m. The speed-spacing relationship was determined by experiments on the bridge. Control vehicles were driven at constant predetermined speeds across the bridge, and from time-lapse photographs the following empirical relationship between vehicle density and speed was found:

$$D = 1.6/[\bar{\ell} + 1.5 + \bar{\ell} (V/16.1)]$$
 (1)

where

D = density in vehicles per kilometer,

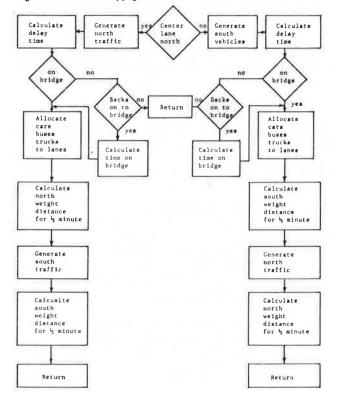
 \bar{l} = average vehicle length in meters, and

V = vehicle stream speed in kilometers per hour.

This relationship conforms reasonably well to envelope curves of the same variables presented by Wheeler and Troy (8) and others (9). The only unknown in the relationship is the vehicle speed, which is assumed here to be constant over various sections of the bridge and causeway. The speed of vehicles moving in the same direction as the stopped vehicle was reduced to 70 percent of the observed speed, and the speed of vehicles in opposing lanes was reduced by 20 percent. Once the blocked lane was cleared stopped vehicles flowed at 1500 vehicles/h (10).

Stoppages were classified into event categories such as single-lane stoppages (including flat tires and running out of fuel) and more serious events including head-on, sideswipe, and rear-end collisions. The type of event was assumed to determine both the number of lanes

Figure 1. Two-lane stoppage.



blocked and the time to clear the stoppage. The majority of events were assumed to be single lane; only head-on and sideswipe collisions blocked two lanes simultaneously.

The direction (and lane, if applicable) stopped was randomly generated from a cumulative probability distribution describing the amount of vehicle flow in each lane. The time to clear the traffic backup at a stoppage was expressed as

$$t = t_1 + t_2 = t_1(x/x - y)$$
 (2)

where

- t = time to clear all backed-up traffic in minutes,
- t₁ = time needed to remove the stopped vehicle in minutes.
- t₂ = additional time needed to remove vehicles added to queue after stopped vehicle is removed, in minutes.
- x = vehicle flow from a stopped situation in vehicles per minute, and
- y = vehicle flow at the time of stoppage in vehicles per minute.

The value of y may be reduced to reflect the traffic that trickles past a single-lane blockage for the two lanes flowing in the same direction. The trickle was estimated at 1200 vehicles/h (11) and the flow after a stop at 1500 vehicles/h. Equation 2 then simplifies to

$$t = t_1 (50/y - 20) \tag{3}$$

 $t_{\rm l}$ was obtained by using a random number generator, with separate upper and lower bounds for mechanical failures and accidents.

Both the time of the day and day of the week of the stoppage influenced the number of heavy vehicles that would be on the bridge. Visual examination of the data films and manual traffic counts indicated that the greatest volume of truck and bus traffic on the bridge was from 7 a.m. to 7 p.m. The data were summarized by hours to give the average upper bound of the vehicle mix by lane. The allocation (percent) of vehicle types in each lane was as follows:

Curb Lane	Center Lane		
50	50		
80	20		
60	40		
	50 80		

The time of day also determined the direction of flow in the center lane. Finally the location of the stoppage indicated to what extent the bridge would be loaded and, combined with the length of time the stoppage was in effect and vehicle flows, whether the vehicles actually backed up on the bridge.

THE SIMULATION

The simulator operated in three distinct steps.

- 1. All significant events within an analysis period (events that created a backup of traffic on the bridge) were stored.
- 2. All such events within the analysis period (90 days) were analyzed to determine the maximum loads for several lengths of bridge and all possible combinations of the three lanes.
- 3. A search produced the maximum of these period maxima, and statistics describing the distribution of the maxima were obtained.

A more detailed accounting of the simulator follows.

There are 12 possible combinations of stopped traffic that may cause a backup on the bridge. The event could be due to mechanical failure, a one-lane accident, a twolane accident, or a head-on collision, each of which occurs with a known frequency.

Two-lane accidents are simple (Figure 1). The direction of traffic flow in the center lane is determined first. For example, in the northbound center lane, northbound traffic is generated in half-minute intervals. The delay time due to the stoppage is calculated, and the location of the blockage is checked. If the stoppage occurs off the bridge, then a further check is made to determine whether the traffic backs up onto the bridge. If not, the event is rejected, and the simulation returns to the main program. If an accident occurs on the bridge or creates a serious backup of traffic on the bridge, the vehicles going north are distributed among the two northbound lanes and the resulting weight distributions are obtained. Southbound traffic is generated in a similar manner, and the weight distribution is obtained. A return to the main program is then effected.

After the analysis period has been covered by a sufficient number of events, the maximum loading for that period is determined. This is obtained by simply aligning the maximum weight in a 45-m moving section with that of one or two stationary 45-m sections. The program then calculates the maximum weights for single, any two, and all three lanes for loaded lengths in multiples of 45 m up to a maximum of 1100 m. Each maximum value and the associated traffic conditions are stored for forty 90-day analysis periods. These maxima are then used to provide statistics for the Gumbel distribution of maximum values, which estimates the maximum load w*, to be expected in Y years as

$$w_{Y,L}^* = \overline{w}_{\max,L} + g\sigma_{\max,L} \tag{4}$$

where

 $\overline{\mathbf{w}}_{\text{max, L}}$ = average of all the maxima observed within each analysis period of Y years,

 $\sigma_{\text{max, L}}$ = standard deviation of the maximum values. g = a factor depending on the return period Y expressed as

$$g = (1/\sigma_n)(\ln Y - \overline{y}_n)$$
 (5)

 $y_n = 0.5436$ for the number of years of simulation,

 $\sigma_n = 1.1413$, obtained from table of the Gumbel distribution.

Thus,

$$g = 0.88 \ln Y - 0.48 \tag{6}$$

This value of g is then used to obtain the maximum weight $w_{Y,L}^*$ expected in a return period of Y years on a loaded length L.

BOUNDARY CONDITIONS

The shape of the curve relating vehicle weight per meter on the vertical axis to loaded length on the horizontal axis was assumed to be a very flat s. At short load lengths, the weight is fixed by the maximum load of single (or a very few) vehicles, and at very long lengths the average vehicle mix determines the weight.

An upper bound to the traffic load is produced by the heaviest vehicle observed on the bridge. Studies in the province (12) and elsewhere (12, 13) found that 1.3 (GVW) is a good upper limit for the weight of overloaded trucks (without special permits). The probability of two or three vehicles of similar weight being on the bridge has been reported by Stephenson (14).

The loads on very long spans are easy to calculate. As the loaded lengths approach infinity, the average load per meter tends to the average mix of vehicles. The limiting value is the number of vehicles in a lane multiplied by their average weight and divided by the length of road occupied. Thus,

$$w_{L\to\infty}^* = (N_C \overline{w}_C + N_B \overline{w}_B + N_T \overline{w}_T) / [N_C (\overline{\ell}_C + 1.5)(1 + R) + N_B (\overline{\ell}_B + 1.5)(1 + R) + N_T (\overline{\ell}_T + 1.5)(1 + R)]$$
(7)

where

 $w_{L^{\infty}}^*$ = average critical weight for a very long length in kilograms per meter,

N = proportion of vehicles,

w = average weight of vehicle in kilograms,

 \bar{l} = average length of vehicle in meters,

C, B, T = subscripts for cars, buses, and trucks,

R = V(0.0621) where V = speed of vehicle in kilometers per hour,

 $N_c = 0.94$ fraction of cars,

 $N_B = 0.04$ fraction of buses,

 $N_T = 0.02$ fraction of trucks,

 $\underline{\underline{\ell}}_{c} = 5 \text{ m},$ $\underline{\underline{\ell}}_{B} = \ell_{T} = 12 \text{ m},$

 $\overline{\underline{\mathbf{w}}}_{c} = 1590 \text{ kg},$

 $\overline{\underline{\mathbf{w}}}_{\mathsf{B}} = 13 \ 600 \ \mathrm{kg}$, and

 $\overline{w}_{T} = 12\ 200 \text{ kg}.$

The maximum critical load over a long length will occur when the traffic is stopped, in which case the average weight per meter for stopped vehicles is given by

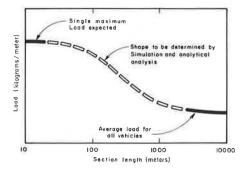
$$\mathbf{w}_{\text{L}\to\infty}^* = 0.94(1590) + 0.04(13\ 600) + 0.02(12\ 200)/0.94(5+1.5) + 0.06(12+1.5) = 330\ \text{kg/m} \text{ on a single lane}$$
 (8)

Similarly the weights for two and three lanes may be calculated to get the extreme boundary value. The hypothesized shape of the vehicle traffic loading as a function of bridge length is shown in Figure 2. The shorter loaded length on the curve is set by the legal load limit and the very long lengths by the average vehicle mix. The curve in between, the most important, is determined by the complex simulation and, eventually, an analytical solution.

ANALYTICAL SOLUTION

The derivation of the analytical solution is rather long and complex; the following is a condensation. The problem was to find the maximum weight on an infinite number of sections of given length on the bridge. The technique

Figure 2. Hypothetical traffic loading on long span bridges (single-lane example).



considered an arbitrary bridge length of 8 meters and how many (Δ) of these make up the required length. For example, if we wish to know the loading on 300 m, then $\Delta = 300/\delta$. The critical load w* is given by

$$w^* = (1.488/h) \ln_e \left\{ \left[D - \sqrt{D^2 - 4(1 - C)} \right] (\sqrt{AB}) \right\}$$
 (9)

where

A = (k/3n)F + [2(n - k + 1)/3n]GB = (n - k + 1/3n)F + (2k/3n)G, C = $(1 - 1/JZ)^{2/S}$

 $D = \sqrt{A/B} + (\sqrt{A/B})^{-1}$

 $F = (Eexp hW_o) (Eexp hM_o) (Eexp hM_o),$ $G = (Eexp hW_o) (Eexp hW_o) (Eexp hM_o),$

k = a value such that $K \le k \le L$,

J = L - K

 $L = K + (length of bridge/\delta)$, and

Table 1. Vehicle traffic loads based on existing operating policy,

Section Length (m)	Single Lane		Two Adjacent Lanes		Three Outside Lanes		Three Lanes	
	Wnun,L	$\sigma_{\text{max},L}$	$\overline{w}_{nia\times_{p}L}$	$\sigma_{nid \times_a L}$	$\overline{\overline{w}}_{n\omega_{N}L}$	$\sigma_{\text{max},L}$	W _{max_L}	$\sigma_{\text{max}_{i}L}$
45	972	146	1092	146	1149	132	1274	132
90	754	89	728	85	921	85	1059	74
180	610	60	737	54	774	60	911	58
360	500	43	628	37	640	40	775	31
720-	432	30	574	37	558	27	711	36
1080	403	25	552	40	524	24	687	42

Notes: Values are in kilograms per meter... 1 kg = 35,3 oz; 1 m = 3,3 ft.

Figure 3. Single-lane traffic loading.

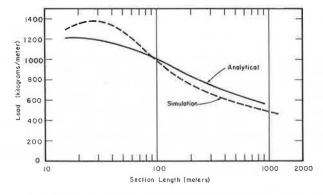
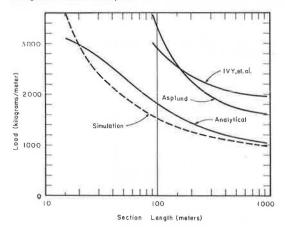


Figure 4. Design traffic load comparison for Lions' Gate Bridge (three-lane example).



$$1/Z = 1 - (1 - Y^{-1}(1 + [(\Delta - 1)(J - 1)/J])^{-1})^{1/J}$$

Before going further the terms and notation must be explained. The bridge and the causeway are composed of n subsections, each of length δ , and numbered K = 1, 2, ..., n. K is the length of the downstream approach affeeting the bridge divided by 5. The unit for measuring △ is 15 m, which was selected because it is approximately the space occupied by a bus, truck, or two cars. The length of the bridge and causeway is approximately 1500 m. For example, if each causeway is 300 m long and Δ = 5 (units of 15 m), each subsection K is 75 m (5 × 15) long. The total number of subsections n is 20. Each subsection is composed of the relevant portions of all three lanes. The variable Y is the return period (in years) and defines the probability that an event will occur as 1/Y. The number of subsections on the bridge is given by J where K is the number of subsections on the causeway to the bridge and L is the number to the far end of the bridge.

The further calculation of F and G requires additional definitions. Stopped lanes are noted by W and moving ones by M with subscripts o and c denoting outside and center lanes respectively. The proportion of units (8) on the average containing vehicles is noted by p, the superscript gives the lane and the subscripts C, B, T denote cars, buses, and trucks. T, when not a subscript, also gives the truck weight in kilograms divided by 453.6. The number of trucks and buses at a given instance is noted by $N_{\text{\tiny T+B}},$ and q is the average number of trucks plus buses divided by N_{T+8} . The value r is the proportion of heavy vehicles that are either trucks or buses in each lane. S is the number of annual events.

Finally the value of h, the constant used to produce a continuous equation from a discrete probability distribution, may be selected to ensure that the boundary conditions are met. The value originally selected as most appropriate was 0.08, but later analysis has shown h to be a function of the loaded length.

With these definitions and the value of h it is possible to calculate F and G. For the outside lane when

$$Eexp(hW_o) = (p_C^o e^{8.38h} + p_B^o e^{33h} + p_T^o Ee^{hT})^{\Delta}$$
 (10)

 $E \exp(hW_0) = (1.955 p_C^0 + 14.013 p_R^0 + Kp_T^0)^{\Delta}$

and

$$K = Ee^{hT} = (12.5/M - 12) [exp(T_{max}) - 2.61]$$

if

$$T_{\text{max}}$$
 = maximum truck weight = 18 000 kg, K = 9.787, and h = 0.08.

The calculations for the inside, middle, stopped lane are

$$E \exp(hW_c) = (p_C^c e^{8.38h} + 14.013 p_B^c + Kp_T^c)^{\Delta}$$

= (1.955p_C^c + 14.013 p_B^c + Kp_T^c)^{\Delta} (11)

For the moving lanes, calculation for the outside lanes is

$$\begin{split} E \exp(hM_o) &= \left\{ e^{3.5h} \left[1 + q^o \left(r_T^o \left\{ E \exp[h(T - 3.5)] - 1 \right\} \right. \right. \\ &+ r_B^o \left(e^{29.5h} - 1 \right) \right]^{N_{T+B}^o} \right\}^{\Delta} \\ &= \left(1.323 \left\{ 1 + q^o \left[9.591 r_B^o + (0.756K - 1) r_T^o \right] \right\}^{N_{T+B}^o} \right)^{\Delta} \end{split} \tag{12}$$

and for the middle lane is

$$\begin{aligned} E \exp(hM_c) &= \left\{ e^{3.5h} \left[1 + q^c \left(r_T^c \left\{ E \exp[h(T - 3.5)] - 1 \right\} \right. \right. \right. \\ &+ r_B^c \left(e^{29.5h} - 1 \right) \right]^{N_{T+B}^c} \right\}^{\Delta} \\ &= \left(1.323 \left\{ 1 + q^c \left[9.591 r_B^c + (0.756K - 1) r_T^c \right] \right\}^{N_{T+B}^c} \right)^{\Delta} \end{aligned} \tag{13}$$

A complete account of the derivation of the analytical formulation will be published shortly.

RESULTS

As stated earlier two methodologies were used to solve the problem of vehicle loads on the Lions' Gate Bridge. The analytical approach involved the development of probability equations representing traffic flow on the three lanes. The traffic conditions used were from an extreme peak hour. The alternative method simulated traffic flow across the bridge under a variety of conditions. It was possible to hunt for the stoppages that produced the maximum load both within individual lanes and across the total bridge. The resulting arithmetic mean and standard deviation of the maximum live loads on the bridge over a period of 10 years based on current operating policies are given in Table 1. The maximum vehicle traffic load can be calculated from equation 4.

The values of g for return periods of 30, 100, and 1000 years are 2.51, 3.57, and 5.58 respectively. Figures 3 and 4 show the vehicular loading results for existing traffic conditions and a return period of 100 years. The lower curves in Figures 3 and 4 are plotted from the data in Table 1 for three- and one-lane vehicle loads with g = 3.57. The analytical approach gave the upper curves in the figures. Visual inspection of the curves developed by the two methodologies shows very good agreement, which tends to add validity to the results. Beyond 75 m, both curves have essentially the same shape. Figure 3 shows an even closer agreement for a single lane. The maximum difference between the two curves at distances beyond 90 m is 18 percent.

The analytical curve should be higher than the simulation since it took slightly more conservative data and is in any case intended to be an upper bound. The reason that the curves for three lanes show a greater disparity than those for one lane is mainly that the third lane is treated differently in the two methods. Both assume that at least one lane has the traffic flowing at all times. The analytical approach tends to overestimate the load in this lane. The total effect on the bridge is small, however, because the critical side of the bridge is the one with the incident in the outer lane. Therefore, the compatibility of the two methods to measure the effective load on the bridge is more nearly represented by Figure 3 than by the simulated-analytical results in Figure 4.

A comparison of outer estimates and the live loads for existing traffic is shown in Figure 4. At lengths less than 150 m the difference between the Lions' Gate Bridge loads and those of Ivy and others (4) and Asplund is a factor of 1.7. At lengths of 900 m the difference is a factor of 2 for Ivy's curves and 1.6 for Asplund's curves. It would appear that Asplund's curve approaches the Lions' Gate curves at extremely long lengths.

The traffic loadings proposed by Ivy and others (4) and Asplund (15) are approximately similar for bridge lengths between 150 and 300 m. Ivy and coworkers studied the traffic on the lower deck of the San Francisco Bay Bridge reserved for trucks and U.S. army regulations for the movement of convoys. Ivy used the maximum expected loads, and the methodology does not allow the varying of vehicle mix or truck sizes. Asplund assumed that cars were mixed with very heavy trucks and that the vehicle load was 59 000 kg. This altered the very conservative estimates made by Ivy. He then ap-

plied simple probability theory to this mixed traffic. Asplund's selection of such a heavy single vehicle is rather arbitrary and does not represent all conditions. Asplund's curve in Figure 4 represents the probability of occurrence as more remote than 1 in 100 000.

The curves developed for the Lions' Gate Bridge reflect a low maximum vehicle load set by legislation over short segments of the bridge. At long lengths, the high percentage of buses using the bridge at rush hours is also apparent. The combination of these two features, unique to the Lions' Gate Bridge, accounts for the relative flatness of the traffic load on loaded lengths.

The resulting traffic load estimates combined with an extensive structural analysis verified the original decision to limit legal truck loads to 12 700 kg. The analysis also indicated that buses, operating under conditions similar to those at present, would not pose any structural problems.

CONCLUSIONS

The Lions' Gate Bridge is a unique three-lane suspension bridge built in 1938. The original traffic loadings for which the bridge was designed reflected vehicle weights and vehicle combinations typical in the late thirties. Since that time, vehicle weights, in particular truck weights, have increased considerably. The structure has also been subjected to many years of corrosive sea atmosphere. The British Columbia highway department required a reevaluation of the structural capabilities of the bridge; trucking companies also took interest in the bridge since any reduction in the legal load limit would greatly increase the distances that trucks had to travel. These two concerns helped to initiate the research presented in this paper.

The methodology developed reflected the need to independently estimate the effect of changing the legal load limit and varying the number of heavy vehicles that might use the bridge. Two methods were developed to estimate the vehicle traffic loads on the bridge: a computer simulation and an analytical formulation.

These methods have been shown to give comparable results. In the most extreme case the absolute difference between the curves is only 18 percent at a length of 60 m on all three lanes. The presence of one large, fully loaded bus or truck would account for this difference. Again for the three-lane case, at long lengths the difference is 75 kg/m. The single-lane agreement is even closer. At a loaded length of 120 m, the difference is only 30 kg/m or 3600 kg total. Although these results do not in themselves confirm the adopted design curve, they do lend credibility to the decision. In the absence of an extensive field survey in which the weight of each vehicle and distance on the highway at the time of an accident were known, the design curve is an optimal choice. The use of existing estimates such as those proposed by Ivy and coworkers, Asplund, or AASHTO would not easily allow the impact of reducing the legal load limit to be studied. Also, the vehicle combination used in those estimates was considerably different from that existing on the Lions' Gate Bridge.

The particular methodology, if generalized, would be particularly useful because it would allow the bridge designer to determine the bridge dimensions to fit a particular vehicle demand specification. If the bridge is along a recreational route, then a small percentage of heavy trucks would be expected. On the other hand, in areas of high industrial concentrations, trucks become more important and thus increased truck usage may be analyzed.

The method allows the traffic engineer and bridge designer to design the approaches and exits from the bridge

in a way that minimizes the potential of vehicles stopping on the bridge. This may be found beneficial in minimizing the vehicular traffic loads on the bridge.

The methodology allows the bridge designer to investigate vehicle traffic loads with a degree of sophistication approaching that used in the structural engineering analysis.

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