Studies of Skewed Concrete Box-Girder Bridges

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Analyses of 51 mathematical models using the finite element analysis program CELL were undertaken to describe the behavior of common skewed, concrete box-girder bridges. An assessment was made of effects on structural behavior of varying superstructure width to span length (aspect) ratio, number of cells, skew angle, type of loading, and depth. Span length and skew angle were found to be the most significant parameters influencing girder shear distribution. Aspect ratio and skew angle affected longitudinal bending moments to the largest extent. Based on the results of this study, methods for shear and bending design of skewed bridges are presented.

Bridge engineers have recognized several peculiarities in skewed concrete box-girder bridge behavior. Shear cracks have developed in exterior girders near obtuse support corners. Transverse cracks have developed at midspan of certain highly skewed, prestressed bridge decks. Tendencies toward uplift at acute support corners of long span bridges have been noted.

Although skewness produces analysis and design complications, it also affords design advantages. Since support reactions tend to concentrate at obtuse corners, reductions occur in effective structure spans. The thick end diaphragms act to reduce longitudinal stresses in the superstructure. Hence, possibilities exist for material savings by refining structural analysis.

University of California researchers developed a finite element analysis program CELL for use in performing elastic analyses of box-girder bridges having a constant depth and an arbitrary plan geometry (1). They further used the program for analyzing skewed bridges. Results of the study enabled them to devise methods whereby the stiffening effects of skewed supports could be considered for reducing superstructure design moments (2). Aluminum models were tested for physical verification (8). Dead-load design moments were reduced 19 percent for 30-deg skews and 41 percent for 45-deg skews.

In an analysis of a highly skewed, two-span, two-celled railroad bridge California Department of Transportation researchers found the following skew effects (3, 4):

1. Significant reductions in longitudinal bending moments from those of comparable straight or normal structures on orthogonal bearings,
2. Severe redistribution of reactions and girder shears toward obtuse support corners,
3. Uplift tendencies at acute corners of the longest span abutments under some loading conditions, and
4. No measurable benefits from including intermediate diaphragms in the superstructure.

Similar effects of skew have also been reported by others (5, 6, 7, 9).

OBJECTIVES, CONCEPTS, AND PROCEDURES

The objectives of the study reported in this paper were to describe the behavior of skewed box-girder bridges, and to suggest design criteria. The CELL program was used to study and analyze 51 mathematical models to assess effects of varying superstructure plan geometry, number of cells, skew angle, type of loading, and depth.

Basic Concepts

The obtuse corner reactions for any symmetrical loading condition will exceed acute corner reactions when a simple span, skewed box-girder superstructure on parallel abutments is supported at its corners. Figure 1 shows this concept. A concentrated load P was applied at the center of a skewed plate (Figure 1a). Since the load was applied nearer the obtuse corners, as opposed to the acute corners, the obtuse reactions were larger. In other words, a structure will behave more stiffly between its obtuse corners than between its acute corners. As a commonly accepted design approach, total abutment reaction R is used to establish a moment diagram for a structure having an assumed span length equal to the center girder length (Figure 1b). In California design practice, a "whole-width" design concept is employed, in which the moment of inertia of all girders comprising the superstructure is lumped in one unit.

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The above procedure, however, leads to a conservative design because the skew itself has the effect of reducing the maximum moment in the structure. One means of demonstrating this effect is to superimpose two conditions that are equivalent to the total left support reaction (Figure 1c). The true reactions on the left may be described as a component that is normal (which sums to R) and a component that is skewed or unbalanced. The normal component affects longitudinal behavior exactly as a normal reaction affects a normal structure. The unbalanced component accounts for the portion of the reaction that is due to skew. It has the effect of applying a couple to the left support that causes a counterclockwise rotation about the moment center (y-axis in Figure 1d), opposing the clockwise rotation caused by the load. Thus, a resisting couple results from the unbalanced longitudinal distribution of support reaction. Its magnitude is calculated as shown at the bottom of Figure 1d. A clockwise couple of the same magnitude occurs at the other end of the span. The magnitude of the couple is a function of the reaction difference, structure width, and skew angle. When the preceding two effects are superimposed, a moment is obtained at midspan that is reduced from the moment of the simple span by the magnitude of the couple as shown in Figure 1e and discussed elsewhere (9).

The above concepts also apply to box-girder bridges; however, they must be modified according to the following indeterminate complexities of box-girder behavior.

1. The end diaphragm contributes measurably to the stiffness of a skewed structure. As its span lengthens, the support rotation increases. When this lengthening occurs, the end diaphragm provides a good mechanism for longitudinal as well as transverse distribution of reaction, carrying an increasingly greater load to the obtuse corner. Increases in skew angle accentuate this redistribution.
2. The unbalanced distribution of support reactions also produces torsional resisting moments at the supports, even under symmetrical loading. If moments are taken about the longitudinal centerline (x-axis) of the structure, a torsional couple is obtained that depends on the reaction unbalance and structure width (Figure 1f).

Most box-girder bridge sections are designed according to the whole-width concept. However, because skewness produces some doubt as to the desirable section for design purposes, the critical moment value had to be determined between that calculated at a total section, normal to the center girder midspan, and that calculated along a nonnormal section, cut somewhere between lines 1 and 2 in Figure 2.

Models and Program Description

The current study was initiated by establishing superstructure width to length (aspect) ratios of the longest spans in each of 395 concrete box-girder structures being designed in California. Based on those data, a range of mathematical models was chosen for assessment of parameter influence and general behavior. Most often, models analyzed were of 4 and 8-celled bridges supported on parallel abutments and containing girders spaced as shown in Figure 3b. Most models were skewed 30 and 45 deg, and all webs were assumed to be vertical.

The support (boundary) conditions used are shown in Figure 4a. Each girder reaction point was restrained against vertical movement. To stabilize each model, transverse movement at center girder reaction points and longitudinal movement at the exact center of the structure were prevented. Figure 4b shows a layout plot of a finite element mesh for a 4-celled box, which is typical of all meshes used in the present study.

Loadings

All model structures were analyzed for dead-load behavior and several for prestress and live-load behavior.

1. Structure dead-load was calculated and applied internally by using the CELL program.
2. Live-load behavior was studied through application of loads appropriate to span lengths. Uniform lane loads and concentrated midspan loads (riders) were applied to spans exceeding 48.8 m (160 ft). Short span behavior was studied through application of one truck (HS 20-44) or the maximum number of trucks a particular structure would be designed to carry.
3. Effects of prestress loads were studied to relate results obtained by using the usual analysis and by using the CELL analysis. Skewed bridges having parallel supports are currently designed by assuming normal supports; therefore, working prestressing forces were calculated for the normal condition. Figure 5 shows the procedure. The forces were assumed to be applied through cables draped as shown in Figure 5a. Equivalent loadings, comprising a combination of concentrated loads at the girder ends and an upward load acting over the deck area, were applied. The equation in Figure 5b was used to calculate the uniform load to balance the dead load.

RESULTS AND OBSERVATIONS

Moment Reduction

Results indicate that substantial savings can be realized by taking advantage of the reduction of design stresses produced by the effects of skew. Figure 6 shows moment-reduction design curves that could be used for arriving at more economical designs for simple span, reinforced concrete, box-girder bridges. These curves represent envelopes of data from the models previously mentioned. The dotted lines indicate extrapolation beyond most of the data.

Initial studies show that similar reductions would exist for multiple span structures. The relative amounts carried by various spans depend on several factors, such as span unbalance and intermediate support conditions. Varying superstructure depth-to-span length (D/L) ratio within the range of 0.0475 to 0.0615 has little influence on moment reduction values.

The curves in Figure 6a could be used to reduce the dead-load design moments in box girders having 6 to 10 cells and spans ranging from 17.1 to 33.5 m (56 to 110 ft). These limits encompass most simple span, reinforced concrete structures. Similar reduction curves for structures having 2 to 5 cells appear in Figure 6b. The curves would be used as follows:

1. Calculate the midspan dead-load moment by assuming that the structure is supported on normal abutments;
2. If the abutment skews are within 15 deg of one another and the cross section is prismatic, proceed to find a reduction factor;
3. Enter Figure 6a or 6b and use the smaller of the two skew angles;
4. Read a factor from the ordinate axis, using the curve for the appropriate girder (normal) span length; and
5. Reduce the dead-load design moment according to the formula $M_D = F \times M_N$, where M_D = design moment and M_N = normal moment.

Aspect ratio was found to be the most significant

Figure 1. Total moment at midspan due to skew.

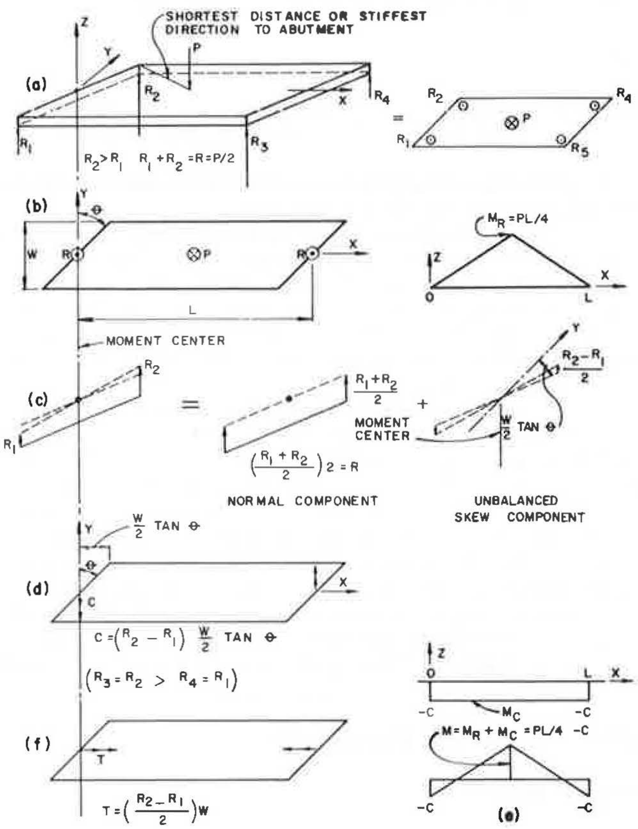


Figure 2. Location of maximum longitudinal stress.

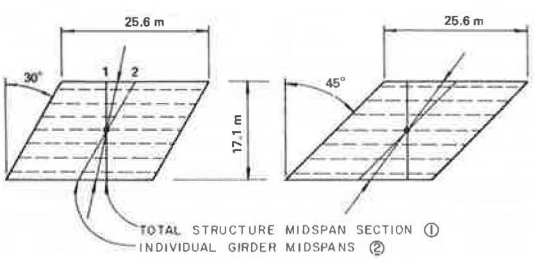


Figure 3. Shear distribution locations and loadings of a typical box girder.

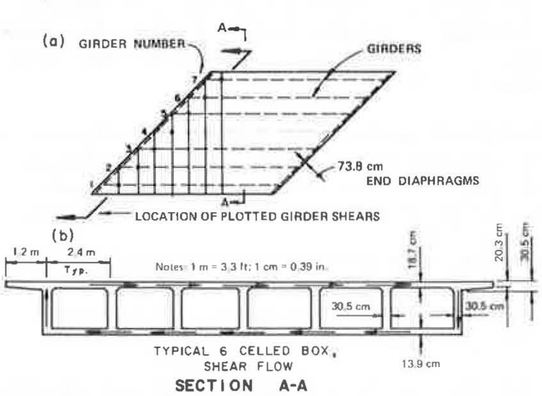


Figure 4. Support conditions for finite element mesh used in models.

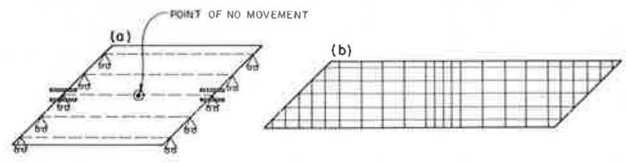


Figure 5. Prestress loading.

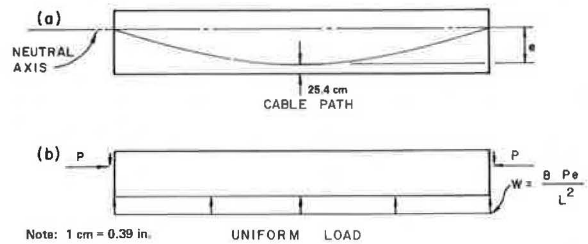


Figure 6. Moment reduction curves.

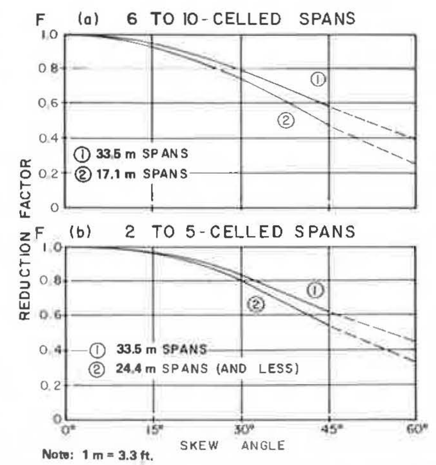
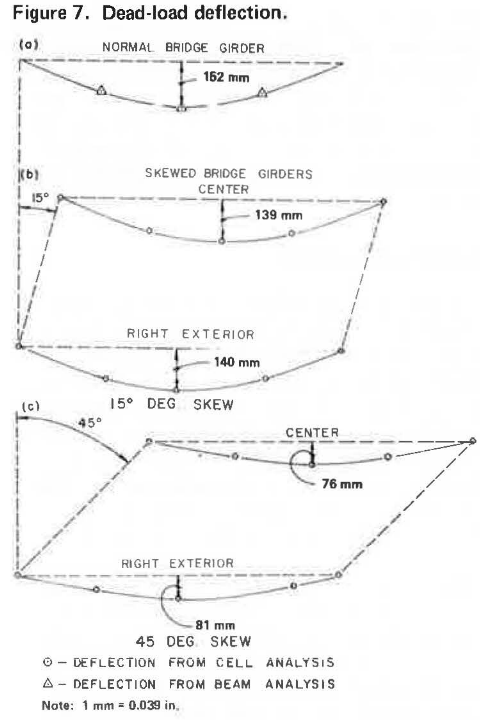


Figure 7. Dead-load deflection.



parameter influencing overall bending behavior. Although aspect ratio is not referred to in the plots themselves, it is reflected in the curves in Figure 6. Notice that the factors decrease with decreasing span length and with an increase in the number of cells. As a general rule, transverse distribution of girder moments for structures skewed less than 45 deg and having aspect ratios of 0.5 and less can be considered the same as that of comparable structures on normal supports. Six to ten-celled structures that have 1:1 aspect ratios and are skewed 45 deg exhibit ratios about 10 percent larger between exterior and center girder moments than do their normally supported counterparts. To verify the analyses, statics checks were made by comparing external and internal moments at midspan. In most cases, moments checked to within 1 percent and vertical forces summed exactly to zero.

Moment Derivation

Figure 2 shows the migration of the maximum longitudinal stress location in various exterior girders. The maximum stress in each individual girder of the structural section is indicated near the lines that are designated by arrows. The largest moment value obtained by summing individual girder moments along any such line was used for establishing reduction curves. The total moment for each skewed structure was divided by the maximum moment value calculated for a normal maximum structure of the same span and section. The result was the reduction factor F.

Deflections

Dead-load deflection data for a CELL model having a 54.9-m (180-ft) span, a 21.9-m (72-ft) width, and 9 girders are presented in Figure 7. This example typifies the study's results. Figure 7a shows the theoretical, elastic, deflected shape of a bridge girder with orthogonal bearings found from simple beam analysis. Results for center and right exterior girders obtained by using CELL analyses are plotted for 15 and 45-deg skews respectively. The following information was noted:

1. A significant reduction in deflection due to skew is evidenced at 15 deg,
2. Deflections are only half as great as normal for 45-deg skews,
3. The discrepancy between exterior and center deflections is larger at 45 deg than at 15 deg, and
4. Narrower structures of the same span as that shown in Figure 7 exhibit smaller discrepancies between exterior and center deflections.

Shear Behavior

The girder shears were calculated at the CELL mesh node locations nearest the left support. The shear distribution location can be thought of as a section cut parallel to the abutment and as close to it as possible (Figure 3a). Transverse shear distribution results under dead load for the simple span box girder depicted in Figure 3 are presented in Figure 8a. The structure is 25.6 m (84 ft) in span length, 17.1 m (56 ft) wide, and skewed 45 deg at each support. In this case, and for all the dead-load analyses, it was evident that the exterior girder framing the obtuse support corner of a structure would carry a much greater portion of the total structure shear than would any other girder. The interior girders carry about the same amounts of shear as they would if the structure were normally supported. Such behavior might be expected because a greater contribution is made to torsional rigidity (shear flow) by the deck and exterior girders than by the interior girders (Figure 3b).

Statics checks indicate a 7 to 9 percent violation of vertical equilibrium between external reactions and internal girder shears. Although this error is tolerable, it could be reduced by mesh refinement. Shear behavior at acute corners is opposite to that at obtuse corners. The acute corner exterior girder carries but a small amount of shear. In some long span and highly skewed structures, the shear at the acute corner girder actually reverses in sign, indicating an uplift tendency.

Six separate live loadings were applied to each of four structural models. Figure 9 shows a plan for live loads 1 and 5. Figure 8b shows that live load 5 is more critical for shear in the obtuse exterior and first interior girders than is live load 1. Live load 1 is resisted by a more uniform shear distribution among the five girders nearest the obtuse corner. Much of live load 1 travels directly to the end diaphragm.

The data indicate that shear distribution depends primarily on span length and that an approximate linear increase occurs in the exterior girder shear as skew angle increases because of relative support rotation. If other parameters are held constant, the longer the span is, the greater is the redistribution of reaction toward the obtuse corner of a skewed support. [These box-girder study results are in agreement with a previously stated theory (9).] Therefore, ranges of design shear factors based on span length were developed from the data. For example, the dead-load shear factors for obtuse corners of a reinforced concrete box girder are shown in Figure 10. The factors were calculated by taking the ratios of the shear results for exterior and first interior girders of skewed and of comparable straight or normal structures ($F = V_s/V_N$). Interpolation within these ranges should lead to acceptable designs, that is, ±5 percent.

The range of shear factors for exterior girder design in multiple span structures is likely to be lower than the comparable range for simple spans. How much lower depends on various factors such as span unbalance, number of spans, and number of columns. Preliminary analyses of a model having two spans of 30.5 m (100 ft) and 24.4 m (80 ft) and skewed 30 deg produced factors for the exterior girder range in the long span about 25 percent lower than those for a simple span of the same length. The short span shears were almost uniformly distributed across the section.

The shear factor range for an exterior girder would be about 13 percent higher for a combined dead and live loading than for a dead loading alone because live load 5 produced a more critical condition in the obtuse exterior girder than live load 1. A converse effect occurs at acute corners and girder shears are also effectively reduced. By interpolating within ranges, one may use the shear factor curves to obtain a factor for a particular structure; depth-to-span length ratio (D/L), span length, and skew are used to obtain a factor to increase design shears from those of identical structural sections supported on normal abutments. The following example illustrates the process. For a bridge that has a span length of 24.4 m (80 ft), a D/L of 0.052, and abutment skews of 30 and 40 deg, find the design shear factors for the exterior and first interior girders at the obtuse corners. The steps in the solution are as follows:

1. Enter the exterior girder range in Figure 10 and interpolate between the 0.045 and 0.060 D/L lines, for a 33.5-m (110-ft) span to establish a line for the desired value of 0.052;
2. Using the span length of 24.4 m (80 ft), interpolate between the new line for a 33.5-m (110-ft) span, D/L of 0.052 and the lower limit line [L = 17.1 m (56 ft)] to es-

tablish the factors of 1.63 for the 30-deg skew and 1.83 for the 40-deg skew;

3. Establish factors of 1.09 and 1.12 for the first interior girder by interpolating within the lower range; and

4. Increase the design shears in the exterior and first interior girders by the appropriate factor F, according to the formula $V_D = F \times V_N$, where V_D = design shear and V_N = normal shear.

High reaction concentrations at the obtuse corner cause about 3 to 6 percent of the total structure dead-load shear to be carried by the end diaphragm as structure aspect ratio increases from 0.5 to 1.0. Most of this dead-load shear is concentrated near the obtuse corner.

Figure 8. Transverse shear distribution.

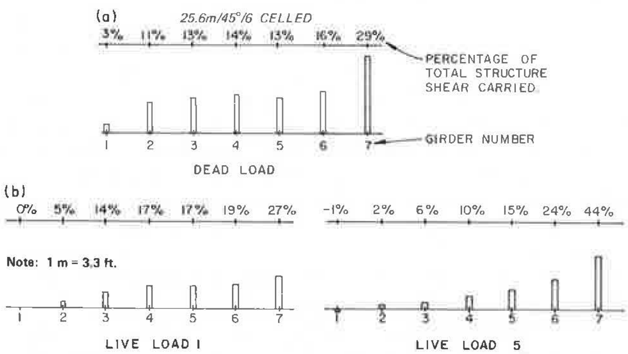


Figure 9. Plan for live loads 1 and 5.

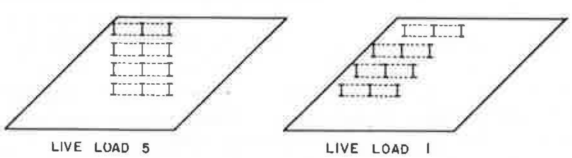
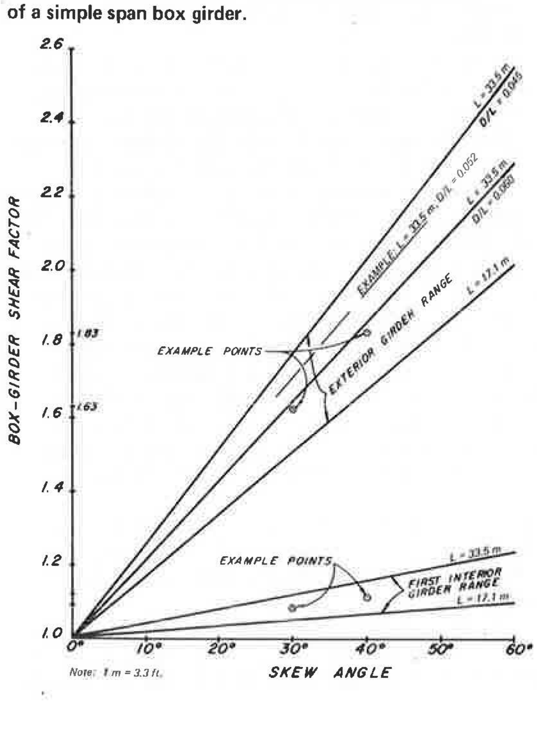


Figure 10. Dead-load shear factors for obtuse corners of a simple span box girder.



Prestress Behavior

Longitudinal stress results indicate that reductions in prestressing forces are possible in wide structures having skews over 45 or 50 deg. For example, a bridge skewed 50 deg, and having 13 girders and an aspect ratio of 0.675, was analyzed by using the CELL program. Results indicate a possible reduction in prestressing forces of 11 percent. This reduction may have occurred because dead-load torque peaks at 45 or 50 deg and tends to reduce dead-load moments most highly at those skews, as discussed by Kollbrunner and Basler (9). However, present design procedures for designing a prestressing system to counteract dead- and live-load stresses in individual girders appear realistic and effective, as far as the total structure is concerned. In producing an effective, uniform, upward load, prestressing has overall effects that oppose the dead load. Since end moments produced by skewed supports resist downward movement under dead loading, they should also resist upward movement due to prestressing. Analysis results for prestress loadings support this notion. Distribution of shear in the models under prestress loading was almost identical to dead-load distribution, though opposite in sign. Thus, draping the cable parabolically with maximum eccentricity at each girder midspan, which is current practice, is the proper general approach. However, a larger torque is produced in the superstructure by dead loadings than that produced by the lower uniform load due to prestressing. Therefore, support design should provide for a larger relative settlement at the obtuse corners resulting from rotations due to combined loading.

Live-Load Bending Behavior

Results indicate that live-load reductions of lesser magnitude than those for dead load can be realized in simple spans. As a structure widens in relation to its length, the indicated live-load reduction lessens relative to the dead-load reduction. The smaller reduction is due to a more localized spanning effect occurring in the individual girder than would exist in a corresponding girder of a narrow bridge. However, the load case studies were not extensive enough to provide general behavioral guidelines.

SUMMARY

This study of skew parameters of simple spans provided the following information.

1. General knowledge regarding the effect of skew on transverse shear distribution to individual girders near abutments of concrete box-girder bridges;
2. A more accurate method for designing shear reinforcement than previously used heuristic methods;
3. Some insight into the overall structural behavior gained from deflection results;
4. Evidence that aspect ratio and skew are the significant parameters influencing bending behavior and that span and skew are significant in affecting shear behavior (depth-to-span length ratio changes within the usual ranges of 0.040 to 0.062 do not appreciably influence overall structural behavior);
5. A proposed method that allows reduction in dead-load design moments in simple span, reinforced concrete bridges and leads to savings in concrete or steel quantities;
6. Illustration that substantial savings are possible in multiple span, reinforced concrete bridges and would far offset the added design expense involved in finite element analyses with the CELL program; and
7. Evidence that shows that prestress forces can be reduced in certain highly skewed structures.

The study did not provide comprehensive live-load analyses, general multiple span design criteria, or refined criteria for prestressing forces required in skewed bridges.

RECOMMENDATIONS

In the design of skewed, simple span, concrete box-girder bridges, the shears in the exterior and first interior girders should be modified according to factors obtained from design charts such as those shown in Figure 10. In the design of abutment bearing pads and end diaphragms for shear, the effects of the high obtuse corner reactions should be considered. Simple span, reinforced concrete bridges should be designed for longitudinal bending according to the following procedure:

1. Analyze each structure for dead loading by assuming that it behaves like a beam on normal supports;
2. If the abutment skews are both greater than 15 deg and within 15 deg of each other and the section contains 2 to 10 cells, proceed to reduce dead-load design moments according to Figure 6;
3. Apply live load to the structural section by analyzing the superstructure as if it were on normal supports (this conservative approach is required because information is limited regarding live-load behavior);
4. Use established methods for designing longitudinal reinforcement but base the design on the reduced dead-load moment obtained in step 1 (the bars should be extended farther toward the obtuse corners than is current practice); and
5. Accept the CELL program for general use in design practice.

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