Fatigue Design of Welded Bridge Details for Service Stresses

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An average stress range histogram for truck loads on short-span highway bridges is derived from 106 strain range records reported in the literature. The histogram is used in conjunction with the concept of an equivalent stress range and the allowable constant amplitude stress range values specified by AASHTO to design bridge details for service stresses. Practical application of the method is illustrated with two design examples.

Highway bridge members are designed statically for the maximum stress due to dead and live loads. The dynamic effect of the live load is considered by adding an impact factor. It is well known, however, that a large number of repeated stress cycles may cause fatigue damage to the structural components at stress levels lower than the allowable maximum stress. A safe fatigue design of the structural components is achieved by limiting the value of the stress range caused by the design live load, including impact.

AASHTO specifications (1, 7), article 1.7.3, fatigue design, give the number of repeated stress cycles for which the bridge must be designed and the allowable stress range depending on the type and location of the detail (Figure 1). The allowable stress range values specified by AASHTO correspond to the 95 percent confidence limit for 95 percent survival of beam specimens tested at constant amplitude stress cycling (5, 6). The number of design load cycles is given by the average daily truck traffic (ADTT) for the traveled artery or the number of expected lane loads.

Fatigue design for truck-induced stress ranges is conservative in two respects. First, loadometer studies and strain history records indicate that few trucks have a gross vehicle weight comparable to that of the design truck. A variety of large but light cargos, partially filled trucks, and empty runs produce a frequency distribution curve for truck weights of concave shape with a peak at about 25 percent of the maximum recorded stress range. This is typical of all bridges surveyed.

To assume for purposes of fatigue design that all trucks are fully loaded is safe but very conservative. A more realistic fatigue design should be based on truck-induced stress histories of variable amplitude, which reflect the actual variation in truck weights. This paper derives an average stress histogram for truck loads on short span highway bridges from published data and shows how it can be used in conjunction with the AASHTO constant amplitude stress range values to design bridge details for service stresses.

Fatigue design is conservative in a second way. Several investigators have matched recorded truck weights with induced strains and have consistently observed lower stresses than those predicted by analysis. This is due mainly to the fact that the various analyses and design rules are conservative. It is not a direct result of fatigue design. However, inasmuch as analysis and design rules may be changed to reduce such discrepancies, it is advisable not to relax the fatigue design specifications.

Special permits may be granted for overloads under the AASHTO operating rating (1, 2). Although a discussion of periodic overload effects on fatigue strength is beyond the scope of this paper, it is helpful to point out that preliminary research findings indicate an enhancement of fatigue life at overload frequencies to which bridges are currently being subjected (25).

STRESS HISTORY OF HIGHWAY BRIDGES

As a first step toward a more realistic fatigue design of highway bridge components, the frequency distribution of stress ranges induced under service conditions, commonly referred to as stress range histogram, must be known. During the last decade, stress range histograms for truck traffic on 29 bridges on Interstate and U.S. highways in semirural and metropolitan areas were recorded (14 through 23). Detailed descriptive information on the bridges and characteristic features of the 106 individual stress range histograms reported in these references is presented elsewhere (13). A collection of all 106 stress range histograms is given by Yamada and Albrecht (24).
Notation

The following symbols are used in this paper:

- \( \text{ADTT} \) = average daily truck traffic,
- \( B_0, B_1 \) = regression coefficients for S-N curve,
- \( f(x) \) = probability density function for stress range histogram,
- \( f_c, f_R, f_t \) = yield point of material, stress range, mean square stress range, respectively,
- \( f_{\text{max}} \) = maximum stress range in stress histogram,
- \( f_{\text{RMC}} \) = root mean cube stress range,
- \( f_{\text{equ}} \) = equivalent stress range,
- \( f_{\text{Rmax}} \) = root mean square stress range,
- \( f_{\text{Rmax}} \) = maximum stress range in stress histogram,
- \( f_{\text{Rho}} \) = fatigue limit or runout level of stress range,
- \( f_{\text{Nu}} \) = fatigue limit or runout level of stress range,
- \( N_{\text{fat}} \) = total number of cycles,
- \( N_{\text{fat}} \) = design fatigue life in years,
- \( N_{\text{fat}} \) = number of design stress cycles above the fatigue limit,
- \( N_{\text{fat}} \) = total number of cycles,
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Figure 1. Allowable constant amplitude stress range values for stress categories D and E.

Figure 2. Typical stress range histograms with two extreme stress range intervals.

Figure 3. Typical stress range histograms with and without cutoff point.

Figure 4. Typical stress range histograms with highest and lowest recorded stress range values, \( f_r, \text{max} \).

Figure 5. Nondimensional cumulative frequency plot of 106 stress range histograms with cutoff point of 0.25 \( f_r, \text{max} \).

Figure 6. Average nondimensional stress range histogram and probability density function.

Table 1. RMS and RMC stress range for the average nondimensionalized stress range histogram.

<table>
<thead>
<tr>
<th>Cutoff Point</th>
<th>Percentage Above Cutoff Point</th>
<th>RMS Stress Range (MPa)</th>
<th>RMC Stress Range (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>100.0</td>
<td>0.435</td>
<td>0.459</td>
</tr>
<tr>
<td>0.30</td>
<td>76.8</td>
<td>0.472</td>
<td>0.493</td>
</tr>
<tr>
<td>0.40</td>
<td>43.0</td>
<td>0.551</td>
<td>0.566</td>
</tr>
<tr>
<td>0.50</td>
<td>22.2</td>
<td>0.633</td>
<td>0.644</td>
</tr>
<tr>
<td>0.60</td>
<td>10.5</td>
<td>0.719</td>
<td>0.728</td>
</tr>
<tr>
<td>0.70</td>
<td>4.5</td>
<td>0.806</td>
<td>0.810</td>
</tr>
<tr>
<td>0.80</td>
<td>1.8</td>
<td>0.887</td>
<td>0.889</td>
</tr>
<tr>
<td>0.90</td>
<td>0.73</td>
<td>0.949</td>
<td>0.950</td>
</tr>
<tr>
<td>1.00</td>
<td>0.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Note: 1 MPa = 145 lbf/in².
average histogram then accounts for the actual live load variation.

Among the curves investigated for the purpose of best fitting the average cumulative frequency distribution, shown as a step function in Figure 6, were Rayleigh functions. They have been used by others (10), presumably because they follow in general the ascending-descending shape of individual histograms such as those shown in Figure 2. The results of the least squares fit analysis revealed a significant lack of correlation for the Rayleigh curve. Its applicability appears to be questionable also for conceptual reasons. A Rayleigh curve would predict gradually vanishing frequencies at very low loads, whereas in reality the addition of groups of cars capable of producing comparable stress fluctuations would cause an upward turn in the frequency distribution. Further, retaining the very low stress ranges reduces the equivalent stress range, as discussed below, and leads to nonconservative estimates of variable amplitude fatigue life.

**FATIGUE LIFE PREDICTION**

The fatigue life of structural components consists of a crack initiation phase and a crack propagation phase. Severe stress concentrations at weldments and the existence of microcracks and slag inclusions at weld borders tend to reduce the crack initiation phase to a small number of cycles compared with the total fatigue life. Therefore, the initiation phase is usually neglected, and the analysis of the useful life is based on crack propagation alone. The assumption is conservative for purposes of design and indeed necessary because undesired weld cracking can preempt entirely the initiation phase.

**Stress Range Versus Fatigue**

The constant amplitude fatigue life of weldments is determined experimentally. The data are usually presented as a log-log plot of stress range versus fatigue life (S-N) with the mean life given by

\[ \log N = B_1 + B_2 \log f \]

(2)

\(B_1\) and \(B_2\) are regression coefficients obtained from a least squares fit. The AASHTO specifications (1) classify all structural details in five stress categories, \(A\) through \(E\), according to their fatigue resistance. For each stress category, the constant amplitude stress range is specified as a function of the design life on the basis of the 95 percent confidence limit for 95 percent survival obtained from the statistical analysis of the pertinent test data. For example, allowable stress ranges for stress categories \(C\) and \(D\) are shown by the solid points and lines in Figure 1.

At low values of stress range, a fatigue limit or a runout level exists for each detail below which fatigue cracking will not occur even after application of a large number of load cycles, usually on the order of 10 million. The safe fatigue limit for details in stress categories \(C\) and \(D\) is shown by solid lines in Figure 1.

**Equivalent Constant Amplitude Stress Range**

Recently, the concept of an equivalent constant amplitude stress range has been advanced (10). It is defined as the constant amplitude stress range that will give the same fatigue life as the variable amplitude stress history. Barsom (3) successfully used the root mean square stress range, \(f_{RMS}\), to correlate fatigue crack propagation rates under constant and variable amplitude cycling.

The RMS stress range is defined as the square root of the mean sum of squares of all stress ranges:

\[ f_{RMS} = \left( \frac{\sum n_i f_i^2}{N} \right)^{1/2} \]

(3)

where

- \(N\) = total number of cycles,
- \(s = \) number of stress range levels, and
- \(n_i = \) number of stress range cycles at each level.

Equation 3 can be derived from the condition that the strain energy induced by a total of \(N\) variable amplitude stress cycles must be the same as the strain energy caused by the application of \(N\) stress cycles of constant amplitude, \(f_{equiv}\).

Combining Miner's theory with the S-N curves fitted to fatigue test data suggests, however, that an exponent of 3 rather than 2 should be used in equation 3. This is shown below.

The Palmgren-Miner theory (8) is an empirical, cumulative damage criterion for evaluation of variable amplitude fatigue life. Its widespread use can be attributed to both the simplicity of the method and the ease of application. The Palmgren-Miner theory states that the damage caused by a number of stress cycles, \(n_i\), can be expressed as a fraction of \(n_i\) to the number of cycles, \(N_i\), required to fail the component at the same stress range level. Failure occurs when the summation of the fractions for each level adds up to unity.

\[ \sum_{i=1}^{s} \left( \frac{n_i}{N_i} \right) = 1.0 \]

(4)

By substituting the anti-log of equation 2 into equation 4, the Palmgren-Miner criterion is given by

\[ \sum_{i=1}^{s} \left( \frac{n_i/10^B_2 f_i^{B_2}}{N_i/10^B_2 f_i^{B_2}} \right) = 1.0 \]

(5)

If the equivalent constant amplitude stress range, \(f_{equiv}\), causes failure, the life, \(N\), can be expressed by the antilog of equation 2 as

\[ N = 10^{B_2} f_{equiv} ^{B_2} \left( N_i/10^B_2 f_i^{B_2} \right)^{-1} \]

(6)

Equating the identities expressed by equations 5 and 6 and solving for the equivalent stress range yield

\[ f_{equiv} = \left( \frac{\sum n_i f_i^{B_2}}{N} \right)^{1/B_2} \]

(7)

Equation 7 is of the same form as equation 3 except the exponent is different. If \(-B_2 = 3\), as found for most structural details \((5, 6)\), is substituted, the equivalent stress range is given by a so-called root mean cube stress.

\[ f_{RMC} = \left( \frac{\sum n_i f_i^3}{N} \right)^{1/3} \]

(8)

As previously explained, a runout level of stress range, \(f_{AO}\) (also called fatigue or endurance limit) is assumed when a large number of cycles, usually on the
order of 10 million, do not produce any fatigue cracking. Therefore the stress range levels below the fatigue limit should be deleted from the calculation of RMS or RMC stresses. Root mean square and root mean cube stresses for the average stress range histogram, equation 1, were computed for several values of fatigue limit, expressed as a fraction of the maximum stress range. The results are given in Table 1. For example, if the fatigue limit of a given detail is 50 percent of the maximum design stress range, only 22.2 percent of all trucks cause stress ranges above the fatigue limit, and its RMS and RMC stress ranges are given by 0.633 and 0.644 of $f_{r,max}$. The difference between the RMS and RMC stress ranges is approximately 5 percent when $f_{r,50} = 0.25 f_{r,max}$, and decreases further as the relative fatigue limit increases.

These small differences explain partially why good correlation between constant and variable amplitudes test data can be obtained by plotting the fatigue lives against the root mean square stress range. The RMC stress range is biased, however, on a sounder theoretical and experimental foundation.

The Fracture Mechanics Approach

In contrast to Miner’s empirical theory, the fracture mechanics approach is based on the physical phenomena of fatigue crack propagation. It has been used with success to correlate observed fatigue lives with the computed number of cycles required to propagate a fatigue crack from an average initial size to failure, for both constant and variable amplitude fatigue. It also explains why the slope of all S-N curves is the same and about numerically equal to the slope of curves in log-log plots of crack growth rate versus range of stress intensity factor. The value of the slope is about 3 for structural details and steels.

It can be shown that both Miner’s theory (equation 4) and the root mean square stress range concept (equation 8) are special cases of the fracture mechanics approach. They give identical variable amplitude fatigue life predictions (13) provided that (a) the crack initiation phase is negligible, (b) no interaction exists between the stress range levels, (c) only stress ranges above the fatigue limit are retained, and (d) the slopes of the S-N curves and the crack growth rate curves are about 3. Substantial experimental evidence can be presented in support of each one of the four conditions.

APPLICATION

A more realistic fatigue design of welded bridge details for service stresses can be performed with the aid of (a) an average stress range histogram for highway bridges, (b) the concept of an equivalent stress range, and (c) the allowable constant amplitude stress range values specified by AASHTO. The following design procedure is recommended.

1. Find the maximum stress range, $f_{r,max}$, at the detail due to live load and impact.

2. Compute $f_{r,50} = f_{r,50}/f_{r,max}$, where the runout stress range, $f_{r,50}$, is defined in the AASHTO specifications (more than 2 million cycles).

3. Find the equivalent RMS stress range, $f_{r,RMS}$.

4. Compute the percentage of frequency occurrence, $P$, of the stress cycles above the fatigue limit by integrating the probability density function, $f(x)$, from $x = x_{fat}$ to $x = 1.0$. This is the percentage of stress range cycles that contribute to crack propagation. Alternatively read the RMC stress range directly from Table 1.

5. From the S-N plot for the appropriate stress category, read the propagation life, $N_{prop}$, corresponding to the equivalent stress range, $f_{r,RMC}$ (Figure 1).

6. Divide the propagation life, $N_{prop}$, by the percentage of frequency occurrence to obtain the total truck traffic, $N_{total} = N_{prop}/P$.

7. Compute the expected fatigue life in calendar years from $N_{year} = N_{total}/(360 \times ADTT)$.

REFERENCES


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